An Interacting Stochastic Models Approach for the Performance Evaluation of DSRC Vehicular Safety Communication

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Abstract—In this paper, an analytic model is proposed for the performance evaluation of vehicular safety related services in the dedicated short range communications (DSRC) system on highways. The generation and service of safety messages in each vehicle is modeled by a generalized M/G/1 queue. The overall model is a set of interacting M/G/1 queues, one queue for each vehicle. The interaction is that the server is shared as it is the contention medium. To make the model scalable, we use semi-Markov process (SMP) model to capture the shared server's behavior from one tagged vehicle's perspective, where the medium contention and back off behavior for this vehicle and influences from other vehicles are considered. Furthermore, this SMP interacts with the tagged vehicle's own M/G/1 queue through fixed-point iteration. The proof for the existence, uniqueness and convergence of the fixed point is provided. Based on the fixed-point solution, performance indices including mean transmission delay, packet delivery ratio (PDR), and packet reception ratio (PRR) are derived. Analytic-numeric results are verified through extensive simulations under various network parameters. Compared with the existing models, the proposed SMP model facilitates the impact analysis of hidden terminal problem on the PDR and PRR computation in a more precise manner.

Index Terms—Analytic model, DSRC, M/G/1 queue, performance evaluation, safety message, SMP model, VANET

1 Introduction

Intervehicle Communication (IVC), as a vital part of Intelligent Transportation System (ITS), has been extensively researched in recent years. Dedicated Short Range Communication (DSRC) [1], adopted by IEEE- and ASTM, is being seriously considered by automotive industry and government agencies as a promising wireless technology for enhancing transportation safety and highway efficiency. The 5.9 GHz DSRC is a secure, high speed, short range wireless interface between vehicles and surface transportation infrastructures that enables rapid communication of vehicle data and other content between On Board Equipment (OBE) and Road Side Equipment (RSE). According to the updated version of the DSRC standard [1], the DSRC physical layer follows the same frame structure, modulation scheme, and training sequences specified by IEEE 802.11a physical layer standard with minor changes; MAC layer of the DSRC is equivalent to the Enhanced Distribution Coordination Access (EDCA) 802.11e that has four different access classes (ACs). In the DSRC-based vehicular ad hoc networks (VANETs), the transportation safety is one of the most crucial features that needs to be addressed. Safety applications usually demand direct vehicle-to-vehicle ad hoc communication due to a highly dynamic network topology and strict delay requirements. Such direct safety communication will involve a broadcast service because safety information can be beneficial to all vehicles around a sender. Broadcasting safety messages is one of the fundamental services in DSRC that is being standardized as IEEE 802.11p.

In the literature, unicast for IEEE 802.11 has been extensively investigated. Bianchi [27] proposed a simple yet accurate discrete-time Markov chain (DTMC) model to evaluate the performance of unicast mechanism under saturation conditions. His paper has inspired many other researchers to develop analytic models of unicast based on DTMC. For example, multiple types of safety messages are evaluated in [28], [29] under saturation conditions. Bianchi's work was later extended to the unsaturated case in [30].

Unique characteristics of the broadcast make its analysis different from that of unicast. First, unicast can have handshake between the sender and the receiver, which provides feedback to the sender for the possible need for retransmission to enhance reliability. By contrast, broadcasting sender cannot obtain feedback information from the receivers and hence it will generally only broadcast once. Second, the hidden terminal area in broadcast can be considerably larger than that in unicast, which makes broadcast very sensitive to hidden terminals. Third, besides throughput and packet transmission delay, distinctly different output measures need to be utilized to characterize broadcast reliability as compared with unicast. Therefore, the analysis methods that have been used for unicast cannot be extended to broadcast in a straightforward manner.

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Performance of vehicular safety communication in DSRC system has been studied in [2], [3], [4], [5]. Two important performance metrics: packet delivery ratio (PDR) and packet reception rate (PRR) are introduced, respectively, in [7] and [3] to quantify the performance of safety message broadcast in DSRC based VANETs. However, the evaluations are mainly based on simulations or experiments. Recently, a few analytic models have been developed to characterize and evaluate the broadcast performance. In [23], an analytic model based on DTMC is constructed to characterize the operation of the IEEE 802.11 MAC backoff counter for broadcast, and closed form solution is obtained for the PDR of wireless LAN. However, saturation assumption is made in this paper, which is not a good approximation for a real situation. Efficiency and reliability of beacon message broadcast in DSRC VANETs is investigated in [24] considering channel fading as the only source of broadcast failures under the saturation condition without accounting for the hidden terminal problem. A DTMC model interacting with an M/G/1 queue was developed in [6], [7], [8] to analyze the performance of the DSRC broadcast services incorporating the backoff counter process, hidden terminals, and unsaturated message generation. Based on [7], a more precise model based on DTMC is constructed in [25] to evaluate the transmission delay and PDR of 1D VANET, and the performance for protocols with and without broadcast retransmissions is compared. Channel switching in current version of IEEE 802.11p was considered and its effects on the VANET performance were evaluated in [26], whereas the hidden terminal effect and unsaturation condition were omitted. The authors in [31], [32], [33] consider multiple types of safety message broadcast using EDCA mechanism. However, most previous research is based on Bianchi's DTMC (i.e., per-slot statistics) model [27] and ignores important aspects of continuous time system behavior leading to approximations. More specifically, due to the characteristic of such discrete models, backoff counter freezing behavior is not accurately captured. Lee and Lee [38] considered such freezing process by adding sub-Markov chain for multiple services using EDCA. However, the backoff counter freezing time triggered by the busy medium (i.e., waiting for the packet in the channel to finish transmission) is still not considered. In a recent paper by Tinnirello and Bianchi [37], an analytic model not based on the per-slot statistics was proposed through a fixed-point computation of the residual backoff counter distribution occurring after a generic transmission attempt. However, this is only for the saturation condition.

In our recent paper [12], an analytic model where a semi-Markov process (SMP) [9] interacting with an M/G/1 queue is developed for the performance evaluation of one-hop direct broadcast service in DSRC safety communication system. The SMP model can accurately capture the continuous time system behavior beyond the per-slot statistics. In addition, it directly incorporates the case of a packet being directly transmitted without going through the backoff process, which is omitted by many previous papers. Moreover, the precise backoff counter freezing process is captured by the SMP model. The hidden terminal problem is also taken into account.

In this paper, we extend the work in [12] and provide a more accurate analytic model for the performance evaluation of one type of safety message in the control channel of

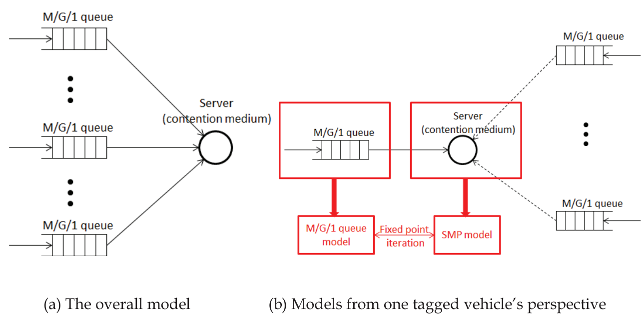

Fig. 1. Models capturing interactions between vehicles.

DSRC. The generation and service of safety messages in each vehicle is modeled by a generalized M/G/1 queue, where two classes of service are considered based on Welch's method [13]. The overall model is a set of interacting M/G/1 queues, one queue for each vehicle, as shown in Fig. 1a. The interaction is that the server is shared as it is the contention medium/channel for the safety message transmissions. To make the model scalable, we use semi-Markov process model to capture the shared server's behavior from a single tagged vehicle's perspective, where the medium/channel contention and backoff behavior for this vehicle and the influence from other vehicles are considered as shown in Fig. 1b. Different from the previous DTMC models capturing the shared server's discrete time behavior, this SMP model directly incorporates the unsaturation condition of the queue in a continuous time fashion. It interacts with the tagged vehicle's M/G/1 queue through the fixed-point iteration as shown in Fig. 1b.

In this paper, we concentrate on the study of fundamental performance issues of IEEE 802.11-based broadcast in DSRC vehicular environment. For simplicity of the analysis, our proposed interacting stochastic model is for one type of safety service in a single channel. However, our model can be easily extended to multiple types of safety messages such as EDCA and multichannel services in current or future IEEE 802.11p.

