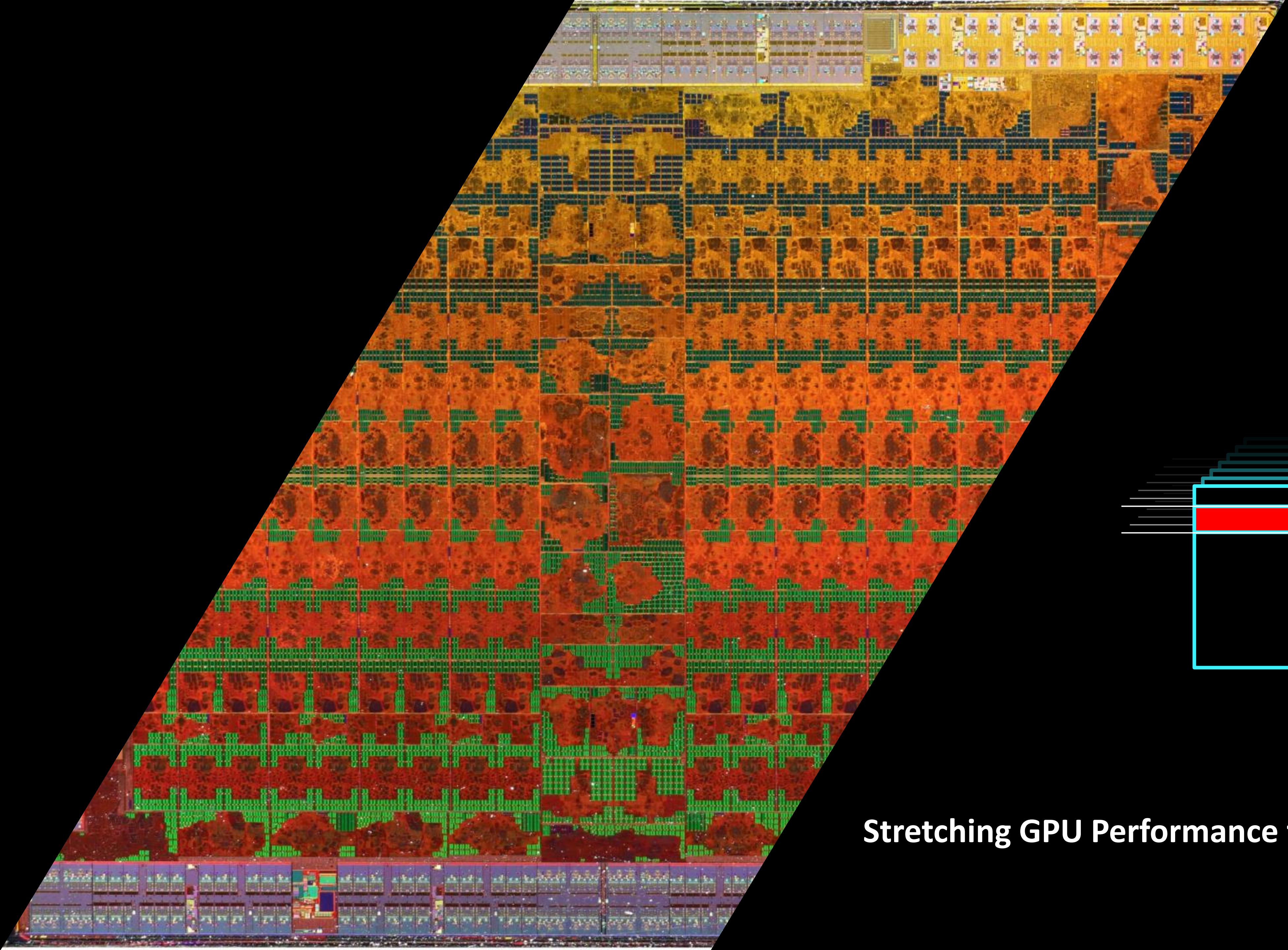
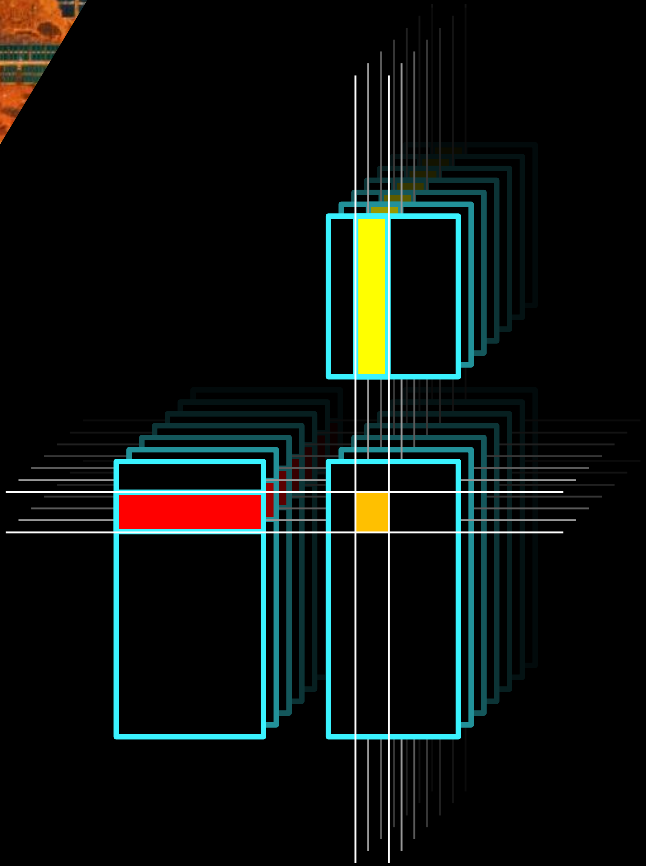


AMD



TENSILE

Stretching GPU Performance for Tensor Contractions

David E Tanner

MOTIVATION

- GEMM API encapsulates trillions of problems, all of which behave differently on GPUs.
  - Precisions, Transpose A, Transpose B, M, N, K, Strides
- Many problem types behave similar to GEMM.
  - $C_{ij} = \sum_k A_{ik} * B_{kj}$ (gemm NN)
  - $C_{ijk} = \sum_l A_{ilk} * B_{jlk}$ (batched gemm NT)
  - $C_{ijk} = \sum_{lmn} A_{ilmnk} * B_{jlmnk}$ (batched gemm w/ 3D summation)
  - $C_{ijk} = \sum_{lmn} A_{inlkm} * B_{mjlkn}$ (batched gemm w/ 3D summation different data layout)
- Goal: auto-generate kernels which achieve peak performance
  - For all problem types.
  - For all problem sizes.
  - On all GPUs.
  - Multiple languages: OpenCL, HIP, Assembly.

