BEAMER SLIDES

Ann Example

Series Results

$$\sum_{x=0}^{\infty} q^x = \frac{1}{1-q}, \ |q| < 1.$$

$$\frac{d}{dx}\left(\sum_{x=0}^{\infty} q^x\right) = \sum_{x=0}^{\infty} x q^{x-1} = \sum_{x=1}^{\infty} x q^{x-1} = \frac{1}{(1-q)^2}, \ |q| < 1.$$

$$\sum_{x=0}^{N} q^x = \frac{1-q^{N+1}}{1-q}.$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = e^x$$

Expectation

Discrete Random Variables

$$E[g(X)] = \sum_{x \in S} g(x) Pr(X = x)$$

$$Var(X) = E[X^2] - E[X]^2.$$

Continuous Random Variables

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$$

$$Var(X) = E[X^2] - E[X]^2.$$

Families of Discrete Random Variables I

Bernoulli random variables, $\boldsymbol{X \sim Bern(p)}$

$$Pr(X = x) = \begin{cases} p, & x = 1, \\ 1 - p, & x = 0. \end{cases}$$

$$E[X] = p \qquad Var(X) = p(1-p).$$

Binomial random variables, $\boldsymbol{X \sim Bin(n, p)}$

$$Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \; x = 0, 1, \ldots, n,$$

$$E[X] = np \qquad Var(X) = np(1-p).$$

Poisson random variables, $\boldsymbol{X \sim Po(\lambda), \; \lambda > 0}$

$$Pr(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, \; x = 0, 1, 2, \ldots .$$

$$E[X] = \lambda \qquad Var(X) = \lambda.$$

Families of Discrete Random Variables II

Geometric random variables, $X \sim Geom(p)$

$$Pr(X = x) = p(1-p)^{x-1}, \; x = 1, 2, \ldots .$$

$$E[X] = \frac{1}{p} \qquad Var(X) = \frac{1-p}{p^2}.$$

Negative Binomial random variables, $X \sim NBin(r, p)$

$$Pr(X = x) = \binom{x-1}{r-1} p^r (1-p)^{n-r}, \; x = r, r+1, \ldots .$$

$$E[X] = \frac{r}{p} \qquad Var(X) = \frac{r(1-p)}{p^2}.$$

Discrete Uniform random variables, $X \sim U(1, 2, \ldots, n)$

$$Pr(X = x) = \frac{1}{n}, \; x = 1, 2, \ldots, n.$$

Families of Continuous Random Variables I

Uniform random variables, $X \sim U(a, b)$,

$$f_X(x) = \frac{1}{b-a},\ a < x < b.$$

$$E[X] = \frac{a+b}{2} \qquad Var(X) = \frac{(b-a)^2}{12}.$$

Exponential random variables, $X \sim Exp(\lambda)$,

$$f_X(x) = \lambda e^{-\lambda x},\ x > 0,\ \lambda > 0.$$

$$E[X] = \frac{1}{\lambda} \qquad Var(X) = \frac{1}{\lambda^2}.$$

Normal random variables, $X \sim N(\mu, \sigma^2)$,

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},\ -\infty < x < \infty,\ -\infty < \mu < \infty,\ \sigma > 0.$$

Families of Continuous Random Variables II

Gamma random variables, $X \sim Ga(n, \lambda)$,

$$f_X(x) = \frac{\lambda^n}{\Gamma(n)} x^{n-1} e^{-\lambda x}, \qquad x > 0,\ n > 0,\ \lambda > 0,\ \Gamma(n) = (n-1)!$$

$$E[X] = \frac{n}{\lambda} \qquad Var(X) = \frac{n}{\lambda^2}.$$

Beta random variables, $X \sim Beta(a, b)$,

$$\begin{aligned} f_X(x) &= \frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1} \\ &= \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}, \qquad 0 < x < 1,\ a > 0,\ b > 0. \end{aligned}$$

$$E[X] = \frac{a}{a+b} \qquad Var(X) = \frac{ab}{(a+b)^2(a+b+1)}.$$