

A STATISTICAL MODEL OF THE GLOBAL CARBON BUDGET

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SUPPLEMENTAL MATERIAL TO EGU VPICO TALK
12. APRIL 2021

TO FIND THE PAPER, SIMPLY GOOGLE "ERIC HILLEBRAND", FIND MY PERSONAL WEB SITE,
AND THE TAB "WORKING PAPERS".



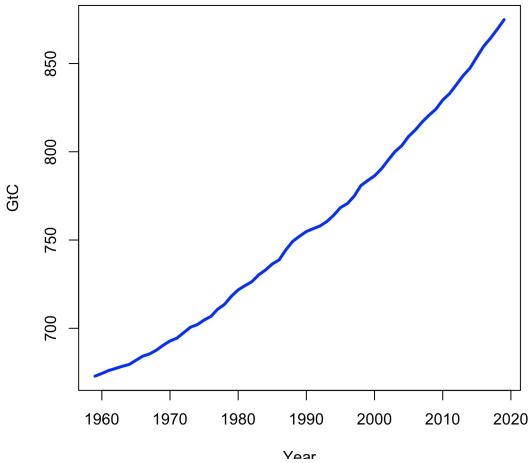
CREATES
INSTITUT FOR ØKONOMI
AARHUS UNIVERSITET

Econometric Models of Climate Change

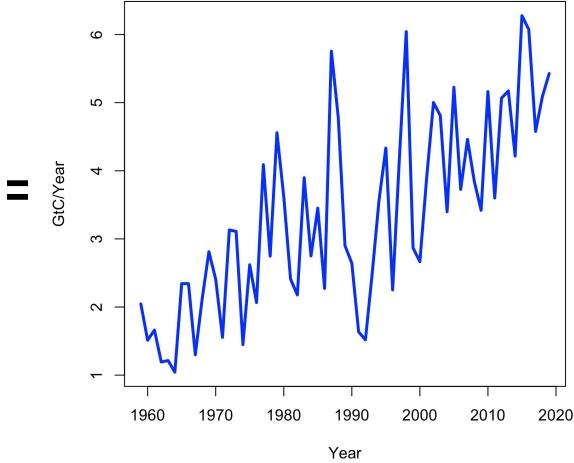
DATA



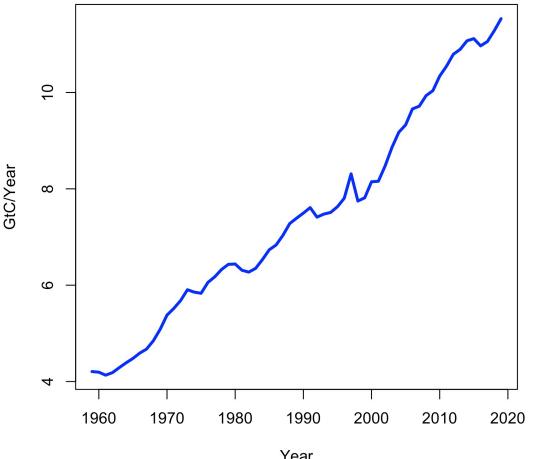
Atmospheric concentrations C



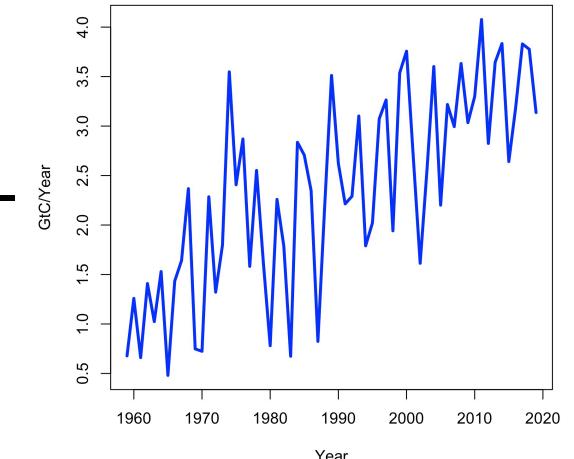
Delta C



Anthropogenic emissions E



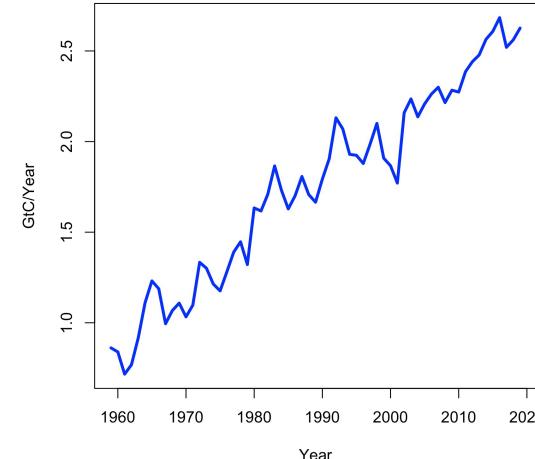
Land sink S_LND



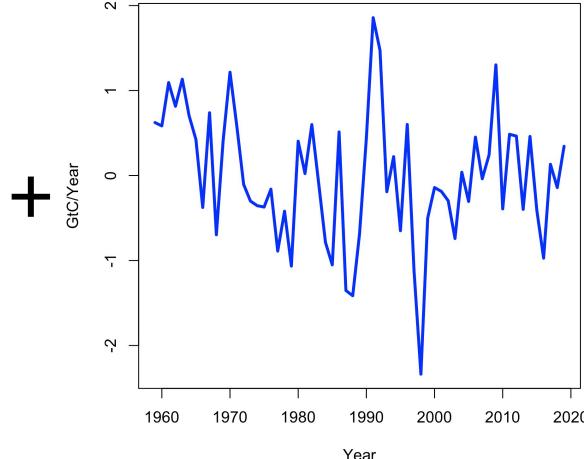
www.globalcarbonproject.org

Friedlingstein et al. (2020),
The global carbon budget 2020,
Earth System Science Data 12,
3269-3340

