2025 r/mathmemes math contest

lets_clutch_this

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1 Information

1. You will be allotted a week (168 hours) of time to solve the problems and submit your answers via the provided Google Form.
2. Since it would be impractical to restrict, any online calculators or aids will be allowed. However, you should subscribe to the honor code of solving the problems by yourself only.
3. Problems range in (roughly increasing) difficulty from mid AMC to late AIME level.

2 Problems

1. Let T be the trapezoid formed by the lines $y = x$, $y = -x$, $y = n$, and $y = 84 - n$, where $0 < n < 84$ is an integer, and let L be the number of lattice points that lie strictly inside T. Given that T has positive area, what is the sum of all possible values of the largest prime factor of L?
2. 2024 keys are fitted on a circular keychain, and all of the keys are either red or blue. It is given that there exists a nontrivial (in other words, non-360°) way you can rotate the keychain such that it looks identical to the original rotation. How many distinct possible values are there for the number of keys out of the 2024 total keys that are red?
3. Suppose there are 7 runners on a circular track, and they all start running clockwise at the same point on the track. For all $1 \leq k \leq 7$, runner k runs at k miles per hour. Given any point in time t, let $\sigma(t)$ be the current permutation of the runners, reading from the start of the track clockwise for one loop. How many distinct possible permutations $\sigma(t)$ are there if the runners can run for an arbitrarily long amount of time? Assume that a valid permutation has all the runners at distinct positions on the track.
4. If N is the number of possible tier lists of all possible tier lists of all possible tier lists of all possible tier lists of all possible tier lists of all possible tier lists of all possible tier lists of all possible tier lists of all possible tier lists of the characters from Lycoris Recoil, what is the remainder when N is divided by 1000? Assume all tier lists have the 6 tiers S, A, B, C, D, and F.
5. To escape from Sukuna's domain expansion, Mahoraga teleports to a random point on the surface of a cube with side length $160\sqrt{3}$. Sukuna's domain expansion forms a sphere with radius $80\sqrt{7}$ around the center point of the cube. What is the probability that the sphere contains the point that Mahoraga picks?
6. 84 Lycoris agents are initially standing in a row. The DA then repeats these two steps 83 times: (1) Clone all existing Lycoris agents (2) Move each the newly cloned Lycoris agents one position in the row to the right of where the previous copies of them were standing. After these steps have been executed, how many Lycoris agents are standing in the 42nd position (from the left) of the row?
7. Let $\triangle ABC$ have circumradius 25, incenter I, and orthocenter H. Suppose $\sin(\angle CAB) = \dfrac{15}{17}$ and $\sin(\angle ABC) = \dfrac{3}{5}$. Find the area of $\triangle HIC$.

8. Points B and C are on the plane such that $BC = 6$. Point A is placed such that $\triangle ABC$ with centroid G and orthocenter H has segment GH parallel to segment BC. Let L be the locus of possible locations of A. There exists a finite set of points P and closed curve $\mathcal{C}$ such that $L \cup P = C$. Find the area enclosed by C.

9. 9 bullets are placed in a row, with the bullet at position 1 being a fake bullet. Chisato is bored and wants to amuse herself, so each second, she randomly chooses two of the 9 bullets and switches their positions in the row. Let E_n be the expected position in the row of the fake bullet after n seconds have elapsed. What is the least value of n such that $E_n \geq 4.999999$?

10. You have a $n \times n$ square in which all of its unit 1×1 tiles (indexed by $1 \leq i, j \leq n$) are initially colored white. For each $1 \leq i \leq n$ let S_i be the set of tiles that have indices $(i,1),(i,2),...,(i,i-1),(i,i+1),...,(i,n),(1,i),(2,i),...,(i-1,i),(i+1,i),...,(n,i)$. Each move, you can choose an $1 \leq i \leq n$, color all black tiles in S_i white, and color all white tiles in S_i black. How many distinct colorings of the $n \times n$ square can you achieve with some finite sequence of moves?

11. Call two integers a and b S-worthy if there exists an integer $m \in S$ such that $am - b$ is divisible by 83. How many subsets $S \subset 1, 2, 3, ..., 82$ are there such that the integers $1, 2, 3, ..., 82$ can be partitioned into two disjoint sets A and B such neither A nor B contains a pair of distinct integers that is S-worthy?

12. Let the graph G comprise of vertices $V_1, V_2, V_3, ..., V_{1023}$ and suppose that each for each $1 \leq i \leq 511$, V_i has an edge connecting it to V_{2i} and an edge connecting it to V_{2i+1}. Let G' be the graph G but with a random subset (chosen with equal probability among all possible subsets) of its edges deleted, and let H be the largest connected subgraph of G' that includes V_1. What is the expected number of leaves (vertices of degree 1) in H?

13. Chisato and Takina play the following game, where there is a token that is initially placed at $(0,0)$ on the 2D coordinate plane, and they take turns moving the token from (x,y) to either $(x+1,y)$ or $(x,y+1)$, with the first player to slide the token to a point on the line $y = 100 - 2x$ considered the winner of the game, in which then the game ends. Note since Takina is cold and analytical, she will always make an optimal move, while Chisato doesn't necessarily make optimal moves since she's just playing the game for fun. Given that Chisato goes first, how many total possible games are there where one of the players wins? Express your answer modulo 1000. (Note that two games are considered identical if and only if they share the exact same sequence of moves.)

14. How many non-empty subsets of the 82-element set $\left\{\pm\binom{83}{1}, \pm\binom{83}{2}, \pm\binom{83}{3}, ..., \pm\binom{83}{41}\right\}$ have the property that the sum of its elements is divisible by 83^2?

15. The number 10007 is prime. For some a, let $a, a^2, ..., a^m$ be a sequence modulo 10007, where m is the least integer ≥ 1 such that $a^m = 1$. Let $\sigma(a)$ denote the permutation of the elements $2, 2^2, 2^3, ..., 2^{13}$ corresponding to the order they appear in the sequence $a, a^2, ..., a^m$. Over all integer choices of $1 < a < 10006$, how many distinct possible permutations $\sigma(a)$ are there?