# A STATISTICAL MODEL OF THE GLOBAL CARBON BUDGET


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20. MAY 2021

# DATA

www.globalcarbonproject.org

Friedlingstein et al. (2020),
The gobal carbon budget 2020,
Earth System Science Data 12,
3269-3340

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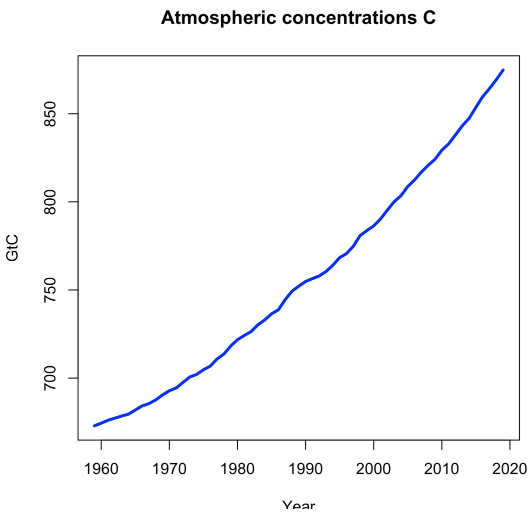


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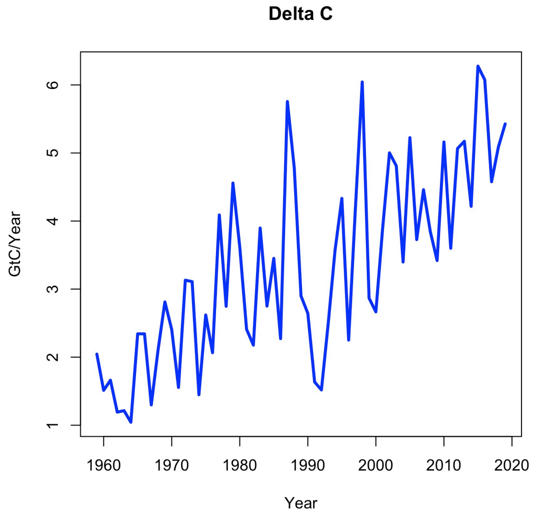


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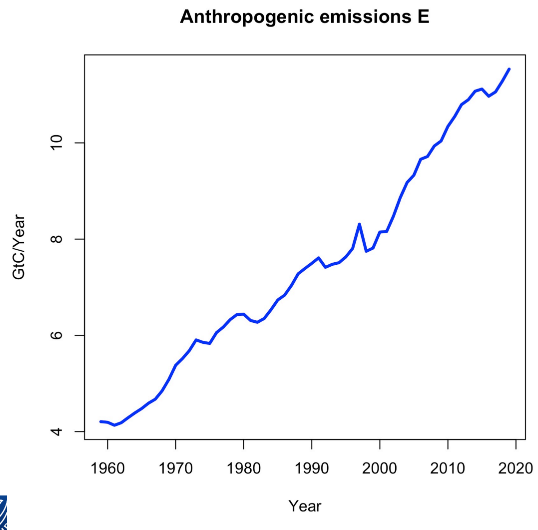


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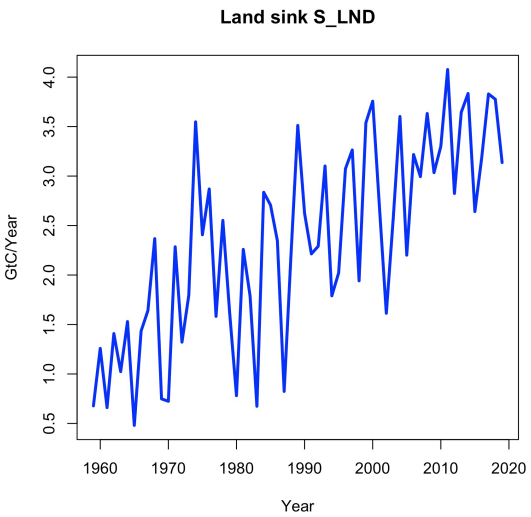


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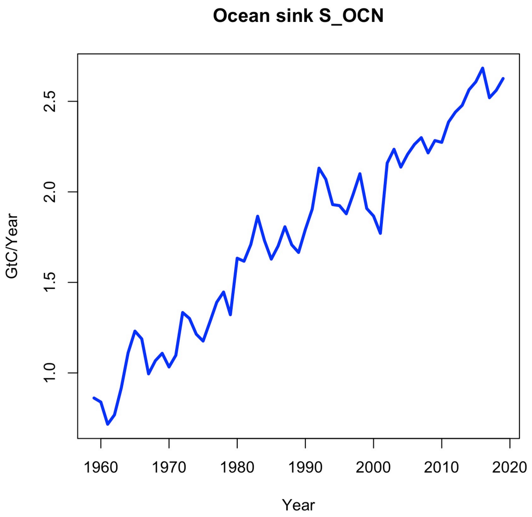


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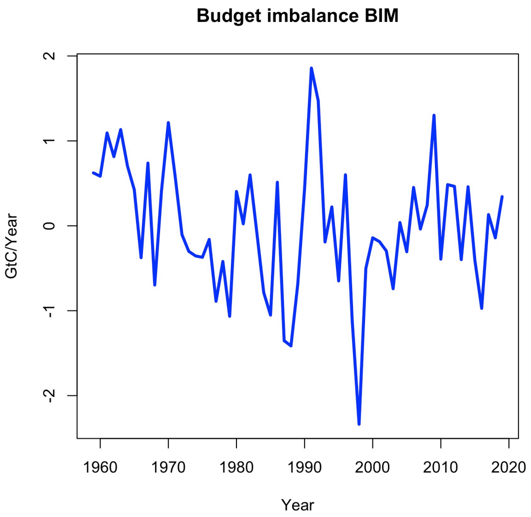
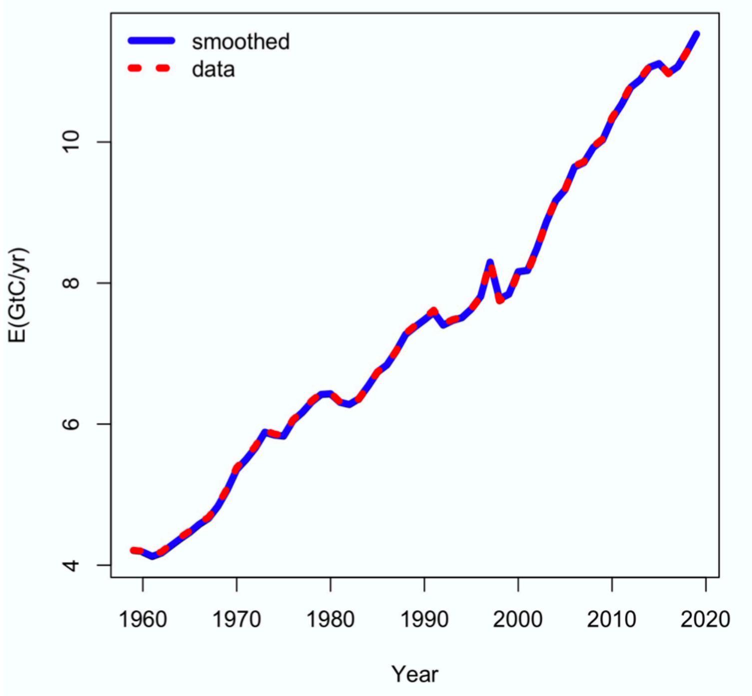
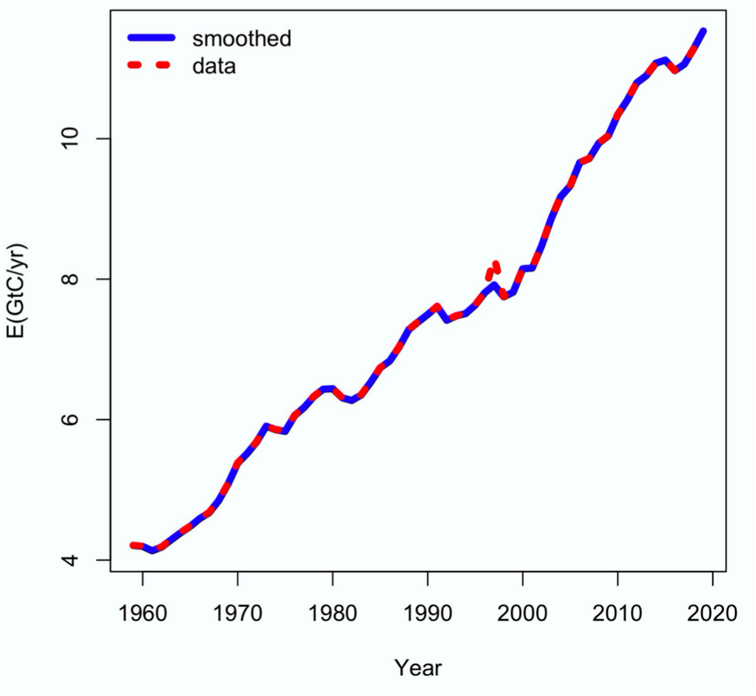