Ocean sink S_OCN



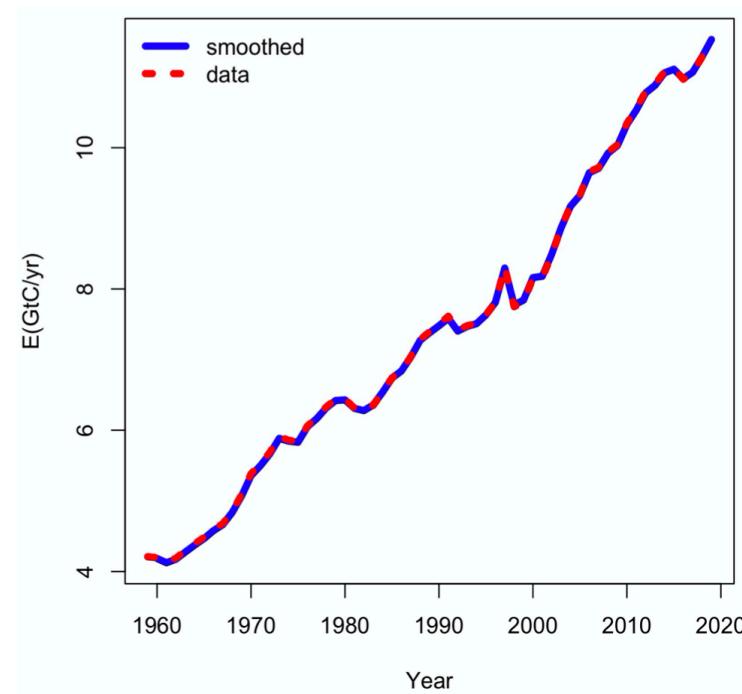
Budget imbalance BIM



OUTLINE

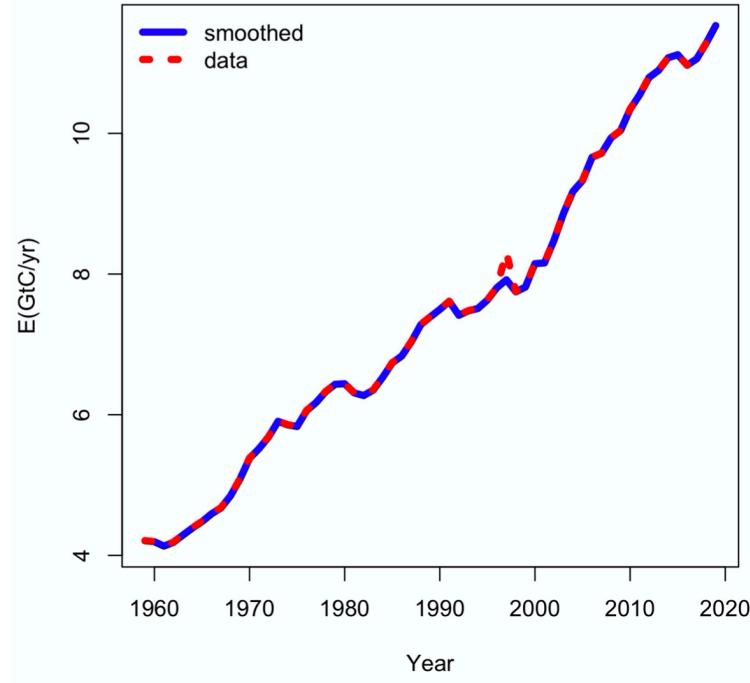
1. Models for the components of the global carbon budget
2. The dynamics of atmospheric concentrations C
3. The system model
4. Estimation: Residual diagnostics, Residual processes, Parameter estimates
5. Discussion: Budget imbalance, airborne fraction, sink rate
6. Validation
7. Nowcasts and forecasts
8. Projections: Long-term scenarios until 2100
9. Conclusions

ANTHROPOGENIC EMISSIONS



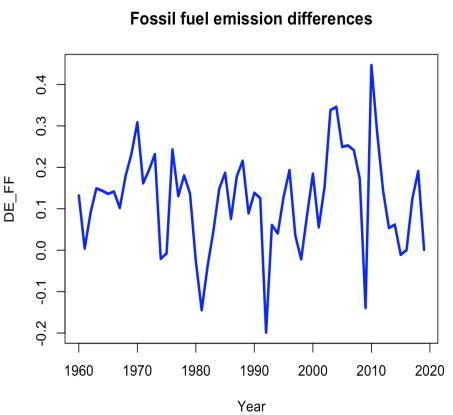
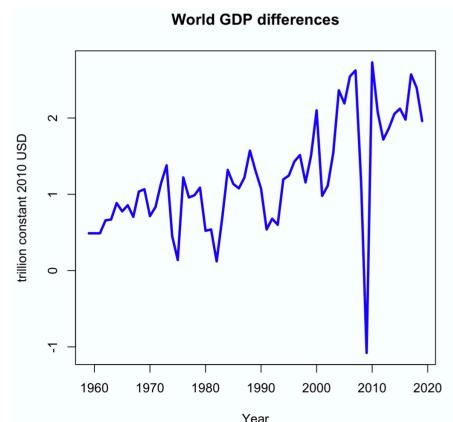
$$\Delta E_t = \frac{0.14}{(0.02)} + X_t^E,$$

$$X_{t+1}^E = \frac{0.06}{(0.26)} X_t^E + \eta_{5,t}, \eta_{5,t} \sim N(0, \frac{0.024}{(0.032)})$$



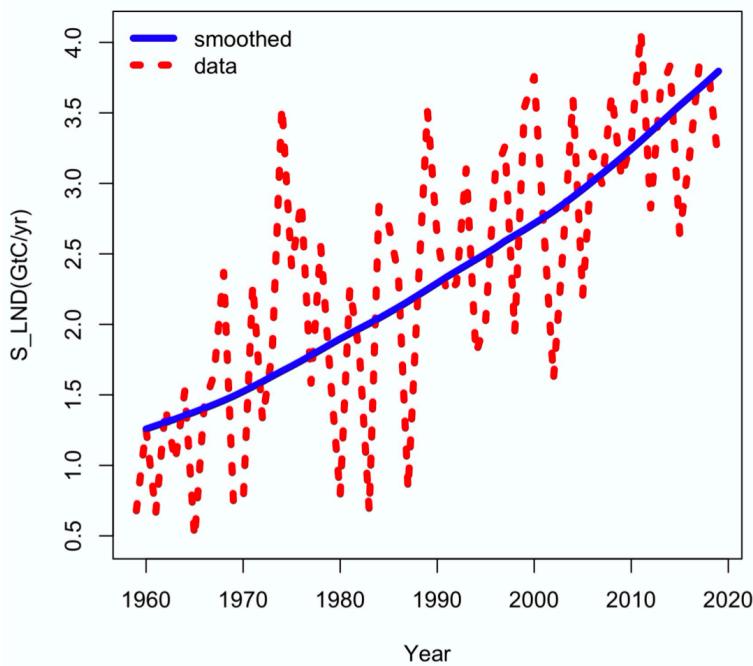
$$\Delta E_t = \frac{0.11}{(0.01)} \Delta GDP_t - \frac{0.29}{(0.06)} I_{1991} + X_t^E,$$

$$X_{t+1}^E = \frac{0.39}{(0.12)} X_t^E + \eta_{4,t}, \eta_{4,t} \sim N(0, \frac{0.004}{(0.001)})$$

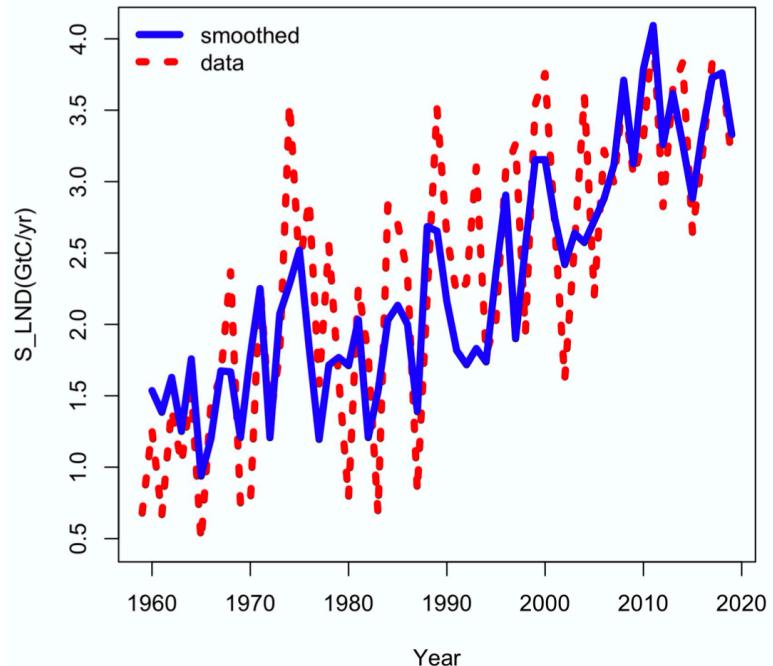


Friedlingstein et al. (2020)
Bennedsen, Hillebrand,
Koopman (2021)

