$$L(t) = \bar{L}_1(t)\, e^{\alpha_T t} = \bar{L}_1(t)\, T(t)$$

$c \qquad \tilde{c}_1$

$k \qquad k$

a

$$k = \frac{K}{L} \qquad k = \frac{K}{\bar{L}_1}$$

$$\begin{aligned} F(K,L) &= A\, K(t)^{\alpha}\, L(t)^{1-\alpha} \\ &= L(t) \quad A\, K(t)^{\alpha}\, L(t)^{-\alpha} \\ &= L(t) \quad A \quad k(t)^{\alpha} \\ &= L(t) \quad f(k(t)) \end{aligned}$$

$$\frac{\partial \pi}{\partial k} = L\left(f'(k) - (r+\delta)\right) \stackrel{!}{=} 0$$

$$\Rightarrow \quad f'(k) = r + \delta$$

$$f'(k) = \alpha A\, k^{\alpha - 1}$$

$$\begin{aligned} L(t) &= \bar{L}_1(t)\, T(t) \\ &= \bar{L}_1(t)\, e^{\alpha_T t} \end{aligned} \qquad \begin{aligned} c &= \tilde{c}_1 e^{-\alpha_T t} \\ k &= k\, e^{-\alpha_T t} \end{aligned}$$

$$H(t, a, \tilde{c}_1, \nu)$$

adjungerede

tilstand kontrol

ν = "ny"

$$H(t, a, c_1, \nu) = u(c_1) \underbrace{L(t)}_{e^{\alpha_L t}} e^{-\rho t} + \nu[(r-\alpha_L)a + w - c_1]$$

Maksimumsprincip i standard form

$$\frac{\partial H}{\partial c_1'} = u'(c_1)e^{-(\rho-\alpha_L)t} - \nu \overset{!}{=} 0 \qquad (1)$$

Differentiér (1) ift. tid:

$$-(\rho-\alpha_L)e^{-(\rho-\alpha_L)t} u'(c_1') + e^{-(\rho-\alpha_L)t} u''(c_1) \dot{c}_1 - \dot{\nu} = 0$$

$$\Rightarrow \dot{\nu} = -(\rho-\alpha_L)e^{-(\rho-\alpha_L)t} u'(c_1) + e^{-(\rho-\alpha_L)t} u''(c_1) \dot{c}_1$$

Anden betingelse:

$$\dot{\nu} = -\frac{\partial H}{\partial a} = -\nu(r-\alpha_L) \qquad (2)$$

Sæt ind fra (1):

$$\dot{\nu} = -u'(c_1)e^{-(\rho-\alpha_L)t}(r-\alpha_L)$$

$$\Rightarrow$$

$$-(\rho-\alpha_L)e^{-(\rho-\alpha_L)t} u'(c_1) + e^{-(\rho-\alpha_L)t} u''(c_1)\dot{c}_1 = -u'(c_1)e^{-(\rho-\alpha_L)t}(r-\alpha_L)$$

gang med $-e^{(\rho-\alpha_L)t}\frac{1}{u'(c)}$:

$$(\rho-\alpha_L) - \frac{u''(c_1)\dot{c}_1}{u'(c_1)} = r-\alpha_L \qquad (*)$$

Fra en opgave fra kapitel 5 ved vi, at

$$-\frac{u''(c_1)\,c_1}{u'(c_1)} = \theta \qquad \text{konstant.}$$

Gang (*) med c_1:

$$(\rho-\alpha_L)\,c_1' + \theta\,\dot{c}_1' = (r-\alpha_L)\,c_1'$$

$$\Rightarrow \boxed{\frac{\dot{c}_1'}{c_1'} = (r-\rho)\frac{1}{\theta}}$$

Bemærk: $r = r(t)$ givet fra firmaets problem.

