Mean, Median, and Mode – Formulae

1. Discrete (Ungrouped) Data

Let the observations be

$$x_1, x_2, \ldots, x_n$$

Mean

$$\bar{x} = \frac{\sum x}{n}$$

Median

Arrange the data in ascending order.

- If n is odd:

$$\text{Median} = \text{value of } \left(\frac{n+1}{2}\right)^{\text{th}} \text{ item}$$

- If n is even:

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{ item} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{ item}}{2}$$

Mode

$$\text{Mode} = \text{value with the highest frequency}$$

2. Discrete Grouped Data

(Data given as values x_i with frequencies f_i)

Mean

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

Assumed Mean Method

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

where

$$d_i = x_i - a$$

and a is the assumed mean.

Median

$$\text{Median} = \text{value whose cumulative frequency } \geq \frac{N}{2}$$

where

$$N = \sum f_i$$

Mode

$$\text{Mode} = \text{value with maximum frequency}$$

3. Continuous Grouped Data

(Data in class intervals with frequencies)

Mean

$$\bar{x} = \frac{\sum f_i m_i}{\sum f_i}$$

where

$$m_i = \frac{\text{lower limit} + \text{upper limit}}{2}$$

Step Deviation Method

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} \times h$$

where

$$d_i = \frac{m_i - a}{h}$$

- h = class width
- a = assumed mean

Median

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f} \right) \times h$$

where

- l = lower boundary of median class
- $N = \sum f$
- cf = cumulative frequency before median class
- f = frequency of median class
- h = class width

Mode

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

where

- l = lower boundary of modal class
- f_1 = frequency of modal class
- f_0 = frequency of class before modal class
- f_2 = frequency of class after modal class
- h = class width

Empirical Relation (Optional)

$$\text{Mode} = 3(\text{Median}) - 2(\text{Mean})$$

2.Discrete (Ungrouped) Data

Let the observations be

$$x_1, x_2, \ldots, x_n$$

Variance

$$\sigma^2 = \frac{1}{n} \sum (x - \bar{x})^2$$

Standard Deviation

$$\sigma = \sqrt{\frac{1}{n}\sum(x-\bar{x})^2}$$

Coefficient of Variation

$$\mathrm{CV} = \frac{\sigma}{\bar{x}} \times 100$$

Discrete Grouped Data

(Data given as values x_i with frequencies f_i)

Variance

$$\sigma^2 = \frac{\sum f_i(x_i-\bar{x})^2}{\sum f_i}$$

Standard Deviation

$$\sigma = \sqrt{\frac{\sum f_i(x_i-\bar{x})^2}{\sum f_i}}$$

Assumed Mean Method

$$\sigma = \sqrt{\frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2}$$

where

$$d_i = x_i - a$$

and a is the assumed mean.

Coefficient of Variation

$$\mathrm{CV} = \frac{\sigma}{\bar{x}} \times 100$$

Continuous Grouped Data

(Data given in class intervals with frequencies)

Variance

$$\sigma^2 = \frac{\sum f_i(m_i-\bar{x})^2}{\sum f_i}$$

Standard Deviation

$$\sigma = \sqrt{\frac{\sum f_i (m_i - \bar{x})^2}{\sum f_i}}$$

where

$$m_i = \frac{\text{lower limit} + \text{upper limit}}{2}$$

Step Deviation Method

$$\sigma = h\sqrt{\frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2}$$

where

$$d_i = \frac{m_i - a}{h}$$

- h = class width
- a = assumed mean

Coefficient of Variation

$$\text{CV} = \frac{\sigma}{\bar{x}} \times 100$$

Bayes' Theorem

If A and B are events with $P(B) \neq 0$, then

$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)}$$

If $A_1, A_2, \ldots, A_n$ are mutually exclusive and exhaustive events, then

$$P(A_i \mid B) = \frac{P(B \mid A_i)\, P(A_i)}{\sum_{j=1}^{n} P(B \mid A_j)\, P(A_j)}$$