LAND SINK

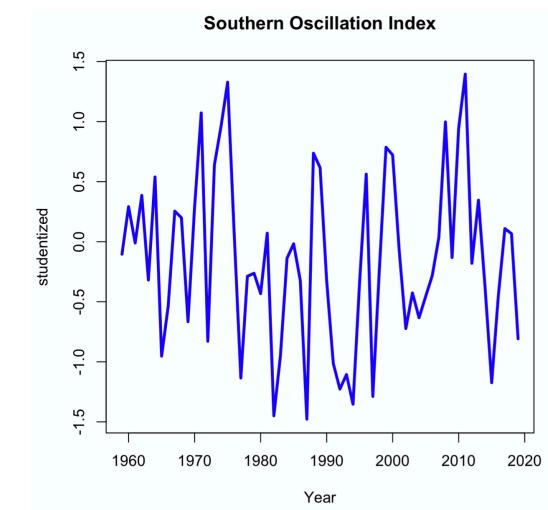


$$S_{LND_t} = \frac{-7.22}{(0.05)} + \frac{7.48}{(0.97)} \frac{C_t}{C_0}$$



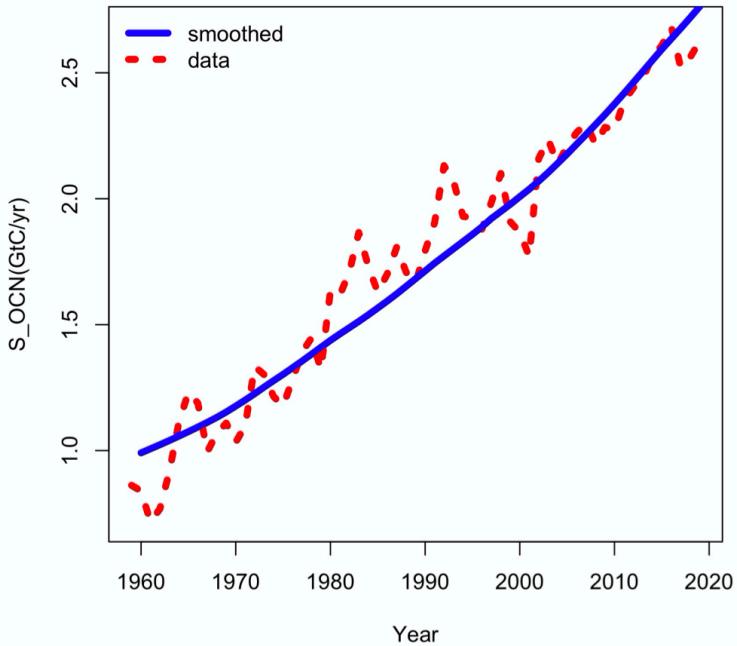
$$S_{LND_t} = \frac{-6.70}{(0.04)} + \frac{7.12}{(0.49)} \frac{C_t}{C_0} + \frac{0.57}{(0.09)} SOI_t$$

Raupach et al. (2014), Raupach (2013), Gloor et al. (2010), Canadell et al. (2007)

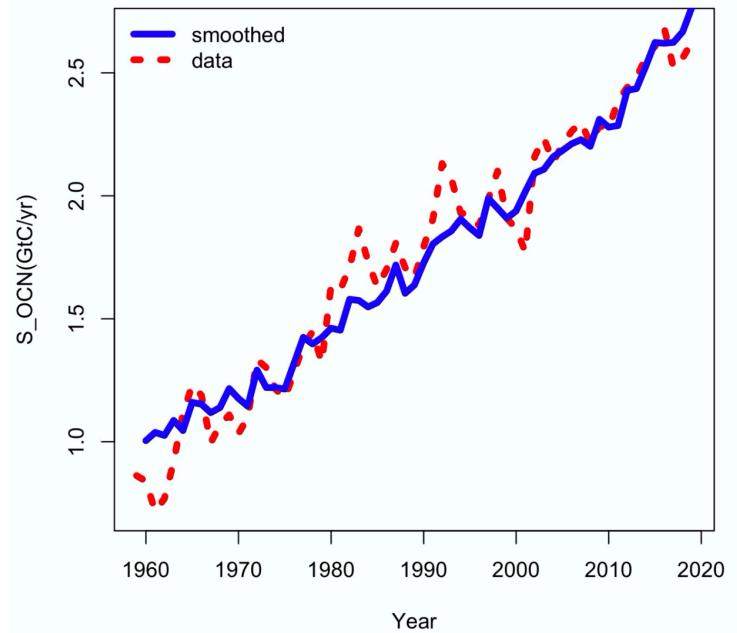


"Moisture sensitivities of both productivity and decomposition are important for capturing the response of the net flux to such [La Niña] events." Haverd et al. (2018, p. 3013)

OCEAN SINK

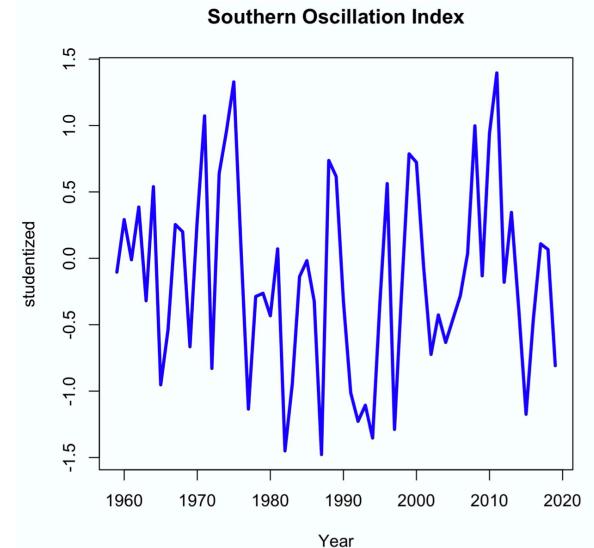


$$S_{OCN_t} = \frac{-4.93}{(0.04)} + \frac{5.22}{(0.41)} \frac{C_t}{C_0}$$



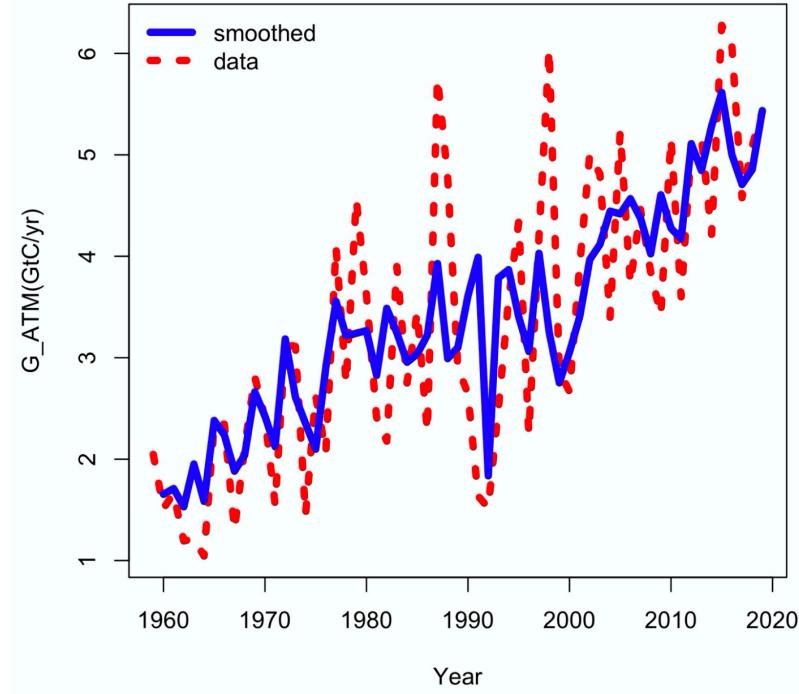
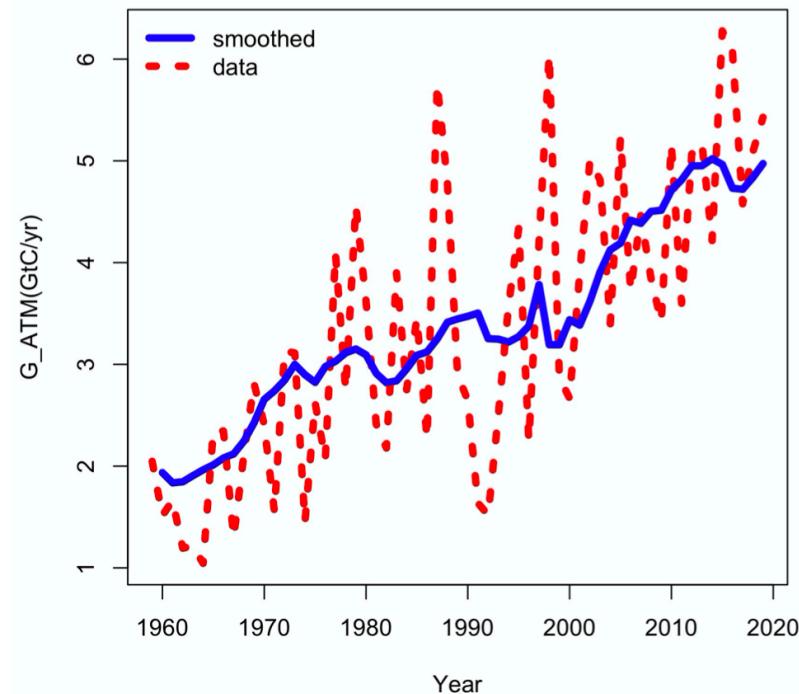
$$S_{OCN_t} = \frac{-4.63}{(0.03)} + \frac{4.99}{(0.29)} \frac{C_t}{C_0} - \frac{0.06}{(0.02)} SOI_t$$

Raupach et al. (2014), Raupach (2013), Gloor et al. (2010), Canadell et al. (2007)