The major contributions made in this paper are three-fold. First, a semi-Markov process model is developed to characterize MAC behavior of IEEE 802.11p-based network with unsaturated message generation and hidden terminals in the continuous time, which is more general and precise compared with the existing DTMC-based models. Second, in order to solve the SMP model, the service time for packets generated from the tagged vehicle is divided into two classes, 1) packets arriving when the tagged vehicle's queue is empty; 2) packets arriving when the tagged vehicle's queue is not empty. Based on Welch's method [13] with two classes of arrivals in an M/G/1 queue, more accurate results can be obtained. Moreover, the proofs for the existence, uniqueness and convergence [14], [15] of the fixed-point iteration [10] between the M/G/1 queue and the SMP model are provided. Third, based on the solution to the interacting SMP and M/G/1 queuing model, expressions for the key performance metrics of DSRC vehicular safety communication are derived. These metrics include the mean transmission delay, packet delivery ratio and packet reception ratio. The analytic results are verified by simulations to show the errors in the decomposition approximations are small. The proposed model is compared with the analytic models published earlier in [7], [39]. The numerical results show that the new model is more accurate.

This paper is organized as follows: Section 2 briefly describes the system behavior in IEEE 802.11 MAC layer protocol and lists the assumptions made to produce a tractable model. Section 3 presents the analytic models and the fixed-point iteration. Performance indices including mean transmission delay, PDR, and PRR are derived in Section 4. The analytic and simulation results are compared in Section 5. Conclusions are presented in the last section.

2 System Description

2.1 Broadcast Protocol

As we know from [1] that the DSRC MAC layer adopts IEEE 802.11 MAC layer specification with minor modifications. In the 802.11 MAC layer protocol [11], distributed coordination function (DCF) is the primary medium access control technique for broadcast services. This section briefly explains the basic access mechanism of DCF for broadcast in the context of the safety communication.

Each vehicle in the network can occasionally generate safety related packets and compete for the channel resource to transmit the packet. For a newly generated packet in a vehicle, the vehicle senses the channel activity before it starts to transmit the packet. If the channel is sensed idle for a time period of distributed interframe space (*DIFS*), the packet can be directly transmitted. Otherwise, the vehicle continues to monitor the channel until channel is detected idle for *DIFS* time period. Subsequently, according to the collision avoidance feature of the protocol, the vehicle generates an initial random backoff counter and goes through the backoff process before transmitting the packet. Moreover, a vehicle must go through the backoff process between two consecutive packet transmissions even if the channel is sensed idle for the duration of *DIFS* time for the second packet. Therefore, a packet generated in a vehicle can directly transmit without undergoing the backoff process only when the following two conditions are satisfied:

- The packet is generated in a vehicle when this vehicle's own queue is empty;
- The channel is sensed idle for *DIFS* time starting from the time instant that the packet is generated.

Regarding the backoff process for a packet transmission, the initial backoff counter is chosen randomly from a uniform probability mass function (pmf) over the range $[0, W_0 - 1]$, where W_0 represents the backoff window size. The backoff time counter is decreased by one if the channel is sensed idle for a time slot of duration σ. The counter is frozen when channel is sensed busy and reactivated when the channel is sensed idle again for more than the *DIFS* duration. The packet is transmitted as soon as the backoff counter reaches zero.

In broadcast services, the transmitting vehicle does not receive any feedback from the receivers and will not retransmit a packet. The detailed descriptions for IEEE 802.11 standard can be found in [11].

2.2 System Assumptions

Several assumptions are made in the broadcast system to produce a simplified yet a high fidelity model.

1. The vehicular ad hoc network is considered to be one-dimensional (1D). The number of vehicles in a line is Poisson distributed with parameter β (vehicle density), i.e., the probability $P(i,\ l)$ of finding i vehicles in a length of l is given by:

$$P(i, l) = \frac{(\beta l)^i}{i!} \cdot e^{-\beta l}. \tag{1}$$

2. All vehicles have the same transmission range, receiving range, and carrier sensing range R.
3. Only one type of safety message in the control channel is considered.
4. Each vehicle is assumed to generate packets as a Poisson stream with rate λ (in packets per second).
5. Each vehicle has an infinite queue to store the packets at the MAC layer. Hence, each vehicle can be modeled as an M/G/1 queue. Queue-length process is independent and identically distributed (i.i.d.) for each vehicle.
6. Channel shadowing or fading, vehicle mobility, and capture effect of transmissions are not considered in this paper.

For assumption 1, the 1D VANET model is a good approximation of ad hoc networks on highway when the distances between lanes on the highway are negligible compared with the length of the highway. In addition, recent statistical analysis of empirical data collected from real-world scenarios [34], [35] show that exponential model is a good fit for sparse highway vehicle traffic in terms of intervehicle distance and intercontact time distribution, which validates the reasonability of assumptions 1 and 4. Heavy traffic scenarios where exponential distributed intervehicle distance does not fit will be considered in our future work. For assumption 2, we currently set three communication ranges the same to simplify the analysis. Extension of the model to more general case with different communication ranges is straightforward with similar strategy in [36]. Assumption 3 can be relaxed by extending our model for multiple types of services as described in the Section 6. The queue model in assumption 5 is a reasonable approach as long as the channel service rate is greater than message arrival rate. Therefore, an infinite queue is a reasonable assumption and can be relaxed easily. Moreover, since the interactions between vehicles are through the channel contention, whose behavior is captured by the SMP model, we assume the queue-length process is independent and identically distributed for each vehicle. Assumption 6 is made because we only concentrate on the impact of packet collisions and the hidden terminal problem on the performance in this paper. In fact, it has been proven in [40], [41] that high mobility of vehicles (up to 120 mph) has very minor impact on the performance of the direct message broadcast network with high data rate (e.g., $\geq$12 Mbps). Channel shadowing/fading will be considered in our later work.

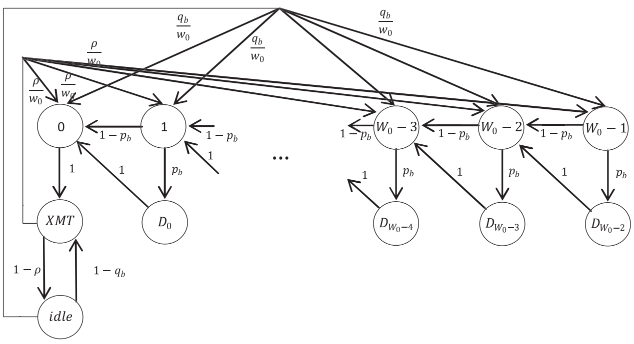

Fig. 2. SMP model for 802.11p broadcast.

3 Analytic Models

Due to the contention medium, the overall problem can be seen as a set of interacting M/G/1 queues. We simplify the problem in this paper by developing an SMP model for the tagged vehicle that does not directly keep track of the queued requests but captures the channel contention and backoff behavior. This SMP model and the tagged vehicle's M/G/1 queue interact with each other and hence we need to use fixed-point iteration to solve the overall model.

3.1 SMP Model

The behavior of a tagged vehicle for packet transmission can be characterized using the SMP model in Fig. 2. The tagged vehicle is in *idle* state if there is no packet in its queue. After a packet is generated, the vehicle senses channel activity for *DIFS* time period. If the channel is detected not busy during this period (with probability $1-q_b$), the vehicle goes from *idle* state to *XMT* state, which means that a packet is transmitting. Otherwise, the vehicle will randomly choose a backoff counter in the range $[0, W_0-1]$, where W_0 is the backoff window size. The backoff counter will be decreased by one if the channel is detected to be idle for a time slot of duration σ (with probability $1-p_b$), which is captured by the transition from state W_0-i to state W_0-i-1. If the channel is busy during a backoff time slot σ (i.e., another vehicle is transmitting a packet), the backoff counter of the tagged vehicle will be suspended and deferred for the duration of packet transmission time T, which presents the transition from state W_0-i to $\mathrm{D_{W_0-i-1}}$ with probability p_b. When the backoff counter reaches zero, the packet will directly be transmitted (an SMP transition occurs from state 0 to state *XMT* with probability one). In *XMT* state, a packet is transmitting. After the packet transmission, if there is no packet left in the queue of the tagged vehicle (with probability $1-\rho$), the vehicle will go from *XMT* to *idle* state and wait for a new incoming packet. If there are packets left in the queue after a packet transmission (with probability ρ), the vehicle will sense the channel again for *DIFS* time and then randomly choose a backoff counter before transmitting the next packet.

