

A STATISTICAL MODEL OF THE GLOBAL CARBON BUDGET

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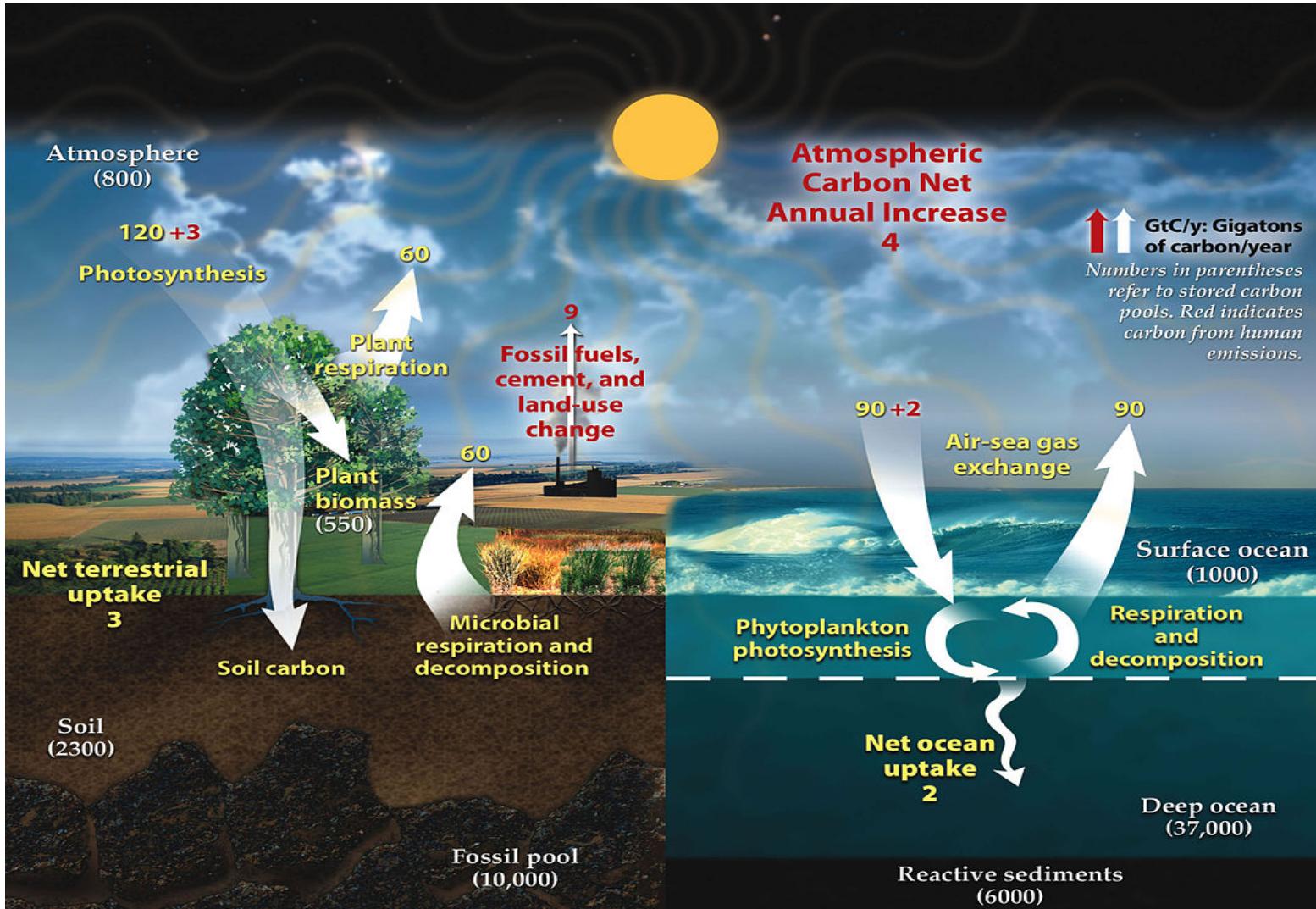
CLIMATE ECONOMETRICS VIRTUAL SEMINAR SERIES
15. DECEMBER 2020



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Econometric Models of Climate Change

GLOBAL CARBON CYCLE

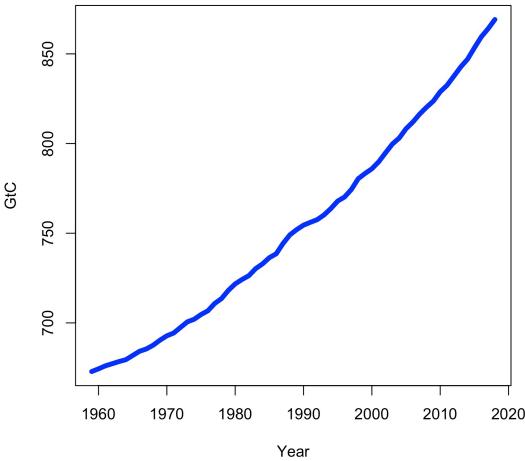


Source:
Wikipedia

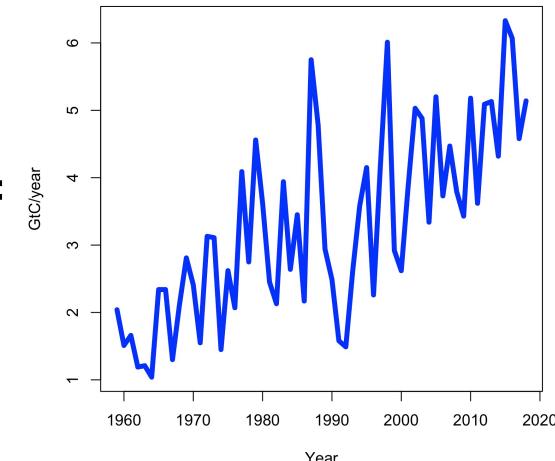
DATA



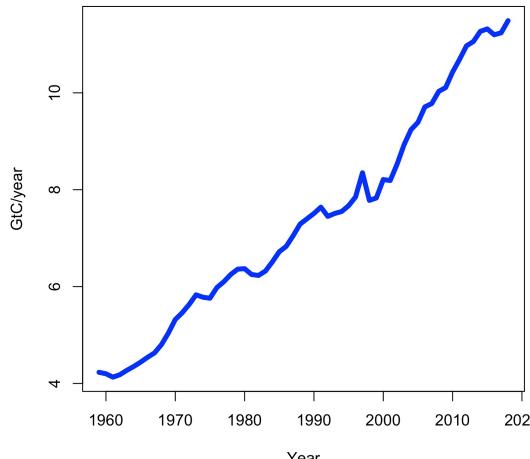
Atmospheric concentrations C



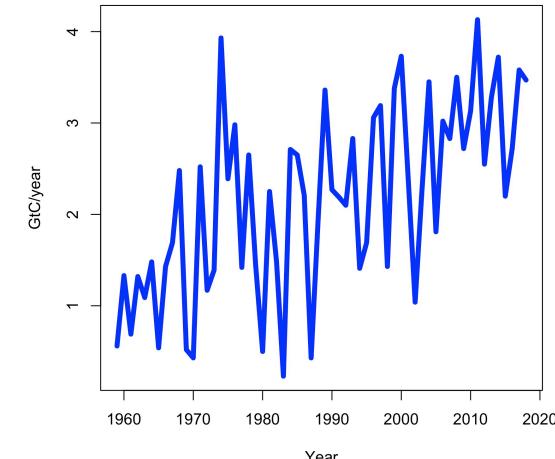
Delta C



Anthropogenic emissions E



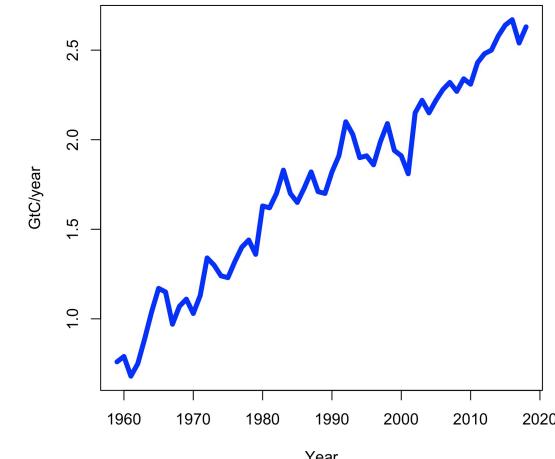
Land sink S_LND



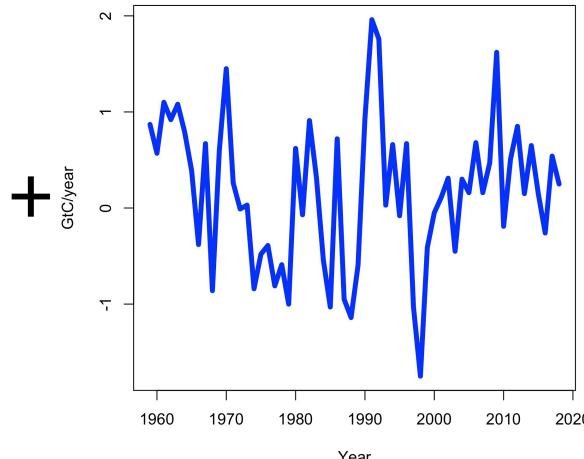
www.globalcarbonproject.org

Friedlingsstein et al. (2019),
The global carbon budget 2019,
Earth System Science Data 11(4),
1783-1838

