ECON 612: MONEY AND BANKING

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FALL 2023

HOMEWORK 1*

SOLUTIONS AND EXPLANATIONS

COLOR LEGEND

- HEADINGS
- GIVEN/PREVIOUSLY FOUND INFORMATION
- CONCEPTS YOU SHOULD ALREADY KNOW
- ANSWER
- ANNOTATIONS AND EXTRA EXPLANATIONS

* A COPY OF THE PROBLEMS IS ATTACHED AT THE END OF THIS DOCUMENT. THERE MAY BE SOME DIFFERENCES BETWEEN THIS VERSION AND THE ONE AVAILABLE ON CANVAS.

1 GIVEN INFORMATION

$n_M = 3$
$n_W = 3$
$\Rightarrow x = 3$ THE NUMBERS ARE THE SAME, SO I'M JUST CALLING THEM "x" TO SIMPLIFY.

$C_N = \$10$

a FINDING C_{NI}

"C_N" IS THE COST WITH NO INTERMEDIARY

$C_{NI} = 2x^2 C_N$ SUBSTITUTE VALUES FROM GIVEN INFORMATION

$= 2(3)^2(10)$

$C_{NI} = \$180$

b GIVEN INFORMATION

$C_F = \$10$

FINDING C_{FI} "C_{FI}" IS THE COST WITH F.I.

$C_{FI} = 2xC_F$ SUBSTITUTE VALUES FROM GIVEN INFORMATION

$= 2(3)(10)$

$C_{FI} = \$60$

c CONCLUSION

TO FIND TOTAL EVALUATION COSTS WITH AN F.I., YOU CAN USE $C_{FI} = 2xC_F$, WHERE x IS THE NUMBER OF INDIVIDUALS OF EITHER GROUP (ASSUMING THEY'RE EQUAL) AND C_F IS THE F.I.'S COST OF EVALUATION.

TO FIND TOTAL EVALUATION COSTS WITHOUT AN F.I., YOU CAN USE $C_{NI} = 2x^2 C_N$, WHERE x IS THE NUMBER OF INDIVIDUALS OF EITHER GROUP (ASSUMING THEY'RE EQUAL) AND C_N IS THE INDIVIDUALS' COST OF EVALUATION.

d FINDING GENERAL EQUATION FOR S

$S = C_{NI} - C_{FI}$ SUBSTITUTE ANSWERS FROM PART c.

$= 2x^2 C_N - 2xC_F$

ASSUMING $C_N = C_F = C$ I MAKE THIS ASSUMPTION TO SIMPLIFY.

$S = 2xC(x-1)$

CONCLUSION

TO FIND EVALUATION COST SAVINGS WITH AN F.I., YOU CAN USE $S = 2xC(x-1)$, WHERE x IS THE NUMBER OF INDIVIDUALS OF EITHER GROUP (ASSUMING THEY'RE EQUAL) AND C IS THE COST OF EVALUATION FOR EITHER GROUP (ASSUMING THEY'RE EQUAL).

2 GIVEN INFORMATION

$n_I = 3$
$n_P = 3^*$
$\Rightarrow x = 3$ THE NUMBERS ARE THE SAME, SO I'M JUST CALLING THEM "x" TO SIMPLIFY.

★ IT'S IMPORTANT TO NOTE THAT PROJECTS AREN'T ENTITIES THAT REQUIRE EVALUATIONS OF THE INVESTORS. THIS AFFECTS OUR EQUATIONS. I'VE PUT A ★ ON THEM TO DRAW YOUR ATTENTION.

$C_N = \$10$

(a) FINDING C_{NI} "C_{NI}" IS THE COST WITH NO INTERMEDIARY

$C_{NI} = X^2 C_N$ ★ SUBSTITUTE VALUES FROM GIVEN INFORMATION

$= (3)^2(10)$

$C_{NI} = \$90$

(b) GIVEN INFORMATION

$C_F = \$10$

FINDING C_{FI} "C_{FI}" IS THE COST WITH F.I.