# ANTHROPOGENIC EMISSIONS



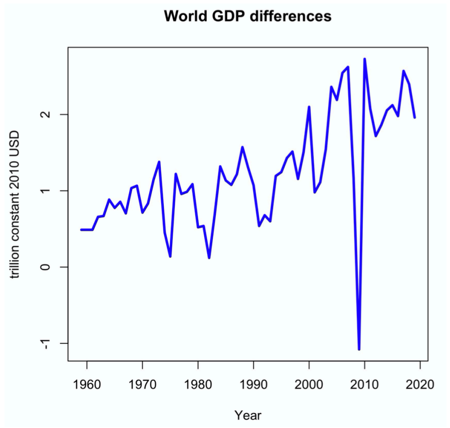

$$\Delta E_t = \underset{(0.02)}{0.14} + X_t^E,$$

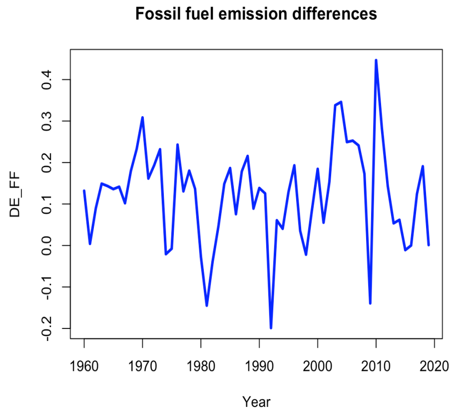
$$X_{t+1}^E = \underset{(0.26)}{0.06} X_t^E + \eta_{5,t},\ \eta_{5,t} \sim N(0, \underset{(0.032)}{0.024})$$




$$\Delta E_t = \underset{(0.01)}{0.11} \Delta GDP_t - \underset{(0.06)}{0.29} I1991 + X_t^E,$$

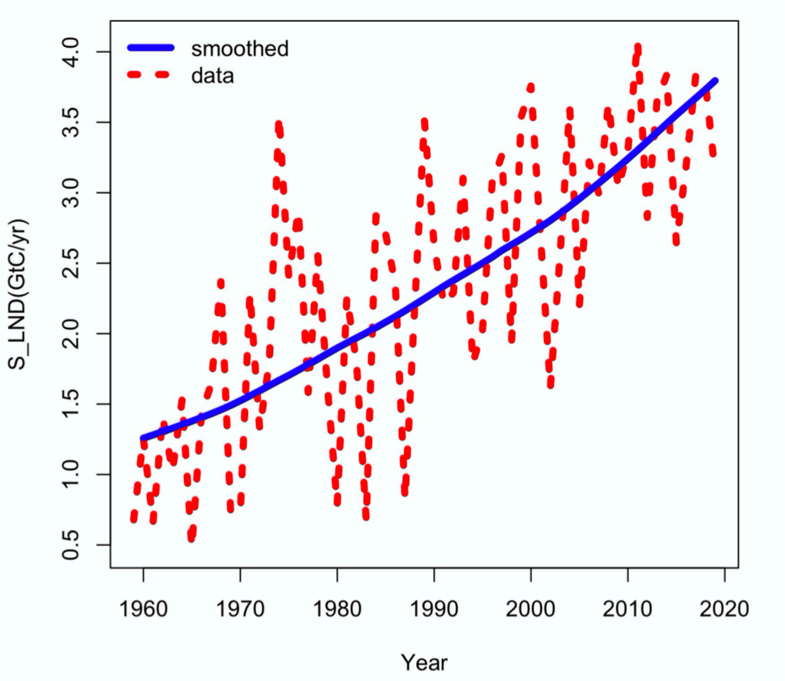
$$X_{t+1}^E = \underset{(0.12)}{0.39} X_t^E + \eta_{4,t},\ \eta_{4,t} \sim N(0, \underset{(0.001)}{0.004})$$

