

$$A \cdot v = \lambda v$$

$$\begin{array}{cc|cc} & & 1 & -1 \\ & & 1 & 1 \\ \hline \frac{5}{2} & -\frac{1}{2} & 2 & -3 \\ -\frac{1}{2} & \frac{5}{2} & 2 & 3 \\ & & 1 & 1 \\ & & 2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} & 3 \begin{pmatrix} -1 \\ 1 \end{pmatrix} \end{array}$$

$$Ax = \lambda x$$

$$Ax - \lambda x = 0$$

$$(A - \lambda I)x = 0$$

$$\det(A - \lambda I) = 0$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (a_{11} - \lambda)(a_{22} - \lambda) - a_{12}a_{21}$$

$$= \lambda^2 - \underbrace{(a_{11} + a_{22})}_{\text{Spur}(A)} \lambda + \underbrace{a_{11}a_{22} - a_{12}a_{21}}_{\det A}$$

$$A = \begin{bmatrix} 6 & 1 \\ 5 & 2 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 6 - \lambda & 1 \\ 5 & 2 - \lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (6 - \lambda)(2 - \lambda) - 5$$

$$= \lambda^2 - \frac{8}{1}\lambda + \frac{7}{1} = 0$$

$$\lambda_{1,2} = -\frac{8}{2} \pm \sqrt{\frac{8^2}{4} - 7}$$

$$= 4 \pm \sqrt{16 - 7} = 4 \pm 3 = \begin{cases} 1 \\ 7 \end{cases}$$

$$(A - \lambda I)x = 0$$

$$A - I: \begin{array}{cc|c} 5 & 1 & 0 \\ 5 & 1 & 0 \end{array}$$

$$\begin{array}{cc|c} 1 & \frac{1}{5} & 0 \\ 0 & 0 & 0 \end{array}$$

$$E(1) = \left\{ \begin{bmatrix} -\frac{1}{5} \\ 1 \end{bmatrix} s \mid s \in \mathbb{R} \right\}$$

$$Ax = \lambda x$$

$$A(\underline{s}x) = s Ax = s \lambda x = \lambda(\underline{s}x)$$

$$A - 7I: \begin{array}{cc|c} -1 & 1 & 0 \\ 5 & -5 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}$$

$$E(7) = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} s \mid s \in \mathbb{R} \right\}$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2 - \lambda & 1 & 1 \\ 2 & 3 - \lambda & 4 \\ -1 & -1 & -2 - \lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (2 - \lambda)[(3 - \lambda)(-2 - \lambda) + 4]$$

$$- 2[(-2 - \lambda) + 1]$$

$$- 1[4 - (3 - \lambda)]$$

$$= 1(2(-3) + 4)$$

$$- 2((-3) + 1)$$

$$- 1(4 - 2)$$

$$= -2 + 4 - 2 = 0$$

$$-\lambda^3 + 3\lambda^2 + \lambda - 3 = 0$$

$$\lambda^3 - 3\lambda^2 - \lambda + 3 = 0 = p_\lambda(\lambda)(\lambda - 1)$$

$$(\lambda^3 - 3\lambda^2 - \lambda + 3) : (\lambda - 1) = \lambda^2 - 2\lambda - 3$$

$$\lambda^3 - \lambda^2$$

$$0 - 2\lambda^2 - \lambda + 3$$

$$-2\lambda^2 + 2\lambda$$

$$0 - 3\lambda + 3$$

$$-3\lambda + 3$$

$$0$$

$$p(\lambda) = (\lambda^2 - 2\lambda - 3)(\lambda - 1)$$

$$\lambda_{2,3} = 1 \pm \sqrt{1 + 3} = 1 \pm 2 = \begin{cases} -1 \\ 3 \end{cases}$$

$$\lambda_1 = 1$$

$$\lambda_2 = -1$$

$$\lambda_3 = 3$$

$$(A - I)x = 0$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 4 \\ -1 & -1 & -3 \end{pmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \end{array}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & -2 \end{pmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \end{array}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \end{array}$$

$$E(1) = \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} s \mid s \in \mathbb{R} \right\}$$

$$(A + I)x = 0$$

$$(3) \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 4 \\ -1 & -1 & -1 \end{pmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \begin{array}{c} 1 \\ 2 \\ (-1) \end{array}$$

$$\begin{pmatrix} 0 & -2 & -2 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \begin{array}{c} 1 \\ 2 \\ (-2) \end{array}$$

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \begin{array}{c} 1 \\ 2 \\ 2 \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \end{array}$$

$$E(-1) = \left\{ \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} s \mid s \in \mathbb{R} \right\}$$

$$(A - 3I)x = 0$$

$$(-1) \begin{pmatrix} -1 & 1 & 1 \\ 2 & 0 & 4 \\ -1 & -1 & -5 \end{pmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \begin{array}{c} 1 \\ 2 \\ 0 \end{array}$$

$$\begin{pmatrix} 1 & -1 & -1 \\ 1 & 0 & 2 \\ 0 & -2 & -6 \end{pmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \begin{array}{c} 1 \\ 2 \\ (-2) \end{array}$$

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 1 & -1 & -1 \end{pmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \end{array}$$

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{pmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \end{array}$$

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \end{array}$$

$$\lambda_1 = 1, \lambda_2 = -1, \lambda_3 = 3$$

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$