Ocean sink S_OCN



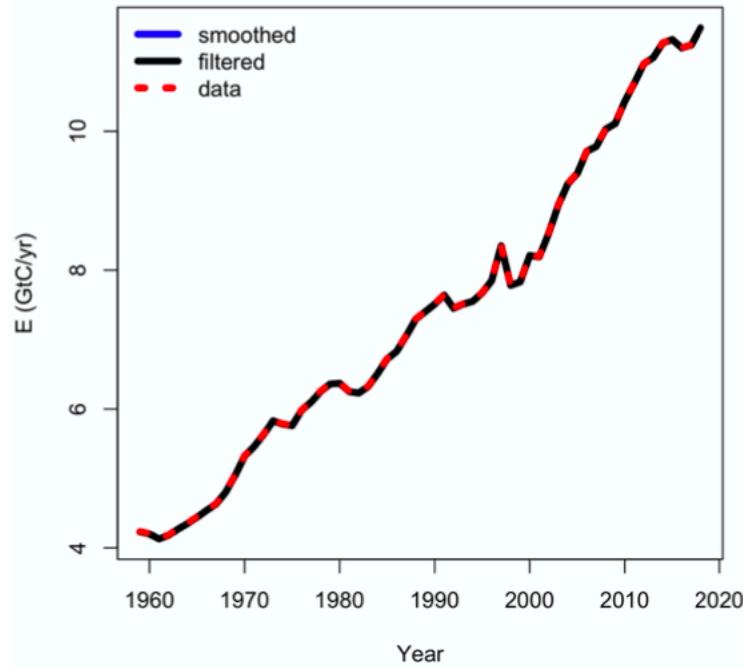
Budget imbalance BIM



OUTLINE OF THE TALK

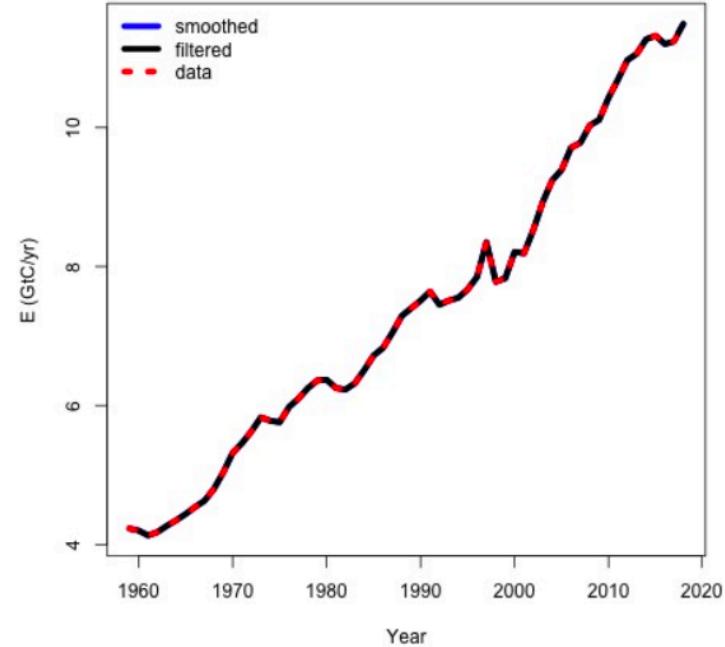
1. Models for the components of the global carbon budget
2. The dynamics of atmospheric concentrations C
3. The system model
4. Estimation: Residual diagnostics, Residual processes, Parameter estimates
5. Simulation
6. Discussion: Budget imbalance, airborne fraction, sink rate
7. Nowcasts and forecasts
8. Projections: Long-term scenarios until 2100
9. Conclusions

ANTHROPOGENIC EMISSIONS



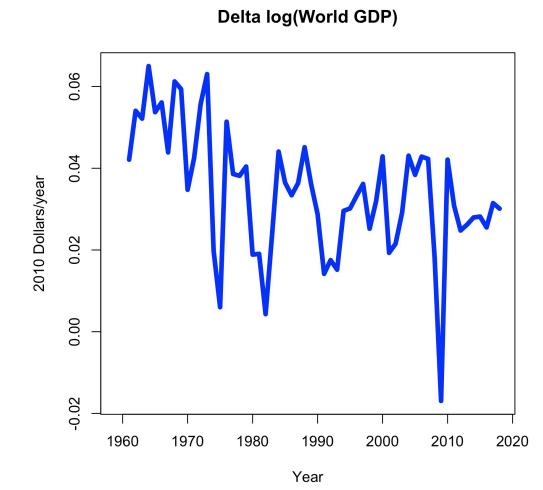
$$\Delta E_t = \frac{0.12}{(0.02)} + \eta_{5,t},$$

$$\eta_{5,t} \sim N(0, \frac{0.026}{(0.004)})$$



$$\Delta E_t = \frac{3.15}{(0.30)} \Delta \log GDP_t^{world} + \frac{-0.11}{(0.08)} I_{1973} + \frac{-0.18}{(0.08)} I_{1980} + \frac{-0.25}{(0.08)} I_{1991} + \frac{-0.65}{(0.18)} I_{1997} + \eta_{5,t}$$

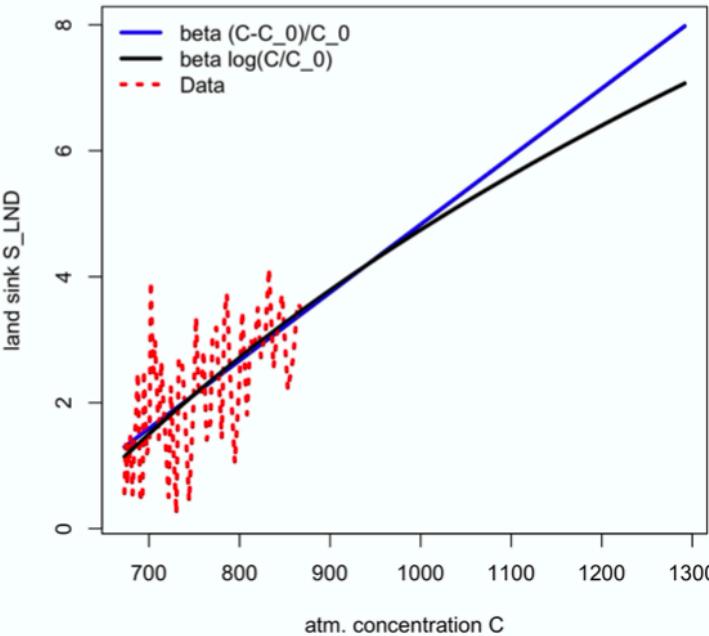
$$\eta_{5,t} \sim N(0, \frac{0.006}{(0.002)})$$



