ECON 612: MONEY AND BANKING

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FALL 2023

EXAMPLE 7.3*

SOLUTIONS AND EXPLANATIONS

COLOR LEGEND

- HEADINGS
- GIVEN/PREVIOUSLY FOUND INFORMATION
- CONCEPTS YOU SHOULD ALREADY KNOW
- ANSWER
- ANNOTATIONS AND EXTRA EXPLANATIONS

* A COPY OF THE PROBLEMS IS ATTACHED AT THE END OF THIS DOCUMENT. THERE MAY BE SOME DIFFERENCES BETWEEN THIS VERSION AND THE ONE AVAILABLE ON CANVAS.

GIVEN INFORMATION

$S(A) = \$100$ THIS IS THE SUCCESSFUL STATE OF PROJECT A.

$P(S|A) = 1$ THIS IS THE PROBABILITY OF SUCCESS FOR PROJECT A.

$\Rightarrow F(A) = \$0$ EVEN THOUGH PROJECT A IS GUARANTEED, WE'LL TREAT THE "FAILURE" STATE AS \$0.

$\Rightarrow P(F|A) = 0$ THIS IS GIVEN IMPLICITLY SINCE $P(F|A) = 1 - P(S|A)$.

$S(B) = \$200$ THIS IS THE SUCCESSFUL STATE OF PROJECT B.

$P(S|B) = 0.5$ THIS IS THE PROBABILITY OF SUCCESS FOR PROJECT B.

$F(B) = \$0$ THIS IS THE UNSUCCESSFUL/"FAILURE" STATE OF PROJECT B.

$P(F|B) = 0.5$ THIS IS THE PROBABILITY OF FAILURE FOR PROJECT B.

$L = \$30$ THIS IS THE LOAN AMOUNT.

$i = 0.10$ THIS IS THE RISKLESS RATE.

$0.90C$ TO THE BANK THIS IS THE VALUE OF COLLATERAL.

1 CONCLUSION

CONTRACT A IS "SAFE" AND CONTRACT B IS "RISKY".

2 CONCLUSION

CONTRACT A: $\{R(A), C\}$

CONTRACT B: $\{R(B), 0\}$

3 DEFINING ICC(A)

$ICC(A): OC(A) | CT(A) \geq OC(A) | CT(B)$ "OC" MEANS OUTCOME AND "CT" MEANS CONTRACT.

$$\begin{array}{l} OC(A)\ P(S|A)[S(A)-R(A)] + \\ CT(A)\ P(F|A)[F(A)-R(A)-C] \end{array} \geq \begin{array}{l} P(S|A)[S(A)-R(B)] + \ OC(A) \\ P(F|A)[F(A)-R(B)]\ CT(B) \end{array}$$

SUBSTITUTE GIVEN VALUES

$$\begin{array}{l} 1[100-R(A)] + \\ 0[0-R(A)-C] \end{array} \geq \begin{array}{l} 1[100-R(B)] + \\ 0[0-R(B)] \end{array}$$

SIMPLIFY ALGEBRAICALLY

$$ICC(A): 100 - R(A) \geq 100 - R(B)$$

DEFINING ICC(B)

$ICC(B): OC(B) | CT(B) \geq OC(B) | CT(A)$

$$\begin{array}{l} OC(B)\ P(S|B)[S(B)-R(B)] + \\ CT(B)\ P(F|B)[F(B)-R(B)] \end{array} \geq \begin{array}{l} P(S|B)[S(B)-R(A)] + \ OC(B) \\ P(F|B)[F(B)-R(A)-C]\ CT(A) \end{array}$$

SUBSTITUTE GIVEN VALUES

$$\begin{array}{l} 0.5[200-R(B)] + \\ 0.5[\underline{0-R(B)}] \end{array} \geq \begin{array}{l} 0.5[200-R(A)] + \\ 0.5[\underline{0-R(A)}-C] \end{array}$$

SIMPLIFY ALGEBRAICALLY

THE BORROWER IS COVERED BY LIMITED LIABILITY, SO THE MOST THEY COULD REPAY IS $F(B) = \$0$.

$$ICC(B): 0.5[200-R(B)] \geq 0.5[200-R(A)] - 0.5C$$

4 CONCLUSION

ICC(B) BINDS. SO, WE'LL CHANGE THE $\geq$ TO $=$.

