

ECON 612: MONEY AND BANKING
ELISE RODRÍGUEZ
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EXAMPLE 7.4*
SOLUTIONS AND EXPLANATIONS

COLOR LEGEND

- ⌘ HEADINGS
- ⌘ GIVEN/PREVIOUSLY FOUND INFORMATION
- ⌘ CONCEPTS YOU SHOULD ALREADY KNOW
- ⌘ ANSWER
- ⌘ ANNOTATIONS AND EXTRA EXPLANATIONS

* A COPY OF THE PROBLEMS IS ATTACHED AT THE END OF THIS DOCUMENT. THERE MAY BE SOME DIFFERENCES BETWEEN THIS VERSION AND THE ONE AVAILABLE ON CANVAS.

GIVEN INFORMATION

$L = \$100$ THIS IS THE LOAN AMOUNT

$S(S) = \$300$ THIS IS THE SUCCESSFUL STATE FOR PROJECT S.

$P(S|S) = 0.9$ THIS IS THE PROBABILITY OF SUCCESS FOR PROJECT S.
 $F(S) = \$0$ THIS IS THE UNSUCCESSFUL/ "FAILURE" STATE FOR PROJECT S.

$P(F|S) = 0.1$ THIS IS THE PROBABILITY OF FAILURE FOR PROJECT S.

$S(R) = \$400$ THIS IS THE SUCCESSFUL STATE FOR PROJECT R.

$P(S|R) = 0.6$ THIS IS THE PROBABILITY OF SUCCESS FOR PROJECT R.

$F(R) = \$0$ THIS IS THE UNSUCCESSFUL/ "FAILURE" STATE FOR PROJECT R.

$P(F|R) = 0.4$ THIS IS THE PROBABILITY OF FAILURE FOR PROJECT R.

$i = 0.10$ THIS IS THE RISKLESS RATE.

$0.90C$ TO THE BANK THIS IS THE VALUE OF COLLATERAL.

1 FINDING ICC

ICC: $OC(S) \geq OC(R)$ "OC" MEANS OUTCOME.

$$\begin{aligned} P(S|S)[S(S) - R] + & \geq P(S|R)[S(R) - R] + \\ P(F|S)[F(S) - R - C] & \geq P(F|R)[F(R) - R - C] \\ 0.9(300 - R) + 0.1(0 - R - C) & \geq 0.6(400 - R) + 0.4(0 - R - C) \end{aligned}$$

SUBSTITUTE GIVEN VALUES

THE BORROWER IS COVERED BY LIMITED LIABILITY,
SO THE MOST THEY COULD REPAY IS $F(S) = F(R) = \$0$.

ICC BINDS:

$$0.9(300) - 0.9R - 0.1C = 0.6(400) - 0.6R - 0.4C \text{ SIMPLIFY.}$$

$$R = C + 100$$

2 DEFINING OTT

OTT: $P(S|S)R + P(F|S)[F(S) + 0.90C] = (1+i)L$ WE'RE USING S VALUES BECAUSE WE WANT THEM TO PICK S.

$$0.9R + 0.1(0 + 0.90C) = (1 + 0.10)(100) \text{ SIMPLIFY.}$$

$$OTT: 0.9R + 0.09C = 110$$

FINDING C

$$0.9R + 0.09C = 110 \text{ SUBSTITUTE GIVEN VALUES AND R FROM PART 1.}$$

$$0.9(C + 100) + 0.09C = 110$$

$$C = 20.202... \text{ ROUND}$$

$$C \approx \$20.20$$

FINDING R

$$R = C + 100 \text{ SUBSTITUTE ANSWER FROM ABOVE.}$$

$$R = 20.202... + 100$$

$$R = 120.202... \text{ ROUND}$$

$$R \approx \$120.20$$

3 CONCLUSION

$$\{R, C\} = \{120.20, 20.20\}$$

Collateral in Moral Hazard: Asking for Collateral to Deter from Risk

Example 7.4

Suppose Brown Bakery needs a \$100 loan to finance a project that will pay off next period. Brown can choose between two projects: S (safe) and R (risky). The bank knows this but is unable to directly control the borrower's choice of project. S will yield a payoff of \$300 with the probability of 0.9 and nothing with the probability of 0.1, and R will yield a payoff of \$400 with the probability of 0.6 and nothing with the probability of 0.4. Everybody is risk neutral and the riskless rate is 10%. How should the bank design its loan contract so that Brown will choose the safer project? Assume once again that collateral worth \$1 to Brown is worth \$0.90 to the bank.

Note: Moral hazard problems differ from adverse selection problems in that here there is only one ICC “forcing” the borrower to take the risky project.

- (1) Write the ICC so that the borrower prefers to take the safe project over the risky project. This will bind because we want to see the “breakeven” R this satisfies.
- (2) Using the zero-profit condition, find the level of repayment and collateral.
- (3) Write the complete loan contract.