Fra (2) får vi, at

$$\dot{\nu} = -(r-\alpha_L)\nu$$

$$\nu(t) = \nu(0)\,e^{-\int_0^t (r(s)-\alpha_L)\,ds}$$

Transversaliteten bliver til:

$$\lim_{t\to\infty} \nu(t)\,a(t) = \lim_{t\to\infty}\left(a(t)\,e^{-\int_0^t (r(s)-\alpha_L)\,ds}\right)$$

$$= 0$$

- $a(t) \geq 0$: Hvis $w(t) = c_1(t)$, så vokser $a(t)$ med raten $(r(t)-\alpha_L)$.

$$\Rightarrow a(t)\,e^{-\int_0^t (r(s)-\alpha_L)\,ds} \xrightarrow[t\to\infty]{} \text{constant} \neq 0$$

$\Rightarrow$ Efter et tidspunkt t' skal $c_1(t) > w(t)$ og aktiverne forbruges.

- $a(t) < 0$: Hvis gælden vokser hurtigere end rentesatsen $r(t) - \alpha_L$, så kan husholdningen på hvert tidspunkt betale renten med ny gæld og gælden vokser til et stort beløb : "pyramidespil" $\square$

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1) $Y(K,L) = AK^\alpha L^{1-\alpha} = LAk^\alpha = L f(k)$

$$\frac{\partial Y}{\partial L} = \frac{\partial L}{\partial L} f(k) + L f'(k) \frac{\partial}{\partial L}\left(\frac{K}{L}\right)$$

$$= e^{\alpha_T t} f(k) + \cancel{L} f'(k)(-1)\left(\frac{K}{L^{\cancel{2}}}\right) e^{\alpha_T t}$$

$$= e^{\alpha_T t}\left(f(k) - f'(k)k\right)$$

$$\stackrel{!}{=} w(t)$$

2) $$a(t) = k(t)$$

$$\Rightarrow \dot{a}(t) = \dot{k}(t)$$

$$\Rightarrow \dot{k}(t) = (r - \alpha_L)k + w - c_1$$

Der gælder også, at $k = k e^{-\alpha_T t}$

$$\Rightarrow \dot{k} = \dot{k} e^{-\alpha_T t} - k \alpha_T e^{-\alpha_T t}$$

$$\Rightarrow \dot{k} = \left[(r - \alpha_L)k + w - c_1\right] e^{-\alpha_T t} - k\alpha_T e^{-\alpha_T t}$$

$$= \left[(r - \alpha_L)k + e^{\alpha_T t}\left(f(k) - f'(k)k\right) - c_1\right] e^{-\alpha_T t} - k\alpha_T e^{-\alpha_T t}$$

$$= (r - \alpha_L)k + f(k) - f'(k)k - c - \alpha_T k$$

$$= \left(f'(k) - \delta - \alpha_L\right)k + f(k) - f'(k)k - c - \alpha_T k$$

$$= f(k) - c - (\delta + \alpha_L + \alpha_T)k$$

$$\Rightarrow \quad \frac{\dot{k}}{k} = \frac{f(k)}{k} - \frac{c}{k} - (\delta + \alpha_L + \alpha_T) \qquad (3)$$

Husholdningens problem resulterede i

$$\frac{\dot{c}_1}{c_1} = (r - \rho)\frac{1}{\theta} = (f'(k) - \delta - \rho)\frac{1}{\theta}$$

Bemærk, at

$$\frac{\dot{c}}{c} = \frac{1}{c}\frac{d}{dt}\left(c_1 e^{-\alpha_T t}\right)$$

$$= \frac{1}{c_1 e^{-\alpha_T t}}\left(\dot{c}_1 e^{-\alpha_T t} - c_1 \alpha_T e^{-\alpha_T t}\right)$$

$$= \frac{\dot{c}_1}{c_1} - \alpha_T$$

$$\Rightarrow \quad \frac{\dot{c}}{c} = (f'(k) - \delta - \rho - \alpha_T \theta)\frac{1}{\theta} \qquad (4)$$

Med $f(k) = Ak^\alpha$ og $f'(k) = \alpha A k^{\alpha - 1}$ får vi, at

$$\frac{d}{dt}\log k(t) = \frac{\dot{k}}{k} = Ak^{\alpha-1} - \frac{c}{k} - (\delta + \alpha_L + \alpha_T)$$

$$\frac{d}{dt}\log c(t) = \frac{\dot{c}}{c} = \left(\alpha A k^{\alpha-1} - (\delta + \rho + \alpha_T\theta)\right)\frac{1}{\theta}$$

$\square$