GPU PERFORMANCE PARAMETERS

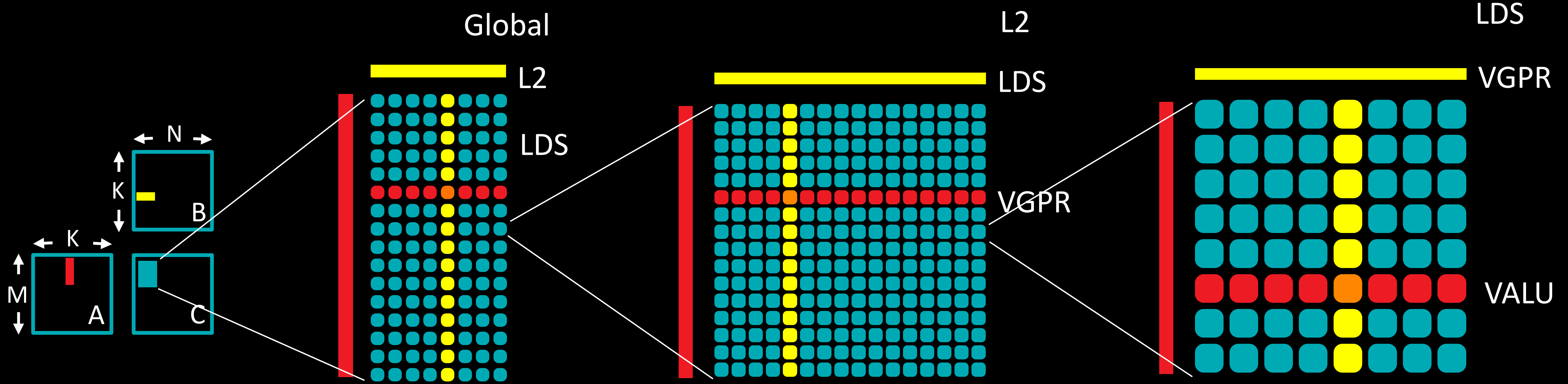
VEGA10 FRONTIER EDITION

- Compute Throughput
  - 13.1 TFlops = 2 (flops/cycle)*64(CUs)*64(lanes/CU)*1600MHz
- Global Memory Bandwidth
  - 480 GB/s
  - coalescing
- Global Memory Latency
  - hundreds of cycles
  - hide using CU-occupancy or ILP
- CU Occupancy
  - resources (VGPRs, LDS) permitting, multiple workgroups are concurrently scheduled on CU
- Whole-GPU Occupancy
  - need enough total workgroups to fill up the GPU
- Caches
  - L2 shared by all CUs
  - L1 dedicated to CU
- LDS Bandwidth
  - TB/s
- LDS Latency
  - tens of cycles
  - hide using CU-occupancy or ILP
- LDS Size
  - 64 kB / CU
- Instruction Divergence
  - all threads within workgroup do same instruction else need to compute and apply execution masks
- Instruction Throughput
  - gemm requires 2*M*N*K instructions, all extras hurt efficiency
  - minimize instructions which don’t count
  - maximize dual-issuing of instructions which don’t count with instructions that do; must be from different wavefronts
    - VALU, SALU, LDS, global memory, branch
- Power
  - VALU, LDS, memory, caches

GENERAL KERNEL-LEVEL STRATEGY

TILING

- Tiling at all memory levels to read from lower-bandwidth memory less and from higher-bandwidth memory more to prevent bandwidth from being bottleneck



(0) GEMM

(1) Grid of 16x8 Workgroups

- Read from global once, store in L2
- Read from L2 and store into LDS 8-16x

(2) Workgroup of 16x16 Threads

- Read from LDS and store in VGPRs 16x

(3) Thread has 8x8 elements

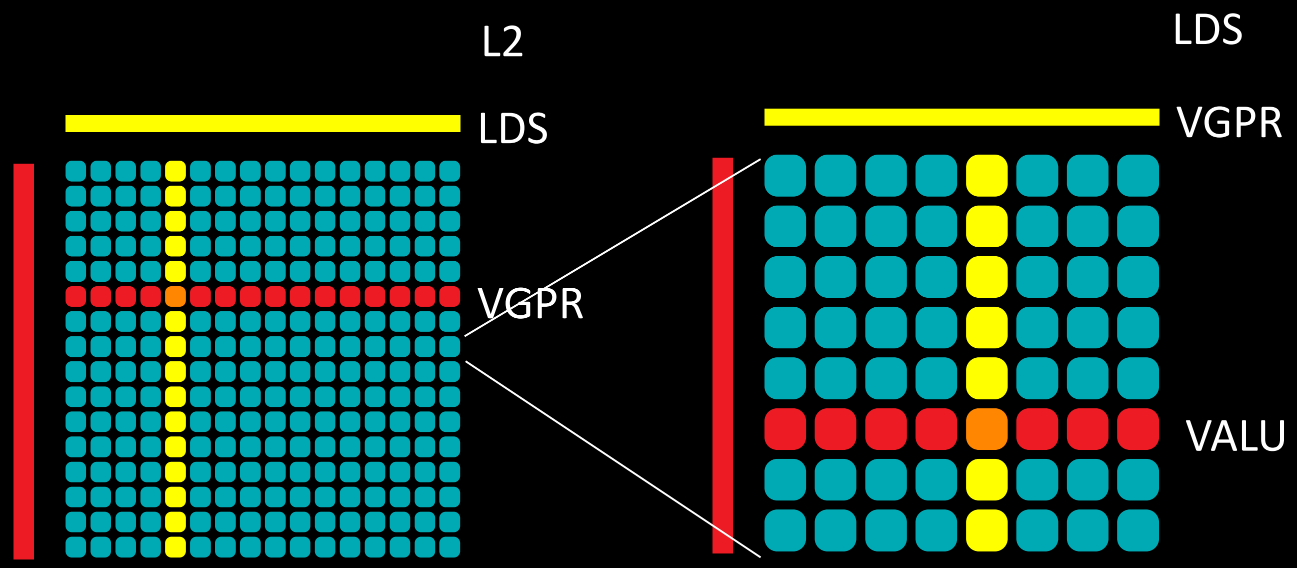
- 8 A VGPRs
- 8 B VGPRs
- 64 C VGPRs, 64 FMAs

THREAD-TILE SUB-ITERATION

WHERE WE WANT TO GET TO

```
// read 8 A elements
ds_read_b128
ds_read_b128

// read 8 B elements
ds_read_b128
ds_read_b128

// 64 MACs
v_mac_f32
v_mac_f32
v_mac_f32
v_mac_f32
+ 60 more

// Note: Prefetching and waits not shown
```

← 16 threads reading exact same address of LDS; coalesced.