DEFINING ICC

$$ICC: 0.5[200-R(B)] = 0.5[200-R(A)] - 0.5C$$ SIMPLIFY

$$ICC: R(B) = R(A) + C$$

5 FINDING R(A)

OTT(A): $P(S|A)R(A) = (1+i)L$ SUBSTITUTE VALUES FROM GIVEN INFORMATION

$(1)R(A) = (1+0.10)(30)$ ISOLATE R(A)

$R(A) = \$33$

FINDING R(B)

OTT(B): $P(S|B)R(B) = (1+i)L$ SUBSTITUTE VALUES FROM GIVEN INFORMATION

$0.5R(B) = (1+0.10)(30)$ ISOLATE R(B)

$R(B) = \$66$

6 FINDING C

$R(B) = R(A) + C$ THIS COMES FROM PART 4.

$C = R(B) - R(A)$ SUBSTITUTE ANSWERS FROM PART 5 ABOVE.

$C = 66 - 33$

$C = \$33$

7 CHECKING ICC(A)

ICC(A): $100 - R(A) \geq 100 - R(B)$ THIS COMES FROM PART 3.

$100 - 33 \geq 100 - 66$ SUBSTITUTE ANSWERS FROM PART 5 ABOVE.

$67 \geq 34$ ✓

8 CONCLUSION

CONTRACT A: $\{33, 33\}$

CONTRACT B: $\{66, 0\}$

9 CONCLUSION

NO. BECAUSE CONTRACT A IS RISKLESS, IT WON'T HAVE TO SURRENDER CAPITAL. NONETHELESS, THE BANK KEEPS COLLATERAL AS A PART OF THE CONTRACT TO DETER RISKIER BORROWERS (I.E., SORTING).

Collateral as a Sorting Device: Overcoming Adverse Selection
Example 7.3

Consider two projects: A and B. Suppose that Project A will be worth \$100 for sure at the end of the period. The end-of-period value of Project B will be \$200 with the probability of 0.5 and \$0 with the probability of 0.5. The project (A or B) requires an investment of \$30 up front and the entire amount is borrowed from the bank. The bank is unable to distinguish between A and B.

Assume that the single-period riskless interest rate is 10% and everybody is risk neutral. Assume that collateral worth \$1 to the borrower is worth only \$0.90 to the bank. The difference of 10 cents on the dollar can be viewed as the bank's cost of taking possession of the collateral.

These repossession costs have two sources. First, assets acquired from a delinquent borrower are often worth less piecemeal to the bank than they are to the borrowers as components of a productive whole. Thus, the mere act of liquidating assets from the borrower to the bank involves legal and other administrative costs. These costs are an important reason why so many bankers see the value of collateral largely in terms of its incentive effects.

The problem is to determine how the bank can design a pair of loan contracts such that each borrower will be induced to truthfully reveal their privately known risk. To do this, the bank will design two risk-adjusted contracts so that the safe borrowers will choose the "safe" contract and the risky borrowers will choose the "risky" contract.

(1) Given the information in the first paragraph, which contract is "safe" and which is "risky"?

(2) Suppose there is collateral for Contract A but none for Contract B. Define the contract in the following terms: Contract X: {R(X), C(X)}. Simplify where possible.

(3) Write the incentive compatibility constraints (ICCs) for each contract.

Note: The ICCs should be set up in a way that dictates each type of borrower always prefers their contract to the other.

(4) Which contract binds? Rewrite the ICC to reflect that. Simplify where possible.

(5) Using the zero-profit condition and new ICC, find the level of repayment expected for each contract.

(6) What is the value of collateral required?

(7) Show that the other (non-binding) ICC is satisfied.

(8) Write the complete loan contract.

(9) Does the bank *actually* take the collateral in these situations? Comment on this.