$$\ddot{x} = \frac{d^2x}{dt^2}$$

$$\dot{x} = at^2 \Rightarrow x = \frac{a}{3}t^3$$

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$$\ddot{x} + a(t)\dot{x} + b(t)x = 0$$

$u_1(t)$, $u_2(t)$ løsninger

$\Rightarrow Au_1 + Bu_2$ er løsning.

$$\frac{d}{dt}(Au_1 + Bu_2) = A\dot{u}_1 + B\dot{u}_2$$

$$\frac{d^2}{dt^2}(Au_1 + Bu_2) = A\ddot{u}_1 + B\ddot{u}_2$$

$$\Rightarrow A\ddot{u}_1 + B\ddot{u}_2 + a(t)(A\dot{u}_1 + B\dot{u}_2) + b(Au_1 + Au_2)$$

$$= A(\underbrace{\ddot{u}_1 + a(t)\dot{u}_1 + bu_1}_{=0})$$

$$+ B(\underbrace{\ddot{u}_2 + a(t)\dot{u}_2 + bu_2}_{=0})$$

$$= 0$$

□

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$$\ddot{x} - x = 0 \Leftrightarrow \ddot{x} = x$$

$$x(t) = Ae^t$$

$$x(t) = Ae^{-t}$$

$$\dot{x} = -Ae^{-t}$$

$$\ddot{x} = Ae^{-t} = x$$

$$\ddot{x} - x = 5$$

$$\Rightarrow u^* = -5 \Rightarrow \dot{u}^* = 0,\ \ddot{u}^* = 0$$

$$\ddot{u}^* - u^* = 5$$

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$$\ddot{x} + a\dot{x} + bx = 0$$

$$x(t) = e^{rt}$$

$$\Rightarrow \dot{x} = re^{rt},\quad \ddot{x} = r^2e^{rt}$$

$$r^2e^{rt} + are^{rt} + be^{rt} = 0 \quad / e^{-rt}$$

$$r^2 + ar + b = 0$$

$$r_{1,2} = -\frac{a}{2} \pm \sqrt{\frac{a^2}{4} - b}$$

$$\Rightarrow u_1 = e^{r_1t},\quad u_2 = e^{r_2t}$$

$$x(t) = Au_1 + Bu_2$$

1) $\frac{a^2}{4} - b > 0 \Rightarrow r_1 \neq r_2,\ r_{1,2} \in \mathbb{R}$

$$x(t) = Ae^{r_1t} + Be^{r_2t}$$

2) $\frac{a^2}{4} - b = 0 \Rightarrow r = -\frac{a}{2}$ dobbelt rod

Betrakt $u_2 = te^{rt}$

$$\dot{u}_2 = e^{rt} + tre^{rt}$$

$$\ddot{u}_2 = re^{rt} + r(e^{rt} + rte^{rt})$$

$$= 2re^{rt} + r^2te^{rt}$$

DL: $\ddot{u}_2 + a\dot{u}_2 + bu_2 = ?$

$$(2re^{rt} + r^2te^{rt})$$

$$+ a(e^{rt} + tre^{rt})$$

$$+ bte^{rt}$$

$$= te^{rt}(r^2 + ar + b) \quad \} = 0$$

$$+ e^{rt}(2\underbrace{r}_{=-\frac{a}{2}} + a) \quad \} = 0$$

$$= 0$$

$$u_1 = e^{rt}$$

$$x(t) = Ae^{rt} + Bte^{rt} = e^{rt}(A + Bt)$$

$$= Au_1 + Bu_2$$

3) $\frac{a^2}{4} - b < 0$

$$r_{1,2} = -\frac{a}{2} \pm \sqrt{\frac{a^2}{4} - b}$$

$$= -\frac{a}{2} \pm i\sqrt{b - \frac{a^2}{4}}$$

$$= \alpha \pm i\beta$$

Løsninger

$$e^{r_1t} = e^{(\alpha + i\beta)t} = e^{\alpha t}e^{i\beta t}$$

$$= e^{\alpha t}(\cos(\beta t) + i\sin(\beta t))$$

$$e^{r_2t} = e^{(\alpha - i\beta)t}$$

$$= e^{\alpha t}(\cos(\beta t) - i\sin(\beta t))$$

$$\frac{e^{r_1t} + e^{r_2t}}{2} = e^{\alpha t}\cos(\beta t)$$

$$\frac{e^{r_1t} - e^{r_2t}}{2i} = e^{\alpha t}\sin(\beta t)$$

$$\Rightarrow x(t) = Ae^{\alpha t}\cos(\beta t) + Be^{\alpha t}\sin(\beta t)$$

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Eks: (a) $\ddot{x} - 3x = 0$

$$r^2 - 3 = 0$$

$$r = \pm\sqrt{3}$$

$$x(t) = Ae^{\sqrt{3}t} + Be^{-\sqrt{3}t}$$

(b) $\ddot{x} - 4\dot{x} + 4x = 0$

$$r^2 - 4r + 4 = 0$$

$$r = 2 \pm \sqrt{4 - 4}$$

$$\Rightarrow x(t) = e^{2t}(A + Bt)$$

(c) $\ddot{x} - 6\dot{x} + 13 = 0$

$$r^2 - 6r + 13 = 0$$

$$r_{1,2} = 3 \pm \sqrt{\frac{36}{4} - \frac{52}{4}}$$

$$= 3 \pm \sqrt{-\frac{16}{4}}$$

$$= 3 \pm i2$$

$$\Rightarrow x(t) = Ae^{3t}\cos(2t) + Be^{3t}\sin(2t)$$