2026 r/mathmemes math contest v4

lets_clutch_this

To be Released Late 2025/Early 2026

1 Information

1. You will be allotted 48 hours of time to solve these 12 problems and submit your answers via the provided Google Form.
2. Since it would be impractical to restrict, any online calculators or aids will be allowed. However, you should subscribe to the honor code of solving the problems by yourself only.
3. Problems range in (roughly increasing) difficulty from early AMC to mid-late Putnam level.

2 Problems (WIP version)

1. James really wants to get a A on his number theory final exam, which is defined as a marked score of 90 or above. He is anxious, as he feels as if his performance was right around borderline. Fortunately for him, his professor is a very lenient grader - for any raw score r, his professor takes $\lceil r \rceil$ as the marked score so as long there exists some sequence of integers $a_1 > a_2 > ... > a_k \geq 0$ such that when r is rounded up to the nearest 10^{-a_1}, then to the nearest 10^{-a_2}, and so on eventually to the nearest 10^{-a_k}, the score rounds to $\lceil r \rceil$. The maximum raw score R that James can score on the final exam yet not get a final grade of A can be expressed as $R = \frac{m}{n}$. Find $m+n$.
2. Let S be the set of all ordered pairs of integers (x, y) such that $1 \leq x, y \leq 2025$ and $x+y$ is even. Let $P(x, y)$ be the probability distribution on x and y defined by the uniform distribution on S. What is the correlation coefficient between x and y?
3. MC Edgar has eight vinyl disks, each of distinct colors and each of radius 1 are placed flat centered at $(0, 2)$, $(2, 0)$, $(0, -2)$, $(-2, 0)$, $(1, -1)$, $(1, 1)$, $(-1, 1)$, and $(-1, -1)$, all with distinct vertical/z-coordinates. Each individual vinyl disk is placed at a fixed (x, y) coordinate, and the colors are fixed as well. He then commemorates his collection of vinyls by using a drone to take a picture of these eight vinyl disks in a birds' eye view centered at $(0, 0)$ in the fixed orientation, where only the tops of the vinyl disks are visible. How many distinct pictures can be taken?
4. Let $a_1, a_2, ..., a_{99}$ be a binary sequence such that for each $1 \leq k \leq 99$, a_k independently has a $k\%$ probability of being 1 and a $(100-k)\%$ probability of being 0. Denote a *run* in a binary sequence as a maximal string of consecutive values. For instance, the binary string 100011100 has 4 distinct *runs*. What is the expected number of *runs* in the binary sequence $a_1, a_2, ..., a_{99}$?
5. You have a deck of 2025 cards. For some fixed $1 \leq k \leq 2025$, let $f(k)$ be the number of possible arrangements of the 2025 cards you can achieve via shuffling this deck by performing some finite number of k-swaps, where a k-swap consists of taking k cards at consecutive positions in the deck and placing the card that was originally the bottommost card among those k cards on the top of the $k-1$ other cards. Find the sum of all distinct possible values of $f(k)$ over the range $1 \leq k \leq 2025$.
6. For an ordered quadruple of integers (a, b, c, d), define $f(0, 0, 0, 0) = 1$, $f(a, b, c, d) = f(a-1, b, c, d) + f(a, b-1, c, d) + f(a, b, c-1, d) + f(a, b, c, d-1)$ for all other nonnegative a, b, c, d, and $f(a, b, c, d) = 0$ if at least one of a, b, c, d are negative. For how many ordered quadruples of integers (a, b, c, d) such that $0 \leq a, b, c, d \leq 15$ is $f(a, b, c, d)$ odd?
7. What is the least value of n such that for any sequence $a_1 < a_2 < ... < a_n$ of distinct positive integers, we have that $\prod_{1 \leq i < j \leq n} (a_j - a_i)$ is divisible by 7^{7^7}?

8. Let $f_k(a,n) = \underbrace{a^{a^{a^{a^{a^{\cdot^{\cdot^{\cdot^{a^n}}}}}}}}}_{\text{k a's}}$, and let $g(a)$ be the least positive integer k such that $f_k(a,n)$ is a constant function modulo $p = 65537$ in n, over all nonnegative integers n. Find $\max_{1 \le a < 65537} g(a)$.

9. Consider the Tree T_5, which consists of a set V of 63 vertices $V_1, V_2, ..., V_{63}$ such that for each $1 \le i \le 31$, vertex V_i connects to vertices V_{2i} and V_{2i+1} via an edge. Call a labeling of the vertices $f : V \to \mathbb{R}$ (a *family-tree labeling* if it satisfies $f(V_i) \in \{0,1\}$ for all $i \ge 32$ and for each $1 \le i \le 31$, $f(V_i) = \frac{f(V_{2i}) + f(V_{2i+1})}{2}$. How many distinct *family-tree labelings* of T_5 satisfy $f(V_1) = \frac{1}{2}$? Two labelings are considered equivalent if there exists an automorphism of T_5 mapping the former labeling to the latter.