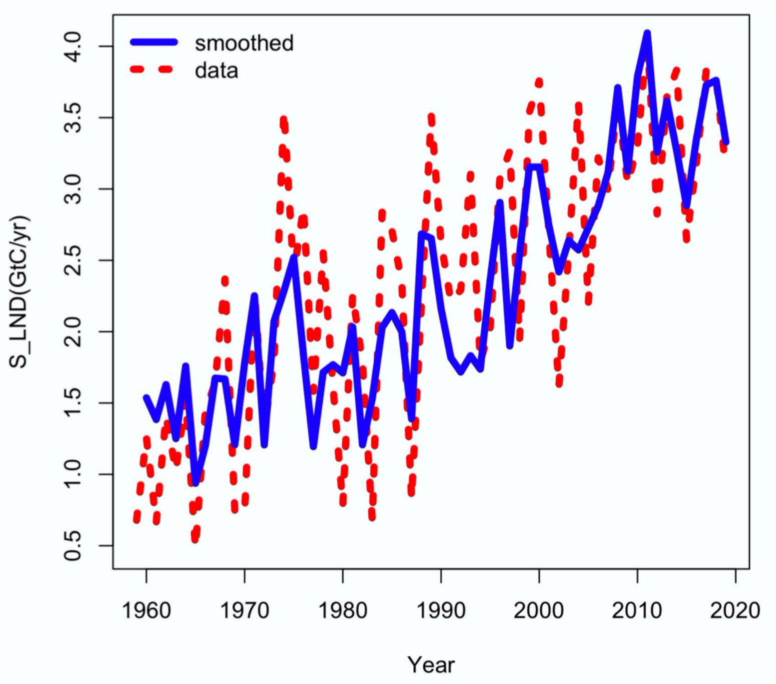


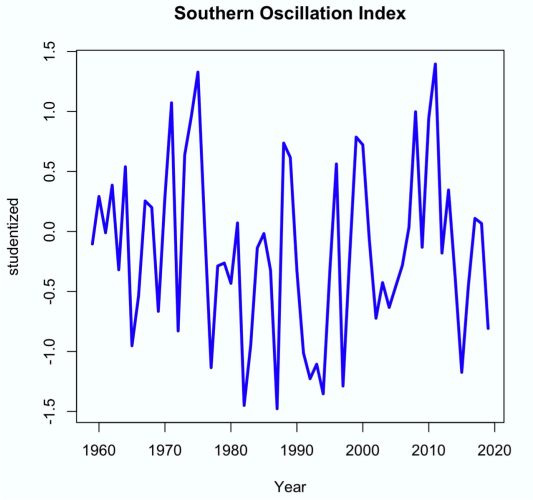




Friedlingstein et al. (2020)
Bennedsen, Hillebrand, Koopman (2021)

# LAND SINK



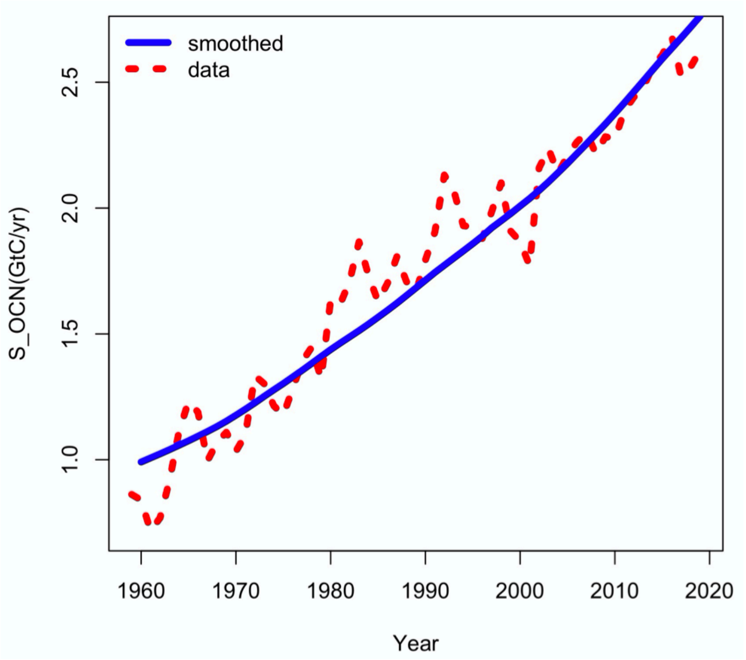

"Moisture sensitivities of both productivity and decomposition are important for capturing the response of the net flux to such [La Nina] events." Haverd et al. (2018, p. 3013)

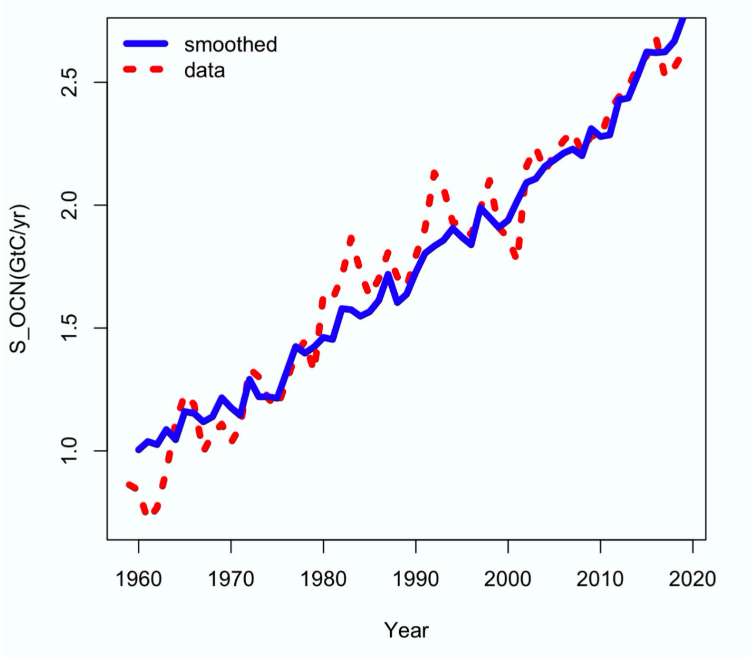
$$S\_LND_t = \underset{(0.05)}{-7.22} + \underset{(0.97)}{7.48}\frac{C_t}{C_0}$$

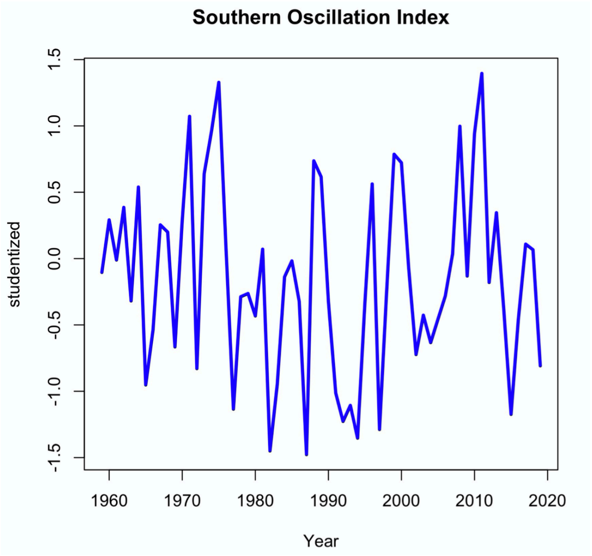
$$S\_LND_t = \underset{(0.04)}{-6.70} + \underset{(0.49)}{7.12}\frac{C_t}{C_0} + \underset{(0.09)}{0.57}SOI_t$$

Raupach et al. (2014), Raupach (2013), Gloor et al. (2010), Canadell et al. (2007)