Next, the analytic method is presented to solve such SMP model to obtain the steady-state probability that a vehicle transmits, which will be used in the fixed-point iteration in Section 3.3.

Define the sojourn time in state j as T_j. The mean and variance of T_j in the SMP model are:

$$E[T_j] = \tau_j = \begin{cases} \sigma & j = 0,1,2,\ldots,W_0-1 \\ T & j = D_0, D_1, \ldots, D_{W_0-2} \\ T & j = XMT \\ \frac{1}{\lambda} + DIFS & j = idle, \end{cases} \tag{2}$$

$$Var[T_j] = \theta_j^2 = \begin{cases} 0 & j = 0,1,2,\ldots,W_0-1 \\ \frac{Var[PA]}{R_d^2} & j = D_0, D_1, \ldots, D_{W_0-2} \\ \frac{Var[PA]}{R_d^2} & j = XMT \\ 1/\lambda^2 & j = idle, \end{cases} \tag{3}$$

where $T = E[PA]/R_d + T_H + DIFS + \delta$. The mean and variance of the packet length are $E[PA]$ and $Var[PA]$, respectively. R_d presents the data rate. Hence, $E[PA]/R_d$ is the time to transmit the packet. T_H presents the time to transmit the packet header including physical layer header and MAC layer header. δ is the propagation delay.

For the model in Fig. 2, the embedded DTMC is first solved for its steady-state probabilities [9]:

$$\begin{cases} v_j = (W_0-j)\cdot v_{W_0-1} \quad j = 0,1,\cdots W_0-1 \\ v_{D_j} = (W_0-j-1)\cdot p_b \cdot v_{W_0-1} \quad j = 0,1,\cdots W_0-2 \\ v_{XMT} = \dfrac{W_0}{\rho + q_b(1-\rho)}\cdot v_{W_0-1} \\ v_{idle} = \dfrac{(1-\rho)W_0}{\rho+q_b(1-\rho)}\cdot v_{W_0-1}, \end{cases} \tag{4}$$

$$\begin{aligned} v_{W_0-1} = \{2[\rho + q_b(1-\rho)]\}/\{[W_0+1+p_b(W_0-1)] \\ [\rho+q_b(1-\rho)]\cdot W_0 + 2(2-\rho)W_0\}. \end{aligned} \tag{5}$$

Taking account of the mean sojourn time in each state, the steady-state probabilities of the SMP are given by Cinlar [22]:

$$\pi_i = \frac{v_i \tau_i}{\sum_j v_j \tau_j}. \tag{6}$$

Therefore, the steady-state probability that a vehicle is in the *XMT* state is given by:

$$\begin{aligned} \pi_{XMT} = \{2T\}/\{[\rho+q_b(1-\rho)][(\sigma+p_b\cdot T)W_0 \\ + (\sigma - p_b\cdot T)] + 2T + 2(1-\rho)(1/\lambda + DIFS)\}. \end{aligned} \tag{7}$$

Although the sojourn time in *XMT* state is T, the real packet transmission only occupies a portion of this sojourn time, which is $E[PA]/R_d + T_H + \delta = T - DIFS$. Hence, the probability that a vehicle transmits in steady state is $\pi_{XMT}(T-DIFS)/T$.

In (7), three unknown parameters are:

- ρ: the probability that there are packets in the queue of the tagged vehicle.
- p_b: the probability that the channel is detected busy in one time slot by the tagged vehicle.
- q_b: the probability that the channel is detected busy in *DIFS* time by the tagged vehicle.

In Section 3.3, we will see that p_b and q_b are functions of ρ, whereas ρ depends on the mean service time to transmit a packet. Therefore, the service time is derived first in the

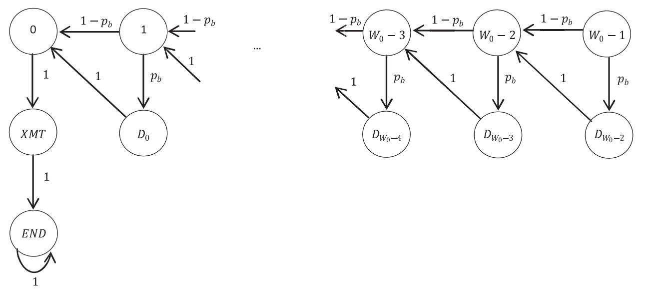

Fig. 3. SMP model for service time computation.

next section. Section 3.3 subsequently illustrates the relationships between these parameters and fixed-point iteration algorithm is utilized to compute the numerical results for these parameters as well as the service time.

3.2 Service Time Computation

As mentioned above, each vehicle in the network can be modeled as an M/G/1 queue. The MAC layer service time is defined as the time interval from the time instant when a packet becomes the head of the queue and starts to contend for transmission, to the time instant when the packet is received.

The SMP model in Section 3.1 describes the behavior of a tagged vehicle continuously transmitting packets in its queue. In this section, the service time for any one packet in the queue needs to be derived. Therefore, the SMP model in Section 3.1 can be modified to contain an absorbing state as shown in Fig. 3. By properly allocating the initial probability, the time to reach the absorbing state will be the service time for a packet transmission.

The mean and variance of the service time conditioned on starting from state $i(S_i)$ will be derived first based on the mean and variance of the sojourn time (T_j) and visit counts (X_{ij}) in each state.

Since the Markov chain contains an absorbing state, the transition probability matrix can be partitioned so that [9]:

$$P = \begin{bmatrix} Q & C \\ 0 & 1 \end{bmatrix}, \tag{8}$$

where Q is a $2W_0$ by $2W_0$ substochastic matrix describing the probabilities of transitions only among the transient states. The fundamental matrix is:

$$M = (I - Q)^{-1}. \tag{9}$$

Let X_{ij} be the random variable denoting the visit counts to state j before entering the absorbing state, given that embedded DTMC started in state i. The expected number of visits to state j starting from state i before absorption is given by the (i, j)th element of the fundamental matrix M. Hence,

$$E[X_{ij}] = m_{ij}. \tag{10}$$

Due to the acyclic nature of the DTMC model in Fig. 3, the fundamental matrix can be easily obtained through the definition of X_{ij} instead of computing (9).

$$M = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ \vdots \\ W_0-2 \\ W_0-1 \\ D_0 \\ D_1 \\ D_2 \\ \vdots \\ D_{W_0-2} \\ XMT \end{array} \begin{array}{c} 0\ 1\ 2\ \cdots\ W_0-2\ W_0-1\ D_0\ D_1\ D_2\ \cdots\ D_{W_0-2}\ XMT \\ \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 1 & 0 & \cdots & 0 & 0 & p_b & 0 & 0 & \cdots & 0 & 1 \\ 1 & 1 & 1 & \cdots & 0 & 0 & p_b & p_b & 0 & \cdots & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 1 & 0 & p_b & p_b & p_b & \cdots & 0 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 1 & p_b & p_b & p_b & \cdots & p_b & 1 \\ 1 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 1 & 0 & \cdots & 0 & 0 & p_b & 1 & 0 & \cdots & 0 & 1 \\ 1 & 1 & 1 & \cdots & 0 & 0 & p_b & p_b & 1 & \cdots & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 1 & 0 & p_b & p_b & p_b & \cdots & 1 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \end{array}. \tag{11}$$

Furthermore, the variance of the number of visits can also be derived using the fundamental matrix. Define $MD = [md_{ij}]$ by:

$$md_{ij} = \begin{cases} m_{ij} & if\ i = j \\ 0 & otherwise. \end{cases} \tag{12}$$

