

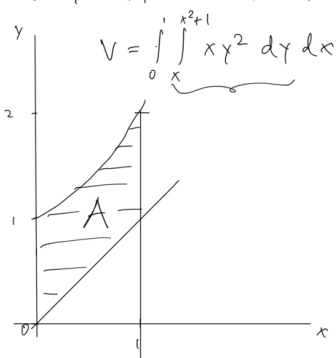
Ekse:
$$\int_1^b \underbrace{\int_1^d \frac{y-x}{(y+x)^3} dy}_{\text{Indre integral}} dx$$

$$\begin{aligned} \int_1^d \frac{y-x}{(y+x)^3} dy &= \int_1^d \frac{y+x-2x}{(y+x)^3} dy \\ &= \int_1^d \frac{1}{(y+x)^2} dy - 2x \int_1^d \frac{1}{(y+x)^3} dy \\ &= -\frac{1}{y+x} \Big|_{y=1}^{y=d} - 2x \left[-\frac{1}{2} \frac{1}{(y+x)^2} \right]_{y=1}^{y=d} \\ &= -\frac{1}{d+x} + \frac{1}{1+x} + \frac{x}{(d+x)^2} - \frac{x}{(1+x)^2} \\ &= \dots \\ &= \frac{1}{(1+x)^2} - \frac{d}{(d+x)^2} \end{aligned}$$

Ydre integral:

$$\begin{aligned} \int_1^b \left(\frac{1}{(1+x)^2} - \frac{d}{(d+x)^2} \right) dx \\ &= -\frac{1}{1+x} \Big|_{x=1}^{x=b} + \frac{d}{d+x} \Big|_{x=1}^{x=b} \\ &= \frac{d}{b+d} - \frac{d}{1+d} - \frac{1}{1+b} + \frac{1}{2} \end{aligned}$$

Ekse: $A = \{(x, y) \mid x \in [0, 1], x \leq y \leq x^2 + 1\}$



Indre integral:

$$\begin{aligned} \int_x^{x^2+1} xy^2 dy &= \frac{x}{3} y^3 \Big|_{y=x}^{y=x^2+1} \\ &= \frac{x(x^2+1)^3}{3} - \frac{x^4}{3} \\ &= \dots \\ &= \frac{x^7}{3} + x^5 - \frac{x^4}{3} + x^3 + \frac{x}{3} \end{aligned}$$

Ydre integral

$$\begin{aligned} \int_0^1 \left(\frac{x^7}{3} + x^5 - \frac{x^4}{3} + x^3 + \frac{x}{3} \right) dx \\ &= \left[\frac{x^8}{24} + \frac{x^6}{6} - \frac{x^5}{15} + \frac{x^4}{4} + \frac{x^2}{6} \right]_0^1 \\ &= \frac{67}{120} \end{aligned}$$

Ek 13: For givet $y \in [c, d]$ er $\frac{\partial^2 F}{\partial y}$ en funktion af x med afledede ift. x lig med $\frac{\partial^2 F}{\partial x \partial y} = f(x, y)$

Betrakt det indre integral:

$$\begin{aligned} \int_a^b f(x, y) dx &= \int_a^b \frac{\partial^2 F(x, y)}{\partial x \partial y} dx \\ &= \frac{\partial F(x, y)}{\partial y} \Big|_{x=a}^{x=b} \\ &= \frac{\partial F(b, y)}{\partial y} - \frac{\partial F(a, y)}{\partial y} \end{aligned}$$

Det ydre integral:

$$\begin{aligned} \int_c^d \left(\int_a^b f(x, y) dx \right) dy \\ &= \int_c^d \left[\frac{\partial F(b, y)}{\partial y} - \frac{\partial F(a, y)}{\partial y} \right] dy \\ &= \left[F(b, y) - F(a, y) \right]_{y=c}^{y=d} \\ &= F(b, d) - F(a, d) - F(b, c) + F(a, c) \end{aligned}$$

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