# OCEAN SINK



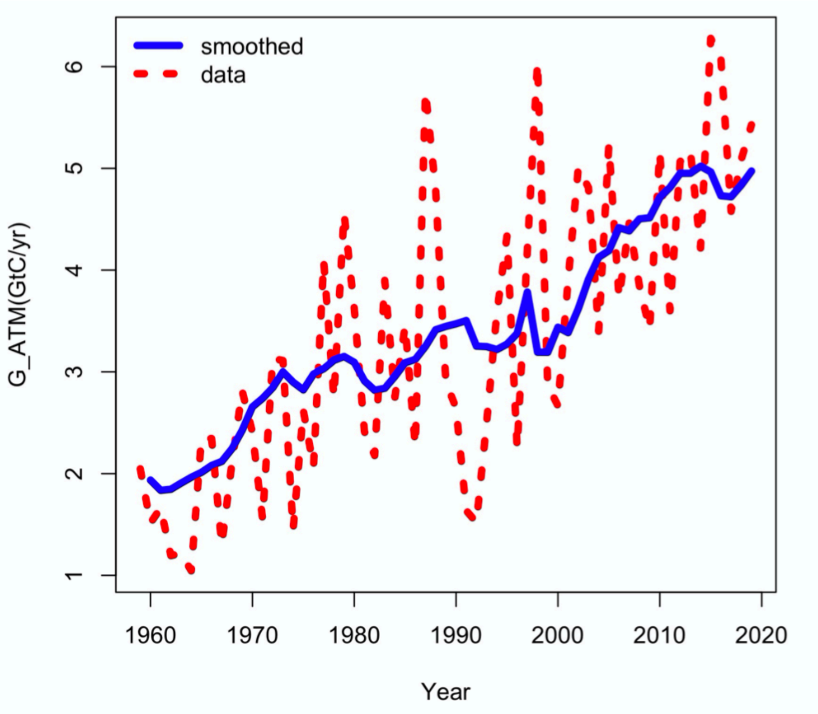

"When the winds are strongest during the cold cycle of ENSO deep upwelling occurs and [ocean CO2 partial pressure] values are at a maximum ." Feely et al. (1999, p. 599)

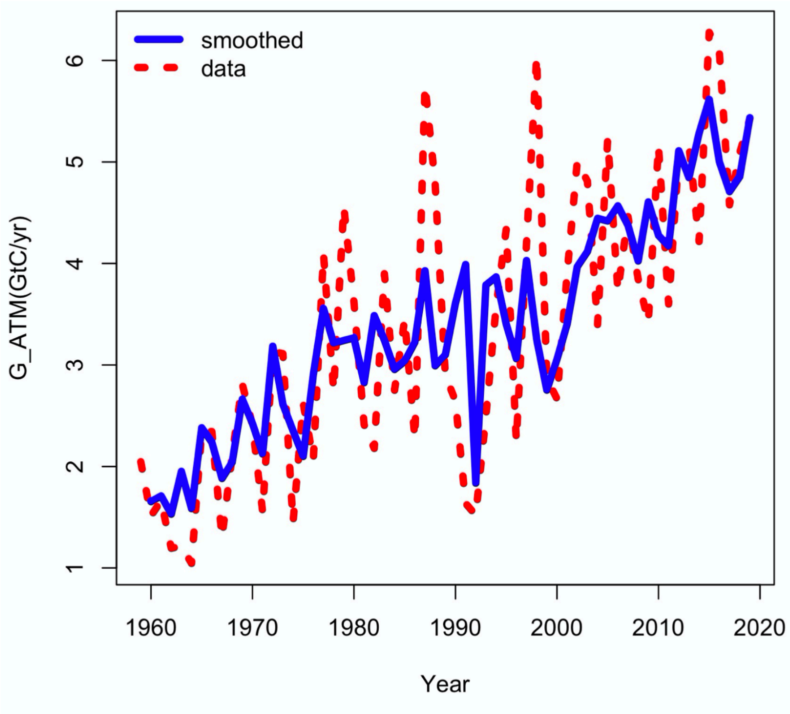
$$S\_OCN_t = \underset{(0.04)}{-4.93} + \underset{(0.41)}{5.22}\frac{C_t}{C_0}$$

$$S\_OCN_t = \underset{(0.03)}{-4.63} + \underset{(0.29)}{4.99}\frac{C_t}{C_0} - \underset{(0.02)}{0.06}SOI_t$$

Raupach et al. (2014), Raupach (2013), Gloor et al. (2010), Canadell et al. (2007)

# ΔC = GROWTH IN ATM. CONCENTRATIONS




$$\Delta C_t = E_t - S\_LND_t - S\_OCN_t$$

# THE DYNAMICS OF C

$$\beta_i^* = \frac{\beta_i}{C_0} \approx 0.01$$

$$x_t = \sum_{i=1}^{t} X_i^E$$

$$q := \frac{1}{1+\beta_1^*+\beta_2^*} \approx \frac{1}{1.02}$$

$$c = E_0 - c_1 - c_2$$

$$\Delta C_t = E_t - S\_LND_t - S\_OCN_t$$

$$= E_t - c_1 - c_2 - \beta_1^* C_t - \beta_2^* C_t + \varepsilon_t, \quad \varepsilon_t \sim I(0)$$

$$(1+\beta_1^*+\beta_2^*)C_t - C_{t-1} = c + dt + x_t + \varepsilon_t$$

$$(1-qL)C_t = qc + qdt + qx_t + q\varepsilon_t$$

Three insights:

$$C_t = q^t\left[C_0 - \frac{qc}{1-q} + \frac{dq^2}{(1-q)^2}\right] + \left[\frac{qc}{1-q} - \frac{dq^2}{(1-q)^2}\right] + \frac{dq}{1-q}t + \sum_{j=0}^{t-1} q^{j+1}x_{t-j} + \sum_{j=0}^{t-1} q^{j+1}\varepsilon_{t-j}$$

$$= o(1) + O(1) + O(t) + I(1) + I(0) = O(t) + I(1)$$

Thus,

$$\Delta C_t = I(0)$$

But,

$$(1-qL)(1-L)C_t = qd + q\Delta x_t + q\Delta\varepsilon_t = I(0)$$

# THE SYSTEM MODEL

**State** equation **Model 1**

$$S\_LND^*_{t+1} = c_1 + \frac{\beta_1}{C_0} C^*_{t+1}$$

$$S\_OCN^*_{t+1} = c_2 + \frac{\beta_2}{C_0} C^*_{t+1}$$

$$E^*_{t+1} = E^*_t + d + X^E_t$$

**State** equation **Model 2**

$$S\_LND^*_{t+1} = c_1 + \frac{\beta_1}{C_0} C^*_{t+1} + \beta_3 SOI_{t+1}$$

$$S\_OCN^*_{t+1} = c_2 + \frac{\beta_2}{C_0} C^*_{t+1} + \beta_4 SOI_{t+1}$$

$$E^*_{t+1} = E^*_t + \beta_5 \Delta\, GDP^{World}_{t+1} + \beta_8 I1991 + X^E_t$$

**Measurement** equation

$$C_t = C^*_t + X_{1,t}$$

$$S\_LND_t = S\_LND^*_t + X_{2,t}$$

$$S\_OCN_t = S\_OCN^*_t + X_{3,t}$$

$$E_t = E^*_t + \beta_6 I1997$$

$$C^*_{t+1} = C^*_t + G\_ATM^*_{t+1}$$

$$G\_ATM^*_{t+1} = E^*_{t+1} - {S_{LND}}^*_{t+1} - {S_{OCN}}^*_{t+1} + \beta_7 I1991$$

$$X_{1,t} = \phi_1 X_{1,t-1} + \eta_{1,t}$$

$$X_{2,t} = \eta_{2,t}$$

$$X_{3,t} = \phi_3 X_{3,t-1} + \eta_{3,t}$$

$$X^E_t = \phi_E X^E_{t-1} + \eta_{4,t}$$

$$\begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \end{bmatrix} \sim N\left[0, \begin{bmatrix} \sigma_1^2 & r_{12}\sigma_1\sigma_2 & r_{13}\sigma_1\sigma_3 & 0 \\ r_{12}\sigma_1\sigma_2 & \sigma_2^2 & 0 & 0 \\ r_{13}\sigma_1\sigma_3 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix}\right]$$

$$\eta_{4,t} \sim N(0, \sigma_4^2\, s_E^2\, I_{t\geq 1996})$$

# RESIDUAL DIAGNOSTICS

Model 1

| Residual | mean | std dev | skew | kurt | JB | LB(1) | DW |
|---|---|---|---|---|---|---|---|
| C | -0.183 | 0.964 | 0.205 | 2.765 | 0.568 | 1.607 | 1.653 |
| E | -0.033 | 1.009 | -1.448 | 8.506 | 98.38*** | 0.073 | 2.060 |
| S_LND | 0.061 | 1.022 | 0.047 | 2.615 | 0.399 | 0.054 | 2.037 |
| S_OCN | 0.093 | 0.978 | 0.377 | 2.988 | 1.447 | 0.116 | 1.908 |

