2025 r/mathmemes math contest

lets_clutch_this

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1 Information

1. You will be allotted a week (168 hours) of time to solve the problems and submit your answers via the provided Google Form.
2. Since it would be impractical to restrict, any online calculators or aids will be allowed. However, you should subscribe to the honor code of solving the problems by yourself only.
3. Problems range in (roughly increasing) difficulty from early AIME to late Putnam level.

2 Problems

1. Given the 5 statements (1) at most 1 of these 5 statements are true, (2) at most 2 of these 5 statements are true, (3) at most 3 of these 5 statements are true, (4) at most 4 of these 5 statements are true, and (5) at most 5 of these 5 statements are true, how many of these 5 statements are true?
2. Mark is standing on Point A on a circular road with two lanes: The outer lane has radius R and the inner lane has radius r. There are two cars that repeatedly drive around the road at constant speeds a and b miles per hour, respectively, where a and b are uniformly random real numbers in the range $[30, 100]$, with a uniformly random circular offset between the two cars. Given that Mark takes $\varepsilon > 0$ time to switch between lanes, what is the probability that Mark is able to survive indefinitely at Point A without getting hit by a car? Express your answer in terms of r, R, and ε.
3. To escape from Sukuna's domain expansion, Mahoraga teleports to a random point on the surface of a cube with side length $160\sqrt{3}$. Sukuna's domain expansion forms a sphere with radius $80\sqrt{7}$ around the center point of the cube. What is the probability that the sphere contains the point that Mahoraga picks?
4. 84 Lycoris agents are initially standing in a row. The DA then repeats these two steps 83 times: (1) Clone all existing Lycoris agents (2) Move each the newly cloned Lycoris agents one position in the row to the right of where the previous copies of them were standing. After these steps have been executed, how many Lycoris agents are standing in the 42nd position (from the left) of the row?
5. Points B and C are on the plane such that $BC = 6$. Point A is placed such that $\triangle ABC$ with centroid G and orthocenter H has segment GH parallel to segment BC. Let L be the locus of possible locations of A. There exists a finite set of points P and closed curve $\mathcal{C}$ such that $L \cup P = C$. Find the area enclosed by C.
6. 9 bullets are placed in a row, with the bullet at position 1 being a fake bullet. Chisato is bored and wants to amuse herself, so each second, she randomly chooses two of the 9 bullets and switches their positions in the row. Let E_n be the expected position in the row of the fake bullet after n seconds have elapsed. What is the least value of n such that $E_n \geq 4.999999$?
7. Denote a path p from $(0, 0)$ to (n, n) as a sequence of points $p_0, p_1, p_2, ..., p_{2n}$ such that $p_0 = (0, 0)$, $p_{2n} = (n, n)$, and for each $0 \leq j < 2n$, $p_{j+1} = (x_j, y_j + 1)$ or $p_{j+1} = (x_j + 1, y_j)$, where $p_j = (x_j, y_j)$. Let P_n be the set of all paths p from $(0, 0)$ to (n, n). For a path p in P_n, denote its corresponding path matrix $M(p)$ as the matrix with the entry $M_{xy} = 1$ for all coordinates $(x, y) \in p$ and 0s everywhere else, where

the rows of $M(p)$ are indexed from 0 to n from bottom to top and the columns are indexed from 0 to n from left to right. For instance, if $p = (0,0),(1,0),(1,1),(1,2),(2,2),(3,2),(3,3)$, then

$$M(p) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}.$$

If p is randomly selected from P_{10}, compute the expected value of $\det(M(p))$.

8. Let the graph G comprise of vertices $V_1, V_2, V_3, ..., V_{1023}$ and suppose that each for each $1 \leq i \leq 511$, V_i has an edge connecting it to V_{2i} and an edge connecting it to V_{2i+1}. Let G' be the graph G but with a random subset (chosen with equal probability among all possible subsets) of its edges deleted, and let H be the largest connected subgraph of G' that includes V_1. What is the expected number of leaves (vertices of degree 1) in H?

