$(a,b) =]a,b[$

Δ Delta
∇ Nabla

$$F : \mathbb{R}^2 \to \mathbb{R}$$
$$(x,y) \mapsto F(x,y)$$

Niveaumængde

$$F(x,y) = c \quad , \quad c \in \mathbb{R}$$

$$\left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \;\middle|\; F(x,y) = c,\ c \in \mathbb{R} \right\}$$

Bevægelser langs niveaukurven: (x_0, y_0), (x,y) således, at

$$F(x_0,y_0) = F(x,y) = c$$
$$x = x_0 + dx$$
$$y = y_0 + dy$$

Totale afledede:

$$\frac{\partial}{\partial x} F(x_0,y_0) \underbrace{(x - x_0)}_{dx} + \frac{\partial}{\partial y} F(x_0,y_0) \underbrace{(y - y_0)}_{dy}$$

$$\boxed{= 0}$$

$$= \left\langle \nabla F(x_0,y_0) \begin{bmatrix} dx \\ dy \end{bmatrix} \right\rangle$$

EMEA p. 412

Eks: $F(x,y) = x^2 + 2xy + y^2$

Find ligningen for tangenten i $(x_0, y_0) = (\frac{1}{4}, \frac{1}{4})$

$$\frac{\partial}{\partial x} F(x,y) = 2x + 2y \quad , \frac{\partial}{\partial x} F(\tfrac{1}{4}, \tfrac{1}{4}) = 1$$

$$\frac{\partial}{\partial y} F(x,y) = 2x + 2y, \quad \frac{\partial}{\partial y} F(\tfrac{1}{4}, \tfrac{1}{4}) = 1$$

Ortogonalitetsbetingelse:

$$\left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} x - \frac{1}{4} \\ y - \frac{1}{4} \end{pmatrix} \right\rangle = 0$$

$$x - \tfrac{1}{4} + y - \tfrac{1}{4} = 0$$

$$y = \tfrac{1}{2} - x$$

Bemærk: $(y - y_0) = - \dfrac{\frac{\partial}{\partial x} F(x_0,y_0)}{\frac{\partial}{\partial y} F(x_0,y_0)} (x - x_0)$

$$y - \tfrac{1}{4} = - \frac{1}{1} (x - \tfrac{1}{4})$$

Lad $x, a \in \mathbb{R}^n$, $h \in \mathbb{R}$, $f : \mathbb{R}^n \to \mathbb{R}$

$$g(h) = f(x + ha)$$

Så gælder

$$\lim_{h \to 0} \frac{g(h) - g(0)}{h} = g'(0)$$

$$= \lim_{h \to 0} \frac{f(x + ha) - f(x)}{h} = f'_a(x)$$

Kædereglen:

$$g'(h) = \sum_{i=1}^{n} \frac{\partial}{\partial x_i} f(x + ha)\, a_i$$

$$x + ha = \begin{bmatrix} x_1 + ha_1 \\ x_2 + ha_2 \\ \vdots \\ x_i + ha_i \\ \vdots \\ x_n + ha_n \end{bmatrix}$$

$$g'(0) = \sum_{i=1}^{n} \frac{\partial}{\partial x_i} f(x)\, a_i$$

$$= \langle \nabla f(x), a \rangle$$

Eks: $f : \mathbb{R}^2 \to \mathbb{R}$

$$(x,y) \mapsto x^2 + 2xy + y^2$$

Retningsafledede i retning $(\frac{1}{2}, \frac{1}{2})'$ i punktet $(\frac{1}{4}, \frac{1}{4})'$:

$$f'_a(x_0,y_0) = \left\langle \nabla f(x_0,y_0), \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \right\rangle$$

$$= \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \right\rangle = 1$$

$\square$

$$f(K,L) = A K^{\alpha} L^{1-\alpha}$$

$$\frac{\partial f}{\partial K} = \alpha A K^{\alpha - 1} L^{1-\alpha}$$

$$= \alpha A \left(\frac{L}{K}\right)^{1-\alpha}$$

$$\frac{\partial f}{\partial L} = (1-\alpha) A K^{\alpha} L^{-\alpha}$$

$$= (1-\alpha) A \left(\frac{K}{L}\right)^{\alpha}$$