ECON 612: MONEY AND BANKING
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EXAMPLE 12.2: THE DIAMOND-DYBVIG MODEL*
SOLUTIONS AND EXPLANATIONS

COLOR LEGEND

- HEADINGS
- GIVEN/PREVIOUSLY FOUND INFORMATION
- CONCEPTS YOU SHOULD ALREADY KNOW
- ANSWER
- ANNOTATIONS AND EXTRA EXPLANATIONS

* A COPY OF THE PROBLEMS IS ATTACHED AT THE END OF THIS DOCUMENT. THERE MAY BE SOME DIFFERENCES BETWEEN THIS VERSION AND THE ONE AVAILABLE ON CANVAS.

GIVEN INFORMATION

$n = 100$

$D = \$1$

$R_1 = \$1$

$R_2 = \$2.25$

$P(D) = 0.40$

$P(L) = 0.60$ THIS ISN'T GIVEN EXPLICITLY. IT COMES FROM $P(L) = 1 - P(D)$.

$U_D = (C_1)^{0.5}$

$U_L = 0.6(C_1 + C_2)^{0.5}$

1 FINDING EACH C_t^i

IF THERE'S NO BANK, THEN $C_1^D = R_1$; $C_2^D = \$0$ AND $C_1^L = \$0$; $C_2^L = R_2$. SO...

$C_1^D = \$1$; $C_2^D = \$0$

$C_1^L = \$0$; $C_2^L = \$2.25$

FINDING EU AT $t=0$

$EU = P(D)U_D + P(L)U_L$ SUBSTITUTE Ps AND Us FROM GIVEN INFORMATION AND SUBSTITUTE THE Cs IN THE Us FROM ABOVE

$EU = 0.40(1)^{0.5} + 0.6[0.6(0+2.25)^{0.5}]$ CALCULATE

$EU = 0.94$

2 GIVEN INFORMATION

* FOR THIS PROBLEM, I DID A LOT MORE WRITING THAN NECESSARY, SO DON'T WORRY IF YOUR APPROACH IS BRIEFER.

$R_{1N} = \$1.10$

$R_{2N} = \$2.10$

a FINDING THE AMOUNT OF DIERS

$D = P(D)n$ SUBSTITUTE FROM GIVEN INFORMATION

$D = 0.40(100)$ CALCULATE

$D = 40$ DIERS

FINDING THE AMOUNT OF PROJECTS TO SATISFY DIERS

$J = DR_{1N}$ SUBSTITUTE FROM GIVEN INFORMATION AND ABOVE

$J = 40(1.10)$ CALCULATE

$J = 44$ PROJECTS REQUIRED

FINDING THE AMOUNT OF LEFTOVER PROJECTS

$J_{LO} = n - J$ SUBSTITUTE FROM GIVEN INFORMATION AND ABOVE

$J_{LO} = 100 - 44$ CALCULATE

$J_{LO} = 56$ PROJECTS LEFTOVER

FINDING THE AMOUNT AVAILABLE TO LIVERS IN $t=2$

$r = J_{LO}R_2$ SUBSTITUTE FROM GIVEN INFORMATION AND ABOVE

$r = 56(2.25)$ CALCULATE

$r = \$126$ AVAILABLE

FINDING THE AMOUNT OF LIVERS COVERED

$J_C = \frac{r}{R_{2N}}$ SUBSTITUTE FROM GIVEN INFORMATION AND ABOVE

$J_C = \frac{126}{2.10}$ CALCULATE

$J_C = 60$ PROJECTS COVERED

CONCLUSION

SINCE THE BANK CAN SATISFY 60 DEPOSITORS IN $t=2$, AND THEY'LL ONLY HAVE 56 LIVERS, THE BANK WILL BE ABLE TO SATISFY THE WITHDRAWALS AND RETURNS PROMISED.

(b) FINDING EACH C_t^i

IF THERE'S NO BANK, THEN $C_1^D = R_{1N}$; $C_2^D = \$0$ AND $C_1^L = \$0$; $C_2^L = R_{2N}$. SO...

$C_1^D = \$1.10$; $C_2^D = \$0$

$C_1^L = \$0$; $C_2^L = \$2.10$

FINDING EU

$EU = P(D)U_D + P(L)U_L$ SUBSTITUTE VALUES FROM GIVEN INFORMATION

$EU = 0.4(1.10)^{0.5} + 0.6[0.6(2.10)^{0.5}]$ CALCULATE

$EU = 0.9412\ldots$ ROUND

$EU \approx 0.941 > 0.94$, SO U IS IMPROVING

3 (a) FINDING C_t^L

IF ONE LIVER DOESN'T WAIT, $C_1^L = R_{1N}$; $C_2^L = 0$. SO...

$C_1^L = \$1.10$; $C_2^L = \$0$

FINDING U_L^{NW}

$U_L^{NW} = 0.6(C_1^L + C_2^L)^{0.5}$ USE U_L FROM GIVEN INFORMATION AND SUBSITUTE VALUES FROM ABOVE

$U_L^{NW} = 0.6(1.10+0)^{0.5}$ CALCULATE

$U_L^{NW} = 0.62928\ldots$ ROUND

$U_L^{NW} \approx 0.629$

(b) FINDING C_t^L

IF THAT LIVER INSTEAD WAITS, $C_1^L = 0$; $C_2^L = R_{2N}$. SO...

$C_1^L = \$0$; $C_2^L = \$2.10$

FINDING U_L^W

$U_L^W = 0.6(C_1^L + C_2^L)^{0.5}$ USE U_L FROM GIVEN INFORMATION AND SUBSITUTE VALUES FROM ABOVE

$U_L^W = 0.6(0+2.10)^{0.5}$ CALCULATE

$U_L^W = 0.86948\ldots$ ROUND

$U_L^W \approx 0.869 > U_L^{NW}$, SO IT'S BETTER TO WAIT (N.E.)