$$\Delta \log GDP_t^{world} \approx 0.034$$

SINKS LINEAR IN CONCENTRATIONS

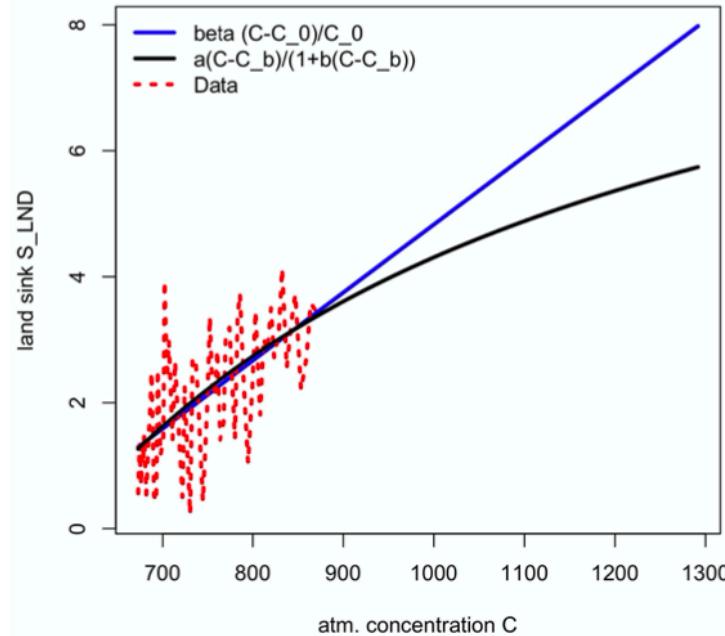
Bacastow-Keeling formula



$$S_{LND_t} = \beta \log\left(\frac{C_t}{C_0}\right)$$

C_0 pre-industrial concentration 593GtC or 279ppm

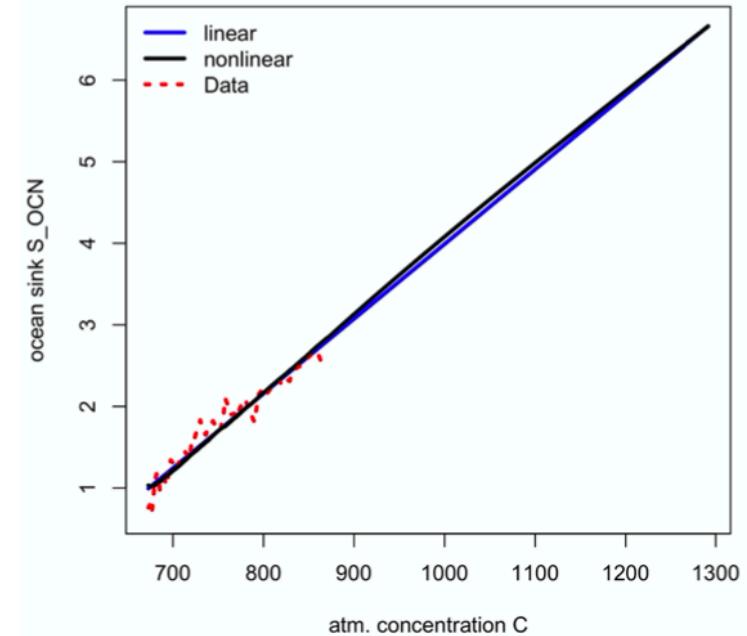
Gifford formula



$$S_{LND_t} = \frac{a(C_t - C_b)}{1 + b(C_t - C_b)}$$

$C_b = 80$ GtC NPP–zerolevel, $a, b > 0$

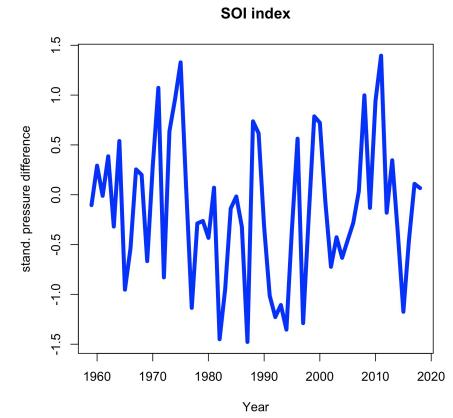
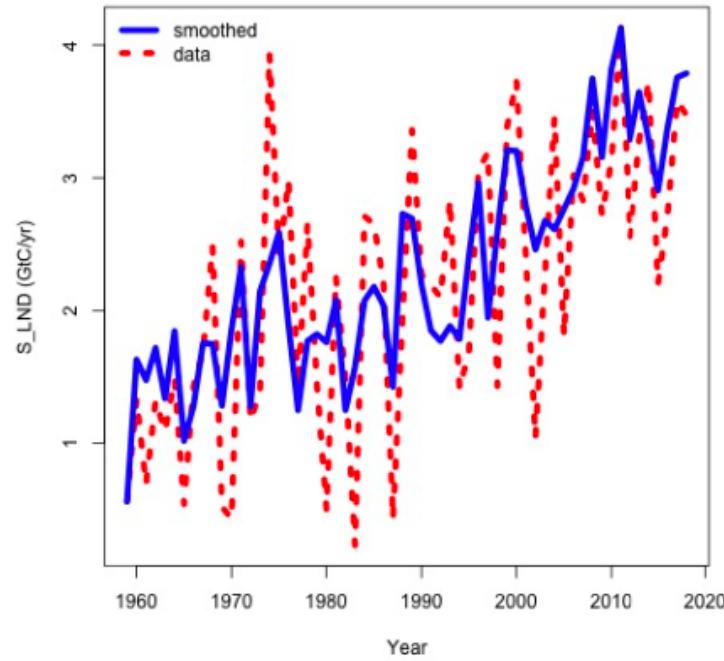
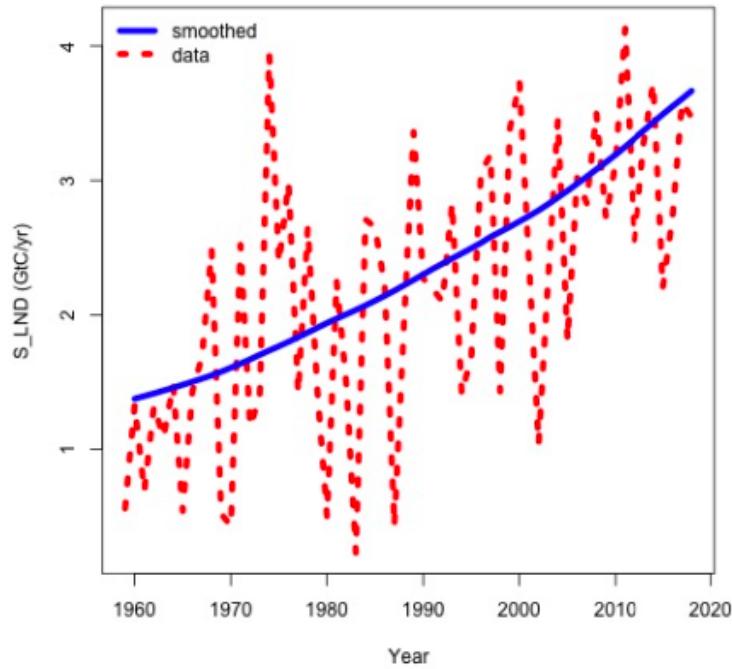
S_{OCN} linear and non-linear fits



$$S_{OCN_t} = k_o(pCO2_t^a - pCO2_t^s)$$

Joos et al. (1996, 2001)
Meinshausen et al. (2011)

LAND SINK

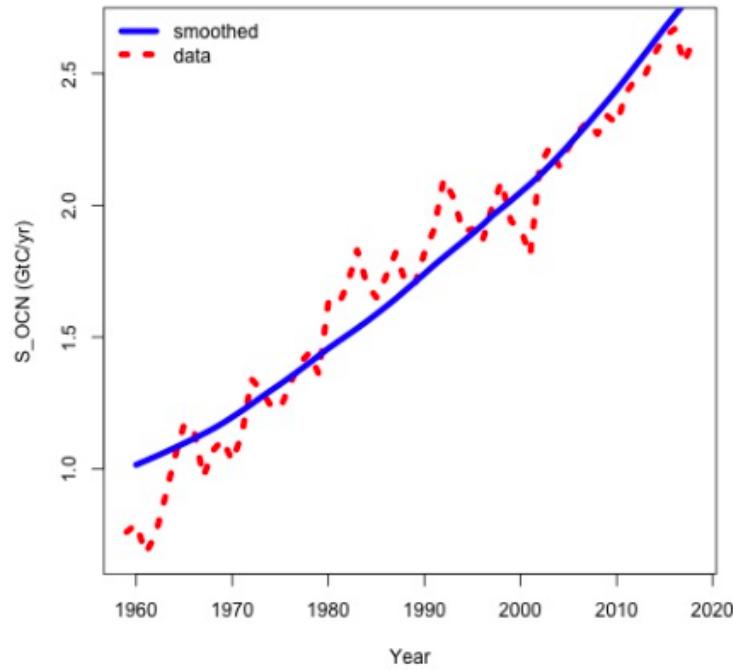


"Moisture sensitivities of both productivity and decomposition are important for capturing the response of the net flux to such [La Niña] events." Haverd et al. (2018, p. 3013)