"When the winds are strongest during the cold cycle of ENSO deep upwelling occurs and [ocean CO₂ partial pressure] values are at a maximum." Feely et al. (1999, p. 599)

$\Delta C = \text{GROWTH IN ATM. CONCENTRATIONS}$



$$\Delta C_t = E_t - S_{LND}t - S_{OCN}t$$

THE DYNAMICS OF C

$$\Delta C_t = E_t - S_LND_t - S_OCN_t$$

$$= E_t - c_1 - c_2 - \beta_1^* C_t - \beta_2^* C_t + \varepsilon_t, \quad \varepsilon_t \sim I(0)$$

$$(1 + \beta_1^* + \beta_2^*)C_t - C_{t-1} = c + dt + x_t + \varepsilon_t$$

$$(1 - qL)C_t = qc + qdt + qx_t + q\varepsilon_t$$

$$\beta_i^* = \frac{\beta_i}{C_0} \approx 0.01$$

$$x_t = \sum_{i=1}^t X_i^E$$

$$q := \frac{1}{1 + \beta_1^* + \beta_2^*} \approx \frac{1}{1.02}$$

$$c = E_0 - c_1 - c_2$$

Three insights:

$$\begin{aligned} C_t &= q^t \left[C_0 - \frac{qc}{1-q} + \frac{dq^2}{(1-q)^2} \right] + \left[\frac{qc}{1-q} - \frac{dq^2}{(1-q)^2} \right] + \frac{dq}{1-q} t + \sum_{j=0}^{t-1} q^{j+1} x_{t-j} + \sum_{j=0}^{t-1} q^{j+1} \varepsilon_{t-j} \\ &= o(1) + O(1) + O(t) + I(1) + I(0) = O(t) + I(1) \end{aligned}$$

Thus,

$$\Delta C_t = I(0)$$

But,

$$(1 - qL)(1 - L)C_t = qd + q\Delta x_t + q\Delta \varepsilon_t = I(0)$$

THE SYSTEM MODEL

State equation Model 1

$$S_LND_{t+1}^* = c_1 + \frac{\beta_1}{C_0} C_{t+1}^*$$

$$S_OCN_{t+1}^* = c_2 + \frac{\beta_2}{C_0} C_{t+1}^*$$

$$E_{t+1}^* = E_t^* + d + X_t^E$$

$$C_{t+1}^* = C_t^* + G_ATM_{t+1}^*$$

$$G_ATM_{t+1}^* = E_{t+1}^* - S_{LND_{t+1}}^* - S_{OCN_{t+1}}^* + \beta_7 I1991$$

$$X_{1,t} = \phi_1 X_{1,t-1} + \eta_{1,t}$$

$$X_{2,t} = \eta_{2,t}$$

$$X_{3,t} = \phi_3 X_{3,t-1} + \eta_{3,t}$$

$$X_t^E = \phi_E X_{t-1}^E + \eta_{4,t}$$

State equation Model 2

$$S_LND_{t+1}^* = c_1 + \frac{\beta_1}{C_0} C_{t+1}^* + \beta_3 SOI_{t+1}$$

$$S_OCN_{t+1}^* = c_2 + \frac{\beta_2}{C_0} C_{t+1}^* + \beta_4 SOI_{t+1}$$

$$E_{t+1}^* = E_t^* + \beta_5 \Delta GDP_{t+1}^{World} + \beta_8 I1991 + X_t^E$$

Measurement equation

$$C_t = C_t^* + X_{1,t}$$

$$S_LND_t = S_LND_t^* + X_{2,t}$$

$$S_OCN_t = S_OCN_t^* + X_{3,t}$$

$$E_t = E_t^* + \beta_6 I1997$$

$$\begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \end{bmatrix} \sim N \left(0, \begin{bmatrix} \sigma_1^2 & r_{12}\sigma_1\sigma_2 & r_{13}\sigma_1\sigma_3 & 0 \\ r_{12}\sigma_1\sigma_2 & \sigma_2^2 & 0 & 0 \\ r_{13}\sigma_1\sigma_3 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix} \right)$$

$$\eta_{4,t} \sim N(0, \sigma_4^2 s_E^2 I_{t \geq 1996})$$

RESIDUAL DIAGNOSTICS

Model 1

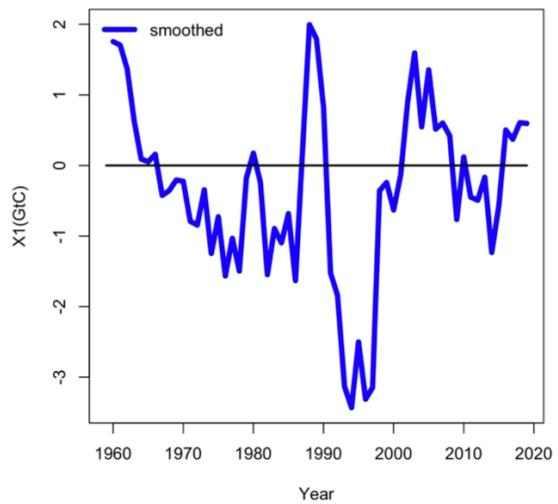
Residual	mean	std dev	skew	kurt	JB	LB(1)	DW
C	-0.183	0.964	0.205	2.765	0.568	1.607	1.653
E	-0.033	1.009	-1.448	8.506	98.38***	0.073	2.060
S_LND	0.061	1.022	0.047	2.615	0.399	0.054	2.037
S_OCN	0.093	0.978	0.377	2.988	1.447	0.116	1.908

Model 2

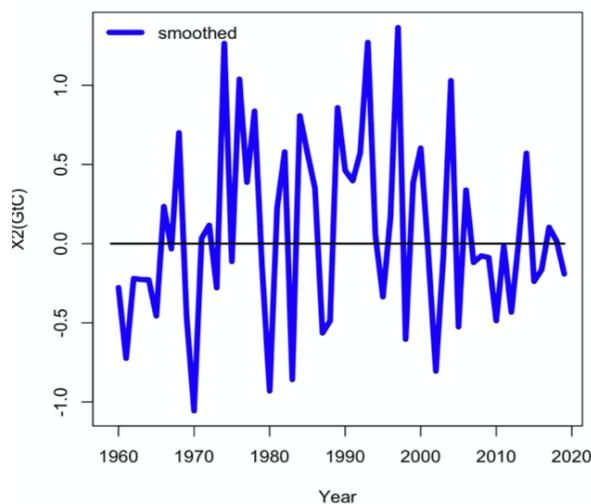
Residual	mean	std dev	skew	kurt	JB	LB(1)	DW
C	-0.140	0.979	0.167	3.272	0.471	0.350	1.840
E	0.004	0.983	-0.269	3.095	0.759	0.062	1.928
S_LND	0.164	0.995	0.285	2.530	1.387	0.223	2.111
S_OCN	0.120	0.951	0.440	3.394	2.360	0.648	2.191