4 (a) GIVEN INFORMATION

ALL 100 DEPOSITORS WITHDRAW

FINDING THE AMOUNT EXPECTED FOR WITHDRAWALS

$W = nR_{1N}$

$W = 100(1.10)$

$W = \$110$, BUT THE BANK ONLY HAS \$100 AVAILABLE

FINDING THE BREAKDOWN OF WITHDRAWALS

DEPOSITORS	AMOUNT	LEFTOVER
1–90 $w_1 = 90$	\$1.10 r_1	\$1
91 $w_2 = 1$	\$1 r_2	\$0
92–100 $w_3 = 9$	\$0 r_3	\$0

FINDING EU_L^{SSC}

$$EU_L^{SSC} = \frac{w_1}{n} 0.6(r_1)^{0.5} + \frac{w_2}{n} 0.6(r_2)^{0.5} + \frac{w_3}{n} 0.6(r_3)^{0.5}$$ USE U_L FROM GIVEN INFORMATION AND SUBSITUTE VALUES FROM ABOVE

$$EU_L^{SSC} = \frac{90}{100} 0.6(1.10)^{0.5} + \frac{1}{100} 0.6(1)^{0.5} + \frac{9}{100} 0.6(0)^{0.5}$$ CALCULATE

$EU_L^{SSC} = 0.57264\ldots$ ROUND

$EU_L^{SSC} \approx 0.573$

b GIVEN INFORMATION

1 LIVER WAITS

CONCLUSION

IF ONE LIVER DECIDES TO WAIT, THEY'LL BE PART OF THE 9 DEPOSITORS WHO GET \$0, AND THEIR $U = 0$. SINCE $EU_L^{SSC} > U$, IT'S BEST TO WITHDRAW (N.E.).

c CONCLUSION

SO, THE NASH EQUILIBRIUM IS $\{w, w\}$.

5 CONCLUSION

IF YOU SWITCH TO THE NON-BANK CONTRACT (LIKE IN 1), THEN THE BANK WOULDN'T NEED TO LIQUIDATE BECAUSE IT CAN COVER ALL CONTRACTS; HOWEVER, THIS DOESN'T PROVIDE CONSUMPTION SMOOTHING ADVANTAGE OF THE BANKING CONTRACT.

6 CONCLUSION

WITH INSURANCE, AT $t=1$, A LIVER WON'T WORRY ABOUT WHAT OTHER LIVERS WILL DO. THIS MEANS THAT NO LIVER HAS AN INCENTIVE TO WITHDRAW EARLY, SO A BANK RUN ISN'T AN EQUILIBRIUM ANYMORE AND WON'T HAPPEN.

The Diamond-Dybvig Model
Example 12.2

There are 100 risk averse individuals with \$1 to invest at $t = 0$ in a project that is worth \$1 at $t = 1$, but yields \$2.25 at $t = 2$. At $t = 0$, each person knows that they have a 40% chance of dying at $t = 1$ and will otherwise live till $t = 2$. Each person will know their type (dier or liver) at time $t = 1$ and can decide whether to liquidate their project at $t = 1$. They have time to withdraw and enjoy consumption in case they find out they are dying at time $t = 1$. Use the utility functions below to answer the following questions:

Utility of a "dier" type: $U_D = (C_1)^{0.5}$

Utility of a "liver" type: $U_L = 0.6(C_1 + C_2)^{0.5}$

(1) What is the expected utility of each individual at time $t = 0$?

Note: The expected utility of an individual you need to first find of what will the allocation be with no bank and then use ONE expected utility function weighted by probabilities. This is an ex-ante utility when individual will not know if they are dier or liver.

(2) Show that a mutually owned bank that collects \$1 deposits at $t = 0$ can improve welfare by promising \$1.10 if deposit is withdrawn at $t = 1$ and \$2.10 if deposit is withdrawn at $t = 2$.

- **(a)** Show that with this scheme, the bank can satisfy withdrawal and returns promised.
- **(b)** Calculate the new expected utility. Same as **(1)** but different numbers. It's not obvious it will give higher utility because, even though at $t = 1$ they are promising more, at $t = 2$, they are promising less.

Note: A Nash Equilibrium (N.E.) is a best response, given what others do. So, we fix others' response and decide what we should do.

(3) ***Bank Contract, Good Equilibrium.*** Show that if all livers believe that other livers will be patient and wait until $t = 2$ to withdraw their money, they are better off being patient too. Fix the other livers' response as that they will be patient and wait.

- **(a)** Suppose one liver does not wait. What is their utility?

  Here they already know they are a liver or conditional on being a liver.
- **(b)** Suppose one liver does wait. What is their utility?

  Here they already know they are a liver or conditional on being a liver.

(4) ***Bank Contract, Bad Equilibrium.*** Show that, if all livers believe that other livers will not be patient and withdraw their money at $t = 1$, they are better off withdrawing early too. Fix the other livers' response so that they will withdraw at $t = 1$.

- **(a)** Suppose one L withdraws. Remembering the promised bank contract and SSC, find out who will get money bank, who will not. Using the probabilities, find the EU of this contract.

  Here the EU weights the probabilities of getting a return.
- **(b)** Suppose one L does not withdraw, show that this is not the best response.
- **(c)** So, the Nash Equilibrium strategy is $\{\ \ ,\ \ \}$.

(5) A way to eliminate the bank run is for the bank to switch bank to the non-bank contract. Why? But this does not do what?

(6) ***Deposit Insurance.*** Explain how a government intervenes and produces deposit insurance that can eliminate the bad equilibrium. Just explain logically, no need to compute it all.