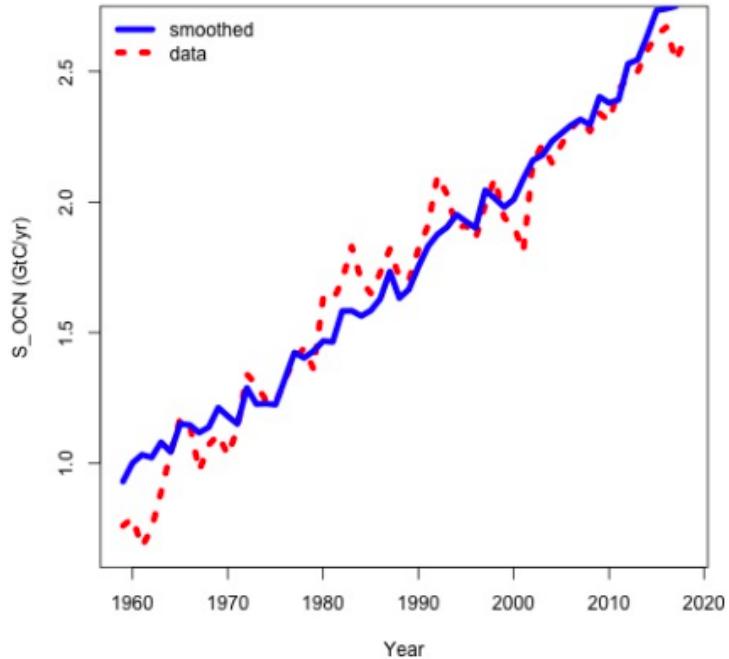
$$S_{LND_t} = \frac{7.23}{(0.88)} \frac{C_t}{C_0}$$

$$S_{LND_t} = \frac{7.20}{(0.90)} \frac{C_t}{C_0} + \frac{0.57}{(0.12)} SOI_t$$

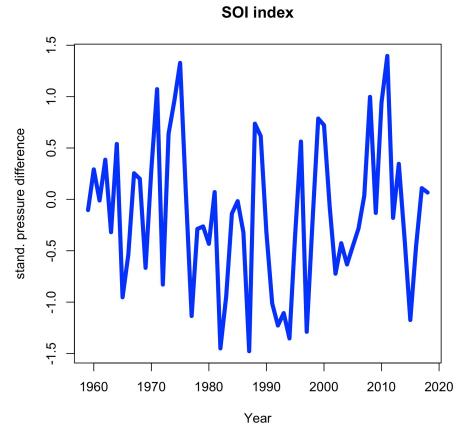
OCEAN SINK



$$S_{OCN_t} = \frac{5.53}{(0.51)} \frac{C_t}{C_0}$$

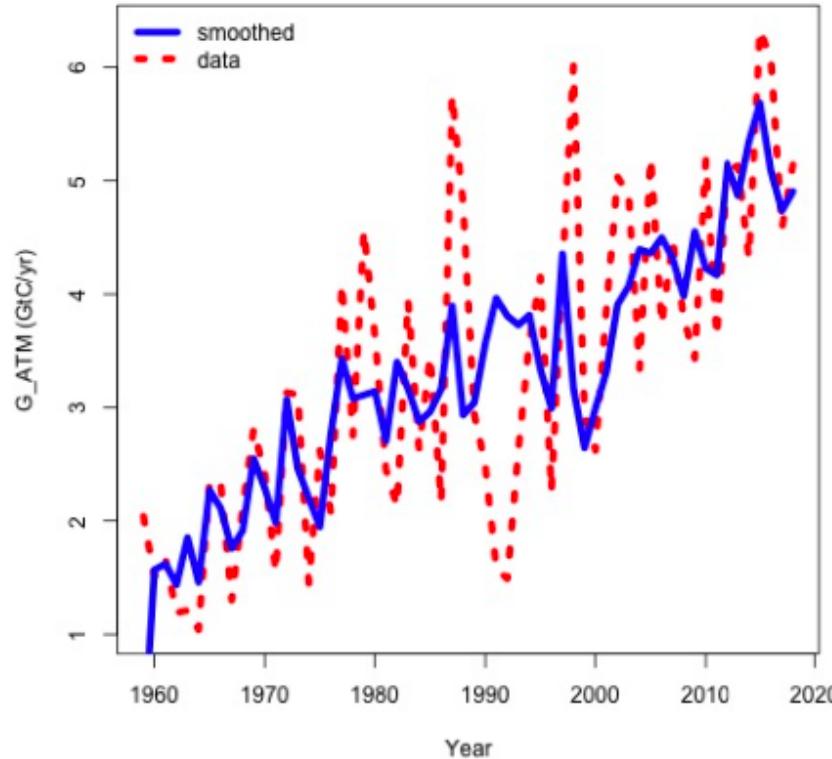
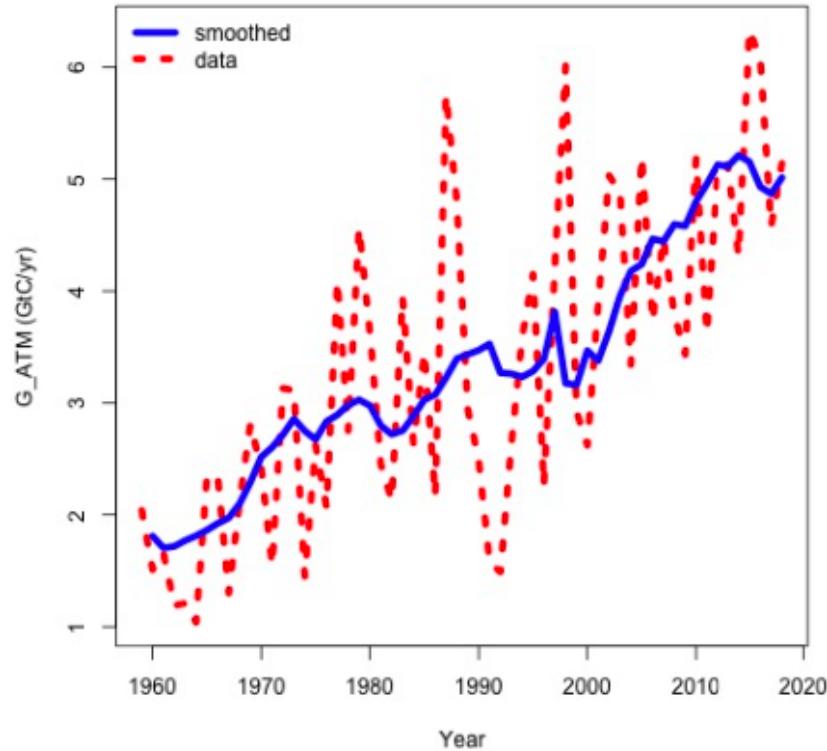


$$S_{OCN_t} = \frac{5.57}{(0.48)} \frac{C_t}{C_0} - \frac{0.05}{(0.01)} SOI_t$$



"When the winds are strongest during the cold cycle of ENSO deep upwelling occurs and [ocean CO₂ partial pressure] values are at a maximum." Feely et al. (1999, p. 599)

$\Delta C = \text{GROWTH IN ATM. CONCENTRATIONS}$



$$\Delta C_t = E_t - S_{LND}t - S_{OCN}t$$

THE DYNAMICS OF C

$$\Delta C_t = E_t - S_LND_t - S_OCN_t$$

$$= E_t - \beta_1^* C_t - \beta_2^* C_t + \varepsilon_t, \quad \varepsilon_t \sim I(0)$$

$$(1 + \beta_1^* + \beta_2^*)C_t - C_{t-1} = E_0 + dt + x_t + \varepsilon_t$$

$$(1 - qL)C_t = qE_0 + qdt + qx_t + q\varepsilon_t$$

$$\beta_i^* = \frac{\beta_i}{C_0} \approx 0.01$$

$$x_t = \sum_{i=1}^t \eta_{5,i}$$

$$q := \frac{1}{1 + \beta_1^* + \beta_2^*} \approx \frac{1}{1.02}$$

Three insights:

$$\begin{aligned} C_t &= q^t \left[C_0 - \frac{qE_0}{1-q} + \frac{dq^2}{(1-q)^2} \right] + \left[\frac{qE_0}{1-q} - \frac{dq^2}{(1-q)^2} \right] + \frac{dq}{1-q} t + \sum_{j=0}^{t-1} q^{j+1} x_{t-j} + \sum_{j=0}^{t-1} q^{j+1} \varepsilon_{t-j} \\ &= o(1) + O(1) + O(t) + I(1) + I(0) = O(t) + I(1) \end{aligned}$$