9. Chisato and Takina play the following game, where there is a token that is initially placed at $(0,0)$ on the 2D coordinate plane, and they take turns moving the token from (x,y) to either $(x+1,y)$ or $(x,y+1)$, with the first player to slide the token to a point on the line $y = 100-2x$ considered the winner of the game, in which then the game ends. Note since Takina is cold and analytical, she will always make an optimal move, while Chisato doesn't necessarily make optimal moves since she's just playing the game for fun. Given that Chisato goes first, how many total possible games are there where one of the players wins? Express your answer modulo 1000. (Note that two games are considered identical if and only if they share the exact same sequence of moves.)

10. A quartic polynomial, $f(x)$, has two distinct inflection points. Let l_1 and l_2 be the tangent lines to $f(x)$ at these inflection points. There exists a third line l_3, called the bitangent, which is tangent to $f(x)$ at two distinct points. Suppose the 3 points formed by the pairwise intersections of l_1, l_2, l_3 are the vertices of an equilateral triangle with area $\dfrac{3\sqrt{3}}{4}$. Find the maximum possible value of $\displaystyle\lim_{x\to\infty} \frac{f(x)}{x^4}$.

11. Let $f(t) = 2|t-3|-2$ for $t \in [1,5]$ and $f(t+4) = f(t)$ for all real t. If a and $\{b_k\}_{k=0}^{\infty}$ are real numbers such that $f(t) = \displaystyle\sum_{k=0}^{\infty} b_k \sin(akt)$ with $a > 0$, then the value of $\displaystyle\sum_{k=0}^{\infty} b_k^2 = \frac{p}{q}$ for relatively prime positive integers p and q. Find $p+q$.

12. The number 10007 is prime. For some a, let $a, a^2, ..., a^m$ be a sequence modulo 10007, where m is the least integer ≥ 1 such that $a^m = 1$. Let $\sigma(a)$ denote the permutation of the elements $2, 2^2, 2^3, ..., 2^{13}$ corresponding to the order they appear in the sequence $a, a^2, ..., a^m$. Over all integer choices of $1 < a < 10006$, how many distinct possible permutations $\sigma(a)$ are there?

13. A Reddit post initially has a score of 0. Whenever someone upvotes the post, the score is increased by 1, and whenever someone downvotes the post, the score is decreased by 1. It is given that when the post's score is at n, the probability that the next person who sees the post will upvote it is $\frac{1}{|n|+2}$ if $n \leq 0$ and $\frac{|n|+1}{|n|+2}$ if $n \geq 0$. If the next person doesn't upvote the post, they will downvote it. Let $P(n)$ be the probability that given that a post has a score of n, it will eventually reach a score of 0. What is the least value of $|n|$ such that $P(n) \leq \frac{1}{10^{12}}$?

14. Let X be the set of all points in the hypercube $[0,1]^{10}$, and let G be the group of (rotational/reflectional) symmetries on X. For each point x, define its orbit $Gx = gx : g \in G$. How many distinct possible values are there for $|Gx|$?

15. Let n be a positive integer. Let U be a discrete uniform random variable with support $[0,n]$. An integer k is sampled from U. Let $V \sim \mathcal{B}(k+1,1)$. Two numbers, a and b are independently sampled from V. Two other numbers α and β are then sampled independently and uniformly at random from the interval $(0,1)$. The intervals I_1 and I_2 are then constructed such that $I_1 = \left(\alpha - \dfrac{a}{2}, \alpha + \dfrac{a}{2}\right)$ and $I_2 = \left(\beta - \dfrac{b}{2}, \beta + \dfrac{b}{2}\right)$. Let $P(n) = P((I_1 \cap I_2 = \emptyset) \cap (I_1 \cup I_2 \subset (0,1))$ as a function of n.

Find $\lim_{n\to\infty} n \times P(n)$.