$C_{FI} = XC_F$ ★ SUBSTITUTE VALUES FROM GIVEN INFORMATION

$= (3)(10)$

$C_{FI} = \$30$

(c) CONCLUSION

TO FIND TOTAL EVALUATION COSTS WITH AN F.I., YOU CAN USE $C_{FI} = XC_F$, WHERE X IS THE NUMBER OF INDIVIDUALS OF EITHER GROUP (ASSUMING THEY'RE EQUAL) AND C_F IS THE F.I.'S COST OF EVALUATION. YOU CAN JUST USE C INSTEAD OF C_F.

TO FIND TOTAL EVALUATION COSTS WITHOUT AN F.I., YOU CAN USE $C_{NI} = X^2 C_N$, WHERE X IS THE NUMBER OF INDIVIDUALS OF EITHER GROUP (ASSUMING THEY'RE EQUAL) AND C_N IS THE INDIVIDUALS' COST OF EVALUATION. YOU CAN JUST USE C INSTEAD OF C_N.

(d) FINDING GENERAL EQUATION FOR S

$S = C_{NI} - C_{FI}$ SUBSTITUTE ANSWERS FROM PART (c).

$= X^2 C_N - XC_F$

ASSUMING $C_N = C_F = C$ I MAKE THIS ASSUMPTION TO SIMPLIFY.

$S = XC(X-1)$

CONCLUSION

TO FIND EVALUATION COST SAVINGS WITH AN F.I., YOU CAN USE $S = XC(X-1)$, WHERE X IS THE NUMBER OF INDIVIDUALS OF EITHER GROUP (ASSUMING THEY'RE EQUAL) AND C IS THE COST OF EVALUATION FOR EITHER GROUP (ASSUMING THEY'RE EQUAL).

(e) CONCLUSION

THESE COSTS DON'T INCUR ANY EXTRA, RESOURCE-CONSUMING COSTS. IF THEY DID, THEY WOULD BE DISSIPATIVE.

(f) FINDING $\frac{ds}{dx}$

$\frac{\delta s}{\delta x} = \frac{\delta}{\delta x} S$ SUBSTITUTE ANSWER FROM PART d.

$= \frac{\delta}{\delta x}[xC(x-1)]$ EXPAND S SO YOU CAN USE THE POWER RULE.

$= \frac{\delta}{\delta x}(x^2C - xC)$ USE THE POWER RULE.

$= 2xC - C$ FACTOR OUT C.

$\frac{\delta s}{\delta x} = C(2x-1)$

CONCLUSION

THE SLOPE OF SAVINGS HAS A POSITIVE, LINEAR, AND DIRECT RELATIONSHIP WITH X. THUS, ASSUMING $x > 1$, SAVINGS INCREASE AS X INCREASES.

(g) GIVEN INFORMATION

$C_N > C_F$

FINDING S

$S = C_{NI} - C_{FI}$ SUBSTITUTE ANSWERS FROM PART (c).

$= x^2C_N - xC_F$ FACTOR OUT X.

$S = x(xC_N - C_F)$

CONCLUSION

IN THE EQUATION ABOVE, C_F IS BEING SUBTRACTED FROM THE TERM WITH C_N. IF...

- $C_N > C_F$, SAVINGS ARE HIGHER.
- $C_N = C_F$, SAVINGS ARE LOWER.
- $C_N < C_F$, SAVINGS ARE EVEN LOWER.

THUS, IF THE BANK HAS A COST ADVANTAGE, SAVINGS ARE GREATER.

(h) CONCLUSION

THIS PROBLEM UNDERSCORES HOW F.I.s' BROKERAGE ROLE LOWERS COSTS VIA INFORMATION REUSABILITY AND AVOIDING DUPLICATED SCREENING.

(3) GIVEN INFORMATION

ONLY GOOD CARS ARE SOLD

$RV(B|G) = \$2,500$ THIS IS THE RESERVATION VALUE OF A BUYER FOR A GOOD CAR. "|" MEANS "CONDITIONAL ON".

$RV(S|G) = \$2,000$ THIS IS THE RESERVATION VALUE OF A SELLER FOR A GOOD CAR.