Thus,

$$\Delta C_t = I(0)$$

But,

$$(1 - qL)(1 - L)C_t = qd + q\Delta x_t + q\Delta \varepsilon_t = I(0)$$

THE SYSTEM MODEL

State equation Model 1

$$S_LND_{t+1}^* = \frac{\beta_1}{C_0} C_{t+1}^*$$

$$S_OCN_{t+1}^* = \frac{\beta_2}{C_0} C_{t+1}^*$$

$$E_{t+1}^* = E_t^* + d + \eta_{5,t}$$

$$C_{t+1}^* = C_t^* + G_ATM_{t+1}^*$$

$$G_ATM_{t+1}^* = E_{t+1}^* - S_LND_{t+1}^* - S_OCN_{t+1}^*$$

$$X_{1,t} = \phi_1 X_{1,t-1} + \eta_{1,t}$$

$$X_{2,t} = \eta_{2,t}$$

$$X_{3,t} = \phi_3 X_{3,t-1} + \eta_{3,t}$$

$$X_{4,t} = \eta_{4,t}$$

State equation Model 2

$$S_LND_{t+1}^* = \frac{\beta_1}{C_0} C_{t+1}^* + \beta_3 SOI_{t+1}$$

$$S_OCN_{t+1}^* = \frac{\beta_2}{C_0} C_{t+1}^* + \beta_4 SOI_{t+1}$$

$$E_{t+1}^* = E_t^* + \beta_5 \Delta \log GDP_{t+1}^{World} + \text{dummies} + \eta_{5,t}$$

$$C_{t+1}^* = C_t^* + G_ATM_{t+1}^*$$

$$\begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \\ \eta_{5,t} \end{bmatrix} \sim N \left(0, \begin{bmatrix} \sigma_1^2 & r_{12}\sigma_1\sigma_2 & r_{13}\sigma_1\sigma_3 & 0 & 0 \\ r_{12}\sigma_1\sigma_2 & \sigma_2^2 & 0 & 0 & 0 \\ r_{13}\sigma_1\sigma_3 & 0 & \sigma_3^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5^2 \end{bmatrix} \right)$$

Measurement equation

$$C_t = C_t^* + X_{1,t}$$

$$S_LND_t = S_LND_t^* + X_{2,t}$$

$$S_OCN_t = S_OCN_t^* + X_{3,t}$$

$$E_t = E_{t-1}^* + X_{4,t}$$

$$\eta_{4,t} \sim N(0, \sigma_4^2 s_E^2 I_{t \geq 1996})$$

RESIDUAL DIAGNOSTICS

Model 1

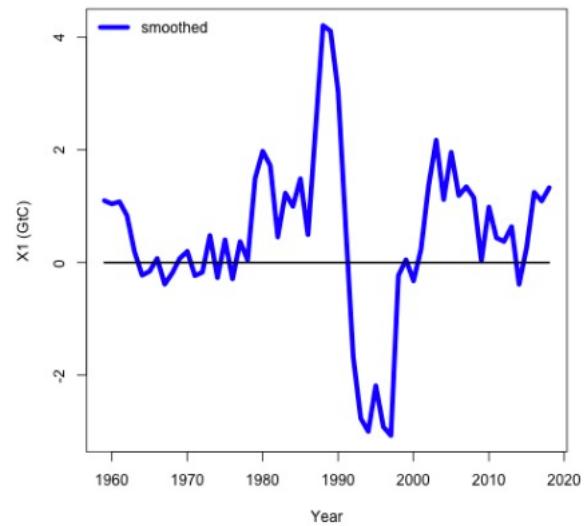
Residual	mean	std dev	skew	kurt	LB(1)	JB	DW
C	0.030	0.953	0.313	3.061	1.671	0.955	1.659
E	0.204	0.988	-1.372	8.084	0.002	80.66***	1.897
S_LND	-0.152	0.985	0.033	2.960	0.202	0.014	2.064
S_OCN	0.051	0.997	0.263	2.843	0.050	0.729	1.906

Model 2

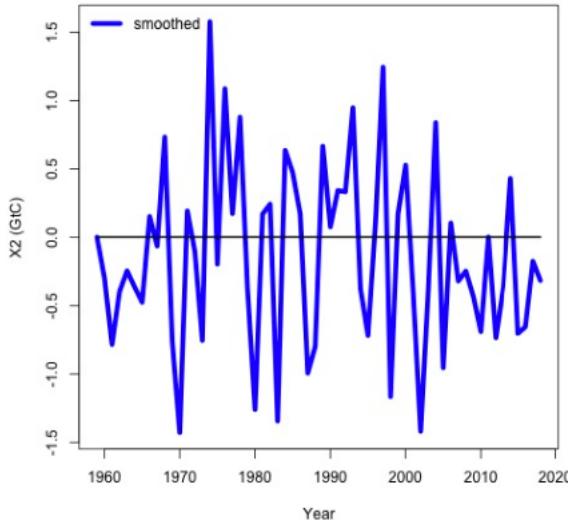
Residual	mean	std dev	skew	kurt	LB(1)	JB	DW
C	-0.031	0.879	-0.239	3.666	0.212	1.569	1.836
E	0.592	0.769	0.351	3.147	1.880	1.198	1.636
S_LND	-0.074	0.983	0.042	2.343	0.478	1.023	2.172
S_OCN	0.032	0.961	0.135	3.441	0.242	0.622	2.093