(2) Workgroup of 16x16 Threads

- Read from LDS and store in VGPRs 16x

(3) Thread has 8x8 elements

- 8 A VGPRs
- 8 B VGPRs
- 64 C VGPRs, 64 FMAs

SUMMATION LOOP

no prefetching

```
calculate addresses

BEGIN LOOP
read global
wait global
barrier
write lds
barrier
read lds 0
wait lds 0
MACs 0
read lds 1
wait lds 1
MACs 1
…
read lds N-2
wait lds N-2
MACs N-2
read lds N-1
wait lds N-1
MACs N-1
END LOOP

write vgprs to C
```

subiter 0

subiter 1

…

subiter N-2

subiter N-1

```
calculate addresses
prefetch iter 0

BEGIN LOOP
read global iter 1
read lds 1
wait lds read 0
MACs 0
read lds 2
wait lds read 1
MACs 1
…
read lds N-1
wait global read
write lds
swap lds red / black ptrs
wait read lds N-2
MACs N-2
wait write lds & N-1
barrier
read lds iter 1 subiter 0
MACs N-1
END LOOP

write vgprs to C
```

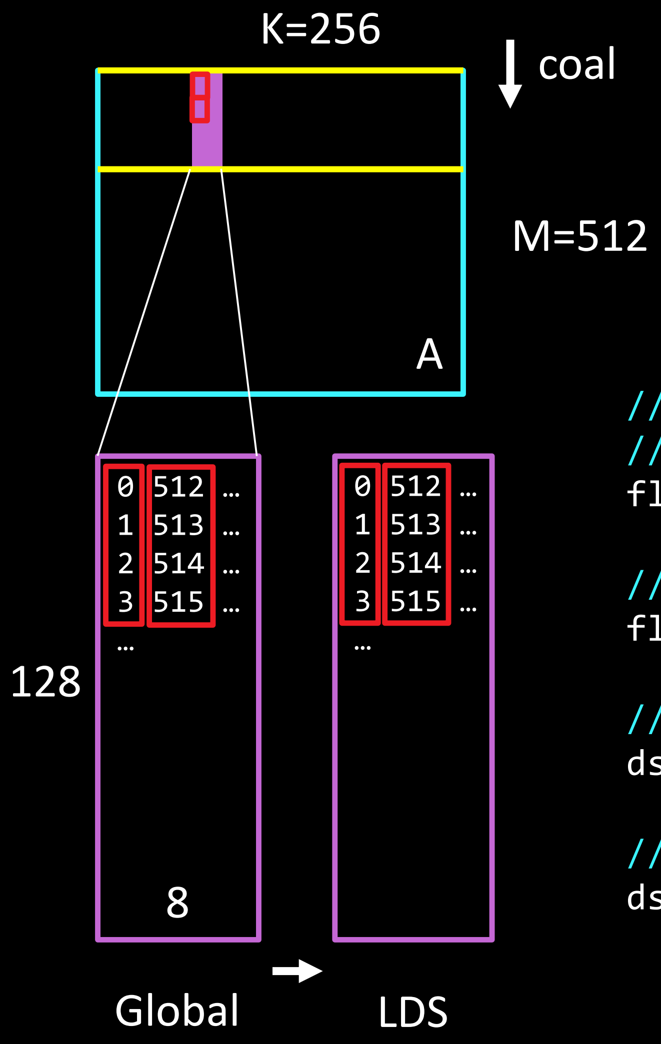
prefetch global

prefetch local

GLOBAL TO LDS

AMD

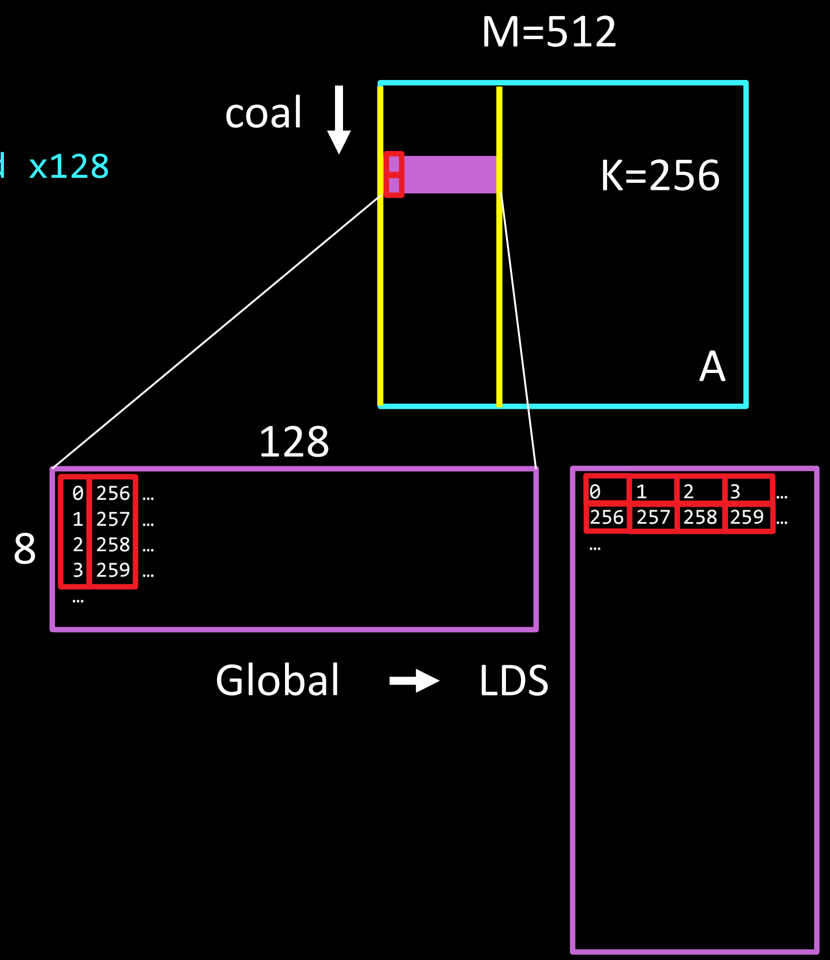
Don't Transpose Data




```
// read 4 A elements
// 512 bytes coalesced x8
flat_load_dwordx4

// read 4 B elements
flat_load_dwordx4

// write A to LDS
ds_write_b128

// write B to LDS
ds_write_b128
```