Multiplication Theorem of Probability

For two events A and B with $P(B) \neq 0$,

$$P(A \cap B) = P(A)\, P(B \mid A)$$

Similarly,

$$P(A \cap B) = P(B)\, P(A \mid B)$$

For Three Events

$$P(A \cap B \cap C) = P(A)\,P(B \mid A)\,P(C \mid A \cap B)$$

If A, B, C are independent events, then

$$P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$$

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Inclusion–Exclusion Principle

For Two Events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

For Three Events

$$\begin{aligned} P(A \cup B \cup C) = {} & P(A) + P(B) + P(C) \\ & - P(A \cap B) - P(B \cap C) - P(C \cap A) \\ & + P(A \cap B \cap C) \end{aligned}$$

General Form (n Events)

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum P(A_i) - \sum P(A_i \cap A_j) + \sum P(A_i \cap A_j \cap A_k) - \cdots + (-1)^{n-1} P(A_1 \cap \cdots \cap A_n)$$

Probability Mass Function (PMF)

Let X be a discrete random variable.

The probability mass function is defined as

$$P(X = x) = p(x)$$

Properties:

- $p(x) \geq 0$ for all x
- $\sum p(x) = 1$

Cumulative Distribution Function (CDF)

The cumulative distribution function of a random variable X is

$$F(x) = P(X \leq x)$$

For Discrete Random Variables

$$F(x) = \sum_{t \le x} P(X = t)$$

For Continuous Random Variables

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

Properties:

- $0 \le F(x) \le 1$
- $F(-\infty) = 0$
- $F(\infty) = 1$

Relation Between PDF and CDF

If $F(x)$ is differentiable, then

$$f(x) = \frac{d}{dx} F(x)$$

Mathematical Expectation, Variance and Standard Deviation

Let X be a random variable.

Mathematical Expectation (Mean)

For a Discrete Random Variable

$$E(X) = \sum x\,P(X = x)$$

For a Continuous Random Variable

$$E(X) = \int_{-\infty}^{\infty} x\,f(x)\,dx$$

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Variance of a Random Variable

Definition

$$\mathrm{Var}(X) = E\big[(X - \mu)^2\big]$$

where $\mu = E(X)$.

Alternate Formula

$$\text{Var}(X) = E(X^2) - \big[E(X)\big]^2$$

Discrete Case

$$\text{Var}(X) = \sum (x-\mu)^2 \, P(X = x)$$

Continuous Case

$$\text{Var}(X) = \int_{-\infty}^{\infty} (x-\mu)^2 \, f(x) \, dx$$

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Standard Deviation of a Random Variable

$$\text{SD}(X) = \sqrt{\text{Var}(X)}$$

Bernoulli Trials and Binomial Distribution

Bernoulli Trial

A Bernoulli trial is a random experiment with only two possible outcomes:

- Success (with probability p)
- Failure (with probability $q = 1 - p$)

Each trial is independent and the probability of success remains constant.

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Binomial Distribution

If a random variable X denotes the number of successes in n independent Bernoulli trials, then X follows a binomial distribution.

Probability Mass Function

$$P(X = k) = f(k; n, p) = \binom{n}{k} p^k q^{n-k}$$

where

- $k = 0, 1, 2, \ldots, n$
- $p =$ probability of success
- $q = 1 - p$

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Mean, Variance and Standard Deviation

Mean

$$E(X) = np$$

Variance

$$\mathrm{Var}(X) = npq$$

Standard Deviation

$$\mathrm{SD}(X) = \sqrt{npq}$$

where $q = 1 - p$.

Poisson Distribution

A discrete random variable X is said to follow a Poisson distribution with parameter $\lambda > 0$ if its probability mass function is given by

Probability Mass Function

$$P(X = k) = f(k; \lambda) = \frac{e^{-\lambda}\lambda^k}{k!}, \quad k = 0, 1, 2, \ldots$$

where

- λ = average rate of occurrence
- $e = 2.71828$ (base of natural logarithm)

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Mean, Variance and Standard Deviation

Mean

$$E(X) = \lambda$$

Variance

$$\mathrm{Var}(X) = \lambda$$

Standard Deviation

$$\mathrm{SD}(X) = \sqrt{\lambda}$$

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Relation with Binomial Distribution

The Poisson distribution is obtained as a limiting case of the binomial distribution when

$$n \to \infty, \quad p \to 0, \quad \text{such that } np = \lambda$$