RESIDUAL PROCESSES

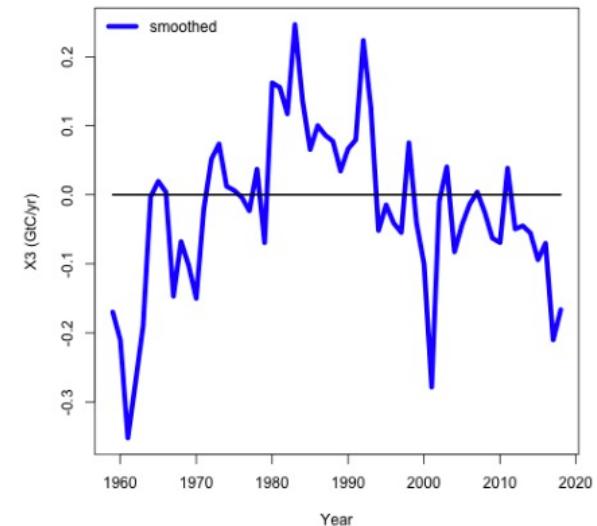
$X_{1,t}$ in C_t



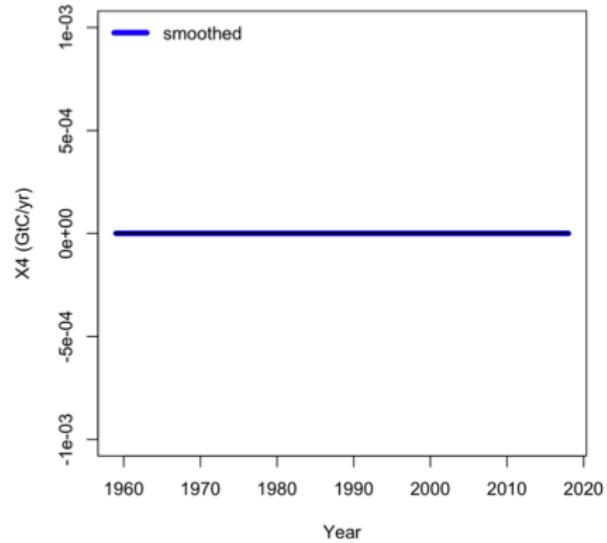
$X_{2,t}$ in S_LND_t



$X_{3,t}$ in S_OCN_t



$X_{4,t}$ in E_t

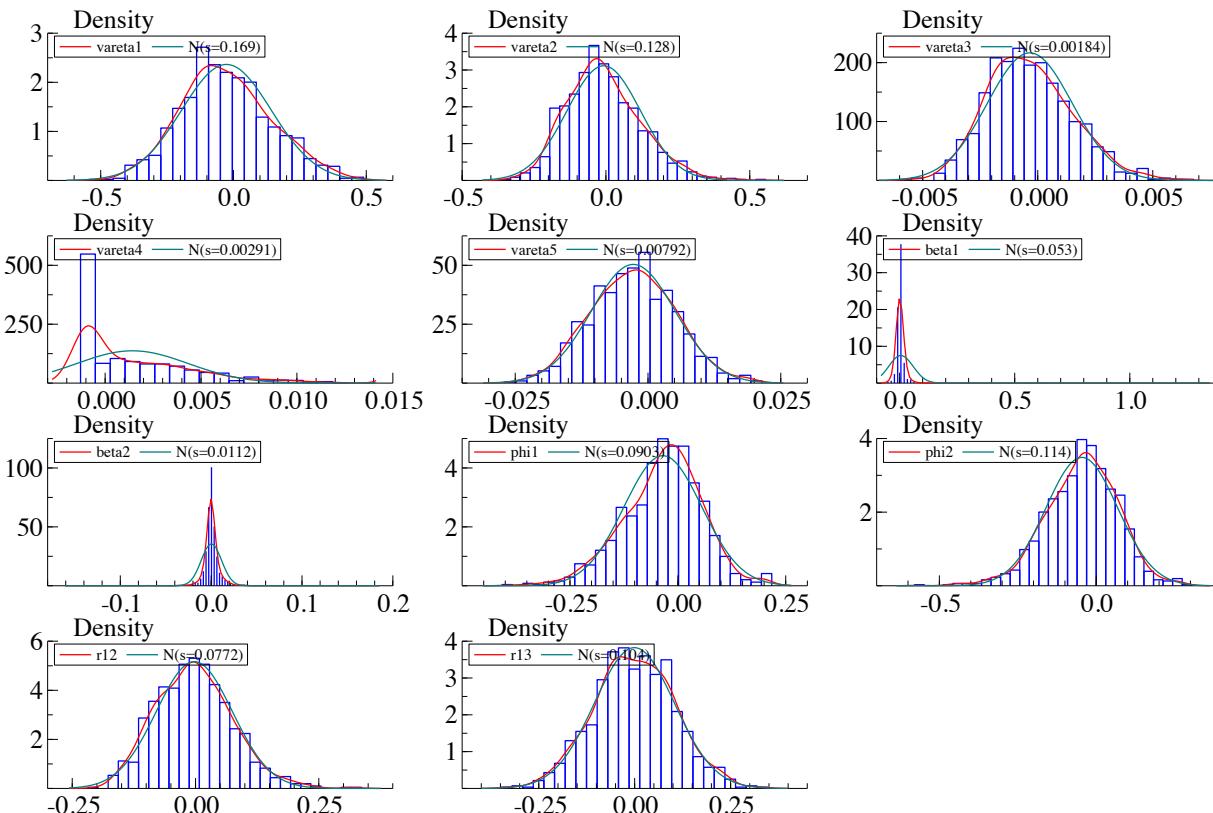


PARAMETER ESTIMATES

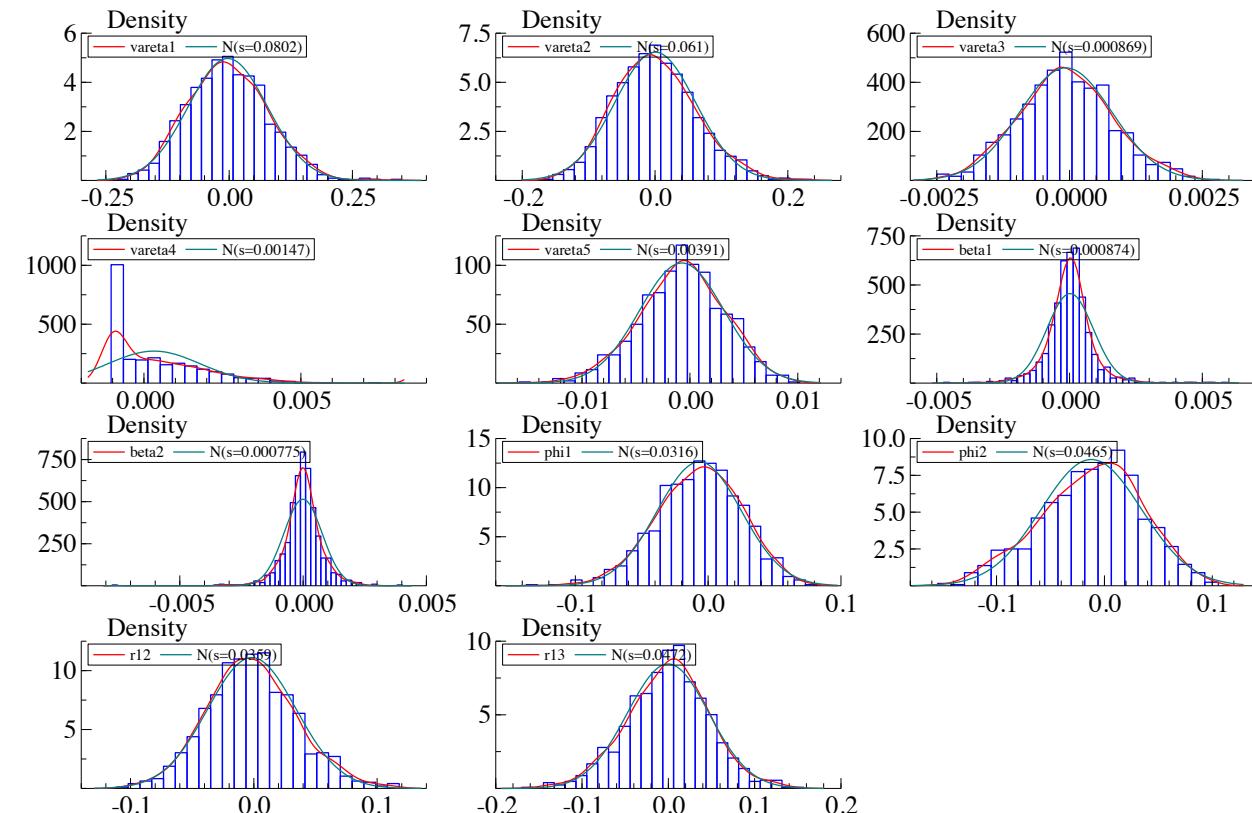
	Coefficients			Variances		
		estimate	std err		estimate	std err
Model 2	c_1 (filt.)	-6.77	0.05	$\sigma_{\eta_1}^2$	0.83	0.20
	c_2 (filt.)	-5.35	0.04	$\sigma_{\eta_2}^2$	0.49	0.16
	β_1	7.20	0.90	$\sigma_{\eta_3}^2$	0.008	0.002
	β_2	5.57	0.48	$\sigma_{\eta_4}^2$	0.006	0.002
	β_3 (filt.)	0.57	0.12	r_{12}	-0.63	0.15
	β_4 (filt.)	-0.05	0.01	r_{13}	-0.08	0.13
	β_5 (filt.)	3.15	0.30	s_E	2.38	0.57
	β_6 (filt.)	-0.11	0.08			
	β_7 (filt.)	-0.18	0.08			
	β_8 (filt.)	-0.25	0.08			
	β_9 (filt.)	-0.65	0.18			
	ϕ_1	0.86	0.07			
	ϕ_3	0.74	0.11			

SIMULATIONS OF MODEL 1

T=60



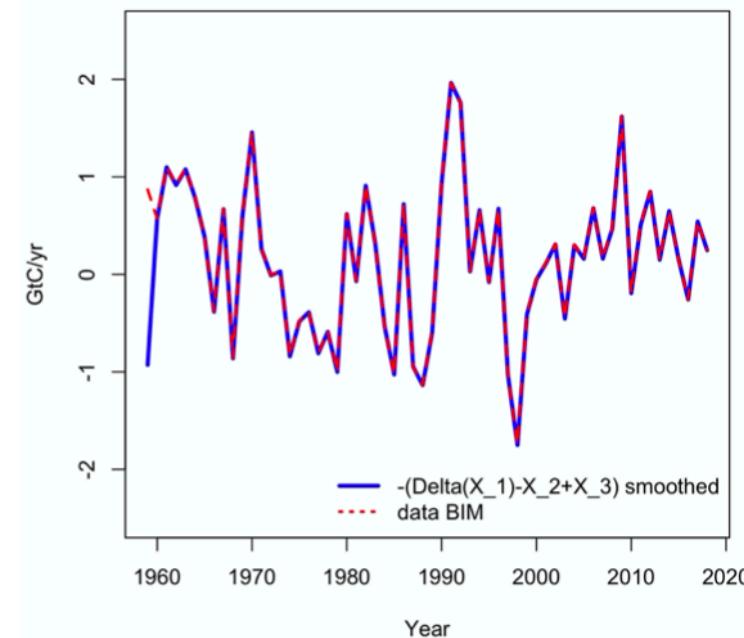
T=250



σ_4^2 exhibits “pile-up” problem (Stock and Watson 1998)