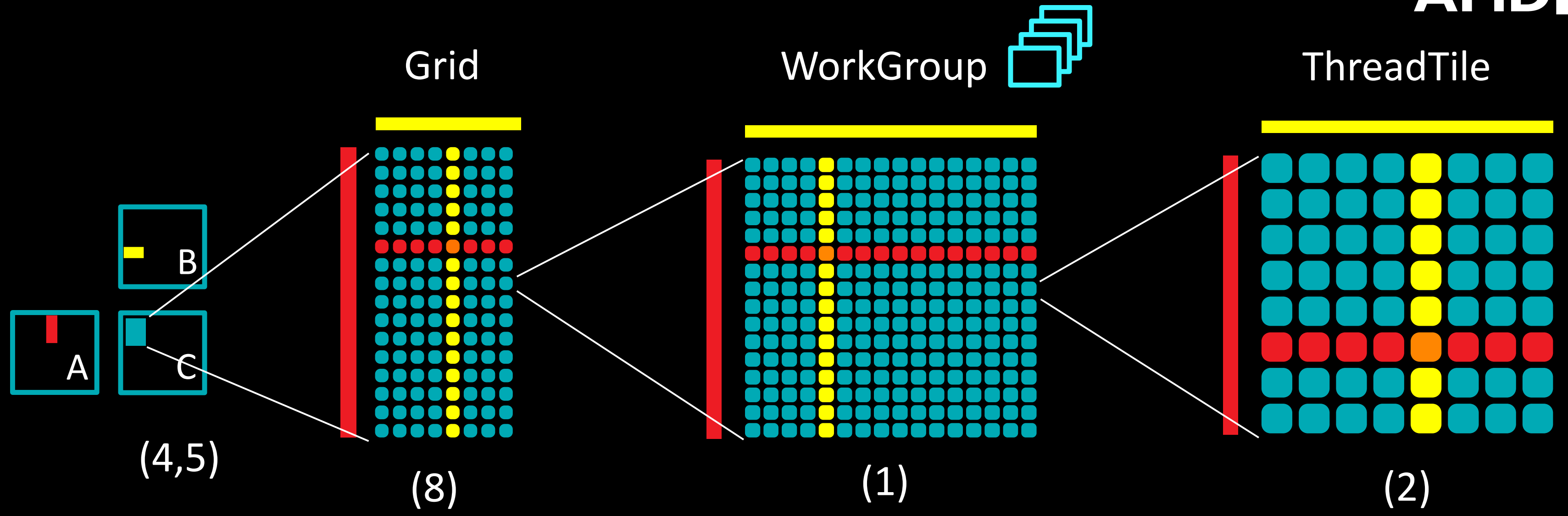
Do Transpose Data

```
// read 4 A elements
// 32 bytes coalesced x128
flat_load_dwordx4

// read 4 B elements
flat_load_dwordx4

// write A to LDS
ds_write_b32
ds_write_b32
ds_write_b32
ds_write_b32

// write B to LDS
ds_write_b32
ds_write_b32
ds_write_b32
ds_write_b32
```



KERNEL PARAMETERS

1) WorkGroup, LocalSplitU
2) ThreadTile
3) VectorWidth
4) GlobalSplitU
5) GlobalSplitUWGM
6) PrefetchGlobalRead
7) PrefetchLocalRead
8) WorkGroupMapping
9) LoopUnroll
10) NumLoadsCoalesced
11) GlobalReadCoalesceGroup
12) GlobalReadCoalesceVector
13) KernelLanguage
14) NonTemporal



(3,4,10,11,12)

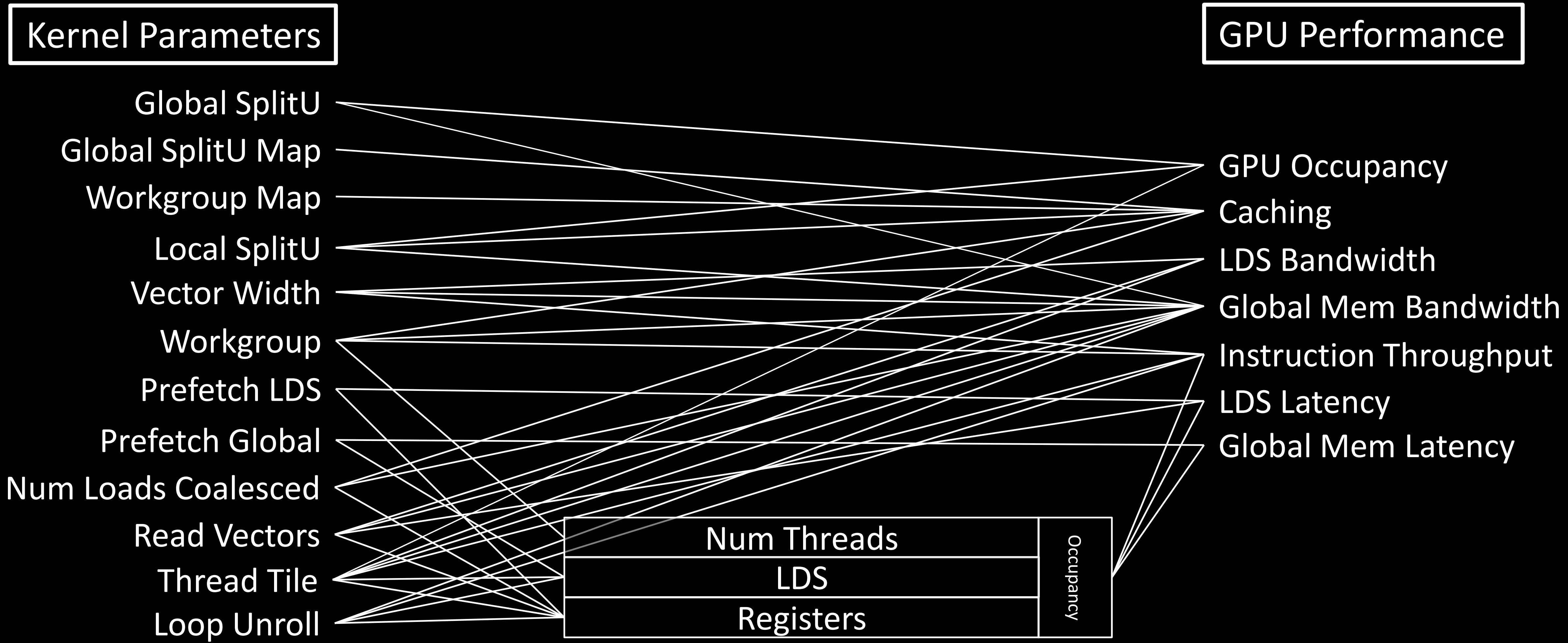
(3,6)

(7)

(9)

```
calculate addresses
prefetch iter 0

BEGIN LOOP
load global iter 1
read lds 1
wait lds read 0
MACs 0
read lds 2
wait lds read 1
MACs 1
…                   (13)

read lds N-1
wait global read
write lds
swap lds red / black ptrs
wait read lds N-2
MACs N-2
wait write lds & N-1
barrier
read lds iter 1 subiter 0
MACs N-1
END LOOP

write vgprs to C       (14)
```

KERNEL PARAMETERS AFFECT GPU PERFORMANCE



MAPPING PROBLEM SIZES TO KERNELS

```
// Exact Sizes
if (sizeI==101 && sizeJ==202 && sizeK==303)
        return function_ptr_0;
if (sizeI==4004 && sizeJ==5005 && sizeK==6006)
        return function_ptr_1;
```

```
// Range Sizes – Recursive Size Splitting
if (sizeI < threshold_I_0) {
        if (sizeJ < threshold_J_0) {
                if (sizeK < threshold_K_0)
                        return function_ptr_2;
                if (sizeK < threshold_K_1)
                        return function_ptr_3;
                // more K thresholds
                return function_ptr_4;
        }
        if (sizeJ < threshold_J_1) {
                // more K thresholds
        }
        // more J, K thresholds
}
if (sizeI < threshold_I_1) {
        // more J, K thresholds
}
// more I, J, K thresholds
```

PERFORMANCE EXPERIMENTS

- Set of 100 kernels
  - ThreadTile = 8x8, 8x4, 8x2, 4x8, 4x4, 4x2, 2x8, 2x4, 2x2
  - WorkGroup = 16x16x1, 16x8x2, 8x16x2, 8x8x4
  - GlocalSplitU = 1, 2, 4, 6, 8
  - DepthU = 8
  - VectorWidth = max
  - PrefetchGlocalRead = True
  - PrefetchLocalRead = True
  - WorkGroupMapping = 8
- Benchmark kernels against 2D range of sizes for sgemm NT
  - M = 16 – 5632; N = 16 – 5632; K = 3104 (moderate)
  - M = 128 – 100,000; N = 16 – 256; K = 3104 (skinny N)
  - M = 64 – 512; N = 16 – 512; K = 1,048,576 (small MxN) GSU=16, 32, 64, 128
- Analyze performance and kernel properties for each data point.

