$$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad e_1, e_2$$

$$\begin{pmatrix} 1.1 \\ 5.7 \end{pmatrix} = 1.1 e_1 + 5.7 e_2$$

$$e_1, e_2, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1.1 \\ 5.7 \end{pmatrix} = (1.1 + x) e_1 + (5.7 + x) e_2 - x \begin{pmatrix} 1 \\ 1 \end{pmatrix} , \quad x \in \mathbb{R}$$

$$\underset{\mathbb{R}^2}{\underset{\cap}{0}} = e_1 + e_2 - \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = e_1 + e_2$$

$$e_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - e_2$$

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \qquad v_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = x_1 v_1 + x_2 v_2 + x_3 v_3$$

$$x_i \in \mathbb{R}$$

$$= \sum_{i=1}^{3} x_i v_i$$

$$f(u+v) = f(u) + f(v)$$

$$f(rv) = r f(v)$$

$$r \in \mathbb{R}, \quad v \in V$$

$$f\left(\sum_{i=1}^{3} x_i v_i\right) = \sum_{i=1}^{3} x_i f(v_i)$$

$$x_i \in \mathbb{R}, \quad v_i \in V$$

$f: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ linear

$$x = \sum_{i=1}^{n} x_i e_i$$

$$f(x) = y \in \mathbb{R}^m$$

$$f\left(\sum_{i=1}^{n} x_i e_i\right) = \sum_{i=1}^{n} x_i \overbrace{f(e_i)}^{\in \mathbb{R}^m}$$

$$\begin{array}{c|c} & \begin{matrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{matrix} \\ \hline m\left\{\begin{matrix} | & | & & | \\ f(e_1) & f(e_2) & \cdots & f(e_n) \\ | & | & & | \end{matrix}\right. & \begin{matrix} \overset{\mathbb{R} \atop \cup}{x_1} \overset{\mathbb{R}^m \atop \cup}{f(e_1)} + x_2 f(e_2) \\ + \cdots + x_n f(e_n) \end{matrix} \end{array}$$

$$\underbrace{\hspace{5cm}}_{n} \qquad \underbrace{\hspace{5cm}}_{Ax \in \mathbb{R}^m}$$

A