Model 2

| Residual | mean | std dev | skew | kurt | JB | LB(1) | DW |
|---|---|---|---|---|---|---|---|
| C | -0.140 | 0.979 | 0.167 | 3.272 | 0.471 | 0.350 | 1.840 |
| E | 0.004 | 0.983 | -0.269 | 3.095 | 0.759 | 0.062 | 1.928 |
| S_LND | 0.164 | 0.995 | 0.285 | 2.530 | 1.387 | 0.223 | 2.111 |
| S_OCN | 0.120 | 0.951 | 0.440 | 3.394 | 2.360 | 0.648 | 2.191 |

# PARAMETER ESTIMATES

Model 2

| Coefficients | estimate | std err | Variances | estimate | std err |
|---|---|---|---|---|---|
| $c_1$ (filt.) | -6.70 | 0.04 | $\sigma^2_{\eta 1}$ | 0.66 | 0.13 |
| $c_2$ (filt.) | -4.63 | 0.03 | $\sigma^2_{\eta 2}$ | 0.32 | 0.06 |
| $\beta_1$ | 7.12 | 0.49 | $\sigma^2_{\eta 3}$ | 0.01 | 0.002 |
| $\beta_2$ | 4.99 | 0.29 | $\sigma^2_{\eta 4}$ | 0.004 | 0.001 |
| $\beta_3$ (filt.) | 0.57 | 0.09 | $r_{12}$ | -0.60 | 0.09 |
| $\beta_4$ (filt.) | -0.06 | 0.02 | $r_{13}$ | -0.01 | 0.12 |
| $\beta_5$ (filt.) | 0.11 | 0.01 | $s_E$ | 2.28 | 0.51 |
| $\beta_6$ (filt.) | 0.39 | 0.06 | | | |
| $\beta_7$ (filt.) | -2.04 | 0.66 | | | |
| $\beta_8$ (filt.) | -0.29 | 0.06 | | | |
| $\phi_1$ | 0.77 | 0.08 | | | |
| $\phi_3$ | 0.62 | 0.12 | | | |
| $\phi_E$ | 0.39 | 0.12 | | | |

$$C^*_{t+1} = C^*_t + G\_ATM^*_{t+1} \qquad C_t = C^*_t + X_{1,t}$$

$$S\_LND^*_{t+1} = c_1 + \frac{\beta_1}{C_0} C^*_{t+1} + \beta_3 SOI_{t+1} \qquad S\_LND_t = S\_LND^*_t + X_{2,t}$$

$$S\_OCN^*_{t+1} = c_2 + \frac{\beta_2}{C_0} C^*_{t+1} + \beta_4 SOI_{t+1} \qquad S\_OCN_t = S\_OCN^*_t + X_{3,t}$$

$$E^*_{t+1} = E^*_t + \beta_5 \Delta GDP^{World}_{t+1} + \beta_8 I1991 + X^E_t$$

$$E_t = E^*_t + \beta_6 I1997$$

$$G\_ATM^*_{t+1} = E^*_{t+1} - S_{LND}{}^*_{t+1} - S_{OCN}{}^*_{t+1} + \beta_7 I1991$$

$$X_{1,t} = \phi_1 X_{1,t-1} + \eta_{1,t}$$

$$X_{2,t} = \eta_{2,t}$$

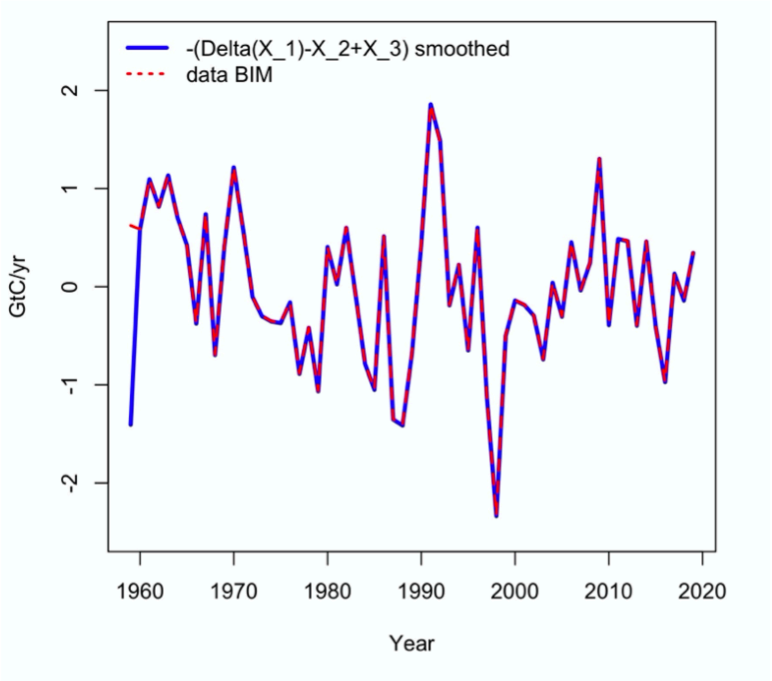
$$X_{3,t} = \phi_3 X_{3,t-1} + \eta_{3,t}$$

$$X^E_t = \phi_E X^E_{t-1} + \eta_{4,t}$$

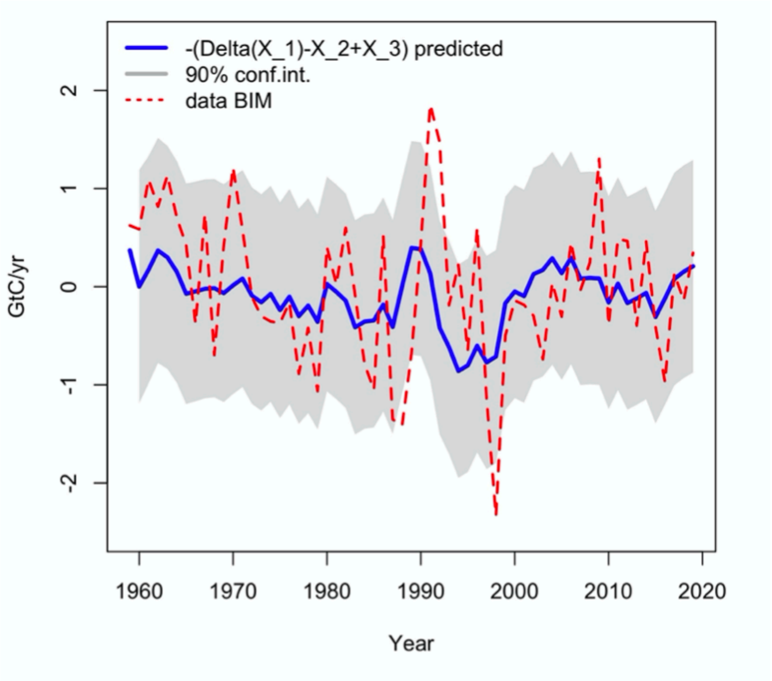
$$\begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \end{bmatrix} \sim N \left[ 0, \begin{bmatrix} \sigma_1^2 & r_{12}\sigma_1\sigma_2 & r_{13}\sigma_1\sigma_3 & 0 \\ r_{12}\sigma_1\sigma_2 & \sigma_2^2 & 0 & 0 \\ r_{13}\sigma_1\sigma_3 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix} \right]$$