PERFORMANCE

AMD

N

M	16	32	64	112	176	256	352	464	592	736	896	1072	1264	1472	1696	1936	2192	2464	2752	3056	3376	3712	4064	4432	4816	5216	5632
16	29	62	118	205	314	448	596	735	890	1,024	1,131	1,204	1,401	1,502	1,562	1,625	1,646	1,750	1,754	1,823	1,833	1,887	1,937	1,890	1,976	1,977	2,036
32	60	116	238	406	615	880	1,137	1,424	1,722	2,006	2,271	2,304	2,628	2,877	2,953	3,120	3,171	3,365	3,452	3,617	3,619	3,654	3,800	3,710	3,848	3,896	3,992
64	121	236	452	774	1,168	1,635	2,030	2,402	2,785	3,173	3,539	3,625	4,206	4,506	4,723	4,959	4,936	5,146	5,483	5,846	5,644	5,979	6,404	5,869	6,251	6,449	6,510
112	206	409	790	1,270	1,831	2,365	2,758	3,275	3,590	3,989	4,353	4,398	5,002	5,487	5,422	6,033	5,625	5,791	6,362	6,879	6,652	6,891	7,436	7,058	7,203	7,016	7,458
176	316	633	1,201	1,848	2,404	3,178	3,457	4,012	4,506	4,695	5,479	5,397	6,223	5,894	6,204	6,908	6,755	7,237	7,001	7,332	7,864	7,630	8,156	7,868	7,704	8,343	8,023
256	456	878	1,575	2,342	3,065	4,021	4,271	4,847	5,052	5,810	5,780	5,968	6,579	6,898	6,811	7,593	7,095	7,458	7,936	8,597	8,012	8,558	9,211	8,357	8,761	8,387	8,835
352	604	1,186	2,141	2,971	3,566	4,566	4,720	5,458	5,924	6,001	6,555	6,683	7,363	7,298	7,224	7,899	7,811	7,934	7,886	8,573	8,636	8,741	9,013	8,483	8,534	9,188	8,854
464	755	1,447	2,489	3,340	4,113	5,100	5,480	5,877	5,810	6,906	7,229	7,075	7,789	7,932	7,927	8,217	8,232	8,403	8,562	8,950	8,526	8,708	9,437	8,882	9,007	8,905	9,378
592	947	1,711	2,892	3,749	4,510	5,561	6,022	5,942	6,393	7,288	7,512	7,550	7,734	8,013	8,120	8,403	8,677	8,430	8,848	9,118	8,947	8,898	9,573	9,414	9,861	9,461	9,707
736	1,073	2,018	3,415	4,254	4,966	6,241	6,099	6,967	7,254	7,666	7,913	8,111	8,476	8,693	8,424	9,010	8,796	9,114	9,079	9,678	9,715	9,801	10,293	9,534	9,505	10,182	9,853
896	1,228	2,225	3,755	4,450	5,394	6,442	6,447	6,794	6,950	7,527	7,974	8,695	8,841	8,860	9,428	9,395	9,785	9,516	9,655	9,758	10,327	10,173	10,338	10,442	10,216	10,456	10,786
1072	1,304	2,380	3,895	4,616	5,536	6,392	6,786	7,242	7,635	8,173	8,763	8,442	9,037	9,050	9,180	9,259	9,446	9,620	9,836	10,038	9,745	10,054	10,334	10,281	10,016	10,399	10,338
1264	1,427	2,716	4,490	5,222	6,367	7,212	7,401	7,782	7,755	8,474	9,031	9,012	9,125	9,373	9,506	9,739	9,929	9,614	10,071	10,300	10,133	10,140	11,079	10,544	11,055	10,580	11,158
1472	1,509	2,924	4,700	5,562	6,147	7,643	7,476	7,889	7,948	8,619	8,981	8,968	9,299	9,585	9,790	9,854	9,897	9,804	10,213	10,185	10,124	10,565	10,824	10,323	10,434	10,564	10,728
1696	1,608	3,128	4,896	5,594	6,562	7,673	7,503	8,015	8,196	8,409	9,484	9,100	9,521	9,792	9,641	9,605	9,943	9,926	10,056	10,052	10,381	10,304	10,753	10,358	10,234	10,841	10,593
1936	1,673	3,162	5,085	6,223	6,961	8,097	7,939	8,244	8,373	9,018	9,442	9,245	9,734	9,894	9,617	10,001	10,076	10,313	10,328	10,756	10,301	10,183	10,874	10,575	10,411	10,514	11,049
2192	1,666	3,291	5,152	6,069	7,078	8,025	7,870	8,285	8,692	8,773	9,876	9,419	9,929	9,894	9,946	10,081	9,841	10,083	10,276	10,515	10,485	10,543	10,970	10,806	10,726	10,663	10,651
2464	1,746	3,438	5,511	6,385	7,275	8,370	8,098	8,424	8,438	9,103	9,541	9,608	9,661	9,778	9,924	10,247	10,056	10,339	10,607	10,577	10,396	10,549	11,076	11,013	11,005	11,025	11,065
2752	1,805	3,501	5,703	6,353	7,109	8,311	8,062	8,530	8,825	9,046	9,846	9,765	10,032	10,185	10,053	10,301	10,260	10,617	10,726	10,485	10,647	11,294	11,298	10,608	10,669	10,732	10,868
3056	1,848	3,623	5,879	6,856	7,724	8,901	8,576	8,932	9,107	9,681	9,918	10,008	10,310	10,268	10,040	10,734	10,520	10,516	10,508	11,474	10,751	11,497	11,552	10,864	11,024	11,207	11,393
3376	1,836	3,570	5,910	6,907	7,934	8,733	8,671	8,510	8,930	9,739	10,468	9,701	10,125	10,145	10,398	10,296	10,481	10,394	10,669	10,722	10,744	10,801	10,989	11,199	10,775	11,037	11,290
3712	1,890	3,701	6,048	6,773	7,709	8,632	8,643	8,630	8,760	9,770	10,161	9,957	10,041	10,525	10,275	10,136	10,441	10,462	11,227	11,372	10,682	10,935	11,159	11,420	11,065	11,343	11,700
4064	1,948	3,824	6,511	7,413	8,252	9,440	8,982	9,380	9,553	10,282	10,555	10,289	11,005	10,828	10,748	10,806	10,915	11,081	11,257	11,483	10,935	11,228	11,521	11,210	11,548	11,328	11,685
4432	1,938	3,700	6,049	7,225	7,945	9,047	8,473	8,873	9,381	9,555	10,561	10,279	10,599	10,338	10,382	10,563	10,819	11,061	10,643	10,856	11,197	11,554	11,277	11,100	11,480	11,371	11,310
4816	1,966	3,819	6,334	7,244	7,879	9,041	8,608	9,070	9,868	9,534	10,245	9,994	11,052	10,466	10,211	10,420	10,701	11,044	10,694	11,023	10,786	11,199	11,605	11,480	11,416	11,375	11,396
5216	1,976	3,920	6,601	7,198	8,310	8,942	9,198	8,960	9,429	10,168	10,722	10,314	10,655	10,538	10,822	10,464	10,578	11,008	10,755	11,146	10,974	11,427	11,338	11,310	11,311	11,364	11,433
5632	2,034	3,996	6,180	7,169	7,800	9,051	8,528	9,295	9,440	9,765	10,740	10,183	10,998	10,577	10,521	10,906	10,524	10,999	10,797	11,250	11,160	11,654	11,602	11,166	11,244	11,355	11,513

