$$\max_{x,y,z} \quad f(x,y,z) = xyz$$
$$f: \mathbb{R}^3 \to \mathbb{R}$$

s.a.
$$g_1(x,y,z) = x^2 + y^2 = 4$$
$$g_2(x,y,z) = x + z = 2$$
$$g_i: \mathbb{R}^3 \to \mathbb{R}$$

$$\mathcal{L}(x,y,z) = xyz - \lambda_1(x^2+y^2-4) - \lambda_2(x+z-2)$$

① $\frac{\partial \mathcal{L}}{\partial x} = yz - 2\lambda_1 x - \lambda_2 = 0$

② $\frac{\partial \mathcal{L}}{\partial y} = xz - 2\lambda_1 y = 0$

③ $\frac{\partial \mathcal{L}}{\partial z} = xy - \lambda_2 = 0$

④ $\frac{\partial \mathcal{L}}{\partial \lambda_1} = x^2 + y^2 - 4 = 0$

⑤ $\frac{\partial \mathcal{L}}{\partial \lambda_2} = x + z - 2 = 0$

② $\Rightarrow \lambda_1 = \frac{xz}{2y}$

③ $\Rightarrow \lambda_2 = xy$

① $\Rightarrow yz - \frac{x^2 z}{y} - xy = 0 \quad / \cdot y$

④ $\Rightarrow y = \pm\sqrt{4-x^2}, \quad y^2 = 4 - x^2$

⑤ $\Rightarrow z = 2 - x$

① $\Rightarrow y^2 z - x^2 z - xy^2 = 0$

$$(4-x^2)(2-x) - x^2(2-x) - x(4-x^2) = 0$$

$x_1 = 2$ er rod.

$$8 - 4x - 2x^2 + x^3 - 2x^2 + x^3 - 4x + x^3 = 0$$

$$3x^3 - 4x^2 - 8x + 8 = 0$$
$$= (\text{polynomium af 2en grad})(x-2)$$

$$(3x^3 - 4x^2 - 8x + 8) : (x-2) = \underbrace{3x^2 + 2x - 4}$$
$$\frac{3x^3 - 6x^2}{\quad 2x^2 - 8x + 8}$$
$$\frac{2x^2 - 4x}{\quad -4x + 8}$$
$$\frac{-4x + 8}{0}$$

$$3x^3 - 4x^2 - 8x + 8 = (\underbrace{3x^2 + 2x - 4})(x-2)$$

$$3x^2 + 2x - 4 = 0 \quad / : 3$$
$$x^2 + \underset{p}{\tfrac{2}{3}}x \underset{q}{- \tfrac{4}{3}} = 0$$

$$x_{2,3} = -\frac{1}{3} \pm \sqrt{\frac{1}{9} + \frac{12}{9}}$$
$$= -\frac{1}{3} \pm \frac{\sqrt{13}}{3}$$

$$3x^3 - 4x^2 - 8x + 8 = (x-2)\left(x + \frac{1}{3} - \frac{\sqrt{13}}{3}\right) \cdot \left(x + \frac{1}{3} + \frac{\sqrt{13}}{3}\right)$$

Kandidat punkter:

$x = 2 \overset{④}{\underset{x^2+y^2=4}{\Longrightarrow}} y = 0$, $\overset{⑤}{\underset{x+z=2}{\Longrightarrow}} z = 0$ $\Bigg\}$ $(x,y,z) = (2,0,0)$ ①

$x = \frac{\sqrt{13}-1}{3} \overset{④}{\Longrightarrow} y = \pm\sqrt{4 - \left(\frac{\sqrt{13}-1}{3}\right)^2}$

$\overset{⑤}{\Longrightarrow} z = 2 - \frac{\sqrt{13}-1}{3}$

② $(x,y,z) = \left(\frac{\sqrt{13}-1}{3}, \sqrt{4-\left(\frac{\sqrt{13}-1}{3}\right)^2}, 2 - \frac{\sqrt{13}-1}{3}\right)$