$$\eta_{4,t} \sim N(0, \sigma_4^2\, s_E^2\, I_{t\geq 1996})$$

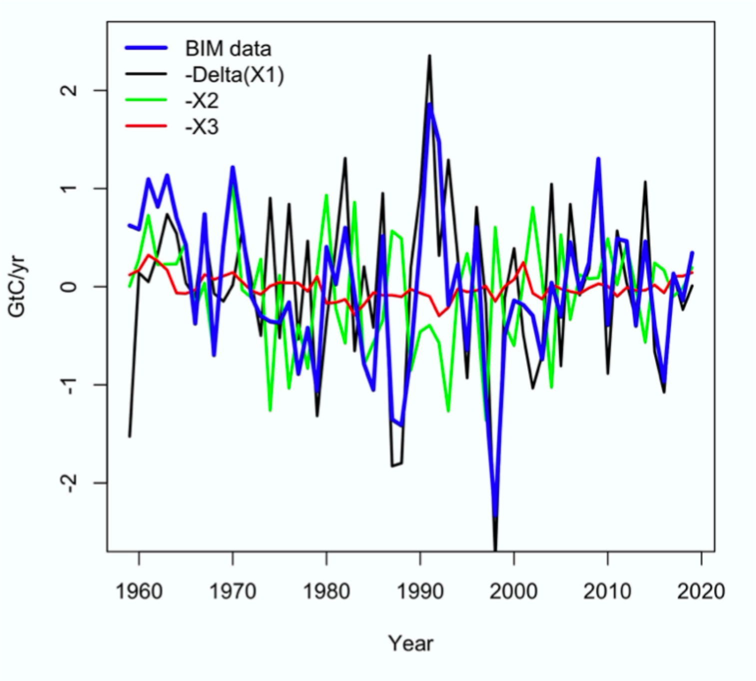
# BUDGET IMBALANCE




(A) Smoothed $-(\Delta X_1 + X_2 + X_3) + \beta_6 I1997 - \beta_7 I1991$

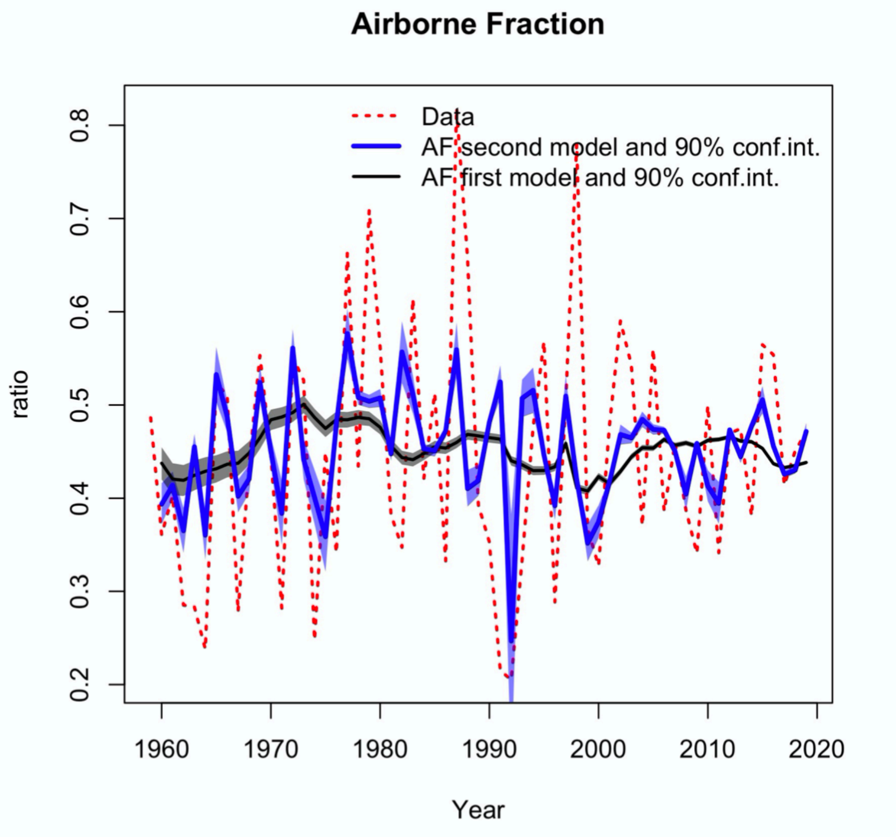


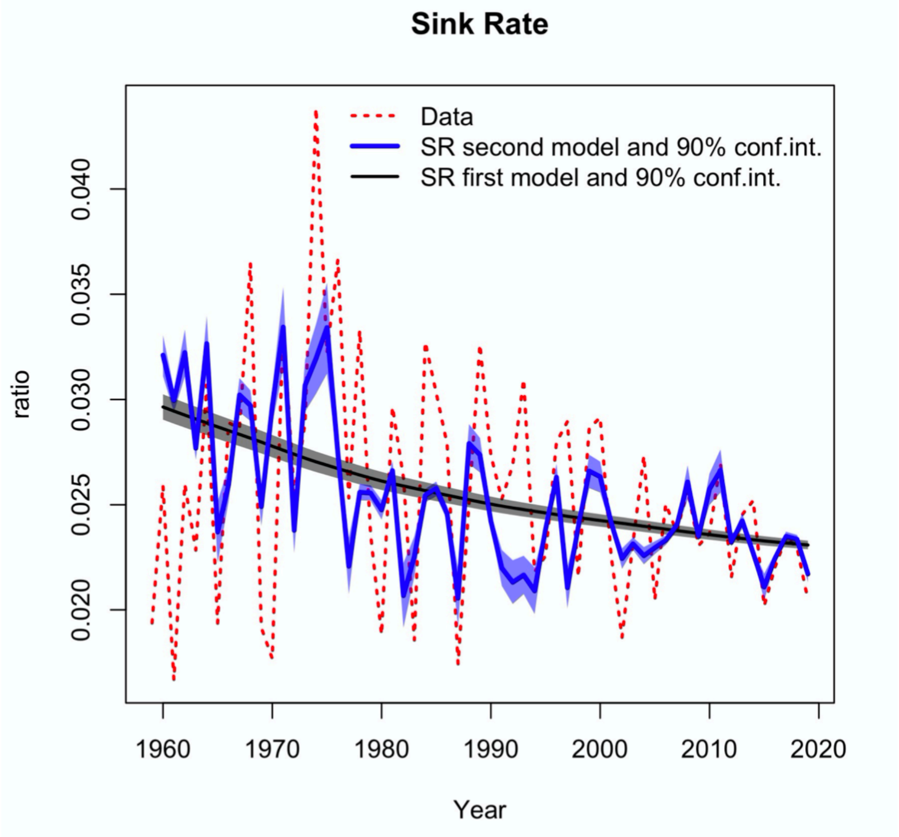

(B) One-year ahead predictions




(C) Components $-\Delta X_1$, $-X_2$, $-X_3$

# AIRBORNE FRACTION AND SINK RATE




$$AF = \frac{\Delta C}{E}$$

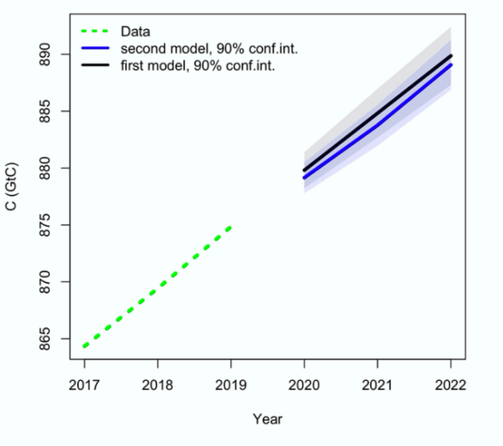
$$SR = \frac{S\_LND + S\_OCN}{C}$$

Raupach et al. (2014), Bennedsen, Hillebrand, Koopman (2019)

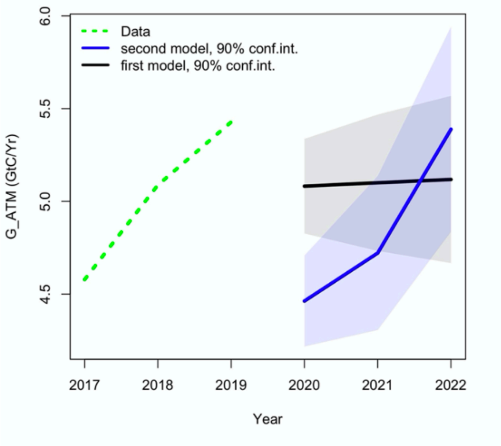
# NOW-/FORECASTS

Forecasts of World GDP growth from IMF and World Bank

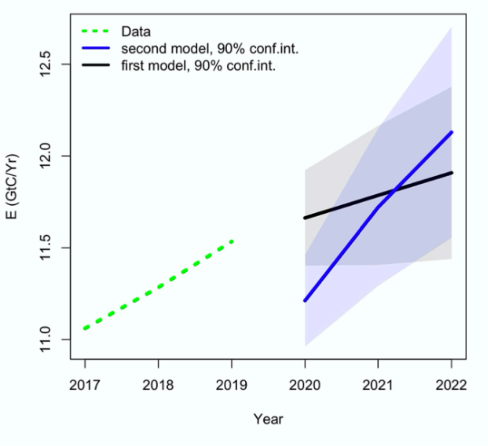
| | 2020 | 2021 | 2022 |
|---|---|---|---|
| IMF | -3.5% | 5.5% | 4.2% |
| World Bank | -4.3% | 4.0% | 3.8% |