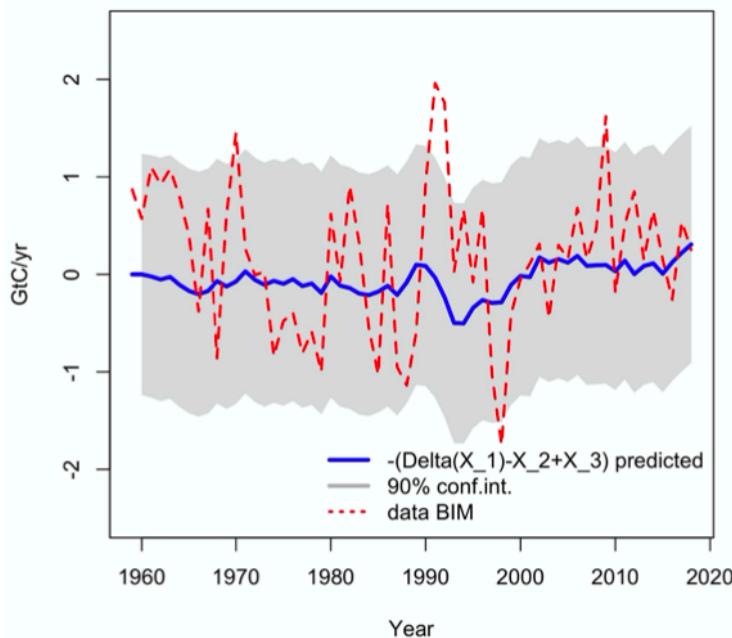
BUDGET IMBALANCE

Budget Imbalance BIM



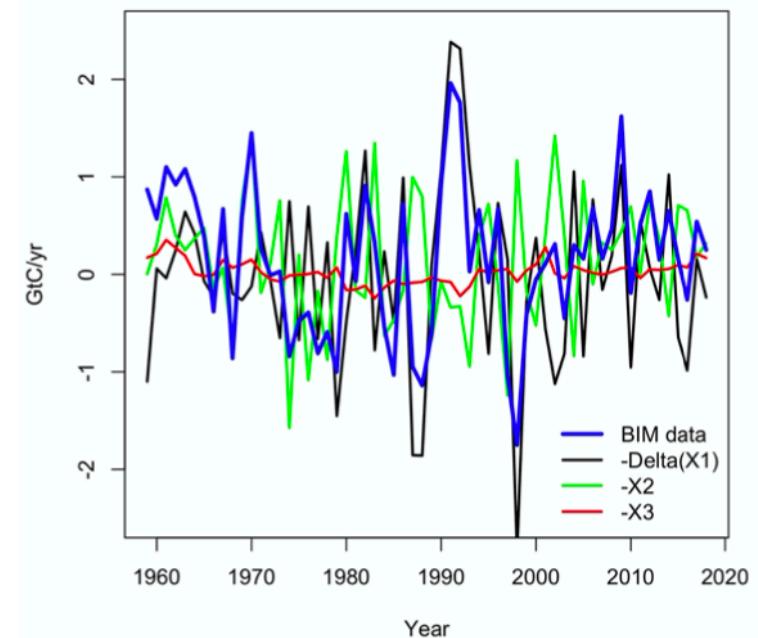
(A) Smoothed
 $-(\Delta X_1 + X_2 + X_3)$

Budget Imbalance BIM



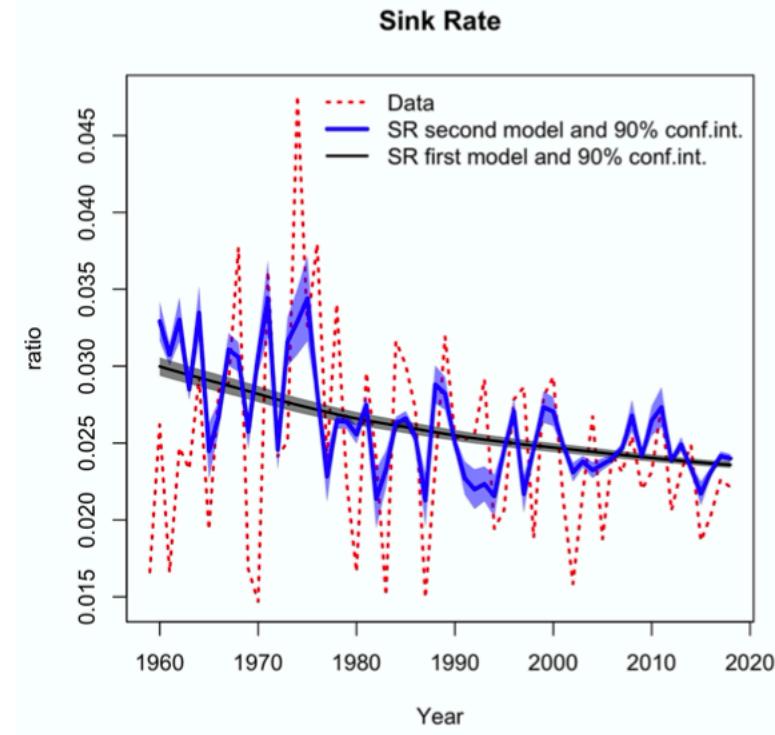
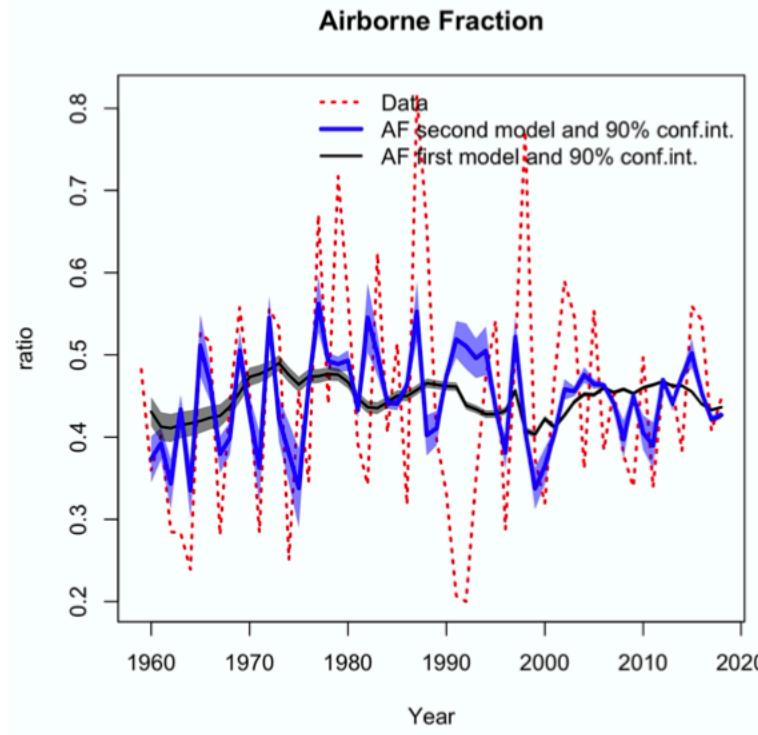
(B) One-year ahead
predictions

