$q_A(x)$

Bevis: Tag en vilkårlig $x \in \mathbb{R}^n$, betragt

$$x^T A x = x^T P D P^T x$$
$$= z^T D z \qquad \text{med } z = P^T x$$

og $z^T D z > 0$

hvis og kun hvis alle diagonalelementer i D er streng positive. $\square$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Eks.:

$$\underbrace{\begin{bmatrix} \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ -\frac{1}{\sqrt2} & \frac{1}{\sqrt2} \end{bmatrix}}_{P^T} = A$$

$$\begin{array}{cc|cc} & & \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ & & -\frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ \hline \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & 1 & 0 \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & 1 \end{array} \quad \checkmark$$

$$\begin{array}{cc|cc|cc} & & -1 & 0 & \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ & & 0 & -2 & -\frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ \hline \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & \sqrt2 & -\frac{3}{2} & \frac{1}{2} \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & -\sqrt2 & \frac{1}{2} & -\frac{3}{2} \end{array}$$

$$\Rightarrow A = \begin{bmatrix} -\frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{3}{2} \end{bmatrix}$$

$$\left.\begin{array}{l} \det A_1 = -\frac{3}{2} \\ \det A_2 = \frac{9}{4} - \frac{1}{4} = 2 \end{array}\right\} \Rightarrow A \text{ neg. def.}$$