③ $(x,y,z) = \left(\frac{\sqrt{13}-1}{3}, -\sqrt{4-\left(\frac{\sqrt{13}-1}{3}\right)^2}, 2 - \frac{\sqrt{13}-1}{3}\right)$

$x = -\frac{\sqrt{13}+1}{3} \overset{④}{\Longrightarrow} y = \pm\sqrt{4 - \left(\frac{\sqrt{13}+1}{3}\right)^2}$

$\overset{⑤}{\Longrightarrow} z = 2 + \frac{\sqrt{13}+1}{3}$

④ $(x,y,z) = \left(-\frac{\sqrt{13}+1}{3}, \sqrt{4-\left(\frac{\sqrt{13}+1}{3}\right)^2}, 2 + \frac{\sqrt{13}+1}{3}\right)$

⑤ $(x,y,z) = \left(-\frac{\sqrt{13}+1}{3}, -\sqrt{4-\left(\frac{\sqrt{13}+1}{3}\right)^2}, 2 + \frac{\sqrt{13}+1}{3}\right)$

① $f(x,y,z) = xyz = 0$

② $xyz \approx 1{,}77$

③ $xyz \approx -1{,}77$

④ $xyz \approx -6{,}96$

⑤ $xyz \approx 6{,}96$

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$$\nabla f = \lambda_1 \nabla g_1 + \lambda_2 \nabla g_2$$

$$\begin{pmatrix} yz \\ xz \\ xy \end{pmatrix} = \lambda_1 \begin{bmatrix} 2x \\ 2y \\ 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$y = 0 \overset{④}{\Longrightarrow} x = \pm 2 \overset{⑤}{\Longrightarrow} z = 0 \quad x = 2$

$z = 4 \quad x = -2$

$$\left.\begin{aligned}(x,y,z) &= (2,0,0) \\ (x,y,z) &= (-2,0,4)\end{aligned}\right\} \Rightarrow f(x,y,z) = 0$$

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$\Rightarrow (x,y,z)$ fra ⑤ er maksimumspunktet.

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Lokale anden-ordens betingelser

Teorem 3.4.1

Afgraensede Hesse-matrix: I maksimumet må afgr. Hesse matrix vaere negativ definit, men kun i retninger af tilladte vektorer, dvs. vektorer $(x, y, z)'$, der opfylder bibetingelserne.

$$\mathcal{L}(x,y,z) = xyz - \lambda_1 x^2 - \lambda_1 y^2 + 4\lambda_1 - \lambda_2 x - \lambda_2 z + 2\lambda_1$$

$\frac{\partial^2 \mathcal{L}}{\partial x^2} = -2\lambda_1 \qquad \frac{\partial^2 \mathcal{L}}{\partial x \partial y} = \frac{\partial^2 \mathcal{L}}{\partial y \partial x} = z$

$\frac{\partial^2 \mathcal{L}}{\partial y^2} = -2\lambda_1 \qquad \frac{\partial^2 \mathcal{L}}{\partial x \partial z} = y$

$\frac{\partial^2 \mathcal{L}}{\partial z^2} = 0 \qquad \frac{\partial \mathcal{L}}{\partial y \partial z} = x$

$$\text{Hes } \mathcal{L} = \begin{bmatrix} -2\lambda_1 & z & y \\ z & -2\lambda_1 & x \\ y & x & 0 \end{bmatrix}$$

$$\nabla g_1 = \begin{bmatrix} 2x \\ 2y \\ 0 \end{bmatrix} \qquad \nabla g_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\underset{\substack{\uparrow \\ ⑤}}{B_3(x^*)} = \begin{bmatrix} 0 & 0 & 2x & 2y & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 2x & 1 & -2\lambda_1 & z & y \\ 2y & 0 & z & -2\lambda_1 & x \\ 0 & 1 & y & x & 0 \end{bmatrix}$$

$$\det B_3(x^*) < 0$$
$$= -130.71$$

□