RESIDUAL PROCESSES

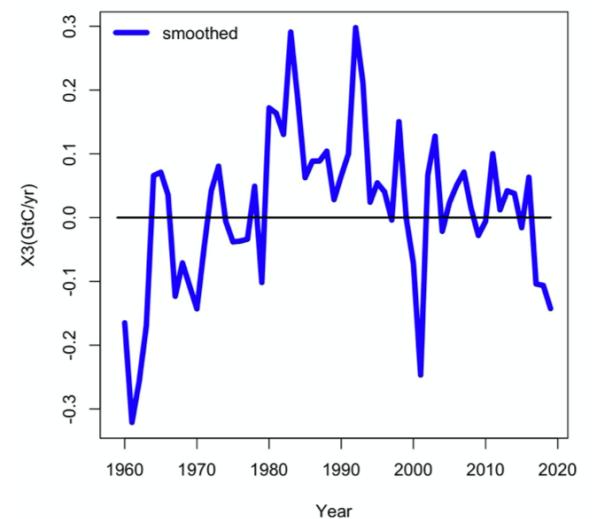
$X_{1,t}$ in C_t



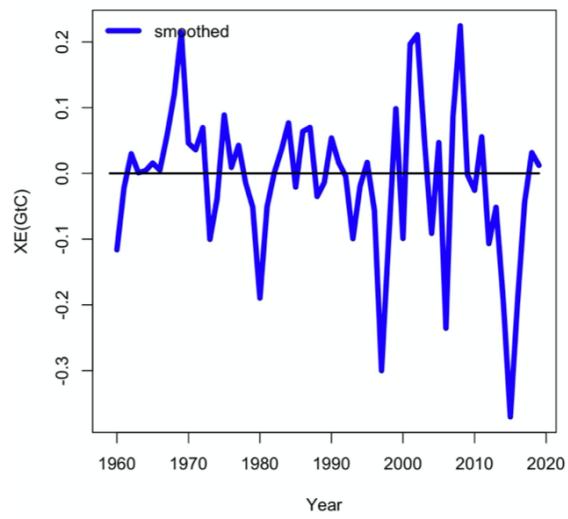
$X_{2,t}$ in S_LND_t



$X_{3,t}$ in S_OCN_t



X_t^E in E_t^*



PARAMETER ESTIMATES

Model 2

	Coefficients		Variances		
	estimate	std err	estimate	std err	
c_1 (filt.)	-6.70	0.04	$\sigma_{\eta_1}^2$	0.66	0.13
c_2 (filt.)	-4.63	0.03	$\sigma_{\eta_2}^2$	0.32	0.06
β_1	7.12	0.49	$\sigma_{\eta_3}^2$	0.01	0.002
β_2	4.99	0.29	$\sigma_{\eta_4}^2$	0.004	0.001
β_3 (filt.)	0.57	0.09	r_{12}	-0.60	0.09
β_4 (filt.)	-0.06	0.02	r_{13}	-0.01	0.12
β_5 (filt.)	0.11	0.01	s_E	2.28	0.51
β_6 (filt.)	0.39	0.06			
β_7 (filt.)	-2.04	0.66			
β_8 (filt.)	-0.29	0.06			
ϕ_1	0.77	0.08			
ϕ_3	0.62	0.12			
ϕ_E	0.39	0.12			

$$C_{t+1}^* = C_t^* + G_ATM_{t+1}^*$$

$$S_LND_{t+1}^* = c_1 + \frac{\beta_1}{C_0} C_{t+1}^* + \beta_3 SOI_{t+1}$$

$$S_OCN_{t+1}^* = c_2 + \frac{\beta_2}{C_0} C_{t+1}^* + \beta_4 SOI_{t+1}$$

$$S_OCN_t = S_OCN_t^* + X_{3,t}$$

$$E_{t+1}^* = E_t^* + \beta_5 \Delta GDP_{t+1}^{World} + \beta_8 I1991 + X_t^E$$

$$E_t = E_t^* + \beta_6 I1997$$

$$G_ATM_{t+1}^* = E_{t+1}^* - S_LND_{t+1}^* - S_OCN_{t+1}^* + \beta_7 I1991$$

$$X_{1,t} = \phi_1 X_{1,t-1} + \eta_{1,t}$$

$$X_{2,t} = \eta_{2,t}$$

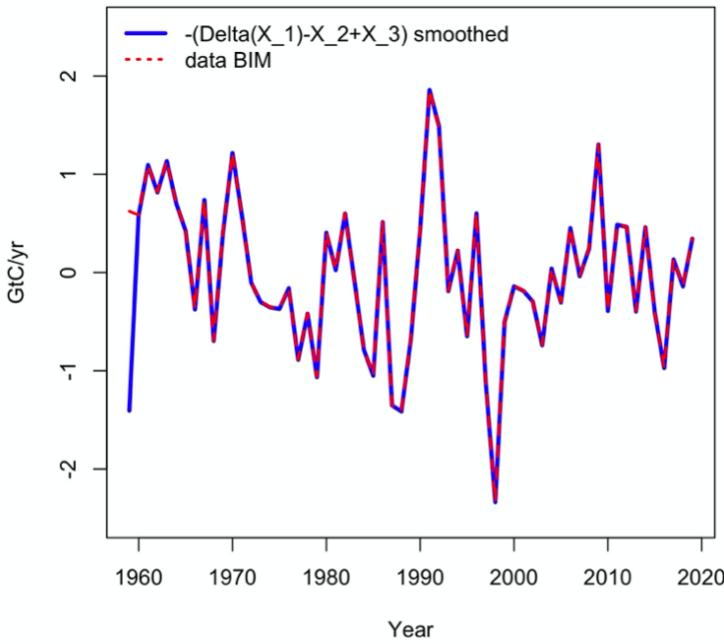
$$X_{3,t} = \phi_3 X_{3,t-1} + \eta_{3,t}$$

$$X_t^E = \phi_E X_{t-1}^E + \eta_{4,t}$$

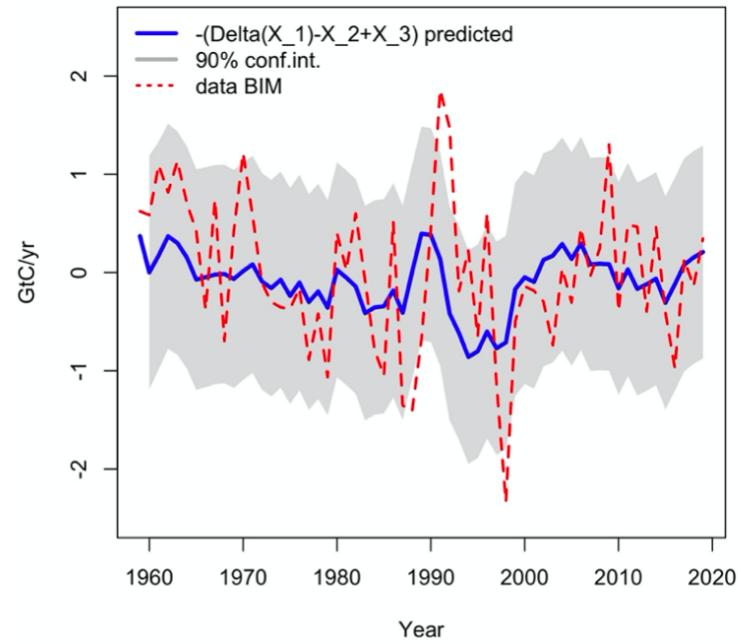
$$\begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \end{bmatrix} \sim N \left(0, \begin{bmatrix} \sigma_1^2 & r_{12}\sigma_1\sigma_2 & r_{13}\sigma_1\sigma_3 & 0 \\ r_{12}\sigma_1\sigma_2 & \sigma_2^2 & 0 & 0 \\ r_{13}\sigma_1\sigma_3 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix} \right)$$

