ECON 612: MONEY AND BANKING

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HOMEWORK 5*

SOLUTIONS AND EXPLANATIONS

COLOR LEGEND

- HEADINGS
- GIVEN/PREVIOUSLY FOUND INFORMATION
- CONCEPTS YOU SHOULD ALREADY KNOW
- ANSWER
- ANNOTATIONS AND EXTRA EXPLANATIONS

* A COPY OF THE PROBLEMS IS ATTACHED AT THE END OF THIS DOCUMENT. THERE MAY BE SOME DIFFERENCES BETWEEN THIS VERSION AND THE ONE AVAILABLE ON CANVAS.

1 GIVEN INFORMATION

$n_L = 600$

$n_H = 400$

$L = \$100$

$S(L) = \$150$

$P(S|L) = 0.8$

$F(L) = \$0$

$P(F|L) = 0.2$

$S(H) = \$155$

$P(S|H) = 0.7$

$F(H) = \$0$

$P(F|H) = 0.3$

$P(L) = 0.6$

$P(H) = 0.4$

$M = \$50{,}000$

$r = 0.08$

BORROWER REQUIRES AT LEAST \$5 IN NET PROFIT

1,000 LOAN APPLICATIONS

$i = 0.45$

$\ell = 500$

FINDING $\pi(i^*)$

LET $i^* = i = 0.45$

$$\pi(i^*) = \frac{ER(L) + ER(H)}{1+r} - M$$

$$= \frac{\overbrace{P(L)\ell \cdot P(S|L)[(1+i^*)L]}^{ER(L)} + \overbrace{P(H)\ell \cdot P(S|H)[(1+i^*)L]}^{ER(H)}}{1+r} - M$$

$$= \frac{(0.6)(500)(0.8)(145) + (0.4)(600)(0.7)(145)}{1+0.08} - 50000$$

$\pi(i^*) = 1018.518\ldots$

$\pi(i^*) \approx \$1{,}018.52$

FINDING i^{**}

MINIMUM NET PROFITS = \$5

$$5 = S(H) - (1+i^{**})L$$

$$5 = 155 - (1+i^{**})(100)$$

$$(1+i^{**})(100) = 150$$

$$1+i^{**} = 1.50$$

$i^{**} = 0.50$ OR $i^{**} = 50\%$

FINDING $\pi(i^{**})$

$$\pi(i^{**}) = \frac{ER(L) + ER(H)}{1+r} - M$$

$$= \frac{\overbrace{P(L)\cdot P(S|L)[(1+i^{**})L]}^{ER(L)} + \overbrace{P(H)\cdot P(S|H)[(1+i^{**})L]}^{ER(H)}}{1+r} - M$$

0% OF BORROWERS ARE LOW-RISK; 100% OF BORROWERS ARE HIGH-RISK

$$= \frac{(0)(500)(0.8)(150) + (1)(500)(0.7)(150)}{1+0.08} - 50000$$

$\pi(i^{**}) = -1388.888...$

$\pi(i^{**}) \approx -\$1,388.89$

CONCLUSION

AS SEEN ABOVE, THE BANK'S PROFITS ARE HIGHER WITH CREDIT RATIONING (WHEN $i = i^*$) THAN WITHOUT RATIONING (WHEN $i = i^{**}$). THUS, THE BANK PREFERS TO RATION ITS CREDIT.

(2) GIVEN INFORMATION

SEE EXAMPLE 8.4

(a) SEE (3)(a) FROM EXAMPLE 8.4.

(b) FINDING R_1^{OTT}

OTT(A): $ER_1(A) + ER_2(A) = C_1 + C_2$

$$P(S|A)R_1 + P(S|A)[P(S|A)R_2] = (L+K) + P(S|A)(L+K)$$

$$0.8R_1 + 0.8(0.8)(130) = (100+5) + 0.8(100+5)$$

$$0.8R_1 + 83.2 = 189$$

$$0.8R_1 = 105.8$$

$$R_1^{OTT} = \$132.25$$

CONCLUSION

SINCE $R_1^{OTT} < R_1^*$, THIS DOES NOT VIOLATE THE ICC.

(c) GIVEN INFORMATION

BANK CAN CHARGE 150 BASIS POINTS ABOVE BREAKEVEN

$\Rightarrow$ ABE = 0.015

FINDING i_1^{OTT}

$$i_1^{OTT} = \frac{R_1^{OTT}}{L} - 1$$

$$i_1^{OTT} = \frac{132.25}{100} - 1$$

$i_1^{OTT} = 0.3225$ OR $i_1^{OTT} = 32.25\%$ "BREAK-EVEN INTEREST RATE"

FINDING i_1^{ABE}

$$i_1^{ABE} = i_1^{OTT} + ABE$$

$= 0.3225 + 0.015$

$i_1^{ABE} = 0.3375$

(i) FINDING R_1^{ABE}

$R_1^{ABE} = (1 + i_1^{ABE})L$

$= (1 + 0.3375)(100)$

$R_1^{ABE} = \$133.75$

FINDING

$E\Pi = ER(A) - C_1 - C_2$

$= \overbrace{ER_1(A) + ER_2(A)}^{ER(A)} - \overbrace{(L+K)}^{C_1} - \overbrace{P(S|A)(L+K)}^{C_2}$

$= P(S|A)R_1^{ABE} + P(S|A)P(S|A)R^* - (L+K) - P(S|A)(L+K)$

$= 0.8(133.75) + 0.8(0.8)(130) - (100+5) - 0.8(100+5)$

$E\Pi = \$1.20$

(ii) FINDING BS_2

$BP = P(S|A)[S(A) - R_1^{ABE}] + P(S|A)P(S|A)[S(A) - R^*]$

$= 0.8(150 - 133.75) + 0.8(0.8)(150 - 130)$

$BP = \$25.80$

(3) (a) SEE (2) FROM EXAMPLE 8.5.

