

ECON 612: MONEY AND BANKING
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HOMEWORK 7*
SOLUTIONS AND EXPLANATIONS

COLOR LEGEND

- ⌘ HEADINGS
- ⌘ GIVEN/PREVIOUSLY FOUND INFORMATION
- ⌘ CONCEPTS YOU SHOULD ALREADY KNOW
- ⌘ ANSWER
- ⌘ ANNOTATIONS AND EXTRA EXPLANATIONS

* A COPY OF THE PROBLEMS IS ATTACHED AT THE END OF THIS DOCUMENT. THERE MAY BE SOME DIFFERENCES BETWEEN THIS VERSION AND THE ONE AVAILABLE ON CANVAS.

3 GIVEN INFORMATION

$$n(NA) = 2$$

$$B(NA) = \$250$$

$$P(B|NA) = 0.8$$

$$W(NA) = \$125$$

$$P(W|NA) = 0.2$$

$$n(SC) = 2$$

$$B(SC) = \$250$$

$$P(B|SC) = 0.5$$

$$W(SC) = \$125$$

$$P(W|SC) = 0.5$$

$$P(NA) = 0.4$$

$$P(SC) = 0.6$$

$$C = 0.08TV$$

FINDING MV_{NC}

$$\begin{aligned} MV_{NC} &= n(NA)P(NA)[P(B|NA)B(NA) + P(W|NA)W(NA)] \\ &\quad + n(SC)P(SC)[P(B|SC)B(SC) + P(W|SC)W(SC)] \\ &= 2(0.4)[0.8(250) + 0.2(125)] \\ &\quad + 2(0.6)[0.5(250) + 0.5(125)] \end{aligned}$$

$$MV_{NC} = \$405$$

FINDING MV_C

$$\begin{aligned} MV_C(NA) &= n(NA)[P(B|NA)B(NA) + P(W|NA)W(NA)] - C \\ &= (1 - 0.08)2[0.8(250) + 0.2(125)] \end{aligned}$$

$$MV_C(NA) = \$414$$

DETERMINING COMMUNICATION PREFERENCE

SINCE $MV_C(NA) > MV_{NC}$, NA PREFERENCES TO COMMUNICATE.

4 GIVEN INFORMATION

$$n(FU) = 2$$

$$B(FU) = \$300$$

$$P(B|FU) = 0.8$$

$$W(FU) = \$150$$

$$P(W|FU) = 0.2$$

$$n(TA) = n(FU) = 2$$

$$B(TA) = \$300$$

$$P(B|TA) = 0.5$$

$$W(TA) = \$150$$

$$P(W|TA) = 0.5$$

$$P(FU) = 0.5$$

$$C = 0.075TV$$

a FINDING MV_{NC}

$$\begin{aligned} MV_{NC} &= n(FU)P(FU)[P(B|FU)B(FU) + P(W|FU)W(FU)] \\ &\quad + n(TA)P(TA)[P(B|TA)B(TA) + P(W|TA)W(TA)] \\ &= 2(0.5)[0.8(300) + 0.2(150)] \\ &\quad + 2(0.5)[0.5(300) + 0.5(150)] \end{aligned}$$

$$MV_{NC} = \$495$$

FINDING MV_C

$$\begin{aligned} MV_C(FU) &= n(FU)[P(B|FU)B(FU) + P(W|FU)W(FU)] - C \\ &= (1 - 0.075)2[0.8(300) + 0.2(150)] \end{aligned}$$

$$MV_C(FU) = \$499.50$$

DETERMINING COMMUNICATION PREFERENCE

SINCE $MV_C(FU) > MV_{NC}$, FU PREFERENCES COMMUNICATE.

b GIVEN INFORMATION

$$R(A|FU) = \$300$$

i FINDING $Pa(A|FU)$

INTUITIVELY, WE KNOW EITHER PORTFOLIO RETURNS \$150 IN THE "WORSE" STATE. SINCE $2 \cdot \$150 = \300 , CLASS A BONDHOLDERS ARE GUARANTEED THEIR RESIDUAL CASH FLOW. THUS, THERE'S NO NEED FOR COMMUNICATION OR ITS COSTS.

