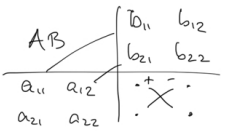
$$\det(v_1, v_2, \dots, rv_i + sv_j, \dots, v_n)$$
$$= r \det(v_1, v_2, \dots, v_i, \dots, v_n) + s \det(v_1, v_2, \dots, v_j, \dots, v_n)$$

(i) $\det(v_1, rv_2, v_3, \dots, v_n) \qquad r \in \mathbb{R}$
$$= r \det(v_1, v_2, \dots, v_n) = r \det A$$

(ii) $\det(v_1 + v_2, v_1 + v_2, v_3, \dots, v_n) = 0$
$$= \det(v_1, v_1 + v_2, v_3, \dots, v_n) + \det(v_2, v_1 + v_2, v_3, \dots, v_n)$$
$$= \det(v_1, v_1, v_3, \dots, v_n) + \det(v_1, v_2, v_3, \dots, v_n) + \det(v_2, v_1, v_3, \dots, v_n) + \det(v_2, v_2, v_3, \dots, v_n) = 0$$
$$\det(v_2, v_1, \dots, v_n) = -\det(v_1, v_2, \dots, v_n)$$

(iii) $\det(v_1, rv_1 + v_2, v_3, \dots, v_n)$
$$= r \det(v_1, v_1, v_3, \dots, v_n) = 0 + \det(v_1, v_2, \dots, v_n)$$
$$= \det A$$

(v) : $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \qquad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$



$\underline{A}x = b \qquad \det A = |A|$

$= [v_1, v_2, \dots, v_n]$

$(v_1, v_2, \dots, v_n)$ lin. unafh. $\iff \det A \neq 0$

$(v_1, v_2, \dots, v_n)$ lin. afh. $\iff \det A = 0$

lin. afh. : $A = [v_1, v_2, \dots, v_{n-1}, rv_1 + sv_2]$

$\det A = r \det(v_1, v_2, \dots, v_1) = 0 + s \det(v_1, v_2, \dots, v_2) = 0$
$= 0$

$A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$

$$A = \begin{pmatrix} 3 & 1 & 1 & 2 \\ 1 & 4 & 3 & 4 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 4 \end{pmatrix} = \begin{bmatrix} B & M \\ 0 & C \end{bmatrix}$$

$$\det A = \det(BC) = \det B \det C$$

$x_1 v_1 + x_2 v_2 + \dots + x_n v_n = b$
$Ax = b$

$\det(b, v_2, v_3, \dots, v_n)$
$= \det(x_1 v_1 + x_2 v_2 + \dots + x_n v_n, v_2, v_3, \dots, v_n)$
$= x_1 \det(v_1, v_2, \dots, v_n) + x_2 \det(v_2, v_2, \dots, v_n) + x_3 \det(v_3, v_2, v_3, \dots, v_n) + \dots + x_n \det(v_n, v_2, \dots, v_n) = 0$
$= x_1 \det(v_1, v_2, \dots, v_n)$

$$\Rightarrow x_1 = \frac{\det(b, v_2, \dots, v_n)}{\det(v_1, v_2, \dots, v_n)}$$

$$x_i = \frac{\det(v_1, \dots, b, \dots, v_n)}{\det A} \quad (i)$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$\det A = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$= a_{11}a_{22} - a_{12}a_{21}$$