$$\eta_{4,t} \sim N(0, \sigma_4^2 s_E^2 I_{t \geq 1996})$$

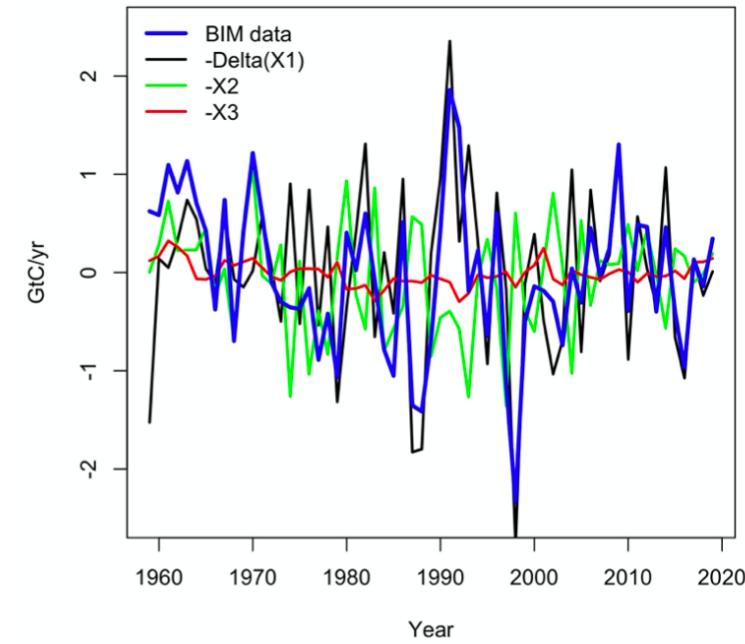
BUDGET IMBALANCE



(A) Smoothed
 $-(\Delta X_1 + X_2 + X_3) + \beta_6 I(1997) - \beta_7 I(1991)$



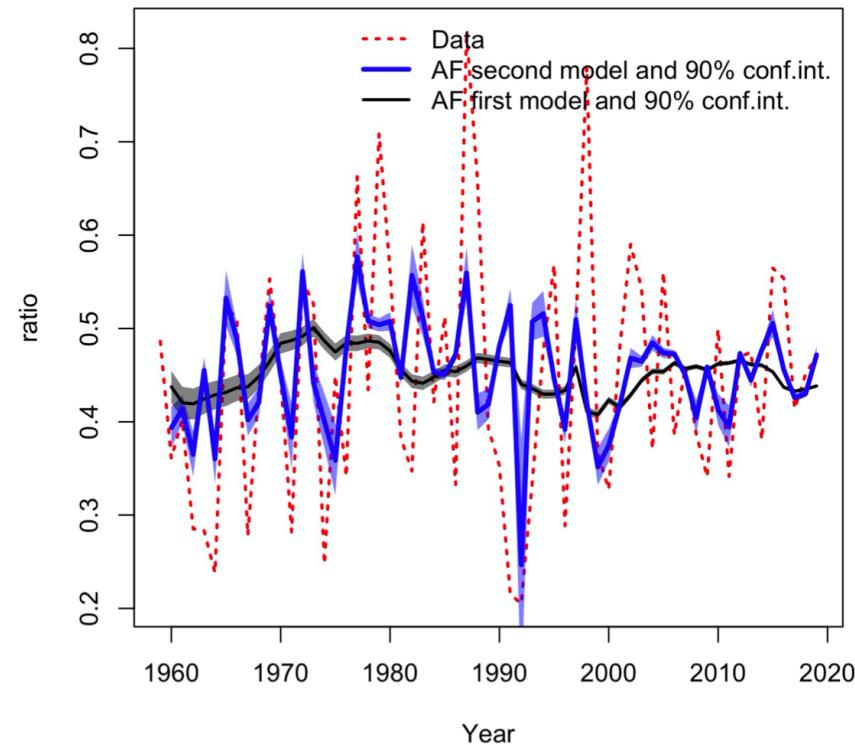
(B) One-year ahead
predictions



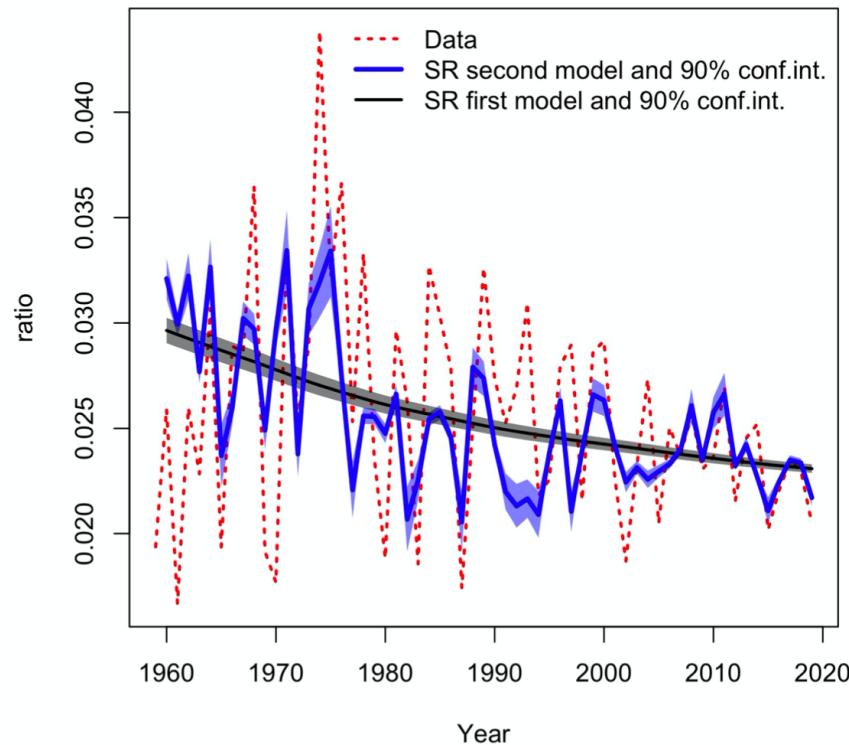
(C) Components
 $-\Delta X_1, -X_2, -X_3$

AIRBORNE FRACTION AND SINK RATE

Airborne Fraction



Sink Rate



$$AF = \frac{\Delta C}{E}$$

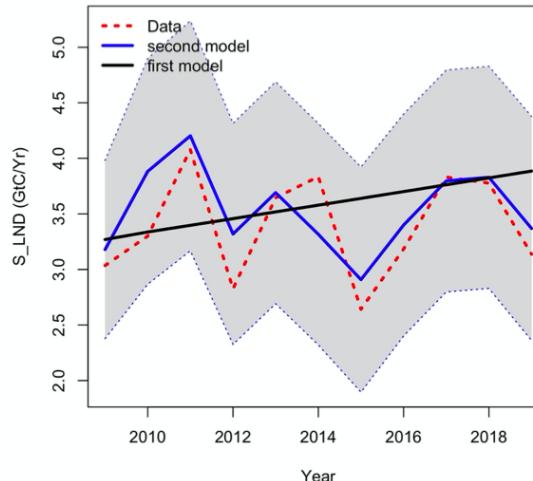
$$SR = \frac{S_{LND} + S_{OCN}}{C}$$

Raupach et al. (2014), Bennedsen, Hillebrand, Koopman (2019)

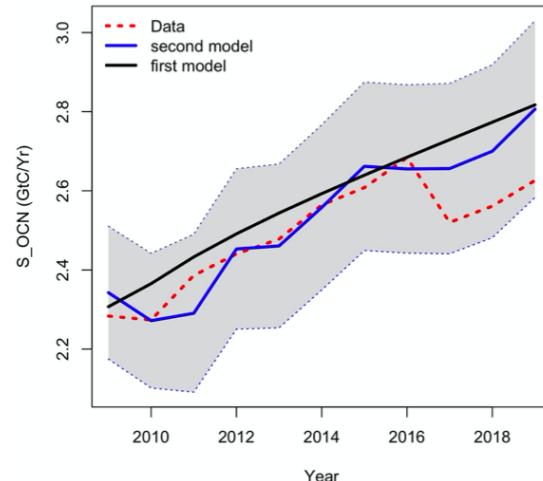
VALIDATION

Estimation of models on subsample
1959-2009

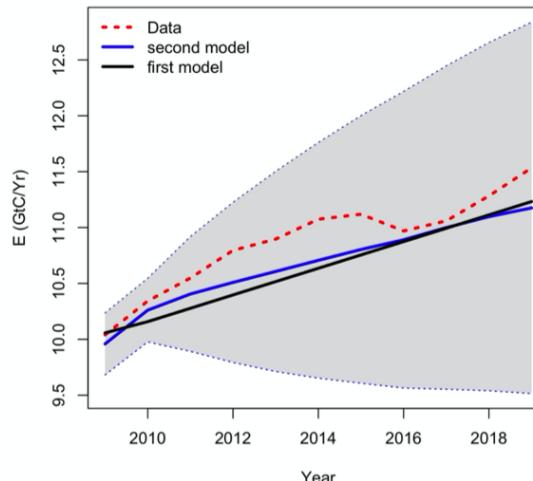
Iterating the models on subsample
2010-2019
given observations on World GDP and SOI