(A) C - IMF

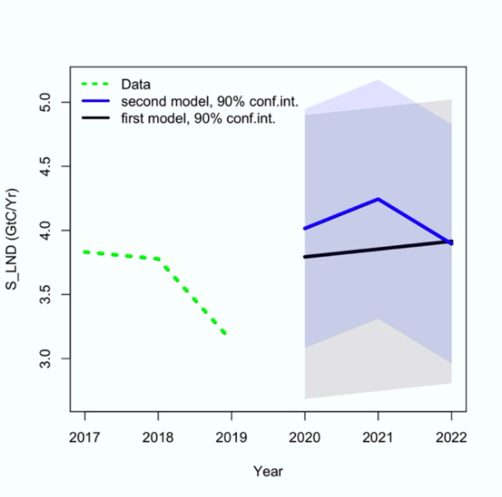



(B) G_ATM* - IMF

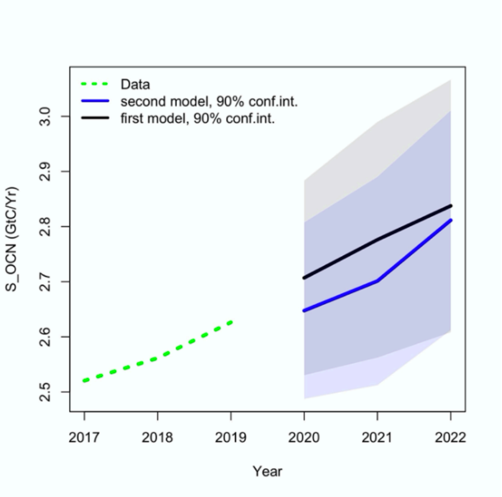



(C) E - IMF

Forecasts of SOI from forecast model of monthly SOI data 1866-2020, with trigonometric seasonal and second-order trigonometric cycle w/ period about 4 years




(D) S_LND - IMF




(E) S_OCN - IMF

# PROJECTIONS TO 2050

$$E^*_{t+1} = E^*_t + \beta_{5,t}\Delta GDP_{2010,t+1} + \beta_8 I1991 + X^E_t,$$

$$\beta_{5,t+1} = \beta_{5,t} + \eta_{5,t},$$

Scenarios:

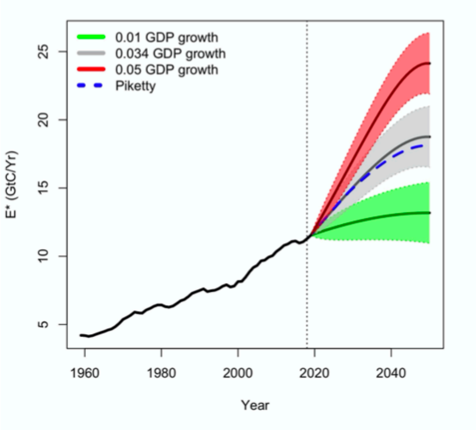
$\beta_{5,t}$ decreases linearly to 0 until 2050

No other abatement

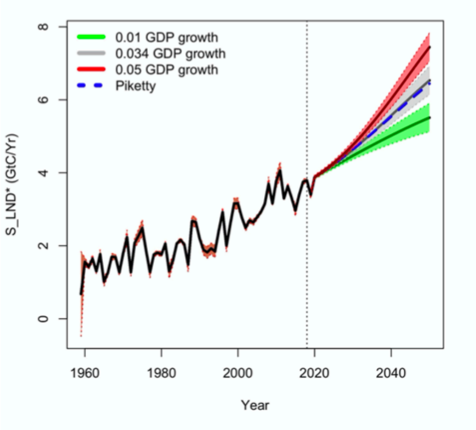
GDP

- 1% annual growth
- 3.4% annual growth
- 5% annual growth

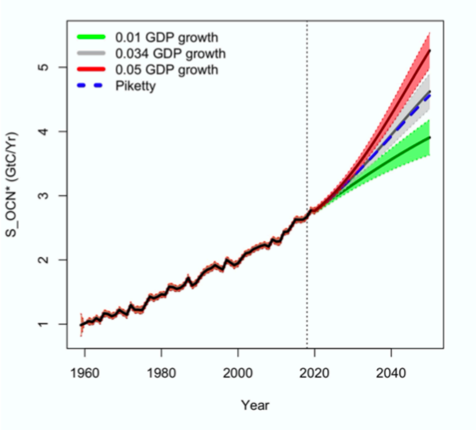
IPCC SR15 (2018)
UNFCCC NDC Synthesis Report (2021)