10. The string $ABCDEFGHIJ$ is initially written on the board. Every iteration, one of the 10 letters in the string is uniformly randomly chosen, and this letter replaces an independently and uniformly random letter out of one of the 10 letters. What is the expected value for the number of iterations until $ABCDEFGHIJ$ becomes a 10-letter string of only one unique letter?

11. Yvonne has 70 miniature houses, each of distinct dimensions in feet (i,j,k), where $1 \le i \le j \le k \le 24$ are each integers. It is given that her 70 miniature houses can be arranged in an order $H_1, H_2, ..., H_{70}$ such that for each $1 \le n < 70$, House H_n is fully contained inside House H_{n+1}. For any such set of 70 miniature houses $(H_1, H_2, ..., H_{70})$ of distinct dimensions with those properties, let $M(H_1, H_2, ..., H_{70})$ be the number of ways to arrange them such that (1) for each $1 \le n < 70$, House H_n is fully contained inside House H_{n+1} and (2) for each miniature house, all of its edges are aligned with integer coordinates. Find the number of distinct possible values of $M(H_1, H_2, ..., H_{70})$, assuming the coordinates of H_{70} are fixed. (Two arrangements are considered the same if in both arrangements, each of the 70 miniature houses $H_1, H_2, ..., H_{70}$ occupy pairwise the same space.)

12. Denote a permutation $\sigma \in S_n$ as *cute* if it can be written as $\sigma = T_1 T_2 ... T_k$ where $0 \le k < n$, each of the T_is is a transposition in the form $(j\ j+1)$ for some $1 \le j < n$, and $T_1, T_2, ..., T_k$ are all distinct. Let $f(n)$ be the number of *cute* permutations σ in S_n. Find the nearest integer to $\frac{f(2027)}{f(2019)}$.

13. Call a function $f : \mathbb{F}_2^8 \to \mathbb{F}_2$ *XOR-like* if there exists some basis $\mathcal{B} = \{b_1, b_2, ..., b_8\}$ of $\mathbb{F}_2^8$ such that for every $x, y \in \mathbb{F}_2^8$, $f(x) = f(y)$ if and only if the Hamming distance from $[x]_{\mathcal{B}}$ to $[y]_{\mathcal{B}}$ is even. How many such *XOR-like* functions are there?

14. Donnie is tiling his 4 foot by 10 foot floor with 20 non-overlapping 1 foot by x foot rectangular tiles whose union fits in the floor exactly, and where x could vary tile by tile. Each tile must be placed in the horizontal orientation, with its 1-foot-long edge parallel to the 4 foot edge of the floor. Also, no four tiles are allowed to meet at the same vertex. How many possible distinct such satisfactory tilings that are there that use all 20 tiles, where two tilings are considered equivalent if and only if (1) the number of tiles on each row is the same comparing both configurations and (2) comparing both configurations, each tile in each row is adjacent to the same set of other tiles? For example, the two tilings on the left of this diagram are equivalent, while both being distinct from the third tiling on the right of this diagram.

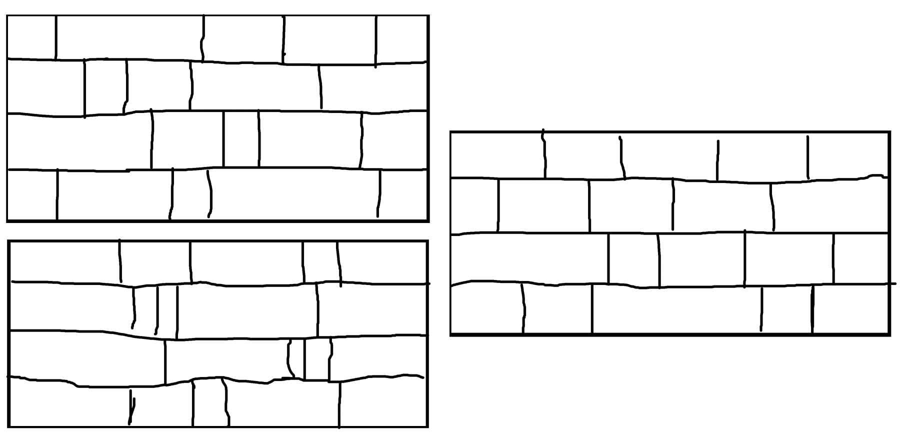

Figure 1: Example Tilings for Problem #14

Note that any rotations and reflections of a tiling arrangement (with respect to the whole 4-by-10 floor) still count as a distinct arrangement.

15. Call a nonnegative rational number *semi-dyadic* if it can be expressed as

$$\frac{a}{2^b - 2^c}$$

where a, b, and c are nonnegative integers, and $b > c$. How many distinct *semi-dyadic* numbers with $0 \leq b, c \leq 10$ are there in the range $[0, 1]$?