$$A = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = [v_1 \; v_2]$$

result of $\{ r_1 v_1 + r_2 v_2 \mid 0 \le r_{1,2} \le 1 \}$

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$$

$$h = v_2 - r v_1$$

$$\langle h, v_1 \rangle = 0 = \langle v_2 - r v_1, v_1 \rangle$$

$$= \langle v_2, v_1 \rangle - r \langle v_1, v_1 \rangle$$

$$= 7 - r\,10$$

$$\Rightarrow r = \frac{7}{10}$$

$$h = \begin{bmatrix} 1 \\ 4 \end{bmatrix} - \frac{7}{10} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{10}{10} - \frac{21}{10} \\ \frac{40}{10} - \frac{7}{10} \end{bmatrix} = \begin{bmatrix} -1.1 \\ 3.3 \end{bmatrix}$$

$$a = \|v_1\| \, \|h\| = \sqrt{10}\sqrt{1.1^2 + 3.3^2} = 11$$

$$v_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad v_1' = \frac{1}{\sqrt{10}} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \qquad v_2' = \frac{1}{\sqrt{17}} \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\|h'\| = \sin(\beta - \alpha) = \frac{\|h'\|}{\underbrace{\|v_2'\|}_{=1}}$$

$$v_1' = \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$$

$$v_2' = \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix}$$

$$\|h'\| = \sin(\beta - \alpha)$$

$$= \sin\beta\cos\alpha - \cos\beta\sin\alpha$$

$$a' = \underbrace{\|v_1'\|}_{1} \|h'\| = \sin(\beta - \alpha)$$

$$A' = [v_1' \; v_2'] = \begin{bmatrix} \cos\alpha & -\cos\beta \\ \sin\alpha & \sin\beta \end{bmatrix}$$

$$= \cos\alpha\sin\beta - \cos\beta\sin\alpha$$

$$v_1 = \rho v_1' = \sqrt{10} \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$$

$$v_2 = \sigma v_2' = \sqrt{17} \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix}$$

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} \rho\cos\alpha & \sigma\cos\beta \\ \rho\sin\alpha & \sigma\sin\beta \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\sin(\beta - \alpha) = \frac{\|h\|}{\|v_2\|} \Rightarrow \|h\| = \|v_2\| \sin(\beta - \alpha)$$

$$a = \|v_1\| \, \|h\| = \|v_1\| \, \|v_2\| \sin(\beta - \alpha)$$

$$= \sqrt{10}\sqrt{17}\sin(\beta - \alpha)$$

$$= \rho\sigma\sin(\beta - \alpha)$$

$$= \rho\sigma(\cos\alpha\sin\beta - \sin\alpha\cos\beta)$$

$$= \rho\cos\alpha\,\sigma\sin\beta - \rho\sin\alpha\,\sigma\cos\beta$$

$$= a_{11}a_{22} - a_{21}a_{12} = 11$$