$$p(\lambda) = \overline{a_n \lambda^n + a_{n-1}\lambda^{n-1} + \dots + a_1\lambda + a_0} = \overline{0}$$

kar. poly. af

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \in \mathbb{R}^{n\times n}$$

$$\overline{0} = 0 = \overline{a_n \lambda^n} + \overline{a_{n-1}\lambda^{n-1}} + \dots + \overline{a_1 \lambda} + \overline{a_0}$$

$$= a_n \overline{\lambda^n} + a_{n-1}\overline{\lambda^{n-1}} + \dots + a_1\overline{\lambda} + a_0$$

$$= a_n \overline{\lambda}^n + a_{n-1}\overline{\lambda}^{n-1} + \dots + a_1\overline{\lambda} + a_0$$

Bevis (ved modstrid):

$$\underbrace{\text{$(v_1,\dots,v_k)$ er egenvektorer med $(\lambda_1,\dots,\lambda_k)$ forskellige egenværdier}}_{A} \Rightarrow \underbrace{\text{$(v_1,\dots,v_k)$ lineært uafhængige}}_{B}$$

Antag $(\neg B \wedge A) \Rightarrow (Z \wedge \neg Z)$ ↯

Antag at $(v_1,\dots,v_k)$ er lineært afhængige.

$\Rightarrow$

$$Z \begin{cases} \text{Der findes en minimal repræsentation} \\ \text{af nul-vektoren (der tages det mindste} \\ \text{antal af vektorer } v_i\text{):} \\ \qquad \sum_i s_i v_i = 0, \quad s_i \neq 0 \text{ for alle } i \end{cases}$$

Vælg et indeks i_1: v_{i_1} egenvektor:

$$A v_{i_1} = \lambda_{i_1} v_{i_1}$$

Gang nul med denne egenværdi:

$$\sum_i (s_i \lambda_{i_1}) v_i = 0$$

Gang nul med A:

$$A \sum_i s_i v_i = \sum_i (s_i \lambda_i) v_i = 0$$

Tag de to nuller fra hinanden:

$$\sum_i s_i \underbrace{(\lambda_i - \lambda_{i_1})}_{=0 \text{ for } i = i_1} v_i = 0$$

$\Rightarrow$ Vi får en ikke-triviel repræsentation af nul, der bruger en vektor (v_{i_1}) mindre end $\sum_i s_i v_i = 0$.

$(\neg Z)$ $\square$

$$A = \begin{bmatrix} 6 & 1 \\ 5 & 2 \end{bmatrix} \qquad \lambda_1 = 1 \quad \lambda_2 = 7 \qquad v_1 = \begin{pmatrix} -\frac{1}{5} \\ 1 \end{pmatrix} \quad v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$A\,[v_1\ v_2]$:

$$\begin{array}{c|cc} & v_1 & v_2 \\ \hline A & Av_1 & Av_2 \\ & \| & \| \\ & \underbrace{1v_1 \quad 7v_2}_{=} & \end{array}$$

$$A[v_1\ v_2] = [v_1\ v_2]\begin{bmatrix} 1 & 0 \\ 0 & 7 \end{bmatrix}$$

$$AP = PD$$

$$P = [v_1\ v_2] \qquad D = \operatorname{diag}(\lambda_1, \lambda_2)$$

$$APP^{-1} = PDP^{-1}$$

$$\boxed{A = PDP^{-1}}$$ egenværdi-dekomposition

$$P^{-1}AP = P^{-1}PD = D$$

$$P = \begin{bmatrix} -\frac{1}{5} & 1 \\ 1 & 1 \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{bmatrix} 2-\lambda & 1 & 0 \\ 0 & 3-\lambda & 0 \\ -1 & 1 & 3-\lambda \end{bmatrix}$$

$$= (3-\lambda)(2-\lambda)(3-\lambda)$$

$$(A - 3I)x = 0$$

$$\left[\begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{array}\right]$$

$$E(3) = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} t \;\middle|\; s, t \in \mathbb{R} \right\}$$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}^{-1}$$