K = 3104

GFlops

WINNERS

N

M

	16	32	64	112	176	256	352	464	592	736	896	1072	1264	1472	1696	1936	2192	2464	2752	3056	3376	3712	4064	4432	4816	5216	5632
16	3	3	3	3	3	3	3	3	1	3	1	7	11	26	1	26	1	26	26	18	18	14	18	15	15	1	15
32	3	3	3	3	3	3	3	3	3	3	1	7	1	18	1	18	1	18	14	18	18	14	18	15	15	29	15
64	3	3	3	3	3	3	3	12	7	12	7	12	12	14	23	30	12	0	30	30	23	30	30	0	30	5	5
112	8	3	3	3	3	1	12	23	10	0	28	25	25	28	28	28	25	22	5	5	28	5	5	28	5	28	28
176	3	3	3	3	23	4	23	13	28	25	25	28	28	28	5	5	28	5	28	28	5	28	5	5	6	6	5
256	8	3	3	1	26	14	9	10	17	22	22	17	22	5	22	24	22	5	24	24	5	6	6	5	24	5	24
352	8	3	3	4	4	2	25	28	28	25	5	28	5	28	28	5	5	6	5	5	5	5	20	24	27	27	5
464	3	3	1	7	4	10	28	28	30	5	28	28	5	5	5	20	5	5	5	20	24	5	27	24	5	5	24
592	3	3	7	4	25	25	28	25	30	5	28	28	28	20	5	5	5	5	24	27	5	5	20	24	27	24	20
736	3	3	4	2	25	28	25	5	5	28	28	28	20	5	5	20	24	5	5	24	24	24	27	20	5	27	24
896	11	1	4	2	17	2	21	17	17	28	28	6	5	5	24	24	27	24	24	24	27	24	6	27	24	6	27
1072	8	1	4	25	28	25	25	28	28	28	6	5	5	5	27	5	5	5	5	24	5	6	24	27	24	6	6
1264	11	1	4	25	28	28	5	5	28	6	5	5	5	27	5	20	24	5	20	24	24	27	27	20	27	20	27
1472	11	18	4	28	25	5	25	5	6	5	28	5	27	5	24	27	24	5	6	20	6	24	27	6	6	6	6
1696	1	1	7	10	25	28	25	5	28	28	24	27	5	24	5	20	24	24	27	24	27	24	27	6	20	27	27
1936	11	1	17	28	28	5	5	20	5	20	24	5	20	27	20	27	24	27	24	27	24	24	27	27	24	6	27
2192	1	26	12	25	25	28	5	5	5	24	27	5	24	24	24	24	6	16	24	27	24	6	27	27	27	27	27
2464	26	26	17	28	5	5	28	5	5	27	24	5	5	5	5	27	6	24	27	20	24	6	27	27	27	27	27
2752	26	26	28	5	28	28	28	5	24	5	5	5	20	24	27	24	24	27	24	24	24	27	27	20	20	20	27
3056	18	26	22	28	28	24	5	20	27	24	5	24	24	20	24	27	27	20	24	27	16	27	27	27	27	27	27
3376	11	11	28	25	5	5	5	5	5	6	27	5	24	16	27	24	24	24	24	6	24	27	27	27	27	27	27
3712	3	11	22	5	28	6	5	27	5	24	24	24	27	24	24	24	24	24	27	27	27	27	27	27	27	27	27
4064	18	18	22	5	5	6	20	27	20	27	20	24	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27
4432	8	15	17	28	5	5	24	24	24	20	27	27	20	16	16	27	27	27	16	27	27	27	27	27	27	27	27
4816	26	15	22	5	28	28	5	5	27	5	24	24	27	16	24	20	27	27	16	27	27	27	27	27	27	27	27
5216	11	19	5	28	5	28	27	5	24	27	20	24	20	27	27	16	27	27	20	27	27	27	27	27	27	27	27
5632	19	19	22	24	5	5	24	24	6	24	27	24	27	24	27	27	27	27	27	27	27	27	27	27	27	27	27

Conclusion 1: Chaotic behavior necessitates recursive size splitting for map.

FLOPS/BYTE OF TILE

Flops/Byte = 2*MT0*MT1 / (MT0+MT1)/4

For example 128x128 tile:

2*128*128/(128+128)/4 = 32 F/B

2* 64* 64/(64+ 64)/4 = 16 F/B

M \ N	16	32	64	112	176	256	352	464	592	736	896	1072	1264	1472	1696	1936	2192	2464	2752	3056	3376	3712	4064	4432	4816	5216	5632
16	8	8	8	8	8	8	8	8	6	8	6	6	6	4	6	4	6	4	4	4	4	4	4	2	2	6	2
32	8	8	8	8	8	8	8	8	8	8	8	11	8	8	8	8	8	8	11	8	8	11	8	8	8	11	8
64	8	8	8	8	8	8	8	11	11	11	11	11	11	11	11	13	11	11	13	13	11	13	13	11	13	16	16
112	8	8	8	8	8	8	11	11	13	11	16	16	16	16	16	16	16	13	16	16	16	16	16	16	16	16	16
176	8	8	8	8	11	11	11	11	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	21	21	16
256	8	8	8	8	8	11	11	13	13	13	13	13	13	16	13	16	13	16	16	16	16	21	21	16	16	16	16
352	8	8	8	11	11	13	16	16	16	16	16	16	16	16	16	16	16	21	16	16	16	16	32	16	32	32	16
464	8	8	8	11	11	13	16	16	13	16	16	16	16	16	16	32	16	16	16	32	16	16	32	16	16	16	16
592	8	8	11	11	16	16	16	16	13	16	16	16	16	32	16	16	16	16	16	32	16	16	32	16	32	16	32
736	8	8	11	13	16	16	16	16	16	16	16	16	32	16	16	32	16	16	16	16	16	16	32	32	16	32	16
896	8	8	11	13	13	13	13	13	13	16	16	21	16	16	16	16	32	16	16	16	32	16	21	32	16	21	32
1072	8	8	11	16	16	16	16	16	16	16	21	16	16	16	32	16	16	16	16	16	16	21	16	32	16	21	21
1264	8	8	11	16	16	16	16	16	16	21	16	16	16	32	16	32	16	16	32	16	16	32	32	32	32	32	32
1472	8	8	11	16	16	16	16	16	21	16	16	16	32	16	16	32	16	16	21	32	21	16	32	21	21	21	21
1696	8	8	11	13	16	16	16	16	16	16	16	32	16	16	16	32	16	16	32	16	32	16	32	21	32	32	32
1936	8	8	13	16	16	16	16	32	16	32	16	16	32	32	32	32	16	32	16	32	16	16	32	32	16	21	32
2192	8	8	11	16	16	16	16	16	16	16	32	16	16	16	16	16	21	21	16	32	16	21	32	32	32	32	32
2464	8	8	13	16	16	16	16	16	16	32	16	16	16	16	16	32	21	16	32	32	16	21	32	32	32	32	32
2752	8	8	16	16	16	16	16	16	16	16	16	16	32	16	32	16	16	32	16	16	16	32	32	32	32	32	32
3056	8	8	13	16	16	16	16	32	32	16	16	16	16	32	16	32	32	32	16	32	21	32	32	32	32	32	32
3376	8	8	16	16	16	16	16	16	16	21	32	16	16	21	32	16	16	16	16	21	16	32	32	32	32	32	32
3712	8	8	13	16	16	21	16	32	16	16	16	16	32	16	16	16	16	16	32	32	32	32	32	32	32	32	32
4064	8	8	13	16	16	21	32	32	32	32	32	16	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32
4432	8	8	13	16	16	16	16	16	16	32	32	32	32	21	21	32	32	32	21	32	32	32	32	32	32	32	32
4816	8	8	13	16	16	16	16	16	32	16	16	16	32	21	16	32	32	32	21	32	32	32	32	32	32	32	32
5216	8	11	16	16	16	16	32	16	16	32	32	16	32	32	32	21	32	32	32	32	32	32	32	32	32	32	32
5632	11	11	13	16	16	16	16	16	21	16	32	16	32	16	32	32	32	32	32	32	32	32	32	32	32	32	32

TOTAL WORKGROUPS / 64CU

M \ N	16	32	64	112	176	256	352	464	592	736	896	1072	1264	1472	1696	1936	2192	2464	2752	3056	3376	3712	4064	4432	4816	5216	563
16	0	0	0	0	0	1	1	1	1	1	1	1	2	1	2	2	3	2	3	3	3	2	4	2	2	8	3
32	0	0	0	0	1	1	1	2	2	3	3	2	4	3	5	4	6	5	3	6	7	4	8	4	5	3	6
64	0	0	1	1	1	2	3	2	2	3	3	4	5	3	5	2	9	5	3	3	10	4	4	9	5	3	3
112	0	0	1	2	2	3	2	2	2	3	2	3	3	3	3	3	6	4	2	3	6	3	3	8	4	9	10
176	0	1	1	2	1	3	3	2	2	3	4	3	3	4	2	3	6	3	7	8	5	10	5	6	2	2	8
256	1	1	2	3	3	2	3	3	3	3	4	6	5	3	7	2	9	5	3	3	7	2	2	9	5	10	6
352	1	1	3	2	4	3	3	2	3	6	2	6	3	8	9	5	6	2	7	8	9	10	3	6	2	2	15
464	1	2	3	2	5	3	2	3	4	3	6	8	4	5	6	2	8	9	10	3	6	13	2	8	17	18	10
592	1	2	2	4	2	3	3	6	5	3	8	10	11	2	8	9	10	11	6	2	15	17	5	10	3	12	6
736	1	3	3	3	3	3	6	3	3	8	10	12	2	8	10	3	6	14	15	9	9	10	3	6	27	4	16
896	1	3	4	3	4	7	10	10	12	10	12	2	9	10	6	7	2	8	9	10	3	13	7	4	16	9	5
1072	2	3	4	3	3	6	9	8	10	12	2	9	10	12	2	16	18	20	23	12	28	8	17	5	20	11	12
1264	2	4	5	3	3	5	3	4	11	2	9	10	12	2	16	5	11	24	7	15	16	4	5	11	6	13	7
1472	2	3	6	3	6	3	12	5	2	8	20	12	2	17	10	3	12	28	8	9	9	21	6	12	14	15	16
1696	2	5	5	4	7	7	14	6	15	19	6	2	16	10	22	6	14	16	4	20	5	24	7	14	16	8	9
1936	3	6	3	3	5	4	5	2	9	3	7	16	5	3	6	4	16	5	20	6	25	27	8	8	36	19	10
2192	3	4	9	6	9	9	6	8	10	6	2	18	11	12	14	16	9	10	23	6	28	16	8	9	10	11	12
2464	2	5	4	4	3	5	13	9	11	2	8	20	24	28	32	5	10	23	6	14	32	17	10	10	11	12	13
2752	3	5	3	2	7	11	15	10	6	15	19	23	7	15	4	20	23	6	29	32	35	10	11	23	25	27	15
3056	3	6	3	5	8	3	8	3	2	9	21	12	15	9	20	6	6	14	32	9	20	11	12	13	14	15	16
3376	5	10	3	9	5	7	9	12	15	5	3	28	16	9	5	25	28	32	35	20	43	12	13	14	16	17	18
3712	7	11	4	3	10	2	10	2	17	10	13	15	4	21	24	27	31	35	10	11	12	13	14	16	17	18	20
4064	4	8	4	3	5	2	3	2	5	3	7	17	5	6	7	8	8	10	11	12	13	14	16	17	19	20	22
4432	9	4	6	8	6	9	6	8	10	6	4	5	11	12	14	8	9	10	23	13	14	16	17	19	20	22	24
4816	5	5	5	4	13	19	13	17	3	27	16	20	6	14	31	18	10	11	25	14	16	17	19	20	22	24	26
5216	8	3	3	9	7	20	2	18	12	4	9	21	13	7	8	19	11	12	27	15	17	18	20	22	24	26	28
5632	1	3	6	2	8	11	8	10	6	16	5	23	7	32	9	10	12	13	15	16	18	20	22	24	26	28	30