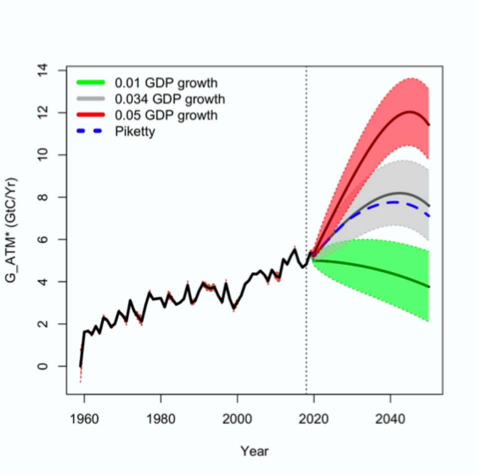

(A) E*



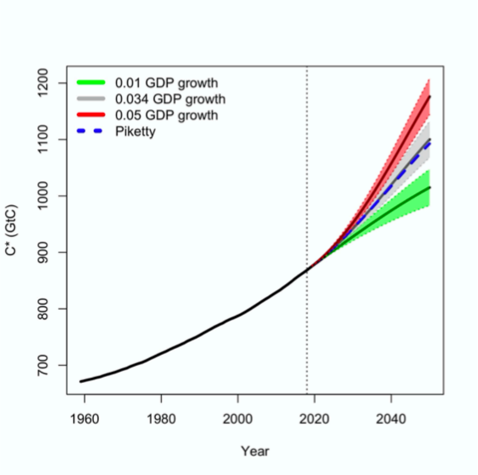

(B) S_LND*




(C) S_OCN*




(D) G_ATM*




(E) C*

# PROJECTIONS TO 2050

$$E_{t+1}^* = E_t^* + \beta_{9,t} + \beta_{5,t}\Delta GDP_{2010,t+1} + \beta_8 I1991 + X_t^E,$$

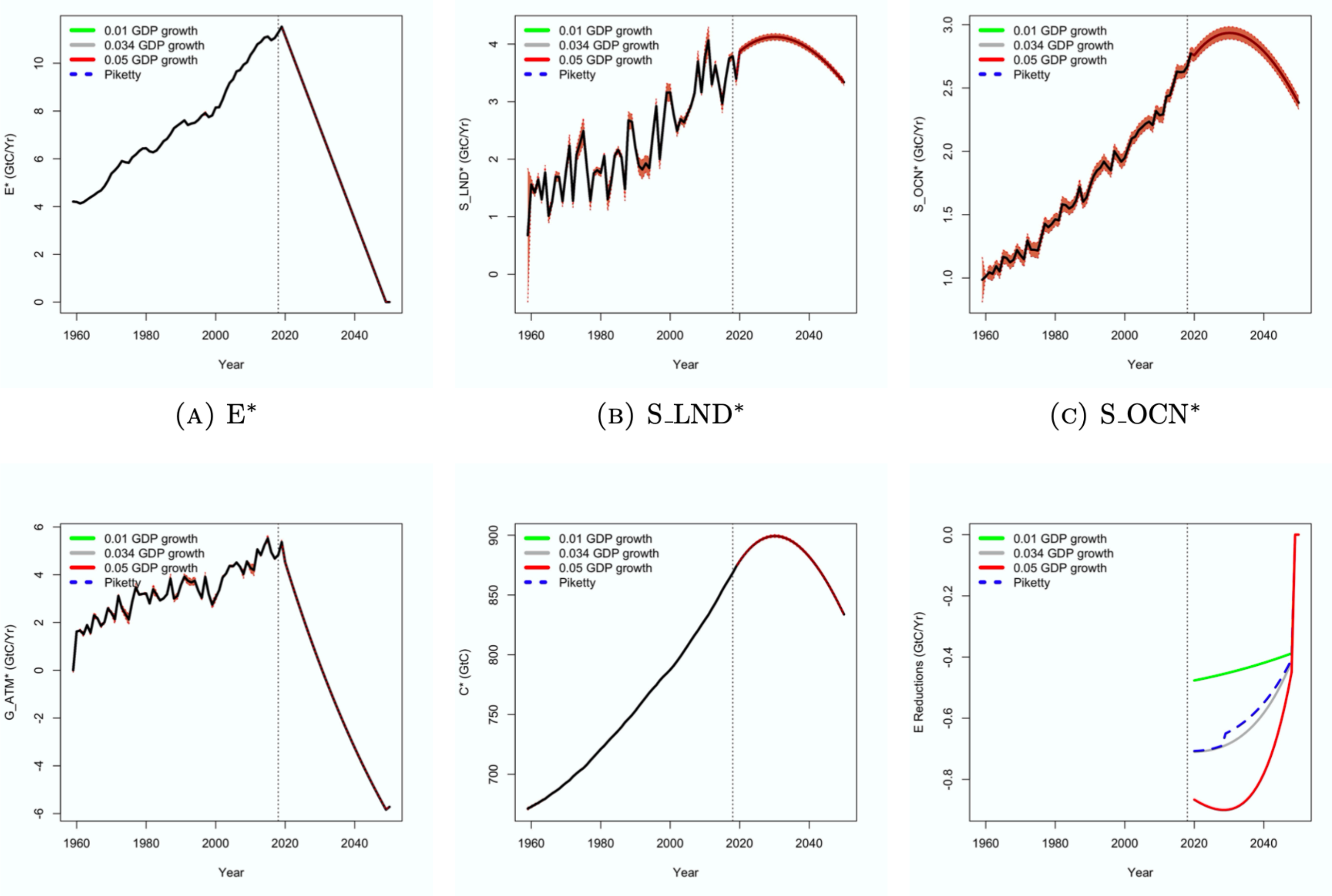
$$\beta_{9,t+1} = \beta_{9,t} + \eta_{6,t},$$

Scenarios:

$\beta_{5,t}$ decreases linearly to 0 until 2050

$E_t$ decreases linearly to 0 until 2050

GDP
- 1% annual growth
- 3.4% annual growth
- 5% annual growth




(A) E* (B) S_LND* (C) S_OCN*

(D) G_ATM* (E) C* (F) Emission reductions

# CONCLUSIONS

- Specification of state-space model for Global Carbon Budget
- World GDP as driver in emissions
- Sinks: linear in CO2 concentrations and in SOI
- CO2 concentrations are I(1) ranging on I(2)
- Model allows for study of key variables such as airborne fraction and sink rate, forecasting, projecting necessary emission reductions

Future directions

- Include ensemble members for S_LND and S_OCN
- Factor model for drift in emissions using large macroeconomic dataset
- Higher resolution on Global Carbon Cycle module (MAGICC)
- Connection to temperatures (Energy Balance Models)
- Cointegration analysis

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