ECON 612: MONEY AND BANKING

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HOMEWORK 4*

SOLUTIONS AND EXPLANATIONS

COLOR LEGEND

- HEADINGS
- GIVEN/PREVIOUSLY FOUND INFORMATION
- CONCEPTS YOU SHOULD ALREADY KNOW
- ANSWER
- ANNOTATIONS AND EXTRA EXPLANATIONS

* A COPY OF THE PROBLEMS IS ATTACHED AT THE END OF THIS DOCUMENT. THERE MAY BE SOME DIFFERENCES BETWEEN THIS VERSION AND THE ONE AVAILABLE ON CANVAS.

1 GIVEN INFORMATION

$L = \$1.5$ MILLION — THIS IS THE LOAN AMOUNT.

$S(A) = \$6.75$ MILLION — THIS IS THE SUCCESSFUL STATE FOR PROJECT A.

$P(S|A) = 0.6$ — THIS IS THE PROBABILITY OF SUCCESS FOR PROJECT A.

$F(A) = \$0$ — THIS IS THE UNSUCCESSFUL/"FAILURE" STATE FOR PROJECT A.

$P(F|A) = 0.4$ — THIS IS THE PROBABILITY OF FAILURE FOR PROJECT A.

$S(B) = \$8$ MILLION — THIS IS THE SUCCESSFUL STATE FOR PROJECT B.

$P(S|B) = 0.5$ — THIS IS THE PROBABILITY OF SUCCESS FOR PROJECT B.

$F(B) = \$0$ — THIS IS THE UNSUCCESSFUL/"FAILURE" STATE FOR PROJECT B.

$P(F|B) = 0.5$ — THIS IS THE PROBABILITY OF FAILURE FOR PROJECT B.

$i = 0.10$ — THIS IS THE RISKLESS RATE.

$0.90C$ TO THE BANK

CONCEPTUALLY, WE WANT TO INDUCE THE BORROWER TO CHOOSE PROJECT A. ACCORDINGLY, WE MAKE THE OUTCOME OF PROJECT A PREFERABLE TO THE OUTCOME OF PROJECT B.

A DEFINING ICC

★ I SPLIT THE LINES IN TWO FOR SPACING, NOT ANY MATHEMATICAL REASON.

ICC: $OC(A) \geq OC(B)$ — "OC" MEANS OUTCOME.

$$P(S|A)[S(A)-R]+P(F|A)[F(A)-R-C] \geq P(S|B)[S(B)-R]+P(F|B)[F(B)-R-C]$$ — SUBSTITUTE GIVEN VALUES

$$0.6(6.75-R)+0.4(0-R-C) \geq 0.5(8-R)+0.5(0-R-C)$$ — REWRITE

THE BORROWER IS COVERED BY LIMITED LIABILITY, SO THE MOST THEY COULD REPAY IS $F(A) = \$0$.

ICC: $0.6(6.75-R)-0.4C \geq 0.5(8-R)-0.5C$

B FINDING OTT

OTT(A): $(1+i)L = ER(A)$ — "ER" REFERS TO THE BANK'S EXPECTED RETURNS.

$$(1+i)L = P(S|A)R + P(F|A)C$$ — SUBSTITUTE GIVEN VALUES

$$(1+0.10)(1.5) = 0.6R + 0.4(0.90C)$$ — GIVEN VALUE OF COLLATERAL

OTT: $(1.10)(1.5) = 0.6R + 0.4(0.90C)$

C SIMPLIFYING THE BINDING ICC TO ISOLATE C

ICC: $0.6(6.75-R)-0.4C = 0.5(8-R)-0.5C$ — FROM PART A.

$$4.05-0.6R-0.4C = 4-0.5R-0.5C$$

$$0.1C = -0.05+0.1R$$

$$C = -0.5+R$$

FINDING R^*

OTT: $(1.10)(1.5) = 0.6R+0.4(0.90C)$ — FROM PART B.

$$(1.10)(1.5) = 0.6R+0.4[0.9(-0.5+R)]$$ — C FROM PART C.

$$1.65 = 0.6R-0.18+0.36R$$

$$1.83 = 0.96R$$

$$R^* = 1.90625 \text{ MILLION}$$

FINDING C^*

$C = -0.5 + R$ FROM PART (c).

$C^* = -0.5 + R^*$

$C^* = -0.5 + (1.90625)$ R^* FROM ABOVE.

$C^* = \$1.40625$ MILLION

FINDING i (FOR FUN!! 🤗) THE QUESTION DIDN'T ASK FOR THIS.

$$i = \frac{R^*}{L} - 1$$

$$i = \frac{1.90625}{1.5} - 1$$

$i = 0.27083...$

$i \approx 27.08\%$

(d) CONCLUSION

THE BANK ASKS FOR AN APPROPRIATE AMOUNT OF COLLATERAL TO DETER THE FIRM FROM TAKING ON PROJECT B, WHICH IS RISKIER. WHEN THE COLLATERAL (C^*) IS HIGH ENOUGH, THE FIRM PREFERS PROJECT A (SAFER) SO THAT IT HAS A LOWER CHANCE OF FORFEITTING THE COLLATERAL.

THE REPAYMENTS (R^*) ALSO HAVE TO BE HIGH ENOUGH TO COVER OPPORTUNITY COSTS BUT NOT SO HIGH THAT THE BANK ENDS UP ENCOURAGING RISKY BEHAVIOR IN THE AGENT.

