Bevis: (a) Fra Taylor følger, at

$$f(x) = f(x_0) + \langle \nabla f(x_0), x - x_0 \rangle + \frac{1}{2} (x-x_0)' \operatorname{Hess} f(x_0 + c(x-x_0)) (x-x_0)$$

for et tal $c \in [0,1]$.

f konkav $\iff$

$$(x-x_0)' \operatorname{Hess} f(x_0 + c(x-x_0)) (x-x_0) \leq 0$$

for alle $x - x_0$ og c. Derfor

$$f(x) - f(x_0) \leq \langle \nabla f(x_0), x - x_0 \rangle \qquad \square$$

(a) Lad $x, y \in P_a$: $x, y \in S$ med

$$f(x) \geq a$$
$$f(y) \geq a$$

For $\lambda \in [0,1]$ lad

$$z = \lambda x + (1-\lambda) y \in S$$

f konkav $\Rightarrow$

$$f(z) \geq \lambda f(x) + (1-\lambda) f(y)$$
$$\geq \lambda a + (1-\lambda) a = a$$

$\Rightarrow z \in P_a \Rightarrow P_a$ konveks. $\square$