(b) SEE (3) FROM EXAMPLE 8.5.

(c) CONCLUSION

SINCE $R_F > S(G)$ AND $R_F > S(B)$, NEITHER BORROWER WILL BORROW AT R_F.

(d) CONCLUSION

IF THERE IS NO LEARNING OVER TIME, THE BANK WOULD CHARGE \$131.25 (R FROM EXAMPLE 8.5). SINCE THERE IS LEARNING (AND COMPETITION FROM OTHER BANKS!), THE LEVEL OF REPAYMENTS DECLINE.

(5) GIVEN INFORMATION

$D_S = \$150$

$D_J = \$1,250$

$DV = \$400$

$S = \$1,400$

$P(S) = 0.8$

$F = \$0$

$P(F) = 0.2$

$C = \$26$

FINDING VE

$VE = P(S)[S-(D_S+D_J)]-C$

$= 0.8[1400-(150+1250)]-25$

$VE = -25$

CONCLUSION

BECAUSE $VE < D$, THE ENTREPRENEUR WILL NOT PARTICIPATE.

FINDING LIQUIDATION PREFERENCE

PREFER TO LIQUIDATE IF $D > P(S)D$

$D_S > P(S)D_S$

$150 > 0.8(160)$

$150 > 120$ ∴ PREFER LIQUIDATION

FINDING BANK'S PAYOFF

$BP = DV - D_S$

$= 400 - 150$

$BP = \$250$ BECAUSE THIS IS LESS THAN D_J, THE BANK MAY PREFER TO RESTRUCTURE IN HOPES OF RECEIVING MORE OF THEIR MONEY.

FINDING D_R

INDIFFERENT WHEN $P(S)(S-D_R)-C = D$

$0.8(1400-D_R)-25 = 0$

$D_R = \$1,368.75$

FINDING d

$d = (D_J+D_S)-D_R$

$= (1250+150)-1368.75$

$d = \$31.25$

FINDING BANK'S PAYOFF

$BP = P(S)(D_S+D_J-d)-D_S$

$= 0.8(1250+150-31.25)-150$

$BP = \$945$

Homework 5

(1) ***Credit Rationing.*** You are a banker and are confronted with a pool of loan applicants, each of whom can be either low risk or high risk. There are 600 low-risk applicants and 400 high-risk applicants and each applicant is applying for a $100 loan. A low-risk borrower will invest the $100 loan in a project that will yield $150 with probability 0.8 and nothing with probability 0.2 one period hence. A high-risk borrower will invest the $100 loan in a project that will yield $155 with probability 0.7 and nothing with probability 0.3 one period hence. You know that 60% of the applicant pool is low risk and 40% is high risk, but you cannot tell whether a specific borrower is low risk or high risk.

You are a monopolist banker and have $50,000 available to lend. Everybody is risk neutral. The current riskless rate is 8%. Each borrower must be allowed to retain a profit of at least $5 in the successful state in order to be induced to apply for a bank loan. You have just learned that 1,000 loan applications have been received after you announced a 45% loan interest rate. You can satisfy only 500.

What should be your optimal (profit-maximizing) loan interest rate? Should it be 45% (at which you must ration half the loan applicants) or a higher interest rate at which there is no rationing? *(This is Review Question 9 from the text, page 201.)*

(2) In Example 8.4, we showed that a two-period contract would not be profitable for a bank. Suppose now the bank makes a three-period loan, a sequence of $100 loans, where the second loan will only be given if the borrower repays the first one. From our work in class, we know that the maximum the banks can charge in the last period so that $R = 130$ and the borrower chooses the safe project ("S").

(a) Find the maximum level of repayments (R) so that the borrower chooses the safe project in the first period too.

(b) Suppose the bank makes zero profits. Compute the level of repayments (R) that it can charge for zero profits. Does it violate the ICC found in **(a)** above?

(c) Suppose the bank can charge 1.5% (150 basis points) above the break-even interest rate so that the borrower will not switch banks.

(i) Calculate bank profits.

(ii) Calculate borrower payoff.

(3) ***Bayes' Rule.*** Consider the continuation of learning over time problem in class using Bayes' Rule (Example 8.5). Suppose the borrower defaults instead of repaying.

(a) Calculate $P(G|F) = \text{Probability}(\text{Good}|\text{Failure})$ using Bayes' Rule.

(b) Now calculate the bank's break-even level of repayments (R) given what you found in **(a)** above.

(c) Show that both borrowers will not borrow (i.e., borrowers are denied further credit).

(d) Return to the situation where the borrower repays their loan. Explain how the repayments will lie between $123.53 and $131.25, depending on the borrower's bargaining strength and how much other banks learn.

(5) A firm is managed by Joe. Joe's firm has two kinds of debt: senior debt, of which it owes $150 to bondholders, and junior debt, which requires a repayment of $1,250. The firm's assets have a current liquidation value of $400, but if the firm continues to operate, it will be worth $1,400 with the probability of 0.8 and $0 with the probability of 0.2 one period after. To manage the firm,

Joe incurs a cost of $25. The bondholders wish to liquidate immediately. What should the bank do? Joe owns all of the firm’s equity.