Define $M_2 = [m_{ij}^2]$. Hence, the variance of the visit counts is [17]:

$$\sigma^2 = M(2M_D - I) - M_2. \tag{13}$$

The service time for a packet transmission starting from state i is given by:

$$S_i = \sum_j T_j \cdot X_{ij}, \tag{14}$$

$$\begin{aligned} E[S_i] &= E\left[\sum_j T_j \cdot X_{ij}\right] = \sum_j E[T_j \cdot X_{ij}] \\ &= \sum_j E[T_j] \cdot E[X_{ij}] = \sum_j \tau_j \cdot m_{ij} \\ &= \begin{cases} (i+1)\sigma + i \cdot p_b \cdot T + T\ for\ i = 0, 1, \ldots, W_0 - 1 \\ T\ for\ i = XMT \end{cases} \\ & j \in \{0, 1 \cdots, W_0 - 1, D_0, D_1, \ldots, D_{W_0-2}, XMT\}. \end{aligned} \tag{15}$$

Since the sojourn time in state 0 is zero in the protocol instead of σ as specified in the model, we adjust the mean of S_i starting from $i = 0, 1, \ldots, W_0 - 1$ by decreasing σ in the results. Hence,

$$E[S_i] = \begin{cases} i \cdot \sigma + i \cdot p_b \cdot T + T & for\ i = 0, 1, \ldots, W_0 - 1 \\ T & for\ i = XMT. \end{cases} \tag{16}$$

The variance of S_i is given by (17).

$$
\begin{aligned}
Var[S_i] &= Var\left[\sum_j T_j \cdot X_{ij}\right] \\
&= \sum_j Var[T_j \cdot X_{ij}] \\
&= \sum_j \{Var[T_j] \cdot E[X_{ij}] + (E[T_j])^2 \cdot Var[X_{ij}]\} \\
&= \sum_j \left(\theta_j^2 \cdot m_{ij} + \tau_j^2 \cdot \sigma_{ij}^2\right) \\
&= \begin{cases} i\left\{\dfrac{Var[PA]}{R_d^2} \cdot p_b + T^2 \cdot p_b \cdot (1-p_b)\right\} + \dfrac{Var[PA]}{R_d^2} \\ \qquad\qquad for \;\; i = 0, 1, \ldots, W_0 - 1 \\ \dfrac{Var[PA]}{R_d^2} \\ \qquad\qquad for \;\; i = XMT \end{cases} \\
& j \in \{0, 1 \cdots, W_0 - 1, D_0, D_1, \ldots, D_{W_0-2}, XMT\}.
\end{aligned} \tag{17}
$$

Based on the mean and variance of the conditional service time starting from state $i(S_i)$ obtained above, we will compute the mean of unconditional service time next to be used in Welch's method. To compute the mean service time, we separate the service time distribution into two classes, 1) for the packet arrivals when the tagged vehicle's queue is empty; 2) for the packet arrivals when the tagged vehicle's queue is not empty.

For the packet that arrives when the tagged vehicle's queue is empty, the service time is given by:

$$
S_e = \begin{cases} S_0 & w.p. \quad q_{e,0} = q_b / W_0 \\ S_1 & w.p. \quad q_{e,1} = q_b / W_0 \\ \quad \vdots & \\ S_{W_0-1} & w.p. \quad q_{e,W_0-1} = q_b / W_0 \\ S_{XMT} & w.p. \quad q_{e,XMT} = 1 - q_b. \end{cases} \tag{18}
$$

The mean and variance of the service time for the packet arrivals when the tagged vehicle's queue is empty are:

$$
\beta_e = E[S_e] = \sum_i E[S_i] \cdot q_{e,i} = \frac{(W_0 - 1)(\sigma + p_b \cdot T) \cdot q_b}{2} + T, \tag{19}
$$

$$
\begin{aligned}
\sigma_e^2 &= Var[S_e] = E\left[S_e^2\right] - \left(E\left[S_e\right]\right)^2 = \sum_i E\left[S_i^2\right] \cdot q_{e,i} - (E[S_e])^2 \\
&= \sum_i \{Var[S_i] + (E[S_i])^2\} \cdot q_{e,i} - (E[S_e])^2 \\
&= \frac{(W_0-1)(2W_0-1)}{6}(\sigma + p_b \cdot T)^2 \cdot q_b \\
&\quad + \frac{W_0 - 1}{2} \cdot \left\{\frac{Var[PA]}{R_d^2} \cdot p_b + T^2 p_b (1 - p_b) \right. \\
&\quad \left. + 2T(\sigma + p_b \cdot T)\right\} \cdot q_b \\
&\quad + \frac{Var[PA]}{R_d^2} + T^2 - (E[S_e])^2 \\
&\quad i \in \{0, 1, \ldots, W_0 - 1, XMT\}.
\end{aligned} \tag{20}
$$

Similarly, for the packet that arrives when the tagged vehicle's queue is not empty, the service time is given by:

$$
S_b = \begin{cases} S_0 & w.p. \quad q_{b,0} = 1/W_0 \\ S_1 & w.p. \quad q_{b,1} = 1/W_0 \\ \quad \vdots & \\ S_{W_0-1} & w.p. \quad q_{b,W_0-1} = 1/W_0. \end{cases} \tag{21}
$$

The mean and variance of the service time for such packets are:

$$
\beta_b = E[S_b] = \sum_i E[S_i] \cdot q_{b,i} = \frac{(W_0 - 1)(\sigma + p_b \cdot T)}{2} + T, \tag{22}
$$

$$
\begin{aligned}
\sigma_b^2 &= Var[S_b] = E\left[S_b^2\right] - (E[S_b])^2 = \sum_i E\left[S_i^2\right] \cdot q_{b,i} - (E[S_b])^2 \\
&= \sum_i \{Var[S_i] + (E[S_i])^2\} \cdot q_{b,i} - (E[S_b])^2 \\
&= \frac{(W_0-1)(2W_0-1)}{6}(\sigma + p_b \cdot T)^2 \\
&\quad + \frac{W_0 - 1}{2} \cdot \left\{\frac{Var[PA]}{R_d^2} \cdot p_b + T^2 p_b (1 - p_b) \right. \\
&\quad \left. + 2T(\sigma + p_b \cdot T)\right\} \\
&\quad + \frac{Var[PA]}{R_d^2} + T^2 - (E[S_b])^2 \\
&\quad i \in \{0, 1, \ldots, W_0 - 1\}.
\end{aligned} \tag{23}
$$

By utilizing Welch's methods [13], the mean service time for a packet unconditioning on the state of the tagged vehicle's queue is:

$$
E[S] = \frac{\beta_e}{1 - \lambda(\beta_b - \beta_e)}. \tag{24}
$$

3.3 Fixed-Point Iteration

In the previous section, the mean service time is shown to depend on two unknown parameters p_b and q_b, whereas channel busy probabilities p_b and q_b depend on the channel utilization ρ of the M/G/1 queue in every vehicle. Therefore, relationships between ρ, p_b, and q_b are determined first in this section, and then the fixed-point iteration algorithm is used to obtain the final solution.

Let N_{cs} denote the average number of vehicles in carrier sensing range of the tagged vehicle, and let N_{tr} denote the average number of vehicles in transmission range of the tagged vehicle. Hence, without loss of generality, we have:

$$
N_{cs} = N_{tr} = 2\beta R. \tag{25}
$$

The average number of vehicles in potential hidden area is:

$$
N_{ph} = 4\beta R - N_{cs} = 2\beta R. \tag{26}
$$

From the tagged vehicle's point of view, p_b is the probability that it senses channel busy during one time slot in the backoff process. Since the channel is detected busy if there is at least one neighbor (i.e., a vehicle in the

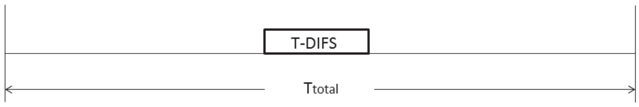

Fig. 4. Abstraction of the packet transmission time.

transmission range of the tagged vehicle) transmitting in a backoff time slot of the tagged vehicle, we have

$$p_b = 1 - \sum_{i=0}^{\infty} (1 - P_{XMT})^i \frac{(N_{tr})^i}{i!} e^{-N_{tr}} = 1 - e^{-N_{tr} \cdot P_{XMT}}, \quad (27)$$

where P_{XMT} is the probability that a neighbor is transmitting in a backoff time slot of the tagged vehicle, to be derived next.

