

## Series Results

$$\sum_{x=0}^{\infty} q^x = \frac{1}{1-q}, \quad |q| < 1.$$

$$\frac{d}{dx} \left( \sum_{x=0}^{\infty} q^x \right) = \sum_{x=0}^{\infty} x q^{x-1} = \sum_{x=1}^{\infty} x q^{x-1} = \frac{1}{(1-q)^2}, \quad |q| < 1.$$

$$\sum_{x=0}^N q^x = \frac{1 - q^{N+1}}{1 - q}.$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = e^x$$

# Expectation

## Discrete Random Variables

$$E[g(X)] = \sum_{x \in \mathcal{S}} g(x) Pr(X = x)$$

$$Var(X) = E[X^2] - E[X]^2.$$

## Continuous Random Variables

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$$

$$Var(X) = E[X^2] - E[X]^2.$$

# Families of Discrete Random Variables I

**Bernoulli random variables,  $X \sim \text{Bern}(p)$**

$$\Pr(X = x) = \begin{cases} p, & x = 1, \\ 1 - p, & x = 0. \end{cases}$$

$$E[X] = p \quad \text{Var}(X) = p(1 - p).$$

**Binomial random variables,  $X \sim \text{Bin}(n, p)$**

$$\Pr(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}, \quad x = 0, 1, \dots, n,$$

$$E[X] = np \quad \text{Var}(X) = np(1 - p).$$

**Poisson random variables,  $X \sim \text{Po}(\lambda)$ ,  $\lambda > 0$**

$$\Pr(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \dots$$

$$E[X] = \lambda \quad \text{Var}(X) = \lambda.$$

## Families of Discrete Random Variables II

**Geometric random variables,  $X \sim \text{Geom}(p)$**

$$\Pr(X = x) = p(1 - p)^{x-1}, \quad x = 1, 2, \dots$$

$$E[X] = \frac{1}{p} \quad \text{Var}(X) = \frac{1 - p}{p^2}.$$

**Negative Binomial random variables,  $X \sim \text{NBin}(r, p)$**

$$\Pr(X = x) = \binom{x-1}{r-1} p^r (1 - p)^{n-r}, \quad x = r, r + 1, \dots$$

$$E[X] = \frac{r}{p} \quad \text{Var}(X) = \frac{r(1 - p)}{p^2}.$$

**Discrete Uniform random variables,  $X \sim U(1, 2, \dots, n)$**

$$\Pr(X = x) = \frac{1}{n}, \quad x = 1, 2, \dots, n.$$

$$E[X] = (n + 1)/2 \quad \text{Var}(X) = (n^2 - 1)/12.$$

# Families of Continuous Random Variables I

**Uniform random variables,  $X \sim U(a, b)$ ,**

$$f_X(x) = \frac{1}{b-a}, \quad a < x < b.$$

$$E[X] = \frac{a+b}{2} \quad \text{Var}(X) = \frac{(b-a)^2}{12}.$$

**Exponential random variables,  $X \sim \text{Exp}(\lambda)$ ,**

$$f_X(x) = \lambda e^{-\lambda x}, \quad x > 0, \quad \lambda > 0.$$

$$E[X] = \frac{1}{\lambda} \quad \text{Var}(X) = \frac{1}{\lambda^2}.$$

**Normal random variables,  $X \sim N(\mu, \sigma^2)$ ,**

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0.$$

$$E[X] = \mu \quad \text{Var}(X) = \sigma^2.$$

## Families of Continuous Random Variables II

**Gamma random variables,  $X \sim Ga(n, \lambda)$ ,**

$$f_X(x) = \frac{\lambda^n}{\Gamma(n)} x^{n-1} e^{-\lambda x}, \quad x > 0, \quad n > 0, \quad \lambda > 0, \quad \Gamma(n) = (n-1)!$$

$$E[X] = \frac{n}{\lambda} \quad \text{Var}(X) = \frac{n}{\lambda^2}.$$

**Beta random variables,  $X \sim Beta(a, b)$ ,**

$$\begin{aligned} f_X(x) &= \frac{1}{B(a, b)} x^{a-1} (1-x)^{b-1} \\ &= \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1} (1-x)^{b-1}, \quad 0 < x < 1, \quad a > 0, \quad b > 0. \end{aligned}$$

$$E[X] = \frac{a}{a+b} \quad \text{Var}(X) = \frac{ab}{(a+b)^2(a+b+1)}.$$

# Column example

Here is some text in a left hand column.

