Betragt $g_i(t) = f(x^* + t e_i)$, $\quad i = 1, \dots, n$, $\quad t \in \mathbb{R}$

e_i: i-te standard basisvektor

f har et ekstremum i x^*

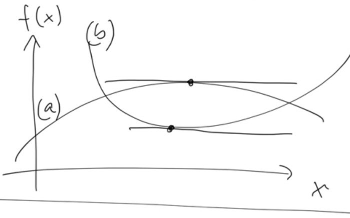
$\Rightarrow$ g_i har et ekstremum i $t = 0$

$\Rightarrow$ $g_i'(0) = 0$

Fordi $\dfrac{\partial f}{\partial x_i} = g_i'(0)$

gælder det, at

$$\nabla f(x^*) = (g_1'(0), g_2'(0), \dots, g_n'(0)) = 0.$$



(a) f konkav

x^* globalt maks $\Rightarrow$ x^* kritisk

x^* kritisk $\Rightarrow$ Ved Teorem 2.4.1

$$f(x) - f(x^*) \le \underbrace{\langle \nabla f(x^*), x - x^* \rangle}_{=0}$$

$$\Rightarrow f(x) \le f(x^*).$$

(b) Samme argument for $-f$, som er konveks.

F Cobb-Douglas

$$F(v_1, \dots, v_n) = A v_1^{a_1} v_2^{a_2} \cdots v_n^{a_n} = x$$

$A, v_i, a_i > 0$, $\quad \sum a_i =: a < 1$

Første-ordens betingelse

$$\frac{\partial}{\partial v_i} \pi = 0$$

$$\Leftrightarrow \quad p A a_i v_1^{a_1} \cdots v_i^{a_i - 1} \cdots v_n^{a_n} = q_i$$

$$p A x^* = \frac{q_i v_i^*}{a_i}$$

$$\Leftrightarrow \quad v_i^* = \frac{a_i p x^*}{q_i}$$

Indsæt v_i^* i $F(v_1, \dots, v_n)$:

$$x^* = A \left[\frac{a_1 p x^*}{q_1}\right]^{a_1} \cdots \left[\frac{a_n p x^*}{q_n}\right]^{a_n}$$

$$= A (p x^*)^a \left[\frac{a_1}{q_1}\right]^{a_1} \cdots \left[\frac{a_n}{q_n}\right]^{a_n}$$

$$\Rightarrow x^* = A^{\frac{1}{1-a}} p^{\frac{a}{1-a}} \left[\frac{a_1}{q_1}\right]^{\frac{a_1}{1-a}} \cdots \left[\frac{a_n}{q_n}\right]^{\frac{a_n}{1-a}}$$

$$\Rightarrow v_i^* = \frac{a_i p x^*}{q_i} = (Ap)^{\frac{1}{1-a}} \left[\frac{a_i}{q_i}\right] \left[\frac{a_1}{q_1}\right]^{\frac{a_1}{1-a}} \cdots \left[\frac{a_n}{q_n}\right]^{\frac{a_n}{1-a}}$$

$y = f(x, r)$ Bogen

Lad

$$\phi(r) = f(x^*(\bar{r}), r) - f^*(r)$$

Da $x^*(\bar{r})$ er et maksimumspunkt for $f(x, r)$ i $r = \bar{r}$, gælder det, at

$$\phi(\bar{r}) = 0$$

Ved definition af f^* gælder

$$f(x^*(r), r) \le f^*(r)$$

$\Rightarrow$ $\phi(r) \le 0$ for alle $r \in B(\bar{r}, \delta)$

$\Rightarrow$ $\phi(r)$ har et indre maksimum i $r = \bar{r}$ og opfylder

$$\left.\frac{\partial \phi(r)}{\partial r_j}\right|_{r=\bar{r}} = 0, \quad j = 1, \dots, k$$

$$= \left.\frac{\partial f(x^*(r), r)}{\partial r_j}\right|_{r=\bar{r}} - \left.\frac{\partial f^*(r)}{\partial r_j}\right|_{r=\bar{r}}$$

$\square$