Equation (7) shows that the probability that a vehicle transmits a packet in steady state is $\pi_{XMT}(T - DIFS)/T$. In addition, the time to transmit a packet is $T - DIFS$. Therefore, we can abstractly define the total time to be T_{total} as shown in Fig. 4. Hence, $\pi_{XMT}(T - DIFS)/T = (T - DIFS)/T_{total}$. Suppose a neighbor of the tagged vehicle transmits a packet as shown in Fig. 4 in time duration T_{total}, a backoff time slot of the tagged vehicle can occupy any one time slot within T_{total}.

For the first backoff time slot of the tagged vehicle, the time duration that can capture the transmission of the neighbor is $T - DIFS + 2\sigma$. One extra time slot σ is the one just before transmission and another is the one just after transmission, which can capture the starting time instant and ending time instant of the packet transmission. Therefore, the probability that a neighbor's transmission is detected in the first backoff time slot of the tagged vehicle is $\pi_{XMT}(T - DIFS + 2\sigma)/T$.

For a backoff time slot that is not the first backoff time slot of the tagged vehicle, the time duration that captures the transmission of the neighbor is 2σ, which captures the starting time instant of the transmission. This is because when the neighbor's transmission is detected in the first backoff time slot by the tagged vehicle, the backoff counter will suspend and wait until the end of this transmission for further decrement. Therefore, if the first backoff time slot detects the transmission, there is no chance for the later backoff time slots to detect the same transmission. As a result, the nonfirst backoff time slot can only detect the transmission when the starting point of the transmission falls within this time slot. Therefore, the probability that a neighbor's transmission is detected in nonfirst backoff time slot of the tagged vehicle is $\pi_{XMT} \cdot 2\sigma/T$.

Since the probability that a backoff time slot is the first backoff time slot is $1/W_0$ and nonfirst backoff time slot is $(1 - 1/W_0)$, the probability that a neighbor's transmission is detected by a backoff time slot of the tagged vehicle is:

$$P_{XMT} = \frac{1}{W_0} \cdot \frac{T - DIFS + 2\sigma}{T} \pi_{XMT} + \left(1 - \frac{1}{W_0}\right) \cdot \frac{2\sigma}{T} \pi_{XMT}. \quad (28)$$

Next, q_b denotes the probability that the channel is detected busy by the tagged vehicle in the *DIFS* duration. Therefore, we can similarly define Q_{XMT} to be the probability that a neighbor's transmission is detected in the *DIFS* duration by the tagged vehicle:

$$Q_{XMT} = \frac{T - DIFS + 2DIFS}{T} \pi_{XMT} = \frac{T + DIFS}{T} \pi_{XMT}. \quad (29)$$

Hence, q_b is given by:

$$q_b = 1 - \sum_{i=0}^{\infty} (1 - Q_{XMT})^i \frac{(N_{tr})^i}{i!} e^{-N_{tr}} = 1 - e^{-N_{tr} \cdot Q_{XMT}}. \quad (30)$$

Based on (27-30), q_b is expressed in terms of p_b to simplify the iteration algorithm:

$$q_b = 1 - (1 - p_b)^{\frac{(T+DIFS)W_0}{T - DIFS + 2\sigma W_0}}. \quad (31)$$

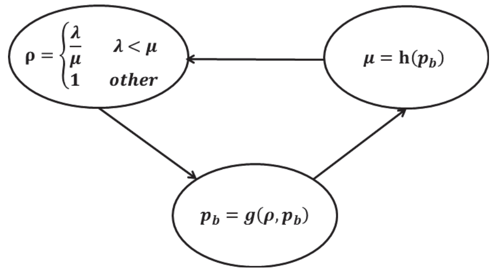

Fig. 5. Import graph for fixed point iteration.

From the above analysis of the relationship between two parameters ρ and p_b (q_b can be expressed in terms of p_b), we notice that p_b depends on ρ and p_b itself. Hence, we denote $p_b = g(\rho, p_b)$ and the reciprocal of mean service time for M/G/1 queue to be $\mu = h(p_b)$. The fixed-point iteration algorithm is outlined as follows according to the import graph [16] in Fig. 5.

Fixed-point iteration algorithm:

Step 1: Initialize $\rho = 1$, which is the saturation condition;

Step 2: With ρ, solve p_b according to (27), (28), (7), (31);

Step 3: With p_b, calculate service rate $\mu = 1/E[S]$ according to (24);

Step 4: If $\lambda < \mu, \rho = \lambda/\mu$; otherwise, set $\rho = 1$; and

Step 5: If ρ converges, then stop the iteration algorithm; otherwise, go to step 2 with the updated ρ.

By utilizing the fixed-point iteration algorithm, the parameters $\rho, p_b, q_b, \pi_{XMT}$ as well as the mean and the variance of the service time can be determined, which are used for the performance indices computation in the next section. The proofs for the existence, uniqueness, and convergence of the fixed-point iteration are given in the appendix, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TC.2012.37.

4 Performance Indices

4.1 Mean Transmission Delay

According to the Welch's method [13], the steady-state expected number of packets in the tagged vehicle's queue (including the packet in the head of the queue that is under service) is:

$$E[Q] = \frac{\lambda\beta_e}{1-\lambda(\beta_b-\beta_e)} + \frac{\lambda^2}{2}\cdot\frac{(\sigma_e^2+\beta_e^2-\sigma_b^2-\beta_b^2)}{1-\lambda(\beta_b-\beta_e)} + \frac{\lambda^2}{2}\cdot\frac{\sigma_b^2+\beta_b^2}{1-\lambda\beta_b}. \quad (32)$$

Using Little's law, the mean delay for a packet transmission is:

$$E[D] = \frac{E[Q]}{\lambda}. \quad (33)$$

4.2 Packet Delivery Ratio

The PDR is defined as [5], [7]: given a broadcast packet sent by the tagged vehicle, the probability that all vehicles in its transmission range receive the packet successfully. Taking account of hidden terminals, we have:

$$PDR = P(N_{cs})P(N_{ph}), \quad (34)$$

where $P(N_{cs})$ is the probability that no vehicles in the transmission range of the tagged vehicle transmits when the tagged vehicle starts transmission, and $P(N_{ph})$ is the probability that no transmissions from the vehicles in the potential hidden terminal area collide with the broadcast packet from the tagged vehicle.

$P(N_{cs})$ can also be interpreted as the nonconcurrent transmission probability, i.e., two packets do not start transmission at the same time. Since DCF employs a discrete-time backoff scheme, if the backoff process is involved, a vehicle is only allowed to transmit at the beginning of each slot time after an idle *DIFS* time duration. Therefore, if the tagged vehicle has not gone through the backoff process before transmitting the packet (with probability $(1-\rho)(1-q_b)$), the concurrent transmission will not occur. Otherwise, the packet transmission is synchronized to the beginning of a slot time, and concurrent transmission may occur if other vehicles' transmission is also synchronized by the backoff process. From the model, we know that the probability that a neighbor starts to transmit a packet at the beginning of the same time slot with the tagged vehicle is $\pi_0 = \pi_{XMT}\cdot\sigma/T$. This is because the sojourn time in state 0 is one time slot σ as shown in the SMP model, hence, π_0 is the probability that a vehicle starts to transmit in the beginning of a time slot immediately after the backoff process. Hence, $P(N_{cs})$ is:

$$\begin{aligned} P(N_{cs}) &= [1-(1-\rho)(1-q_b)]\cdot \\ &\quad \left[\sum_{i=0}^{\infty}(1-\pi_0)^i\frac{(N_{cs}-1)^i}{i!}e^{-(N_{cs}-1)}\right] + (1-\rho)(1-q_b) \\ &= [1-(1-\rho)(1-q_b)]\cdot e^{-(N_{cs}-1)\pi_0} + (1-\rho)(1-q_b). \end{aligned} \quad (35)$$

Since the transmission time for a packet is $T - DIFS = E[PA]/R_d + T_H + \delta$, the event that a transmission from hidden terminals collides with the tagged vehicle's transmission only happens when hidden terminals start to transmit during the vulnerable period $2(T-DIFS) = 2(E[PA]/R_d + T_H + \delta)$ [7]. Using $\pi_{XMT} = T/T_{total}$ as an abstraction of the steady state behavior shown in Fig. 4, the probability that a vehicle starts to transmit during the vulnerable period of hidden terminal transmissions [7] is:

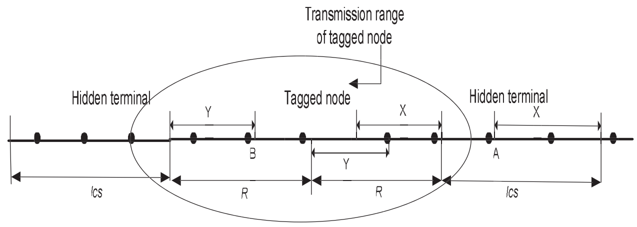

Fig. 6. PRR computation.

$$\frac{2(T-DIFS)}{Ttotal} = \pi_{XMT}\frac{2(T-DIFS)}{T}. \quad (36)$$

Therefore,

$$\begin{aligned} P(N_{ph}) &= \sum_{i=0}^{\infty}\left(1-\pi_{XMT}\cdot\frac{2(T-DIFS)}{T}\right)^i\frac{(N_{ph})^i}{i!}e^{-N_{ph}} \\ &= e^{\frac{-2(T-DIFS)\cdot N_{ph}\cdot\pi_{XMT}}{T}}. \end{aligned} \quad (37)$$

Hence, based on (34), (35), (37), the PDR can be computed. From the analytic results demonstrated in Section 5, we can see that the hidden terminals problem has more impact than concurrent transmissions on the PDR.

4.3 Packet Reception Ratio

Packet reception ratio (PRR) is defined as the percentage of nodes that successfully receive a packet from the tagged node among the receivers being investigated at the moment that the packet is sent out [7].

Similar to the computation for PDR, we consider both the concurrent transmission and hidden terminal effects while computing PRR. Therefore,

$$PRR = PRR_{cc}PRR_{ht}. \quad (38)$$

The impact of the concurrent transmission and hidden terminals will be evaluated in the following two sections.

4.3.1 Impact of the Concurrent Transmission

Transmissions from nodes within a distance R away from the tagged node in the meantime at which the tagged node transmits may cause collisions. When the tagged node starts transmission in the beginning of a slot time, collisions will take place if any node in the transmission range of the tagged node starts transmission in the beginning of the same time slot. As shown in Fig. 6, any node transmitting on the right hand side of the tagged node (i.e., node in $L_1 = \{x|x\in[0,R]\}$) will result in failure of all nodes in L_1 receiving the broadcast packet. Hence, ratio of successfully receiving nodes in the range L_1 can be expressed as

$$PRR_{cc}^1 = \sum_{i=0}^{\infty}(1-\pi_0)^i\frac{(\beta R)^i}{i!}e^{-\beta R} = e^{-\beta R\pi_0}. \quad (39)$$

On the other hand, transmissions from any node on the left hand side of the tagged node (i.e., node in $L_2 = \{x|x\in[-R,0]\}$) will only result in failure of some of the nodes receiving the broadcast packet in L_1. Similar to analysis of the hidden terminal impact, the ratio of successful receiving nodes due to any transmission in L_2 depends on the position of the closest node transmitting in L_2 to the tagged node.

Denote Y as a random variable that represents the distance from the closest node transmitting in L_2 (B in Fig. 6) to the outer boundary of range L_2. Let R_t be the range where no station transmits, so that

$$P(Y \leq y) = \sum_{k=0}^{\infty} [P(\text{none of } k \text{ nodes in } R_t \text{ transmits in a slot})], \tag{40}$$

$R_t = \{x | x \in [-R + y, 0]\}$. Then, the CDF for Y is: (40) is the probability that the closest interfering node in L_2 is at least (R-y) away from the transmitter; i.e., the probability that no nodes within R_t transmit in the meantime the tagged node starts to transmit. Hence, we have:

$$P(Y \leq y) = \sum_{k=0}^{\infty} (1 - \pi_0)^k \frac{(\beta(R - y))^k}{k!} e^{-\beta(R-y)} = e^{-\beta(R-y)\pi_0}. \tag{41}$$

Thus, the expected number of failed nodes in L_1 due to concurrent transmission of nodes in L_2 can be expressed as:

$$\begin{aligned} NF_h &= \int_0^R \beta y P(y \leq Y \leq y + dy) \\ &= \int_0^R \beta^2 y \pi_0 e^{-\beta(R-y)\pi_0} dy = \beta R - \frac{1}{\pi_0}\left(1 - e^{-\beta\pi_0 R}\right). \end{aligned} \tag{42}$$

Therefore, the percentage of receivers in L_1 that are free from collisions caused by the concurrent transmissions of nodes in the range L_2 is:

$$PRR_{cc}^2 = \frac{\beta R - NF_h}{\beta R} = \frac{1}{\beta R \pi_0}(1 - e^{-\beta R \pi_0}). \tag{43}$$

Define PRR_{cc} as the percentage of receivers in L_1 that are free from collisions caused by the concurrent transmissions of nodes in the range $\{x | x \in [-R, R]\}$. If the tagged vehicle transmits the packet without going through the backoff process, with probability $(1 - \rho)(1 - q_b)$, concurrent transmission will not occur. Otherwise, concurrent transmissions may occur. Therefore,

$$\begin{aligned} PRR_{cc} &= PRR_{cc}^1 \cdot PRR_{cc}^2 \cdot [1 - (1 - \rho)(1 - q_b)] \\ &\quad + (1 - \rho)(1 - q_b) \\ &= \frac{e^{-\beta R \pi_0}}{\beta R \pi_0}(1 - e^{-\beta R \pi_0})[1 - (1 - \rho)(1 - q_b)] \\ &\quad + (1 - \rho)(1 - q_b). \end{aligned} \tag{44}$$

4.3.2 Impact of the Hidden Terminals

We observe that the ratio of receivers affected by the hidden terminals only depends on the position of the hidden node (referred as hidden crucial node) that has the closest distance to boundary of the transmitter's sensing range among all the transmitting nodes in the potential hidden terminal area. Denote X as a random variable that represents the distance from the hidden crucial node (A in Fig. 6) to the outer boundary of $\{x | x \in [0, 2R]\}$. Let R_s be the range in the potential hidden terminal area, where no node transmits, such that $R_s = \{x | x \in [l_{cs}, 2R - x]\}$, where $l_{cs} = N_{cs}/(2\beta)$ is carrier sensing range of a node in the network.

Then, the cumulative distribution function (CDF) for X is:

$$\begin{aligned} P(X \leq x) &= \sum_{k=0}^{\infty} P(\text{ none of } k \text{ nodes in } R_s \text{transmits during} \\ &\quad \text{tagged node's transmission}) \\ &= \sum_{k=0}^{\infty} \left(1 - \pi_{XMT} \cdot \frac{2(T - DIFS)}{T}\right)^k \frac{[\beta(2R - l_{cs} - x)]^k}{i!} \\ &\quad \exp(-\beta(2R - l_{cs} - x)) \\ &= \exp\left(-\frac{2\pi_{XMT}\beta(T - DIFS)(2R - l_{cs} - x)}{T}\right) \\ &= \exp(-C(2R - l_{cs} - x)), \end{aligned} \tag{45}$$

where $C = 2\pi_{XMT}\beta(T - DIFS)/T$. Thus, the expected number of failed nodes in [0, R] due to the hidden terminal problem can be expressed as:

$$\begin{aligned} NF_h &= \int_0^{2R - l_{cs}} \beta x f_X(x) dx \\ &= \int_0^{2R - l_{cs}} \beta x d\{P(X \leq x)\} = \beta(2R - l_{cs}) \\ &\quad - \frac{\beta}{C}[1 - \exp(-c(2R - l_{cs}))]. \end{aligned} \tag{46}$$

Therefore, the percentage of receivers that are free from collisions caused by hidden terminal problem is:

$$PRR_{ht} = \frac{\beta R - NF_h}{\beta R} = \frac{l_{cs} - R}{R} + \frac{1}{RC}[1 - \exp(-C(2R - l_{cs}))]. \tag{47}$$

Hence, based on (38), (44), (47), the PRR can be computed. Similar to the case of the PDR, the hidden terminals problem also has more impact than concurrent transmissions on the PRR, as shown in Section 5.