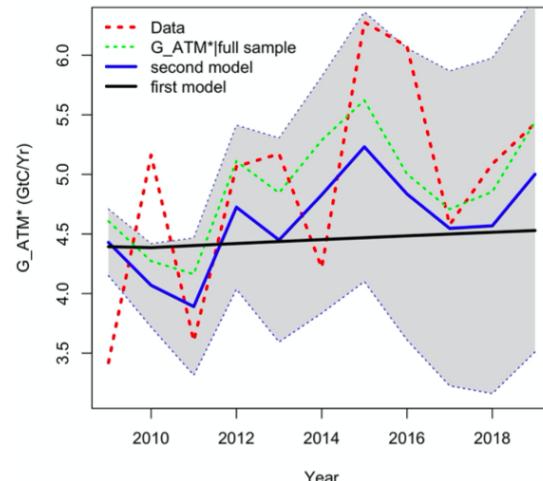
(A) S_LND



(B) S_OCN



(C) E

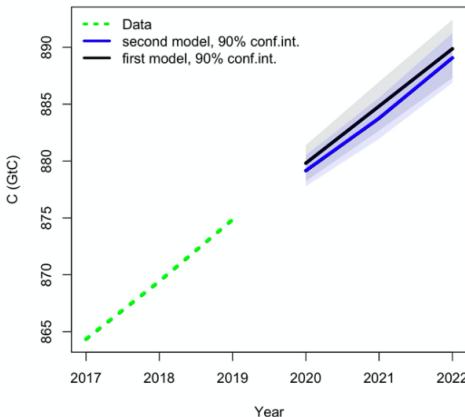


(D) G_ATM*

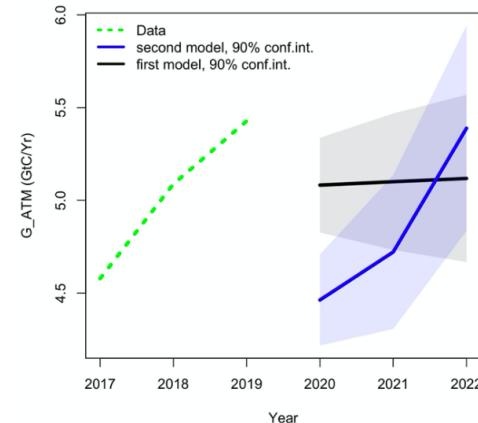
NOW-/FORECASTS

Forecasts of World GDP growth from IMF and World Bank

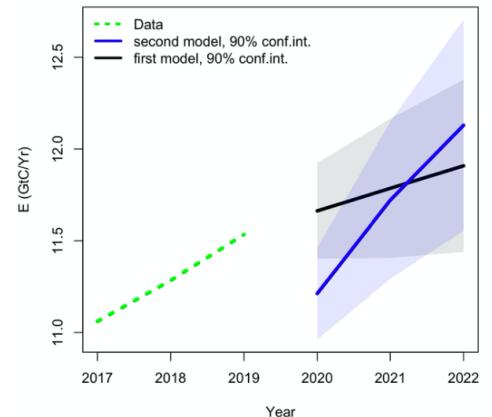
	2020	2021	2022
IMF	-3.5%	5.5%	4.2%
World Bank	-4.3%	4.0%	3.8%



(A) C - IMF

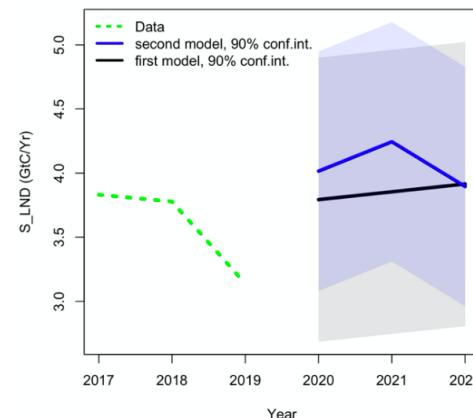


(B) G_ATM* - IMF

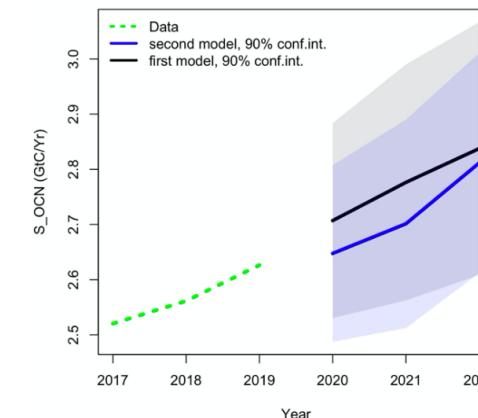


(C) E - IMF

Forecasts of SOI from forecast model of monthly SOI data 1866-2020, with trigonometric seasonal and second-order trigonometric cycle w/ period about 4 years



(D) S_LND - IMF



(E) S_OCN - IMF

PROJECTIONS TO 2100

Scenarios:

GDP

- 1% growth
- 3.4% growth

IPCC SR15 (2018)

UNFCCC NDC Synthesis Report (2021)

1.5° warming

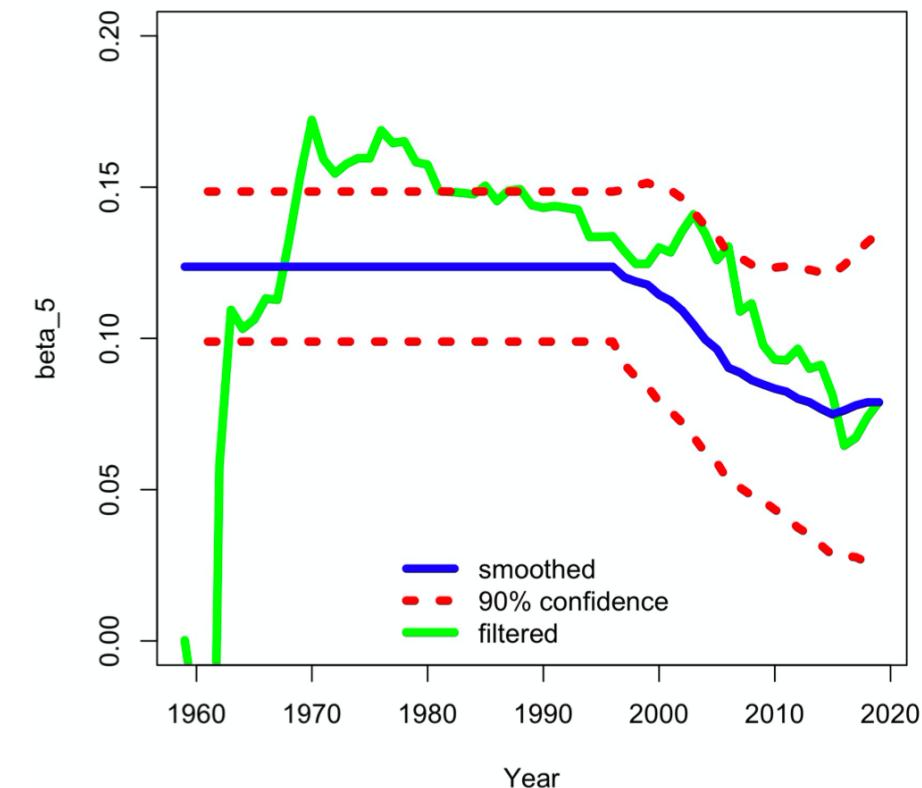
- 55% of 2010 emissions by 2030
- Net-0 by 2050
- $\beta_{5,t}$ decreasing linearly to 0 by 2050

<2° warming

- 75% of 2010 emissions by 2030
- Net-0 by 2070
- $\beta_{5,t}$ decreasing linearly to 0 by 2070

$$E_{t+1}^* = E_t^* + \beta_{5,t} \Delta GDP_{2010,t+1} + \beta_8 I1991 + X_t^E,$$

$$\beta_{5,t+1} = \beta_{5,t} + \eta_{5,t},$$



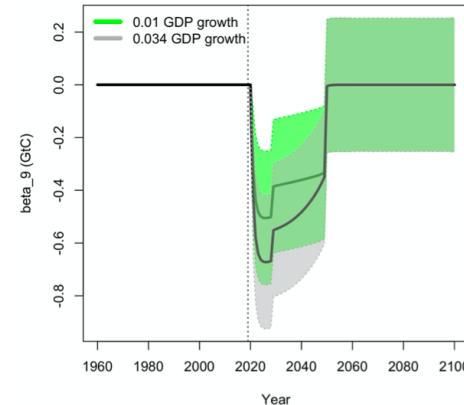
PROJECTIONS TO 2100

$$E_{t+1}^* = E_t^* + \beta_{9,t} + \beta_{5,t} \Delta GDP_{2010,t+1} + \beta_8 I1991 + X_t^E,$$

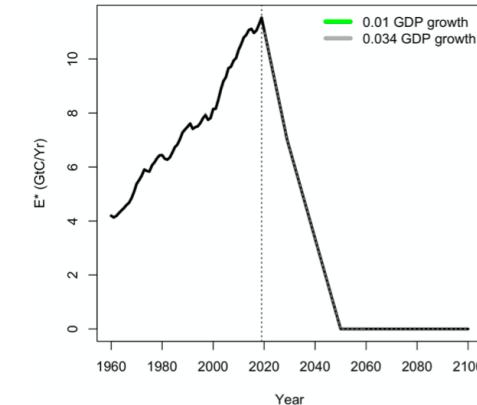
$$\beta_{9,t+1} = \beta_{9,t} + \eta_{6,t},$$