(a) GIVEN INFORMATION

PERFECT INFORMATION

CONCLUSION

THE SELLING PRICE WILL BE BETWEEN \$2,000 AND \$2,500, DEPENDING ON BARGAINING.

(b) GIVEN INFORMATION

LEMONS ARE AVAILABLE

$RV(B|L) = \$1,000$ THIS IS THE RESERVATION VALUE OF A BUYER FOR A LEMON (BAD CAR).

$RV(S|L) = \$800$ THIS IS THE RESERVATION VALUE OF A SELLER FOR A LEMON (BAD CAR).

ASYMMETRIC INFORMATION

$P(L) = 0.5$ THIS IS THE PERCENTAGE OF LEMONS (BAD CARS) ON THE MARKET.

$P(G) = 0.5$ THIS IS THE PERCENTAGE OF GOOD CARS ON THE MARKET. THIS IS GIVEN IMPLICITLY SINCE $P(G) = 1 - P(L)$.

FINDING $BRV(B)$ THIS IS THE BLIND RESERVATION VALUE OF THE BUYER.

$$BRV(B) = P(G)RV(B|G) + P(L)RV(B|L)$$ SUBSTITUTE VALUES FROM GIVEN INFORMATION

$$= 0.5(2500) + 0.5(1000)$$

$$BRV(B) = \$1,750$$

CONCLUSION

THIS BLIND RESERVATION VALUE IS LOWER THAN THE MINIMUM SELL VALUE FOR GOOD CARS. THUS, NO GOOD CAR SELLERS WILL SELL THGIR CARS. AS A RESULT, ONLY LEMONS WILL BE LEFT IN THE MARKET.

(c) CONCLUSION

WITH INFORMATIONAL ASYMMETRY, BUYERS CAN'T BE SURE WHETHER THE CARS AVAILABLE ARE GOOD OR LEMONS. SO THEY'RE WILLING TO PAY LESS. IN THIS EXAMPLE, THAT PRICE IS TOO LOW FOR GOOD CAR SELLERS. SO NO GOOD CARS WILL BE SOLD. THIS IS A MARKET FAILURE.

(d) FINDING $C(S|G)$ THIS IS THE HIGHEST EVALUATION COST A SELLER OF GOOD CARS WOULD BE WILLING TO PAY.

$$C(S|G) = RV(S|G) - BRV(B)$$ SUBSTITUTE VALUES FROM GIVEN INFORMATION AND PART (b).

$$= 2000 - 1750$$

$$C(S|G) = \$250$$

IF A BUYER WERE PAYING FOR THIS SIGNAL (E.G., HAVING A MECHANIC INSPECT THE CAR), THE EQUATION WOULD INSTEAD BE $C(B|G) = RV(B|G) - BRV(B)$.

(4) GIVEN INFORMATION

$S(Sa) = \$150$ THIS IS THE SUCCESSFUL OUTCOME OF A SAFE BORROWER.

$P(S|Sa) = 1$ THIS IS THE PROBABILITY OF A SAFE BORROWER ACHIEVING A SUCCESSFUL OUTCOME.

$S(R) = \$153$ THIS IS THE SUCCESSFUL OUTCOME OF A RISKY BORROWER.

$P(S|R) = 0.9$ THIS IS THE PROBABILITY OF A RISKY BORROWER ACHIEVING A SUCCESSFUL OUTCOME.

$F(R) = \$0$ THIS IS THE "FAILURE"/UNSUCCESSFUL OUTCOME OF A RISKY BORROWER.

$P(F|R) = 0.1$ THIS IS THE PROBABILITY OF A RISKY BORROWER ACHIEVING AN UNSUCCESSFUL OUTCOME.

(a) CONCLUSION

WITH A "SAFE" BORROWER, THE BANK GETS $P(S|Sa)R = R$ WHILE, WITH A "RISKY" BORROWER THE BANK GETS $P(S|R)R = 0.9R$. SINCE $R > 0.9R$, THE BANK PREFERS THE SAFE OPTION.

