

This document is a modified copy of the first five chapters of **Basic Analysis: Introduction to Real Analysis**, a free textbook included here for demonstration purposes.

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See <https://www.jirka.org/ra/> for more information (including contact information, possible updates and errata).

The L<sup>A</sup>T<sub>E</sub>X source for the book is available for possible modification and customization at github: <https://github.com/jirilebl/ra>

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