$$\dot{x}(t) = \frac{dx}{dt}$$

$\dot{x} = x \qquad x(t) = e^t$

$\dot{x} = ax \qquad x(t) = e^{at} \leftarrow$

$a(t) \equiv a \in \mathbb{R}$

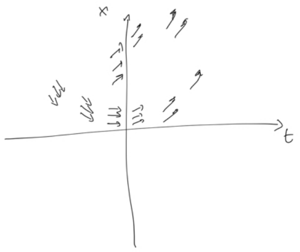
$$\dot{x} = \underset{a(t)}{\underbrace{3t}}\, x(t)$$

$$\ddot{x} = \frac{d^2x}{dt^2} = -x$$

$x(t) = \sin t \qquad\qquad x(t) = \cos t$

$\dot{x} = \cos t \qquad\qquad \dot{x} = -\sin t$

$\ddot{x} = -\sin t \qquad\qquad \ddot{x} = -\cos t$



Eks: $\dot{x} = -2t\,x^2$

med begyndelsesbetingelse

$x(1) = -1$

- Separér: $\frac{dx}{dt} = -2t\,x^2$

$$-\frac{dx}{x^2} = 2t\,dt$$

- Integrér:

$$-\int \frac{1}{x^2}\,dx = \int 2t\,dt$$

$$\frac{1}{x} = t^2 + C, \quad C \in \mathbb{R}.$$

$$x(t) = \frac{1}{t^2 + C}$$

Begyndelsesbetingelse:

$$x(1) = \frac{1}{1+C} = -1$$

$$\Rightarrow C = -2$$

$$x(t) = \frac{1}{t^2 - 2}$$

Eks. 2 Økonomisk vækst

- Cobb-Douglas produktionsfunktion

$$X(t) = A\,K(t)^{1-\alpha}\,L(t)^{\alpha}$$

$$A > 0, \quad \alpha \in (0,1)$$

- Forandring i kapitalapparatet:

$$\dot{K}(t) = s\,X(t)$$

$s = 1 - c$, $c \in (0,1)$, c marginale tilbøjelighed til at forbruge

- Arbejdsstyrke

$$L(t) = L_0 e^{\lambda t}, \quad \lambda \text{ fertilitetsrate}$$

$$\Rightarrow \dot{K}(t) = sA\,K(t)^{1-\alpha} L_0^{\alpha} e^{\alpha\lambda t}$$

- Separér: $K^{\alpha-1}\,dK = sAL_0^{\alpha} e^{\alpha\lambda t}\,dt$
- Integrér: $\int K^{\alpha-1}\,dK = sAL_0^{\alpha} \int e^{\alpha\lambda t}\,dt$

$$\frac{K^{\alpha}}{\alpha} = \frac{sAL_0^{\alpha}}{\alpha\lambda} e^{\alpha\lambda t} + C, \quad C \in \mathbb{R}$$

$$\Rightarrow K(t) = \left[\frac{sAL_0^{\alpha}}{\lambda} e^{\alpha\lambda t} + \alpha C\right]^{\frac{1}{\alpha}}$$

Begyndelsesbetingelse: $K(0) = K_0 \in \mathbb{R}$

$$K(0) = \left[\frac{sAL_0^{\alpha}}{\lambda} + \alpha C\right]^{\frac{1}{\alpha}} \overset{!}{=} K_0$$

$$\Rightarrow C = \frac{K_0^{\alpha}}{\alpha} - \frac{sAL_0^{\alpha}}{\alpha\lambda}$$

$$\Rightarrow K(t) = \left[\frac{sAL_0^{\alpha}}{\lambda}\left(e^{\alpha\lambda t} - 1\right) + K_0^{\alpha}\right]^{\frac{1}{\alpha}} \qquad \square$$

$$\dot{x} = rx$$

$$\frac{dx}{dt} = rx = \underset{r}{\underset{\parallel}{f(t)}}\,\underset{x}{\underset{\parallel}{g(x)}}$$

$$\frac{dx}{x} = r\,dt$$

$$\int \frac{1}{x}\,dx = \int r\,dt$$

$$\log x = rt + C, \quad C \in \mathbb{R}$$

$$x(t) = e^{rt + C} = e^{rt} e^{C}$$

$$X(0) = x_0 \in \mathbb{R}$$

$$x(0) = e^{C} = x_0 \quad \Rightarrow \quad C = \log x_0$$

$$e^{C} = x_0$$

$$\Rightarrow x(t) = x_0 e^{rt}$$

$\square$