The Diamond-Dybvig Model
Example 12.2

There are 100 risk averse individuals with \$1 to invest at $t = 0$ in a project that is worth \$1 at $t = 1$, but yields \$2.25 at $t = 2$. At $t = 0$ each person knows that they have a 40% chance of dying at $t = 1$ and will otherwise live till $t = 2$. Each person will know their type (dier or liver) at time $t = 1$ and can decide whether or not to liquidate their project at $t = 1$. They have time to withdraw and enjoy consumption in case they find out they are dying at time $t = 1$. Use the utility functions below to answer the following questions:

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(1) What is the expected utility of each individual at time $t = 0$?

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(2) Show that a mutually owned bank that collects \$1 deposits at $t = 0$ can improve welfare by promising \$1.10 if deposit is withdrawn at $t = 1$ and \$2.10 if deposit is withdrawn at $t = 2$.

a. Show that with this scheme, the bank can satisfy withdrawal and returns promised.

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(3) ***Bank Contract – Good Equilibrium.*** Show that if all livers believe that other livers will be patient and wait until $t = 2$ to withdraw their money, they are better off being patient too. Fix the other livers' response as that they will be patient and wait.

a. Suppose one liver does not wait. What is their utility?

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GIVEN INFORMATION

$n = 100$	$P(D) = 0.4$
$D = \$1$	$P(L) = 1 - P(D) = 0.6$
$R_1 = \$1$	$U(D) = (C_1)^{0.5}$
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1) DEFINE C IN EACH PERIOD FOR BOTH TYPES

IF THERE'S NO BANK, THEN $C_1^D = R_1$; $C_2^D = \$0$ AND $C_1^L = \$0$; $C_2^L = R_2$. SO...

$C_1^D = \$1$; $C_2^D = \$0$

$C_1^L = \$0$; $C_2^L = \$2.25$

FIND EU AT $t=0$

$EU = P(D)U(D) + P(L)U(L)$

$= 0.4(C_1^D)^{0.5} + 0.6[0.6(C_1^L + C_2^L)^{0.5}]$

$= 0.4(1)^{0.5} + 0.6[0.6(0 + 2.25)^{0.5}]$

$= 0.94$

2) GIVEN INFORMATION

$R_1^B = \$1.10$

$R_2^B = \$2.10$

a) FIND AMOUNT OF DIERS

$n(D) = P(D)n$

$= 0.4(100)$

$= 40$ DIERS

FIND AMOUNT OF PROJECTS REQUIRED TO SATISFY AMOUNT OF DIERS

$J = n(D)R_1^B$

$= 40(1.10)$

$= 44$ PROJECTS REQUIRED

FIND AMOUNT OF LEFTOVER PROJECTS

$J_{LO} = n - J$

$= 100 - 44$

$= 56$ PROJECTS LEFTOVER

FIND AMOUNT AVAILABLE TO LIVERS IN $t=2$

$r = J_{LO} R_2$

$= 56(2.25)$

$= \$126$ AVAILABLE

FIND AMOUNT OF LIVERS COVERED BY THIS AMOUNT

$J_C = \frac{r}{R_2^B}$

$= \frac{126}{2.10}$

$= 60$ LIVERS COVERED

CONCLUDE

SINCE THE BANK CAN COVER 60 PROJECTS IN $t=2$ AND THEY'LL ONLY HAVE 56 LIVERS, THE BANK WILL BE ABLE TO SATISFY.

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$c_1^D = \$1.10;\ c_2^D = 0$

$c_1^L = \$0;\ c_2^L = \$2.10$

FIND EU

$EU = P(D)U(D) + P(L)U(L)$

$= 0.4(1.10)^{0.5} + 0.6[0.6(0 + 2.10)^{0.5}]$

$= 0.9412...$

≈ 0.941 > 0.94, SO IMPROVING :)

3) a) $U(NW|L) = 0.6(c_1^{NW} + c_2^{NW})^{0.5}$

$= 0.6(1.10 + 0)^{0.5}$

$= 0.6292...$

≈ 0.629

b) $U(W|L) = 0.6(c_1^W + c_2^W)^{0.5}$

$= 0.6(0 + 2.10)^{0.5}$

$= 0.8694...$

≈ 0.869 > 0.629, SO BETTER TO WAIT

4) a) FIND AMOUNT EXPECTED IF ALL WITHDRAW

$W = nR_1^B$

$= 100(1.10)$

$= \$1.10$ BUT THE BANK ONLY HAS \$100 AVAILABLE

DEFINE WITHDRAWAL BREAKDOWN

DEPOSITORS	AMOUNT	LEFTOVER
1–90 $w_1 = 90$	\$1.10 $= r_1$	\$1
91 $w_2 = 1$	\$1 $= r_2$	\$0
92–100 $w_3 = 9$	\$0 $= r_3$	\$0

FIND EU

$EU = \sum_1^{z \to 3} \frac{w_z}{n} U(z|L)$

$= \frac{w_1}{n} 0.6(r_1)^{0.5} + \frac{w_2}{n} 0.6(r_2)^{0.5} + \frac{w_3}{n} 0.6(r_3)^{0.5}$

$= \frac{90}{100} 0.6(1.10)^{0.5} + \frac{1}{100} 0.6(1)^{0.5} + \cancel{\frac{9}{100} 0.6(0.5)^{0.5}}$

$= 0.5723...$

≈ 0.572

b) IF ONE LIVER DECIDES TO WAIT, THEY'LL BE PART OF THE 9 DEPOSITORS WHO GET \$0, AND THEIR U=0. SINCE EU > U, IT'S BEST TO WITHDRAW (N.E.).

THIS IS A BAD EQUILIBRIUM THAT RESULTS IN A BANK RUN!

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c) SO, THE NASH EQUILIBRIUM IS $\{W, W\}$.

5) IF YOU SWITCH TO THE NON-BANK CONTRACT (LIKE IN 1)), THEN THE BANK WOULDN'T NEED TO LIQUIDATE BECAUSE IT CAN COVER ALL CONTRACTS; HOWEVER, THIS DOESN'T PROVIDE CONSUMPTION SMOOTHING ADVANTAGE OF THE BANKING CONTRACT.

6) WITH INSURANCE, AT $t=1$, A LIVER WON'T WORRY ABOUT WHAT OTHER LIVERS WILL DO. THIS MEANS THAT NO LIVER HAS AN INCENTIVE TO WITHDRAW EARLY, SO A BANK RUN ISN'T AN EQUILIBRIUM ANYMORE AND WON'T HAPPEN.