Budget Imbalance BIM



(C) Components
 $-\Delta X_1, -X_2, -X_3$

AIRBORNE FRACTION AND SINK RATE



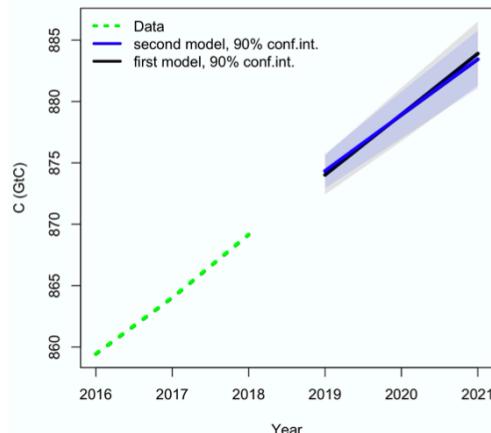
$$AF = \frac{\Delta C}{E}$$

$$SR = \frac{S_{LND} + S_{OCN}}{C}$$

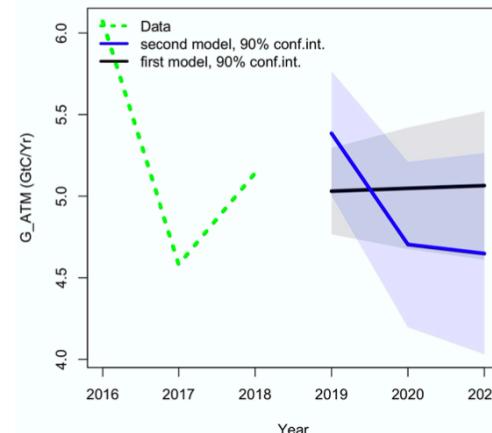
NOW-/FORECASTS

Forecasts of World GDP growth from IMF and World Bank

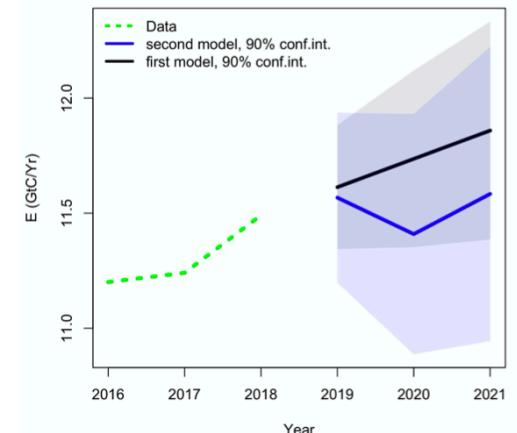
	2019	2020	2021
IMF	3.2%	-4.9%	5.4%
World Bank	2.6%	-5.2%	4.2%
Data	2.4%	N/A	N/A



(A) C

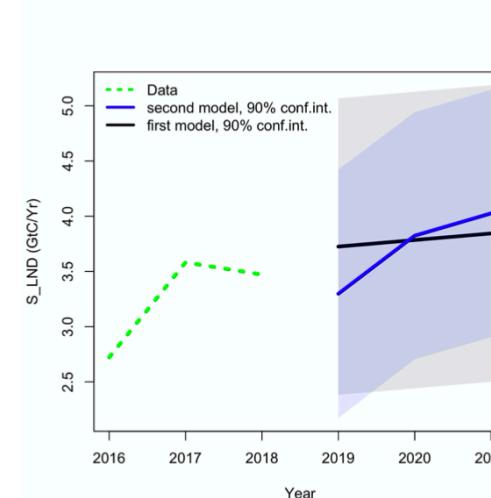


(B) G_ATM*

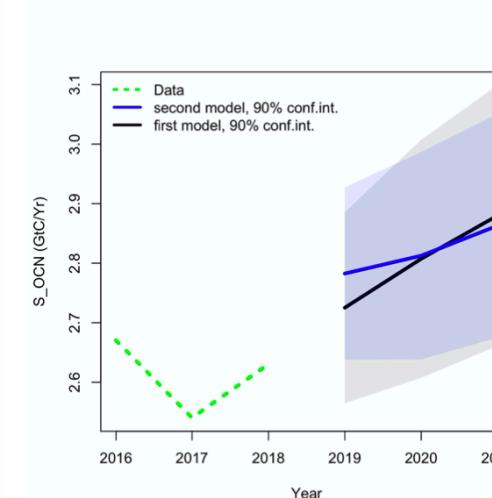


(C) E

Forecasts of SOI from forecast model of monthly SOI data 1866-1920, with trigonometric seasonal and second-order trigonometric cycle w/ period about 4 years



(D) S_LND

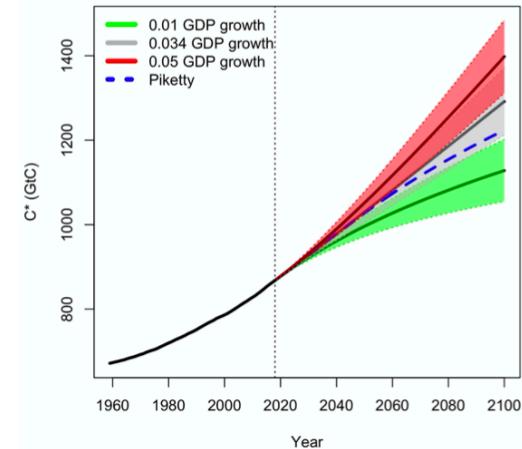


(E) S_OCN

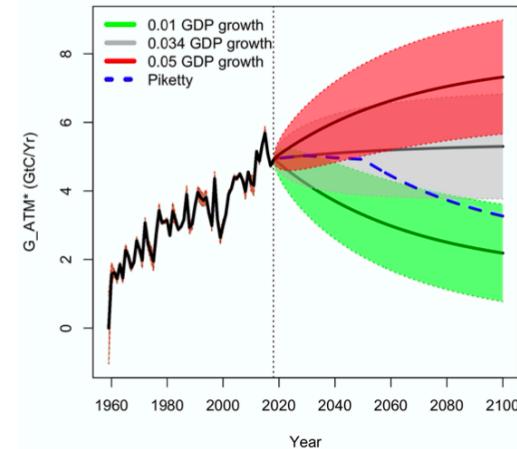
PROJECTIONS TO 2100

Scenarios:

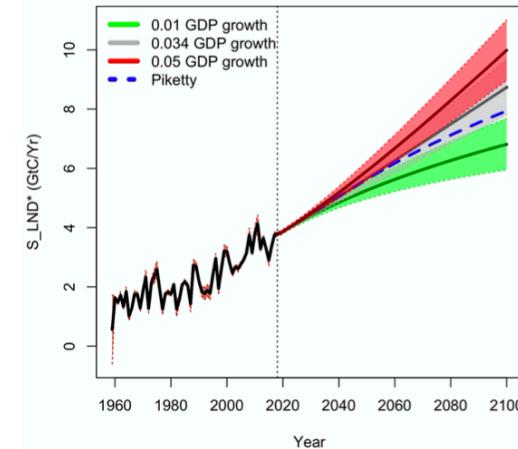
- 1% GDP growth
- 3.4% GDP growth
- 5% GDP growth
- Piketty scenario:
 - 3.4% til 2030
 - 3% til 2050
 - 1.5% til 2100



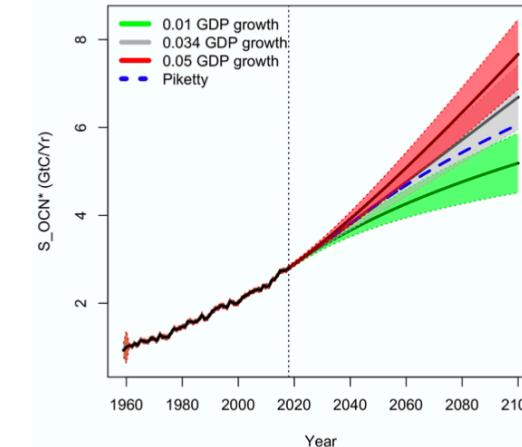
(A) C*



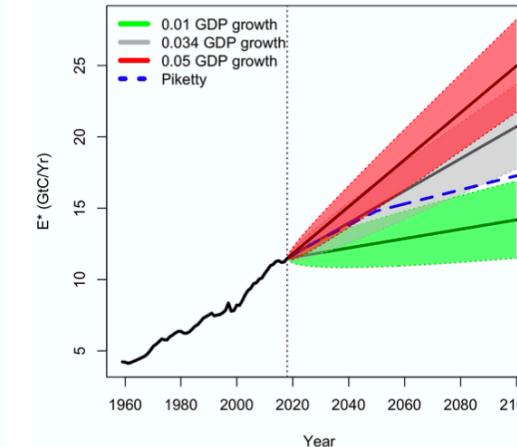
(B) G_ATM*



(C) S_LND*



(D) S_OCN*



(E) E*

CONCLUSIONS

- Specification of state-space model for Global Carbon Budget
- World GDP as driver in emissions
- Sinks: linear in CO₂ concentrations and in SOI
- CO₂ concentrations are I(1) ranging on I(2)
- Model allows for forecasting, projections, study of key variables such as airborne fraction and sink rate

Future directions

- Include ensemble members for S_LND and S_OCN
- Factor model for drift in emissions using large macroeconomic dataset
- Higher resolution on Global Carbon Cycle module (MAGICC)
- Connection to temperatures (Energy Balance Models)
- Cointegration analysis



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