$$Y(L,K) = AL^{\alpha}K^{\beta}$$

$$F(L,K) = Y(L,K) - Y_0 = 0$$

$$K = K(L)$$

Hvad er $K'(L)$?

$$\frac{dK(L)}{dL} = -\frac{D_L F(L,K)}{D_K F(L,K)}$$

$$\left(\left(D_L = \frac{\partial}{\partial L} \qquad D_K = \frac{\partial}{\partial K} \right) \right.$$

$$= -\frac{\alpha A L^{\alpha-1} K^{\beta}}{\beta A L^{\alpha} K^{\beta-1}}$$

$$= -\frac{\alpha}{\beta}\frac{K}{L}$$

$$\frac{\partial}{\partial x} f(x,y(x))$$

$$= \frac{\partial}{\partial x} f(x,y) + \frac{\partial}{\partial y} f(x,y)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{\frac{\partial}{\partial x} f(x,y)}{\underbrace{\frac{\partial}{\partial y} f(x,y)}_{\neq 0}}$$

Bevis: Vi betragter tilfældet $(a,b) = (0,0)$ og lader

$$A := D_x F(0,0) \in \mathbb{R}^{m \times k}$$

$$B := D_y F(0,0) \in \mathbb{R}^{m \times m} \qquad (\text{inv.})$$

Fordi F er diff. i $(0,0)$:

$$F(x,y) = Ax + By + \phi(x,y)$$

således, at

$$\lim_{x,y \to 0} \frac{\phi(x,y)}{\|(x,y)\|} = 0 \in \mathbb{R}^m.$$

Fordi $F(x,g(x)) = 0$ for enhver $x \in X_1$:

$$F(x,g(x)) = Ax + Bg(x) + \phi(x,g(x)) = 0$$

$$\Rightarrow \quad g(x) = \underbrace{-B^{-1}A}_{D_x g}\, x \underbrace{- B^{-1}\phi(x,g(x))}_{=:\,\gamma(x,g(x))}$$

Med sammenlignelige argumenter som i beviset for kædereglen kan man vise, at

$$\lim_{x \to 0} \frac{\gamma(x,g(x))}{\|x\|} = 0$$

(Her bruger vi antagelsen, at g er kontinuert:

$$\|g(x)\| \leq c\|x\|$$

for et $c > 0$ og x tilstrækkeligt lille.)

Vi får:

$$D_x g = -D_y F(0,0)^{-1} D_x F(0,0) \qquad \square$$