ECON 612: MONEY AND BANKING

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EXAMPLE 8.2*

SOLUTIONS AND EXPLANATIONS

COLOR LEGEND

- HEADINGS
- GIVEN/PREVIOUSLY FOUND INFORMATION
- CONCEPTS YOU SHOULD ALREADY KNOW
- ANSWER
- ANNOTATIONS AND EXTRA EXPLANATIONS

* A COPY OF THE PROBLEMS IS ATTACHED AT THE END OF THIS DOCUMENT. THERE MAY BE SOME DIFFERENCES BETWEEN THIS VERSION AND THE ONE AVAILABLE ON CANVAS.

GIVEN INFORMATION

$P(L) = 0.5$ THIS IS THE PROBABILITY THAT THE BORROWER IS LOW-RISK.

$P(H) = 0.5$ THIS IS THE PROBABILITY THAT THE BORROWER IS HIGH-RISK.

$n_L = 1{,}000$ THIS IS THE NUMBER OF LOW-RISK BORROWERS.

$n_H = 1{,}000$ THIS IS THE NUMBER OF HIGH-RISK BORROWERS.

$L = \$100$ THIS IS THE LOAN AMOUNT.

$S(L) = \$130$ THIS IS THE SUCCESSFUL STATE FOR LOW-RISK BORROWERS.

$P(S|L) = 0.9$ THIS IS THE PROBABILITY OF SUCCESS FOR LOW-RISK BORROWERS.

$F(L) = \$0$ THIS IS THE UNSUCCESSFUL STATE FOR LOW-RISK BORROWERS.

$P(F|L) = 0.1$ THIS IS THE PROBABILITY OF FAILURE FOR LOW-RISK BORROWERS.

$S(H) = \$135$ THIS IS THE SUCCESSFUL STATE FOR HIGH-RISK BORROWERS.

$P(S|H) = 0.8$ THIS IS THE PROBABILITY OF SUCCESS FOR HIGH-RISK BORROWERS.

$F(H) = \$0$ THIS IS THE UNSUCCESSFUL STATE FOR HIGH-RISK BORROWERS.

$P(F|H) = 0.2$ THIS IS THE PROBABILITY OF FAILURE FOR HIGH-RISK BORROWERS.

$M = \$100{,}000$ THIS IS THE AMOUNT OF MONEY THE BANK HAS FOR LENDING.

2,000 APPLICATIONS

$r = 0.05$ THIS IS THE RISKLESS RATE. USUALLY, I CALL THIS i.

BORROWER REQUIRES AT LEAST \$1 IN NET PROFIT

1 GIVEN INFORMATION

$i^* = 0.29$

NOTICE THAT THIS IS THE HIGHEST RATE THE BANK CAN CHARGE WITHOUT THE LOW-RISK BORROWERS DROPPING OUT. WHEN $i = 29\%$, NET PROFITS $= 130 - (1+0.29)(100) = \$1$, WHICH IS THEIR GIVEN NET PROFIT REQUIREMENT.

FINDING ℓ

$$\ell = \frac{M}{L}$$

$$= \frac{100{,}000}{100}$$

$$\ell = 1{,}000$$

FINDING $\pi(i^*)$

$$\pi(i^*) = \frac{ER(L) + ER(H)}{1+r} - M$$

$$= \frac{\overbrace{P(L)\ell \cdot P(S|L)[(1+i^*)L]}^{ER(L)} + \overbrace{P(H)\ell \cdot P(S|H)[(1+i^*)L]}^{ER(H)}}{1+r} - M$$

$$= \frac{(0.5)(1000)(0.9)(129) + (0.5)(1000)(0.8)(129)}{1+0.05} - 100000$$

$$\pi(i^*) = 4428.571\ldots$$

$$\pi(i^*) \approx \$4{,}428.57$$

2 a CONCLUSION

YES. THE LENDER ONLY EXTENDS CREDIT TO 1,000 BORROWERS

EVEN THOUGH 2,000 BORROWERS ARE INTERESTED AT i^*.

b CONCLUSION

1,000 BORROWERS

c CONCLUSION

SINCE 1,000 BORROWERS ARE LEFT OUT AND EACH WANTS $100, THE TOTAL UNSATISFIED DEMAND IS $100,000.