5 Numerical and Simulation Results

The computation for analytic models and corresponding simulations are conducted in Matlab. Note that the analytic model consists of decomposition and fixed-point iteration while the simulative solution does not. All other assumptions are the same in the simulation and analytic models. The results show the high accuracy of our decomposition-based analytic approximation. We consider a freeway system where the number of vehicles is Poisson distributed. Each vehicle on the road is equipped with DSRC wireless capability. The control channel of DSRC is exclusively used for safety related broadcast communication. Table 1 shows the parameters used in this paper, which reflect typical DSRC network settings in [1]. In addition, the variables in Table 1 such as the packet arrival rate and the average packet length are chosen according to [42] to satisfy the maximum channel utilization limitation of IEEE 802.11, which is required to be 54-66 percent in the absence of hidden terminals. For unrealistic situations that exceed such limits, our proposed model in this paper may not apply. Under such channel saturation conditions, other models have been developed earlier [23], [43].

TABLE 1
DSRC Communication Parameters

Parameters	Values	Parameters	Values
Tx range R	500 m	Propagation delay δ	0 μs
Average Packet Length E[PA]	variable	Variance of Packet Length Var[PA]	0
PHY preamble	40 μs	PLCP header	4 μs
MAC header	272 bits	CWMin W0-1	15
Packet arrival rate λ	variable	Vehicle density β	variable
Slot time σ	16 μs	DIFS	64 μs

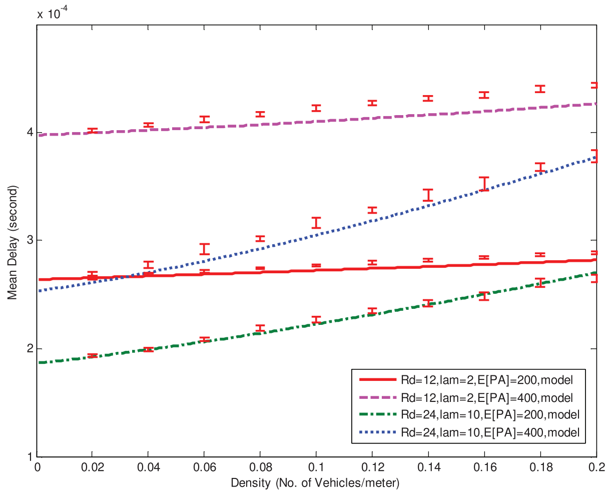

Fig. 7. Mean delay of DSRC highway safety messaging.

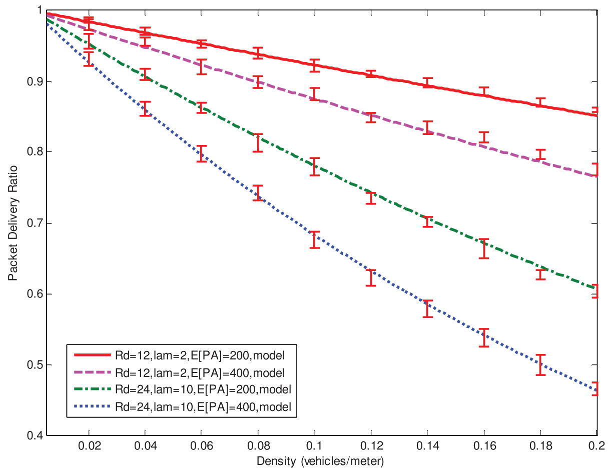

Fig. 8. PDR of DSRC highway safety messaging.

5.1 Numerical Versus Simulation Results

Figs. 7, 8, and 9 present the numerical results of the proposed model including mean transmission delay, PDR, and PRR, respectively, versus the vehicle density β (# vehicles per meter), data rate R_d (*Mbps*), packet arrival rate λ (# packets per second), and average packet length $E[PA]$ (*byte*).

The simulation results for 95 percent confidence intervals are also presented in the Figs. 7, 8, and 9 show that the analytic results from the model have nice match with the simulation results. Another observation from Fig. 7 is that high data rate and shorter packet length facilitate the decrease of the mean transmission delay. The PDR decreases fast as the density β increases as shown in Fig. 8 comparing

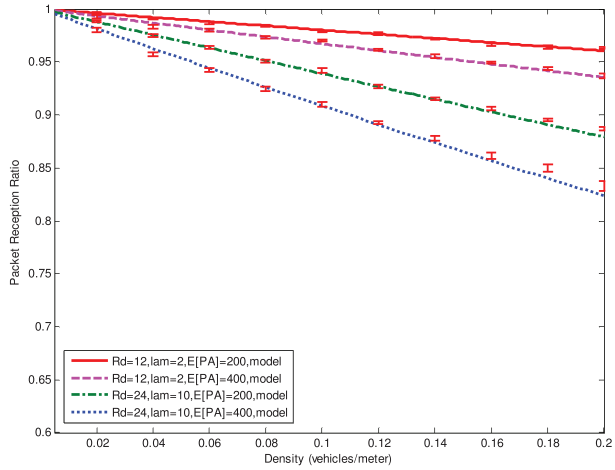

Fig. 9. PRR of DSRC highway safety messaging.

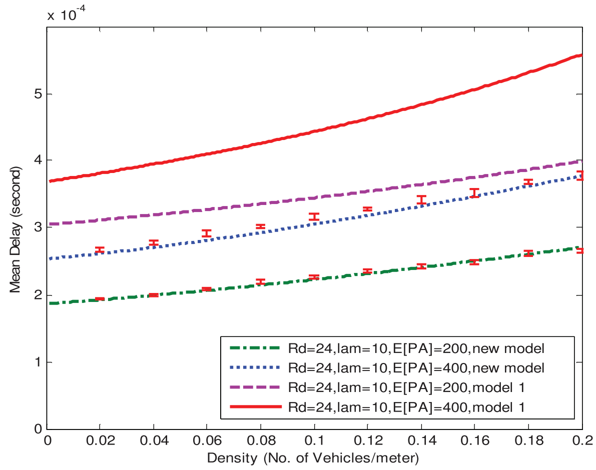

Fig. 10. Mean delay comparison.

to PRR as shown in Fig. 9. Similar to the mean delay, PDRs, and PRRs also benefit from high data rate and short packet length. Comparing Fig. 8 with Fig. 9, we can observe that, given same network parameters, PDRs are less than PRRs. This is because that PDRs count the number of packets that are successfully received by all intended receivers, while *PRRs* count the percentage (or probability) of the intended receivers that successfully receive a packet from a sender.

5.2 Comparison with Previous Models

In this section, in order to show that our proposed model is more precise and general, we compare a few analytic and simulation results from the proposed model with the analytic results from previous models. The model in [7], denoted as Model 1, is used for the mean delay and PDR comparison. The model in [39], denoted as Model 2, is used for PRR (although referred as PDR in (19) of the paper) comparison. Figs. 10, 11, and 12 present the mean transmission delay, PDR, and PRR, respectively for these models along with the simulation results for 95 percent confidence intervals.

Fig. 10 shows that, for the same input parameters, the new model obtains lower mean delay than the model in [7]. This is mainly since the new SMP model considers the fact that a packet can be directly transmitted without

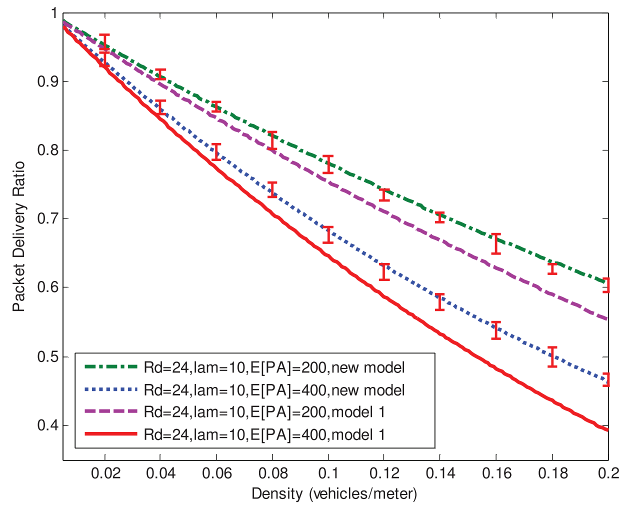

Fig. 11. PDR comparison.