ii FINDING $Pa(B|FU)$

$$\begin{aligned} Pa(B|FU) &= TV(FU) - Pa(A|FU) \\ &= n(FU)[P(B|FU)B(FU) + P(W|FU)W(FU)] \\ &\quad - Pa(A|FU) \\ &= 2[0.8(300) + 0.2(150)] - 300 \\ &= 540 - 300 \end{aligned}$$

$$Pa(B|FU) = \$240$$

iii FINDING $MV_C(B)$

$$\begin{aligned} MV_C(B) &= Pa(B|FU) - C(B|FU) \\ &= 240 - 0.075(240) \end{aligned}$$

$$MV_C(B) = \$222$$

CONCLUSION

THE VALUE NOW IS THE SUM OF THE PAYOFFS (\$522). SINCE THIS IS GREATER THAN THE PRE-

VIOUSLY PREFERRED MV_c(FU), THE STRIPPING METHOD IS BETTER.

GIVEN INFORMATION

Pa(B|FU) = \$300 WITH CREDIT ENHANCER

FINDING PROBABILITIES AND SHORTFALLS

LOAN 1 OUTCOME	LOAN 2 OUTCOME	PROB- ABILITY	SHORT- FALL
B(FU)	B(FU)	P(B FU)P(B FU)	
B(FU)	W(FU)	P(B FU)P(W FU)	FOR ALL: $n(FU)Pa(B FU)$
W(FU)	B(FU)	P(W FU)P(B FU)	$-L_1 - L_2$
W(FU)	W(FU)	P(W FU)P(W FU)	
LOAN 1 OUTCOME	LOAN 2 OUTCOME	PROB- ABILITY	SHORT- FALL
300	300	0.8(0.8)	600 - 300 - 300
300	150	0.8(0.2)	600 - 300 - 150
150	300	0.2(0.8)	600 - 150 - 300
150	150	0.2(0.2)	600 - 150 - 150
LOAN 1 OUTCOME	LOAN 2 OUTCOME	PROB- ABILITY	SHORT- FALL
300	300	0.64	0
300	150	0.16	150
150	300	0.16	150
150	150	0.04	300

FINDING EL

$$EL = \sum_n^n P(SF_n) SF_n \quad \text{HERE } n=4 \text{ SINCE THERE ARE 4 SCENARIOS}$$

$$= 0.64(0) + 0.16(150) + 0.16(150) + 0.04(300)$$

$$EL = \$60$$

FINDING Pa(FU)

$$Pa(FU) = Pa(A|FU) + Pa(B|FU) - EL$$

$$= 300 + 300 - 60$$

$$Pa(FU) = \$540$$

Homework 7

(3) NA Bank has two loans. Each loan will repay \$250 to the bank with a probability of 0.8 and \$125 with a probability of 0.2. This is private information to the bank. But investors cannot tell apart this loan portfolio from SC Bank which has the same number of loans but will repay \$250 with a probability of 0.5 and \$125 with a probability of 0.5. The prior belief of investors is that NA has its actual portfolio with a probability of 0.4 and SC with a probability of 0.6. The cost of communicating the true value of its loans to investors is 8% of the true value.

Find out if NA prefers securitization with communication or securitization without.

(4) Suppose that the First United Bank of America has two loans. Each is due to be repaid one period hence and has independent and identically distributed cash flows. Each loan will repay \$300 with a probability of 0.8 and \$150 with a probability of 0.2. However, while the bank knows this, the investors cannot distinguish this loan from that of the Third TransAmerica Bank which has the same number of loans but will pay \$300 with a probability of 0.5 and \$150 with a probability of 0.5. There is a prior belief of 0.5 that the First United Bank of America has the higher-valued portfolio. The cost of communicating the true value of its loans to investors is 7.5% of the true value.

- (a) Find out if First United prefers securitization with communication to securitization without.
- (b) The First United can create two classes of bondholders in a senior-subordinated structure or junior-senior structure. Class A bondholders will receive the first tranche and are entitled to \$300 or the residual cash flow, whichever is smaller.
 - (i) Argue why the bank does not need to communicate its value to Class A bondholders.
 - (ii) Find out its payoff from Class B bondholders for whom it needs to communicate the true value (and thus pay the communication cost).
 - (iii) Compare the total value now of securitization and compare it to (a) above. Is this “stripping securities” method better? Why?
- (c) Now suppose First United can use a credit enhancer. A credit enhancer will guarantee that Class B bondholders get \$300 for sure. The credit enhancer thus replaces communication from the bank. How much will the bank have to pay a credit enhancer in a competitive market? Calculate the total value now for the bank.