

ECON 612: MONEY AND BANKING
ELISE RODRÍGUEZ
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EXAMPLE 8.5*
SOLUTIONS AND EXPLANATIONS

COLOR LEGEND

- ⌘ HEADINGS
- ⌘ GIVEN/PREVIOUSLY FOUND INFORMATION
- ⌘ CONCEPTS YOU SHOULD ALREADY KNOW
- ⌘ ANSWER
- ⌘ ANNOTATIONS AND EXTRA EXPLANATIONS

* A COPY OF THE PROBLEMS IS ATTACHED AT THE END OF THIS DOCUMENT. THERE MAY BE SOME DIFFERENCES BETWEEN THIS VERSION AND THE ONE AVAILABLE ON CANVAS.

GIVEN INFORMATION

$$L = \$100$$

$$S(G) = \$135$$

$$P(S|G) = 0.9$$

$$F(G) = \$0$$

$$P(F|G) = 0.1$$

$$S(B) = \$150$$

$$P(S|B) = 0.4$$

$$F(B) = \$0$$

$$P(F|B) = 0.6$$

PERFECTLY COMPETITIVE SO ETT = \$0

$$k = 0.05$$

$$\Rightarrow K = kL \Rightarrow K = \$5$$

$$i = 0.05$$

$$P(G) = 0.8$$

$$P(B) = 0.2$$

1 FINDING R

$$\text{OTT: } (1+i)L = ER(G) + ER(B)$$

$$(1+0.05)L = P(G)P(S|G)R + P(B)P(S|B)R$$

$$(1+0.05)(100) = 0.8(0.9)R + 0.2(0.4)R$$

$$105 = 0.72R + 0.08R$$

$$105 = 0.80R$$

$$R = \$131.25$$

2 FINDING P(G|S)

$$\begin{aligned} P(G|S) &= \frac{P(S|G)P(G)}{P(S|G)P(G) + P(S|B)P(B)} \\ &= \frac{(0.9)(0.8)}{(0.9)(0.8) + (0.4)(0.2)} \end{aligned}$$

$$P(G|S) = 0.9$$

FINDING P(B|S)

$$\begin{aligned} P(B|S) &= 1 - P(G|S) \\ &= 1 - 0.9 \end{aligned}$$

$$P(B|S) = 0.1$$

FINDING R_S

$$\text{OTT: } (1+i)L = ER(G) + ER(B)$$

$$(1+0.05)L = P(G|S)P(S|G)R_S + P(B|S)P(S|B)R_S$$

$$(1+0.05)(100) = 0.9(0.9)R_S + 0.1(0.4)R_S$$

$$105 = 0.81R_S + 0.04R_S$$

$$105 = 0.85R_S$$

$$R_S = 123.529\dots$$

$$R_S \approx \$123.53$$

3 FINDING P(G|F)

$$\begin{aligned}P(G|F) &= \frac{P(F|G)P(G)}{P(F|G)P(G) + P(F|B)P(B)} \\&= \frac{(0.1)(0.8)}{(0.1)(0.8) + (0.6)(0.2)}\end{aligned}$$

$$P(G|F) = 0.4$$

FINDING P(B|F)

$$\begin{aligned}P(B|F) &= 1 - P(G|F) \\&= 1 - 0.4\end{aligned}$$

$$P(B|F) = 0.6$$

FINDING R_F

$$\text{OR: } (1+i)L = ER(G) + ER(B)$$

$$(1+i)L = P(G|F)P(S|G)R_F + P(B|F)P(S|B)R_F$$

$$(1+0.05)(100) = 0.4(0.9)R_F + 0.6(0.4)R_F$$

$$105 = 0.36R_F + 0.24R_F$$

$$105 = 0.6R_F$$

$$R_F = \$175$$

Example 8.5

Suppose Midtown Community Bank is faced with two types of borrowers that it cannot distinguish: G and B. The Type G borrower wishes to borrow \$100 to invest in a single-period project that yields \$135 with probability 0.9 and zero with probability 0.1 at the end of the period. The Type B borrower wishes to borrow the same amount in a project that yields \$150 with probability 0.4 and zero with probability 0.6 at the end of the period. If the borrower comes to the bank for a loan in the second period, it will be to finance exactly the same kind of project as in the first period. Assume that Midtown Community Bank is perfectly competitive and there is universal risk neutrality. Midtown's cost of funds is 5%, the riskless rate. Assume that the bank's prior belief is that there is a 0.8 probability that the borrower is of type G and a 0.2 probability that it is of Type B.

- (1) Calculate the level of repayments (R) the bank will charge, considering their stated prior beliefs.
- (2) Suppose that the borrower repays the loan and the bank updates its beliefs using Bayes' Rule.
Find the new level of repayments given this initial success (R_S).
- (3) Suppose instead that the borrower defaults on the loan and the bank updates its beliefs using Bayes' Rule. Find the new level of repayments given this initial failure (R_F).