1.5° warming

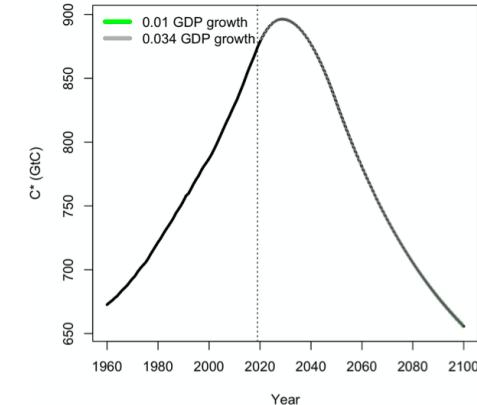
- 55% of 2010 emissions by 2030
- Net-0 by 2050
- $\beta_{5,t}$ decreasing linearly to 0 by 2050



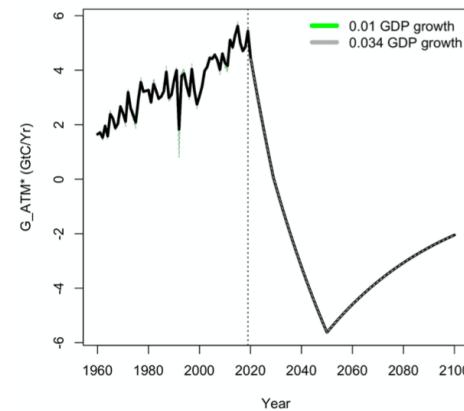
(A) Emission reductions $\beta_{9,t}$



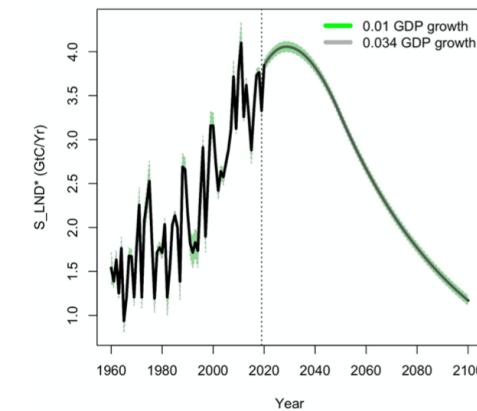
(B) E^*



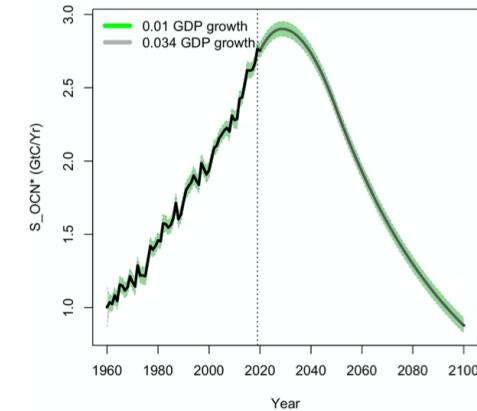
(C) C^*



(D) G_{ATM}^*



(E) S_{LND}^*

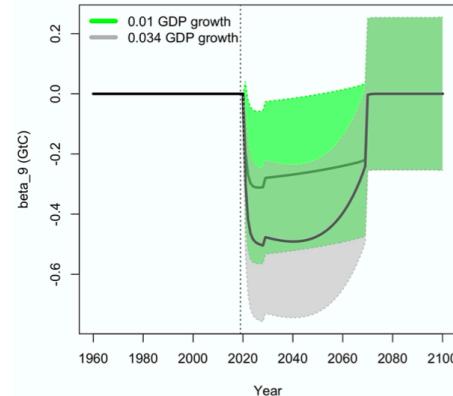


(F) S_{OCN}^*

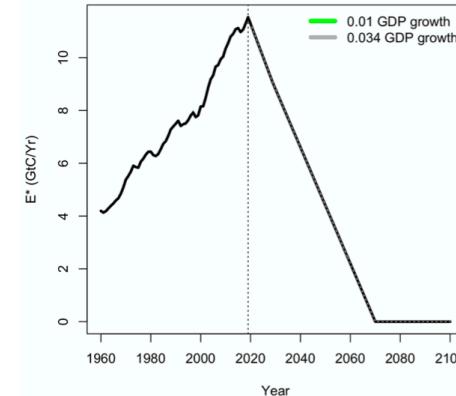
PROJECTIONS TO 2100

<2° warming

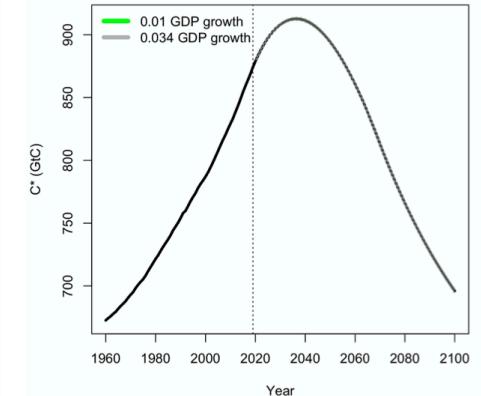
- 75% of 2010 emissions by 2030
- Net-0 by 2070
- $\beta_{5,t}$ decreasing linearly to 0 by 2070



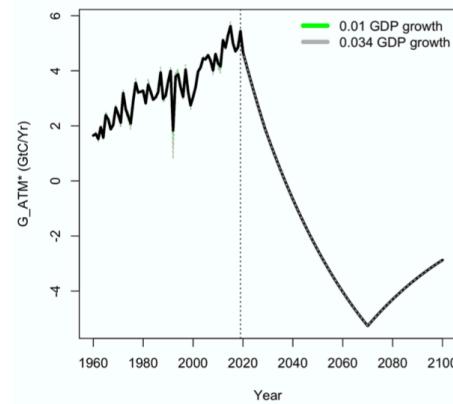
(A) Emission reductions $\beta_{9,t}$



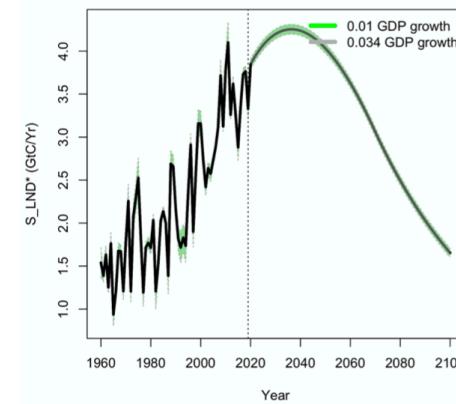
(B) E^*



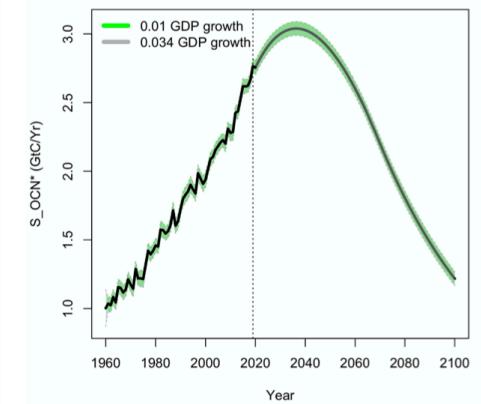
(C) C^*



(D) G_{ATM}^*



(E) S_{LND}^*



(F) S_{OCN}^*

$$E_{t+1}^* = E_t^* + \beta_{9,t} + \beta_{5,t} \Delta GDP_{2010,t+1} + \beta_8 I1991 + X_t^E,$$

$$\beta_{9,t+1} = \beta_{9,t} + \eta_{6,t},$$

CONCLUSIONS

- Specification of state-space model for Global Carbon Budget
- World GDP as driver in emissions
- Sinks: linear in CO₂ concentrations and in SOI
- CO₂ concentrations are I(1) ranging on I(2)
- Model allows for study of key variables such as airborne fraction and sink rate, forecasting, projecting necessary emission reductions

Future directions

- Include ensemble members for S_LND and S_OCN
- Factor model for drift in emissions using large macroeconomic dataset
- Higher resolution on Global Carbon Cycle module (MAGICC)
- Connection to temperatures (Energy Balance Models)
- Cointegration analysis



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