(b) FINDING R(Sa) THIS IS THE NET RETURN OF A SAFE BORROWER.

$$R(Sa) = P(S|Sa)[S(Sa) - R] + \underbrace{P(F|Sa)[F(Sa) - R]}_{\text{SINCE } P(F|Sa) = 1 - P(S|Sa) = 0,\ \text{THIS TERM IS 0.}}$$

SUBSTITUTE VALUES FROM GIVEN INFORMATION

$$= 1(150 - R)$$

$$R(Sa) = 150 - R$$

(c) FINDING R(R) THIS IS THE NET RETURN OF A RISKY BORROWER.

$$R(R) = P(S|R)[S(R) - R] + P(F|R)[F(R) - R]$$

SUBSTITUTE VALUES FROM GIVEN INFORMATION

$$= 0.9(153 - R) + 0.1(\underbrace{0}_{\text{THE BORROWER IS COVERED BY LIMITED LIABILITY, SO THE MOST THEY COULD REPAY IS } F(R) = \$0.} - R)$$

$$R(R) = 0.9(153 - R)$$

(d) FINDING R THIS IS THE LEVEL OF REPAYMENT AT WHICH THE BORROWER IS INDIFFERENT TO BEING SAFE OR RISKY.

INDIFFERENT WHEN $R(Sa) = R(R)$ SUBSTITUTE ANSWERS FROM PARTS (b) AND (c).

$$150 - R = 0.9(153 - R)$$ SIMPLIFY TO FIND R

$$R = \$123$$

(5) GIVEN INFORMATION

$S(Sa) = \$150$ THIS IS THE SUCCESSFUL OUTCOME OF A SAFE BORROWER.

$P(S|Sa) = 0.8$ THIS IS THE PROBABILITY OF A SAFE BORROWER ACHIEVING A SUCCESSFUL OUTCOME.

$F(Sa) = \$0$ THIS IS THE "FAILURE"/UNSUCCESSFUL OUTCOME OF A SAFE BORROWER.

$P(F|Sa) = 0.2$ THIS IS THE PROBABILITY OF A SAFE BORROWER ACHIEVING AN UNSUCCESSFUL OUTCOME.

$S(R) = \$161$ THIS IS THE SUCCESSFUL OUTCOME OF A RISKY BORROWER.

$P(S|R) = 0.5$ THIS IS THE PROBABILITY OF A RISKY BORROWER ACHIEVING A SUCCESSFUL OUTCOME.

$F(R) = \$0$ THIS IS THE "FAILURE"/UNSUCCESSFUL OUTCOME OF A RISKY BORROWER.

$P(F|R) = 0.5$ THIS IS THE PROBABILITY OF A RISKY BORROWER ACHIEVING AN UNSUCCESSFUL OUTCOME.

(a) CONCLUSION

WITH A SAFE BORROWER, THE BANK GETS $P(S|Sa)R = 0.8R$ WHILE, WITH A RISKY BORROWER THE BANK GETS $P(S|R)R = 0.5R$. SINCE $0.8R > 0.5R$, THE BANK PREFERS THE SAFE OPTION.

(b) CONCLUSION

BECAUSE BORROWERS ARE COVERED BY LIMITED LIABILITY AND THE UNSUCCESSFUL OUTCOME FOR BOTH SAFE AND RISKY IS \$0, BORROWERS WILL ONLY REPAY GIVEN SUCCESS. THUS, IN CONTRARY TO BANK MOTIVATIONS, BORRDWERS WILL PREFER TO BE RISKY SINCE REPAYMENTS ARE LOWER ($0.8R > 0.5R$).

