$$H = f e^{-rt} + pg$$

$$H^c = He^{rt} = f + \underbrace{pe^{rt}}_{\lambda} g$$

$$\lambda = e^{rt} p$$

$$\dot{\lambda} = r\underbrace{e^{rt} p}_{\lambda} + e^{rt} \dot{p}$$

$$\dot{\lambda} - r\lambda = e^{rt} \dot{p}$$

$$\dot{p} = e^{-rt} (\dot{\lambda} - r\lambda)$$

$$= -\frac{\partial H}{\partial x}$$

$$\frac{\partial H^c}{\partial x} = e^{rt} \frac{\partial H}{\partial x} \Leftrightarrow \frac{\partial H}{\partial x} = e^{-rt} \frac{\partial H^c}{\partial x}$$

$$\dot{p} = e^{-rt} (\dot{\lambda} - r\lambda)$$

$$= - e^{-rt} \frac{\partial H^c}{\partial x}$$

$$\Rightarrow \dot{\lambda} - r\lambda = -\frac{\partial H^c}{\partial x}$$

Eks: $\max_u \int_0^T \left(-x^2 - \frac{1}{2}u^2\right) e^{-rt}\,dt$

$\dot{x} = x + u \quad , \quad u \in \mathbb{R}$

$x(0) = 1$

$x(T)$ fri $\Rightarrow p(T) = 0$

Løs på alminnelig måde:

$H(t,x,u,p) = \left(-x^2 - \frac{1}{2}u^2\right) e^{-rt} + p(x+u)$

$H'_u = -ue^{-rt} + p \overset{!}{=} 0 \Rightarrow u = pe^{rt}$

$\dot{p} = -H'_x = 2xe^{-rt} - p \quad , \quad p(T) = 0$

$\Rightarrow \quad \dot{x} = x + pe^{rt}$

$\dot{p} = 2xe^{-rt} - p$

Se Problem 6.5.3 (student manual)

og eksempel 9.2.2.

Nu bruger vi H^c i stedet for:

$H^c = He^{rt} = -x^2 - \frac{1}{2}u^2 + \underbrace{pe^{rt}}_{\lambda}(x+u)$

u^* maks. H^c:

$\frac{\partial H^c}{\partial u} = -u + \lambda = 0 \Rightarrow u = \lambda$

$$\dot{\lambda} - r\lambda = -\frac{\partial H^c}{\partial x}$$
$$= 2x - \lambda$$

$$\Rightarrow \quad \dot{x} = x + \lambda$$
$$\dot{\lambda} = 2x + (r-1)\lambda$$

Saet $r=2$ som i eksempel 9.2.2

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix}$$

$$\det \begin{bmatrix} 1-\lambda & 1 \\ 2 & 1-\lambda \end{bmatrix} = (1-\lambda)^2 - 2$$
$$= \lambda^2 - 2\lambda - 1$$
$$\Rightarrow \quad \lambda_{1,2} = 1 \pm \sqrt{2}$$

$$\left[\begin{array}{cc|c} 1-1-\sqrt{2} & 1 & 0 \\ 2 & 1-1-\sqrt{2} & 0 \end{array}\right]$$
$$\left[\begin{array}{cc|c} -\sqrt{2} & 1 & 0 \\ 2 & -\sqrt{2} & 0 \end{array}\right]$$
$$\left[\begin{array}{cc|c} 1 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \underbrace{-\sqrt{2} + \frac{2}{\sqrt{2}}}_{0} & 0 \end{array}\right] \Rightarrow v_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1-1+\sqrt{2} & 1 & 0 \\ 2 & 1-1+\sqrt{2} & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} \sqrt{2} & 1 & 0 \\ 2 & \sqrt{2} & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \sqrt{2}-\frac{2}{\sqrt{2}} & 0 \end{array}\right] \Rightarrow v_2 = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 1 \end{bmatrix}$$

Gang v_1 med $\sqrt{2}$ og v_2 med $-\sqrt{2}$:

$$v_1 = \begin{bmatrix} 1 \\ \sqrt{2} \end{bmatrix} \qquad v_2 = \begin{bmatrix} 1 \\ -\sqrt{2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ \lambda \end{bmatrix} = A \begin{bmatrix} 1 \\ \sqrt{2} \end{bmatrix} e^{(1+\sqrt{2})t} + B \begin{bmatrix} 1 \\ -\sqrt{2} \end{bmatrix} e^{(1-\sqrt{2})t}$$

$$x(0) = 1$$

$$x(0) = A + B = 1$$

$$\lambda(T) = 0$$

$$= \sqrt{2}e^{(1+\sqrt{2})T}A - \sqrt{2}e^{(1-\sqrt{2})T}B$$

$$\begin{bmatrix} 1 & 1 \\ \sqrt{2}e^{(1+\sqrt{2})T} & -\sqrt{2}e^{(1-\sqrt{2})T} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & \underbrace{\dfrac{-\cancel{\sqrt{2}}\, e^{(1-\sqrt{2})T}}{\cancel{\sqrt{2}}\, e^{(1+\sqrt{2})T}}}_{-e^{-2\sqrt{2}T}} & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & -\left(1+e^{-2\sqrt{2}T}\right) & -1 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & \dfrac{1}{1+e^{-2\sqrt{2}T}} \end{array}\right]$$

$$\left[\begin{array}{cc|l} 1 & 0 & 1-\dfrac{1}{1+e^{-2\sqrt{2}T}} = A \\ 0 & 1 & \dfrac{1}{1+e^{-2\sqrt{2}T}} = B \end{array}\right]$$

$$A = 1 - \frac{1}{1+e^{-2\sqrt{2}T}} = \frac{1+e^{-2\sqrt{2}T}}{1+e^{-2\sqrt{2}T}} - \frac{1}{1+e^{-2\sqrt{2}T}}$$

$$= \frac{e^{-2\sqrt{2}T}}{1+e^{-2\sqrt{2}T}} \;\Big|\; \frac{e^{2\sqrt{2}T}}{e^{2\sqrt{2}T}}$$

$$= \frac{1}{1+e^{2\sqrt{2}T}}$$

$$\Rightarrow \quad x(t) = \frac{1}{1+e^{2\sqrt{2}T}}\, e^{(1+\sqrt{2})t} + \frac{1}{1+e^{-2\sqrt{2}T}}\, e^{(1-\sqrt{2})t}$$

$$\lambda(t) = \frac{\sqrt{2}}{1+e^{2\sqrt{2}T}}\, e^{(1+\sqrt{2})t} - \frac{\sqrt{2}}{1+e^{-2\sqrt{2}T}}\, e^{(1-\sqrt{2})t}$$

$$p = \lambda e^{-2t}$$

$$= \frac{\sqrt{2}}{1+e^{2\sqrt{2}T}} e^{(\sqrt{2}-1)t} - \frac{\sqrt{2}}{1+e^{-2\sqrt{2}T}} e^{(-\sqrt{2}-1)t}$$

$$B = \frac{1}{1+e^{-2\sqrt{2}T}} \quad \frac{e^{2\sqrt{2}T}}{e^{2\sqrt{2}T}}$$

$$\square$$