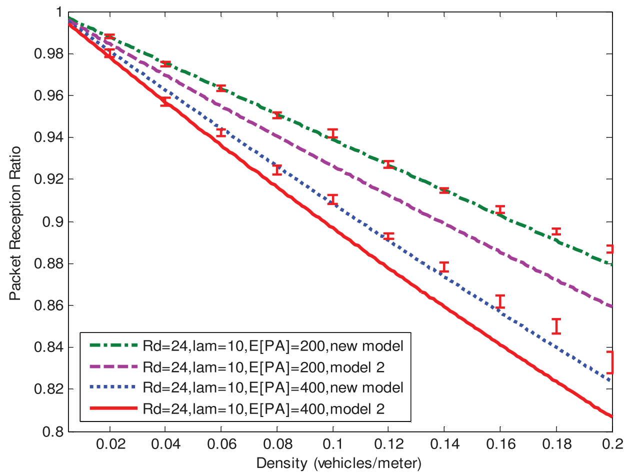

Fig. 12. PRR comparison.

undergoing backoff process, which was ignored in [7]. In addition, the PDR results in Fig. 11 also show that new model has better match with simulations than Model 1. The PRR computation method in [39, (19)] (i.e., Model 2) is shown as below (A few symbol adjustments are made according to our definitions in this paper).

$$PRR = \frac{1-\pi_0}{2\beta R \cdot \pi_{XMT} \cdot \frac{T-DIFS}{T}} \cdot \exp\left(-2\beta R \pi_0\right) \cdot \left[1 - \exp\left(-2\beta R \cdot \pi_{XMT} \cdot \frac{T-DIFS}{T}\right)\right]. \tag{48}$$

The PRR results comparison is illustrated in Fig. 12 and the proposed new model shows better match than Model 2.

For better illustration, the relative errors of these models with simulation results are presented in Tables 2, 3, and 4, with input parameters $R_d = 24$ Mbps, $\lambda = 10$ packets/second, $E[PA] = 200$ bytes. Table 2 shows that the packet mean transmission delay obtained from our proposed new model matches very well with the simulation (relative error is less than 2 percent), whereas the Model 1 without considering the packet transmission not undergoing backoff procedure deviates from the simulation results significantly (relative error is more than 48 percent). Table 3 also shows that the PDRs obtained from the new model (relative error is less than 2 percent) match better with the simulation than Model 1(relative error is less than 8 percent). Table 4 shows that the PRRs from the new model (relative error is less than 1 percent) match better than Model 2 (relative error is less than 2 percent).

TABLE 2
Mean Delay $E[D]$ Comparisons

Density	0.02	0.06	0.1	0.14	0.18	0.2
Simulation(ms)	0.1938	0.2090	0.2265	0.2422	0.2608	0.2651
Model 1(ms)	0.3114	0.3269	0.3444	0.3642	0.3866	0.3989
new model(ms)	0.1924	0.2064	0.2227	0.2407	0.2602	0.2703

TABLE 3
PDR Comparisons

Density	0.02	0.06	0.1	0.14	0.18	0.2
Simulation	0.9568	0.8622	0.7788	0.7018	0.6271	0.6032
Model 1	0.9469	0.8465	0.7539	0.6687	0.5905	0.5540
new model	0.9523	0.8628	0.7809	0.7062	0.6381	0.6065

TABLE 4
PRR Comparisons

Density	0.02	0.06	0.1	0.14	0.18	0.2
Simulation	0.9888	0.9646	0.9440	0.9160	0.8963	0.8884
Model 2	0.9846	0.9549	0.9264	0.8988	0.8723	0.8594
new model	0.9878	0.9633	0.9389	0.9148	0.8909	0.8791

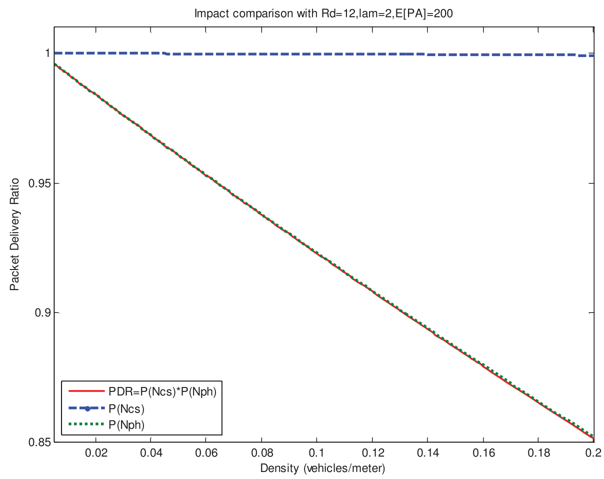

Fig. 13. Impact of concurrent transmissions and hidden terminals on PDR.

5.3 Impact Comparison between Concurrent Transmission and Hidden Terminals

As per Sections 4.2 and 4.3, the PDR and PRR are influenced by both concurrent transmission and hidden terminals. In order to determine which factor is more critical for DSRC safety message transmission, we conduct the impact comparison between the concurrent transmission and hidden terminals in this section. Fig. 13 presents the PDR according to (34), where $P(N_{cs})$ incorporates the impact of

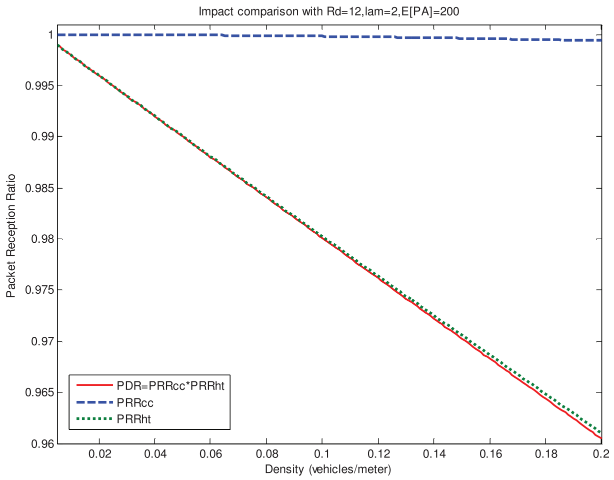

Fig. 14. Impact of concurrent transmissions and hidden terminals on PRR.

concurrent transmission and $P(N_{ph})$ incorporates the impact of hidden terminals. Similarly, Fig. 14 presents the PRR according to (38), where PRR_{cc} incorporates the impact of concurrent transmission and PRR_{ht} incorporates the impact of hidden terminals. As shown in these figures, the PDR and PRR are dominated by the hidden terminals problem, whereas the concurrent transmission has little influence on the PDR and PRR. Therefore, to improve the performance of DSRC-based safety message transmission, the most crucial thing is to reduce the impact of hidden terminals.

6 Conclusions and Future Work

In this paper, a more general and accurate analytic model using SMP interacting with an M/G/1 queue has been developed to characterize the behavior of DSRC for highway safety communications. New proposed SMP model also facilitates the accurate evaluation of hidden terminal impact that is one of major factors for degradation of the reliability, which has not been properly or precisely addressed in the previous work. Both PDR and PRR, very important reliability metrics for DSRC safety-related services, are analytically derived using the new model structure. The model is cross validated against simulations. Moreover, the analysis with different input parameters is used to suggest better parameter settings that will improve the performance by decreasing the mean delay, increasing PDR and PRR. Above all, the proposed model will be beneficial to the analysis, design, and network parameter optimization for required performance and reliability of DSRC VANET for safety-related services. Based on the proposed model, more practical models for more applications can be developed.

In the future, the assumptions made in Section 2.2 will be gradually relaxed to reflect the real world scenario. The analytic model will be extended to incorporate different packet arrival processes such as Markov modulated Poisson process (MMPP), Markovian arrival process (MAP) instead of Poisson arrivals. Different types of safety messages over the control channel will be considered. In a tagged vehicle, we can use different queues to store different types of messages and construct separate SMP models for each type of safety message service. These separated SMP models will interact with each other and also with their own corresponding M/G/1 queue. Fixed-point iteration will then be used to obtain the converged solution. Therefore, the extension of our model for multiple services is quite straightforward. Moreover, channel shadowing/fading, capturing effect, and vehicle mobility will be incorporated. 1D highway will be extended to 2D case to model the intersecting roads, parallel lines in highways, square grid for downtown area, etc. Besides one-hop direct broadcast transmission strategy, multihop and multicycle transmission strategy will also be considered in future.

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