(c) FINDING R(Sa)

$$R(Sa) = P(S|Sa)[S(Sa) - R] + P(F|Sa)[F(Sa) - R]$$

SUBSTITUTE VALUES FROM GIVEN INFORMATION

$$= 0.8(150 - R) + 0.2(\underbrace{0}_{\text{THE BORROWER IS COVERED BY LIMITED LIABILITY, SO THE MOST THEY COULD REPAY IS } F(Sa) = \$0.} - R)$$

$$R(Sa) = 0.8(150 - R)$$

FINDING R(R)

$R(R) = P(S|R)[S(R) - R] + P(F|R)[F(R) - R]$ SUBSTITUTE GIVEN VALUES

$= 0.5(161 - R) + 0.5(0 - R)$

THE BORROWER IS COVERED BY LIMITED LIABILITY, SO THE MOST THEY COULD REPAY IS F(R) = $0.

$R(R) = 0.5(161 - R)$

FINDING R^* THIS IS THE HIGHEST LEVEL OF REPAYMENT BANKS CAN STIPULATE WITHOUT BORROWERS SWITCHING TO RISKY.

INDIFFERENT WHEN $R(Sa) = R(R)$ SUBSTITUTE ANSWERS FROM ABOVE

$0.8(150 - R) = 0.5(161 - R)$ SIMPLIFY TO FIND $R = R^*$

$R^* = 131.666...$ ROUND TO TWO DECIMAL PLACES

$R^* \approx \$131.67$

(d) CONCLUSION

AS EXPLAINED IN PART (a), THE BANK GETS 0.8R FOR A SAFE PROJECT. SINCE $R^* \approx \$131.67$, THE BANK'S REVENUE IS EXPECTED TO BE $0.8(131.67) \approx \$105.34$.

(b) GIVEN INFORMATION

$S = \$5,000$ THIS IS THE SUCCESSFUL STATE OF THE PROJECT.

$P(S) = 0.7$ THIS IS THE PROBABILITY THAT THE PROJECT WILL ACHIEVE A SUCCESSFUL STATE.

$F = \$0$ THIS IS THE FAILURE STATE OF THE PROJECT.

$P(F) = 0.3$ THIS IS THE PROBABILITY THAT THE PROJECT WILL ACHIEVE A "FAILURE" STATE. THIS IS GIVEN IMPLICITLY SINCE $P(F) = 1 - P(S) = 0.3$.

$L = \$4,000$ THIS IS THE LOAN AMOUNT BEING CONSIDERED.

FINDING LENDING DECISION

WILL LEND IF $R \geq L$

$P(S)S + P(F)F \geq L$ SUBSTITUTE VALUES FROM GIVEN INFORMATION

$0.7(5000) + 0.3(0) \geq 4000$ SIMPLIFY AND EVALUATE

$3,500 \not\geq 4,000$

$\therefore$ LENDER WILL NOT LEND

(7) (a) CONCLUSION

YES. THE FINDER (BROKER) FINDS A SELLER (LANDLORD) FOR A BUYER (RENTER) LOOKING FOR AN APARTMENT.

(b) CONCLUSION

NO. THE SALESPERSON IS BOTH A BUYER AND SELLER OF THE CAR.

(c) CONCLUSION

NO. THE STOCKBROKER IS BOTH A BUYER AND SELLER IN THEIR TRANSACTIONS.

(8) CONCLUSION

INFORMATION IS NON-RIVAL SINCE MULTIPLE ENTITIES

CAN HAVE THE SAME INFORMATION. IT'S PARTIALLY EXCLUDABLE SINCE ENTITIES CAN MAKE THEIR INFORMATION INACCESSIBLE TO OTHERS (E.G., FINANCIALLY, LEGALLY).

9 CONCLUSION

CHECK-UPS AND LOAN COVENANTS.

10 CONCLUSION

BANKS TRANSFORM DEPOSITS INTO MORTGAGES. THIS COMES WITH SEVERAL TYPES OF RISK:

- LIQUIDITY
- DURATION
- DIVISIBILITY
- CREDIT
- CURRENCY

Homework 1

(1) Suppose there are three men and three women. They want to find the perfect partner from the other group. Suppose evaluation costs are $10.

- **(a)** Find the total cost of evaluation (without an intermediary).
- **(b)** Suppose a matchmaker can evaluate for a cost of $10. Find the matchmaker's total cost for evaluation.
- **(c)** Write a general equation for the total cost of evaluation, with and without the intermediary for a cost c.
- **(d)** Find a general equation for the evaluation cost savings with an intermediary.