(2) GIVEN INFORMATION

$L = \$100$ THIS IS THE LOAN AMOUNT.

$S = \$400$ THIS IS THE SUCCESSFUL STATE FOR THE PROJECT.

$F = \$0$ THIS IS THE UNSUCCESSFUL/"FAILURE" STATE FOR THE PROJECT.

$C(H) = \$75$ THIS IS THE COST OF HIGH EFFORT.

$P(S|H) = 0.7$ THIS IS THE PROBABILITY OF SUCCESS FOR THE PROJECT IF THE BORROWER PUTS IN HIGH EFFORT.

$\Rightarrow P(F|H) = 0.3$ THIS IS GIVEN IMPLICITLY SINCE $P(F|H) = 1 - P(S|H)$.

$P(S|L) = 0.5$ THIS IS THE PROBABILITY OF SUCCESS FOR THE PROJECT IF THE BORROWER PUTS IN LOW EFFORT.

$\Rightarrow P(F|L) = 0.5$ THIS IS GIVEN IMPLICITLY SINCE $P(F|L) = 1 - P(S|L)$.

$i = 0.05$ THIS IS THE RISKLESS RATE.

$0.90C$ TO THE BANK THIS IS THE VALUE OF COLLATERAL.

CONCEPTUALLY, WE WANT TO INDUCE THE BORROWER TO CHOOSE HIGH EFFORT. ACCORDINGLY, WE MAKE THE OUTCOME OF HIGH EFFORT PREFERABLE TO THE OUTCOME OF LOW EFFORT.

(a) DEFINING ICC

ICC: $OT(H) \geq OT(L)$ "OC" MEANS OUTCOME.

$$P(S|H)(S-R) + P(F|H)(F-C) - C(H) \geq P(S|L)(S-R) + P(F|L)(F-R)$$

$$0.7(400-R) + 0.3(0-C) - 75 \geq 0.5(400-R) + 0.5(0-R)$$

ICC: $0.7(400-R)-0.3C-75 \geq 0.5(400-R)-0.5C$

(b) FINDING OIT

OIT: $(1+i)L = ER(H)$

$(1+0.05)(100) = P(S|H)R + P(F|H)C$

$(1.05)(100) = 0.7R + 0.3(0.90C)$

OIT: $105 = 0.7R + 0.3(0.90C)$

(c) SIMPLIFYING THE BINDING ICC TO ISOLATE C

ICC: $0.7(400-R)-0.3C-75 = 0.5(400-R)-0.5C$

$280-0.7R-0.3C-75 = 200-0.5R-0.5R$

$205-0.7R-0.3C = 200-R$

$-0.3C = -5-0.3R$

$C = R-25$

FINDING R^*

OIT: $105 = 0.7R+0.3(0.90C)$

$105 = 0.7R + 0.3(0.90)(R-25)$

$105 = 0.7R + 0.27R - 6.75$

$111.75 = 0.97R$

$R^* = 115.206...$

$R^* \approx \$115.21$

FINDING C^*

$C = R-25$

$C^* = R^* - 25$

$C^* = 115.206... - 25$

$C^* = 90.206...$

$C^* \approx \$90.21$

(d) CONCLUSION

SIMILARLY TO (1), THE BANK ASKS FOR AN APPROPRIATE LEVEL OF COLLATERAL AND REPAYMENTS TO DETER THE BORROWER FROM CHOOSING LOW EFFORT, WHICH HAS A HIGHER PROBABILITY OF FAILURE (RISKIER).

Homework 4

(1) National Cleaner Corp. needs a $1.5 million loan to finance a project that pays off next period. There are two projects available: A and B. You are a lending officer and know about the projects but cannot control the borrower's project choice. Project A will yield a payoff of $6.75 million with a probability of 0.6 or $0 with a probability of 0.4. Project B will yield a payoff of $8 million with a probability of 0.5 or $0 with a probability of 0.5. Everybody is risk neutral and the riskless interest rate is 10%. You want to make sure the borrower chooses Project A with the use of collateral; however, collateral is costly and $1 of the borrower's collateral is worth only $0.90 to the bank.

(a) Write the binding ICC so that the borrower will choose Project A.

(b) Write the zero-profit condition, assuming the borrower chooses Project A.

(c) Find the repayments and collateral the bank will ask for in the loan contract.

(d) Explain the intuition of what is going on here.

(2) Mr. Joseph Brown would like to borrow $100 from your bank to invest in a project that will pay off one period from now. This project is risky and its payoff depends on Mr. Brown's effort in managing it. If the project is successful, it pays off $400, but if it's unsuccessful, it pays nothing. Mr. Brown can choose two levels of effort: high or low. If he chooses high effort levels, the probability that the project is 0.7 and 0.5 for the high and low effort levels, respectively. The riskless interest rate is 5%, and the use of collateral is costly since, for every $1 of collateral, the bank values it at $0.90. Assume that you cannot observe Mr. Brown's effort.

(a) Write the binding ICC so that Mr. Brown will choose high effort.

(b) Write the zero-profit condition, assuming that Mr. Brown chooses high effort.

(c) Find the repayments and collateral the bank will ask for in its loan contract.

(d) Explain the intuition of what is going on here.