

ECON 612: MONEY AND BANKING
ELISE RODRIGUEZ
FALL 2023

EXAMPLE 7.2*
SOLUTIONS AND EXPLANATIONS

COLOR LEGEND

- ⌘ HEADINGS
- ⌘ GIVEN/PREVIOUSLY FOUND INFORMATION
- ⌘ CONCEPTS YOU SHOULD ALREADY KNOW
- ⌘ ANSWER
- ⌘ ANNOTATIONS AND EXTRA EXPLANATIONS

* A COPY OF THE PROBLEMS IS ATTACHED AT THE END OF THIS DOCUMENT. THERE MAY BE SOME DIFFERENCES BETWEEN THIS VERSION AND THE ONE AVAILABLE ON CANVAS.

GIVEN INFORMATION

$$L = \$150$$

$$S(C) = \$300$$

$$P(S|C) = 0.8$$

$$F(C) = \$0$$

$$P(F|C) = 0.2$$

$$S(D) = \$600$$

$$P(S|D) = 0.5$$

$$F(D) = \$0$$

$$P(F|D) = 0.5$$

$$\tau = 0.30$$

1 CONCLUSION

TYPE C IS SAFER AND TYPE D IS RISKIER.

2 CONCLUSION

CONTRACT C: $\{R(C), E\}$

CONTRACT D: $\{R(D), O\}$

3 a FINDING $R(C)$

$$OTT(C): P(S|C)R(C) = L - E \quad \text{INCLUDED BECAUSE OF THE CONTRACT FROM PART 2.}$$
$$0.8R(C) = 150 - E$$
$$R(C) = \frac{150 - E}{0.8}$$

b FINDING $R(D)$

$$OTT(D): P(S|D)R(D) = L$$

$$0.5R(D) = 150$$

$$R(D) = \$300$$

4 a FINDING $ICC(C)$

$$ICC(C): (1 - \tau)DC(C)|CT(C) \geq (1 - \tau)DC(C)|CT(D)$$

$$(1 - \tau)P(S|C)[S(C) - R(C)] - E \geq (1 - \tau)P(S|D)[S(D) - R(D)]$$

$$(1 - 0.30)0.8[300 - R(C)] - E \geq (1 - 0.30)0.8[600 - R(D)]$$

$$ICC(C): 0.56[300 - R(C)] - E \geq 0.56[600 - R(D)]$$

"OC" MEANS OUT-COME AND "CT" MEANS CONTRACT.

b FINDING $ICC(D)$

$$ICC(D): (1 - \tau)DC(D)|CT(D) \geq (1 - \tau)DC(D)|CT(C)$$

$$(1 - \tau)P(S|D)[S(D) - R(D)] \geq (1 - \tau)P(S|C)[S(C) - R(C)] - E$$

$$(1 - 0.30)0.5[600 - R(D)] \geq (1 - 0.30)0.5[300 - R(C)] - E$$

$$ICC(D): 0.35[600 - R(D)] \geq 0.35[300 - R(C)] - E$$

5 FINDING E

$$ICC: 0.35[600 - R(D)] = 0.35[300 - R(C)] - E$$

$$210 - 0.35(300) = 210 - 0.35 \left[\frac{150 - E}{0.8} \right] - E$$

$$105 = 210 - 65.625 + 0.4375E - E$$
$$E = \$70$$

a FINDING R(C)

$$R(C) = \frac{150 - E}{0.8}$$
$$= \frac{150 - 70}{0.8}$$

$$R(C) = \$100$$

CONCLUSION

THE OPTIMAL CONTRACT FOR BORROWERS WITH TYPE C PROJECTS WOULD HAVE THEM CONTRIBUTE \$70 OF THEIR OWN EQUITY ($E^* = \70) AND REPAY AT $R^*(C) = \$100$, MEANING THEY WOULD ONLY BORROW \$80 ($L^*(C) = \80).

b CONCLUSION

THE OPTIMAL CONTRACT FOR BORROWERS WITH TYPE C PROJECTS WOULD HAVE THEM CONTRIBUTE \$0 OF THEIR OWN EQUITY AND REPAY AT $R^*(D) = \$300$, MEANING THEY WOULD BORROW \$150 ($L^*(D) = \150).

6 CHECKING ICC(C)

$$ICC(C): 0.56[300 - R(C)] - E \geq 0.56[300 - R(D)]$$

$$0.56(300 - 100) - 70 \geq 0.56(300 - 300)$$

$$42 \geq 0 \checkmark$$

NOTE: IN CLASS, WE SAW THAT COLLATERAL CAN SERVE AS A SIGNAL FROM THE BORROWER TO THE LENDER THAT THEY'RE SAFE. IN THIS PROBLEM, WITHOUT COLLATERAL, THE BORROWER "CREDITS" THEIR PROJECT BY PUTTING THEIR OWN INVESTMENT INTO IT (I.E., "SKIN IN THE GAME"). AS WITH COLLATERAL, THE SAFE BORROWER PREFERENCES THE LOWER REPAYMENTS BECAUSE THEY'LL BE MORE LIKELY TO REPAY. THE WILLINGNESS OF THE SAFER BORROWER TO INVEST THEIR OWN MONEY IN THE PROJECT SIGNALS THAT THEIR PROJECT IS SAFER SINCE THEY'RE LESS LIKELY TO LOSE THEIR OWN MONEY.

Signaling Profitability

Example 7.2

Suppose we have a firm that needs \$150 to invest in a project that will yield a random payoff in one period hence. The firm knows the profitability distribution of the project's cash flow, but no one else does. All that outsiders know is that the project can be Type C or Type D. If it is Type C, then the project will yield a cash flow of \$300 with a probability of 0.8 and \$0 with a probability of 0.2. If the project is Type D, it will yield a cash flow of \$600 with a probability of 0.5 and \$0 with a probability of 0.5.

For simplicity, suppose that interest and principal payments on debt are tax-deductible and that the firm can raise equity capital (it currently has negligible equity capital on its books) from those who know the firm's cash flow distribution (e.g., managers who own stock). The firm currently has owners, but the book value of their equity is, for all practical purposes, \$0. However, debt must be acquired in the form of a loan from a bank, which cannot tell whether the borrower has a Type C or a Type D project. The corporate tax rate applicable to the borrower is 30%.

As a banker, how should you deal with such a borrower, assuming that the borrower is locked into either a Type C or Type D project and cannot choose its own project? *Spoiler: Similar to collateral, the bank will ask for equity for the safe project and no equity for the risky project.*

- (1) Which project type is safer and which is riskier?
- (2) Define the loan contract for each project type.
- (3) Using the zero-profit condition, find the level of repayment for each of the following:
 - (a) Type C
 - (b) Type D
- (4) Using the information from (2), write the ICC for each of the following:
 - (a) Type C
 - (b) Type D
- (5) Assuming the risky ICC binds, find the optimal contract (repayment and equity level) for each of the following:
 - (a) Type C
 - (b) Type D
- (6) Check if the other ICC binds.