(2) Suppose there are three investors looking for projects to invest in. There are three potential projects out there: A, B, and C. Suppose evaluation costs are $10.

- **(a)** Find the total cost for evaluation (without an intermediary).
- **(b)** Suppose a bank can evaluate for a cost of $10. Find the bank's total cost for evaluation.
- **(c)** Find general equations for the total cost of evaluation, with and without the bank for a cost c.
- **(d)** Find a general equation for the evaluation cost savings with a bank.
- **(e)** Explain why the cost is redistributive and not dissipative.
- **(f)** Prove that, as the number of people increase, the savings increase.
- **(g)** Show that, if the bank has a relative evaluation cost advantage (i.e., $c_{\text{investor}} > c_{\text{bank}}$), the savings are even greater.
- **(h)** What point about banks does this problem make?

(3) Consider a used car market. Only good cars are sold. The buyer has a reservation value of $2,500. The minimum the seller wants to sell the car for is $2,000.

- **(a)** Without information issues, what price will a car sell for?
- **(b)** The buyer now realizes that there are other "lemons" in the market for which they have a reservation price of $1,000 and the seller is willing to sell the car for at least $800. The buyer cannot distinguish between the lemons and the other cars, but they know that there is a 50% chance that it will be a lemon. Will a transaction take place now?
- **(c)** Explain why there is a market failure.
- **(d)** Suppose the "good" car seller can signal to buyers that they are "good" car sellers at a cost. What is the most they would pay for that information?

(4) ***Moral Hazard.*** Here we aren't interested in the riskiness of types, but if different types of borrower would take on a riskier project than the bank would want given the bank has already approved their loan. The borrower will sign a contract with the bank. To simplify, we'll only worry about R, the gross repayments (including the principal) the borrower has to pay back. The borrower's output is the following:

$$\text{Safe project} = 150 \text{ with } p = 1$$
$$\text{Risky project} = 153 \text{ with } p = 0.9$$
$$\text{Risky project} = 0 \text{ with } p = 0.1$$

- **(a)** The bank prefers the safe project. Why?
  Note: Always assume risk neutrality.
- **(b)** If the borrower picks the safer project, what is their net return?
- **(c)** If the borrower picks the riskier project, what is their net return?

(d) Find the R in which the borrower is indifferent between the two projects.

(5) ***Moral Hazard.*** Using the same set-up from question **(4)**, the borrower's output is the following:

$$\text{Safe project} = 150 \text{ with } p = 0.8$$
$$\text{Safe project} = 0 \text{ with } p = 0.2$$
$$\text{Risky project} = 161 \text{ with } p = 0.5$$
$$\text{Risky project} = 0 \text{ with } p = 0.5$$

(a) Show that the bank prefers that the borrower picks the safer project.

(b) Show that the borrower may prefer the riskier project.

(c) Find the maximum R^* the bank can charge without the borrower picking the riskier project.

(d) Find the bank's revenue at R^*.

(6) Suppose an entrepreneur has a project that will return $5,000 with a probability of 0.7, $0 otherwise. The entrepreneur requires a loan of $4,000. The lender would want to at least break even. Assuming the lender is risk-neutral and there's no interest on the loan, should the lender lend the money?

(7) We define a pure broker as someone who brings together buyers and sellers. In the examples below, identify if the person is a pure broker and explain.

(a) In New York City, it's hard to find an apartment. Some people are compensated with "finder's fees" to find apartments for people.

(b) A used car salesperson

(c) A stockbroker

(8) A non-rival good is a good which two people can use without reducing its value (e.g., a lighthouse). An excludable good is a good which you can exclude others from using (e.g., a ten-foot barrier around a house and garden). Public goods are non-rival and non-excludable. Private goods are rival and excludable. In terms of this terminology, how would you classify information?

(9) How do banks minimize post-contract informational asymmetry (i.e., moral hazard)?

(10) Explain how the bank provides Q.A.T. services with home mortgages. Explain how this in itself creates risk.