3 a FINDING i^{**}

MINIMUM NET PROFITS = $1

$$1 = S(H) - (1+i^{**})L$$

$$1 = 135 - (1+i^{**})(100)$$

$$(1+i^{**})(100) = 134$$

$$1+i^{**} = 1.34$$

$$i^{**} = 0.34 \quad \text{OR} \quad i^{**} = 34\%$$

b FINDING $\pi(i^{**})$

$$\pi(i^{**}) = \frac{ER(L)+ER(H)}{1+r} - M$$

$$= \frac{\overbrace{P(L)\ell \cdot P(S|L)[(1+i^{**})L]}^{ER(L)} + \overbrace{P(H)\ell \cdot P(S|H)[(1+i^{**})L]}^{ER(H)}}{1+r} - M$$

0% OF BORROWERS ARE LOW-RISK

100% OF BORROWERS ARE HIGH-RISK

$$= \frac{(0)(1000)(0.9)(134) + (1)(1000)(0.8)(134)}{1+0.05} - 100000$$

$$\pi(i^{**}) = 2095.238\ldots$$

$$\pi(i^{**}) \approx \$2{,}095.24$$

4 a CONCLUSION

THE PROFITS FROM 1 (WITH RATIONING) ARE HIGHER THAN THE PROFITS FROM 2 (WITHOUT RATIONING). THE RATIONING PROFITS ARE HIGHER BECAUSE THE INCREASE OF THE INTEREST RATE (FROM 29% TO 34%) DOESN'T COMPENSATE FOR THE CHANGE IN THE PROBABILITY OF SUCCESS (FROM A MIX OF 80% AND 90% TO JUST 80%).

b CONCLUSION

YES. THE BANK WILL CHOOSE TO RATION CREDIT BECAUSE IT WILL EXPERIENCE HIGHER PROFITS.

Example 8.2: Credit Rationing

Suppose that you are the loan officer for the Midtown Community Bank and you know that within a particular risk class, there are two types of borrowers: low-risk borrowers and high-risk borrowers. However, you cannot distinguish between them.

You believe that the probability that a randomly chosen borrower is low risk is 0.5 and that the borrower is high risk is 0.5. There are 1,000 potential loan applications of each type within this risk class. Each applicant would like a loan of $100. The low-risk borrower will invest this loan in a project that lasts for one period hence will yield $130 with probability 0.9 and nothing with probability 0.1. The high-risk borrower will invest the loan in a project that will yield $135 with probability 0.8 and nothing with probability 0.2 one period hence.

Midtown Community Bank is a monopolist with respect to these borrowers. Assuming that the only pricing instrument available is the loan interest rate, how should you price a loan to a borrower in this risk class so as to maximize the bank's expected profit? You have only $100,000 available to lend and the junior lending officer who reports to you has advised you that 2000 loan applications were received when it was announced that the bank would charge an interest rate of 29%. The current riskless rate is 5%. Assume that a borrower must have at least 1 dollar of net profit in the successful state in order to apply for a bank loan, and that there is universal risk neutrality.

(1) Find the bank's profits when $i^* = 29\%$. If there is credit rationing, assume it is random.

(2) Given your answer in **(1)**, answer the following questions:

- **(a)** Is there credit rationing?
- **(b)** If so, how many borrowers are left out?
- **(c)** If so, how much demand (in dollars) can you not satisfy?

(3) Consider that the bank raises the interest rate, which is common practice when there is excess demand. The bank raises the interest rate to the maximum it can keep the high-risk borrowers in the market.

- **(a)** What is that interest rate (i^{**})?
- **(b)** Find the banks profits with this new interest rate (i^{**}).

(4) Compare the profits at i^* and i^{**} and answer the following questions:

- **(a)** Which is larger? Why?
- **(b)** In the end, does the bank ration credit?