GLOBAL SPLIT U

AMD

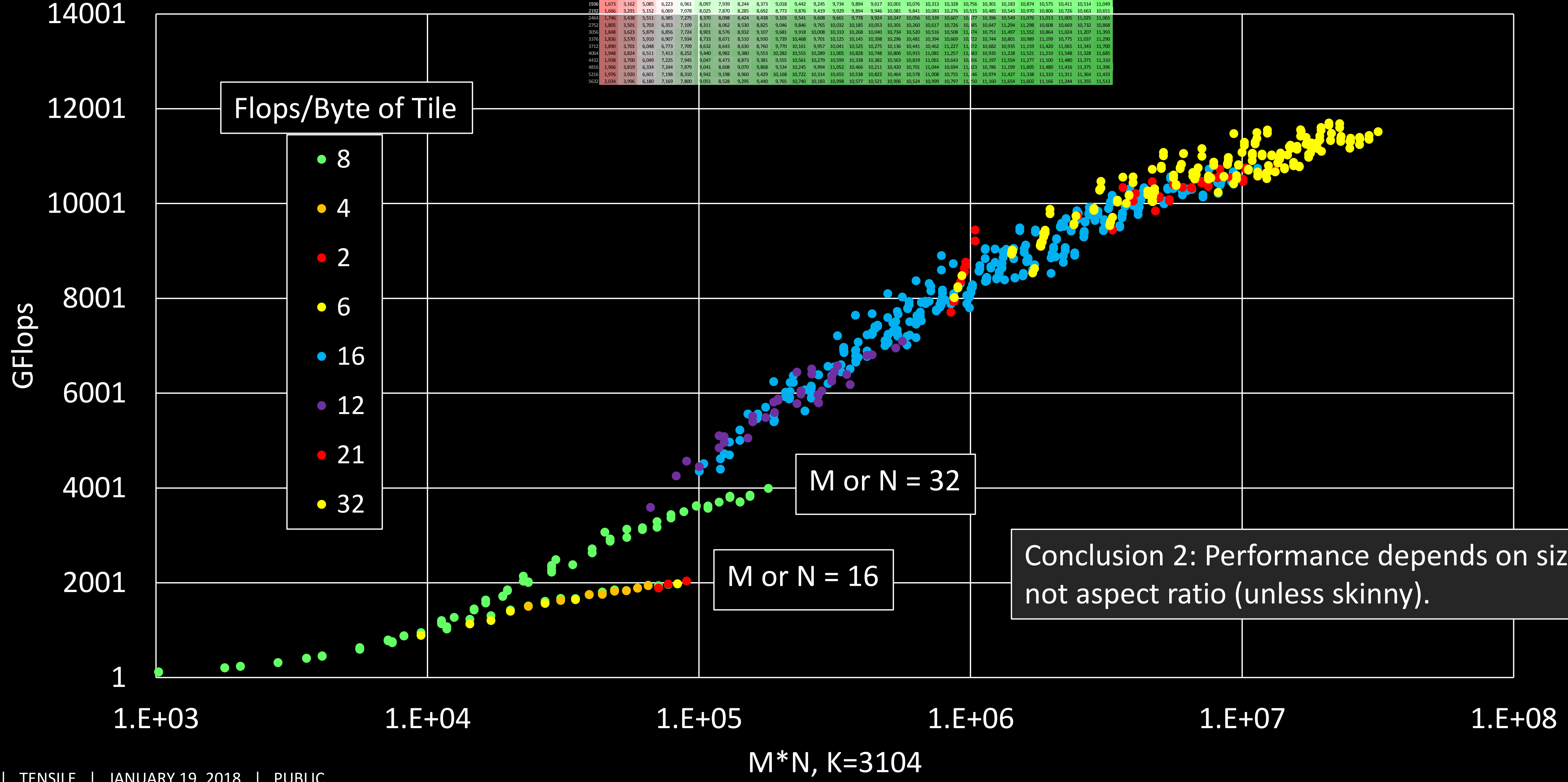
N

M	16	32	64	112	176	256	352	464	592	736	896	1072	1264	1472	1696	1936	2192	2464	2752	3056	3376	3712	4064	4432	4816	5216	563
16	8	8	8	8	8	8	8	8	6	8	6	6	6	4	6	4	6	4	4	4	4	4	4	2	2	6	2
32	8	8	8	8	8	8	8	8	8	8	6	6	6	4	6	4	6	4	4	4	4	4	4	2	2	2	2
64	8	8	8	8	8	8	8	8	6	8	6	8	8	4	6	4	8	4	4	4	6	4	4	4	4	2	2
112	8	8	8	8	8	6	8	6	6	4	4	6	6	4	4	4	6	4	2	2	4	2	2	4	2	4	4
176	8	8	8	8	6	8	6	4	4	6	6	4	4	4	2	2	4	2	4	4	2	4	2	2	1	1	2
256	8	8	8	6	4	4	4	6	6	4	4	6	4	2	4	1	4	2	1	1	2	1	1	2	1	2	1
352	8	8	8	8	8	8	6	4	4	6	2	4	2	4	4	2	2	1	2	2	2	2	2	1	1	1	2
464	8	8	6	6	8	6	4	4	4	2	4	4	2	2	2	2	2	2	2	2	1	2	1	1	2	2	1
592	8	8	6	8	6	6	4	6	4	2	4	4	4	2	2	2	2	2	1	1	2	2	2	1	1	1	2
736	8	8	8	8	6	4	6	2	2	4	4	4	2	2	2	2	1	2	2	1	1	1	1	2	2	1	1
896	6	6	8	8	6	8	8	6	6	4	4	1	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1
1072	8	6	8	6	4	6	6	4	4	4	1	2	2	2	1	2	2	2	2	1	2	1	1	1	1	1	1
1264	6	6	8	6	4	4	2	2	4	1	2	2	2	1	2	2	1	2	2	1	1	1	1	2	1	2	1
1472	6	4	8	4	6	2	6	2	1	2	4	2	1	2	1	1	1	2	1	2	1	1	1	1	1	1	1
1696	6	6	6	6	6	4	6	2	4	4	1	1	2	1	2	2	1	1	1	1	1	1	1	1	2	1	1
1936	6	6	6	4	4	2	2	2	2	2	1	2	2	1	2	1	1	1	1	1	1	1	1	1	1	1	1
2192	6	4	8	6	6	4	2	2	2	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2464	4	4	6	4	2	2	4	2	2	1	1	2	2	2	2	1	1	1	1	2	1	1	1	1	1	1	1
2752	4	4	4	2	4	4	4	2	1	2	2	2	2	1	1	1	1	1	1	1	1	1	1	2	2	2	1
3056	4	4	4	4	4	1	2	2	1	1	2	1	1	2	1	1	1	2	1	1	1	1	1	1	1	1	1
3376	6	6	4	6	2	2	2	2	2	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3712	8	6	4	2	4	1	2	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4064	4	4	4	2	2	1	2	1	2	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4432	8	2	6	4	2	2	1	1	1	2	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4816	4	2	4	2	4	4	2	2	1	2	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1
5216	6	2	2	4	2	4	1	2	1	1	2	1	2	1	1	1	1	1	2	1	1	1	1	1	1	1	1
5632	2	2	4	1	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

PERFORMANCE VS M*N

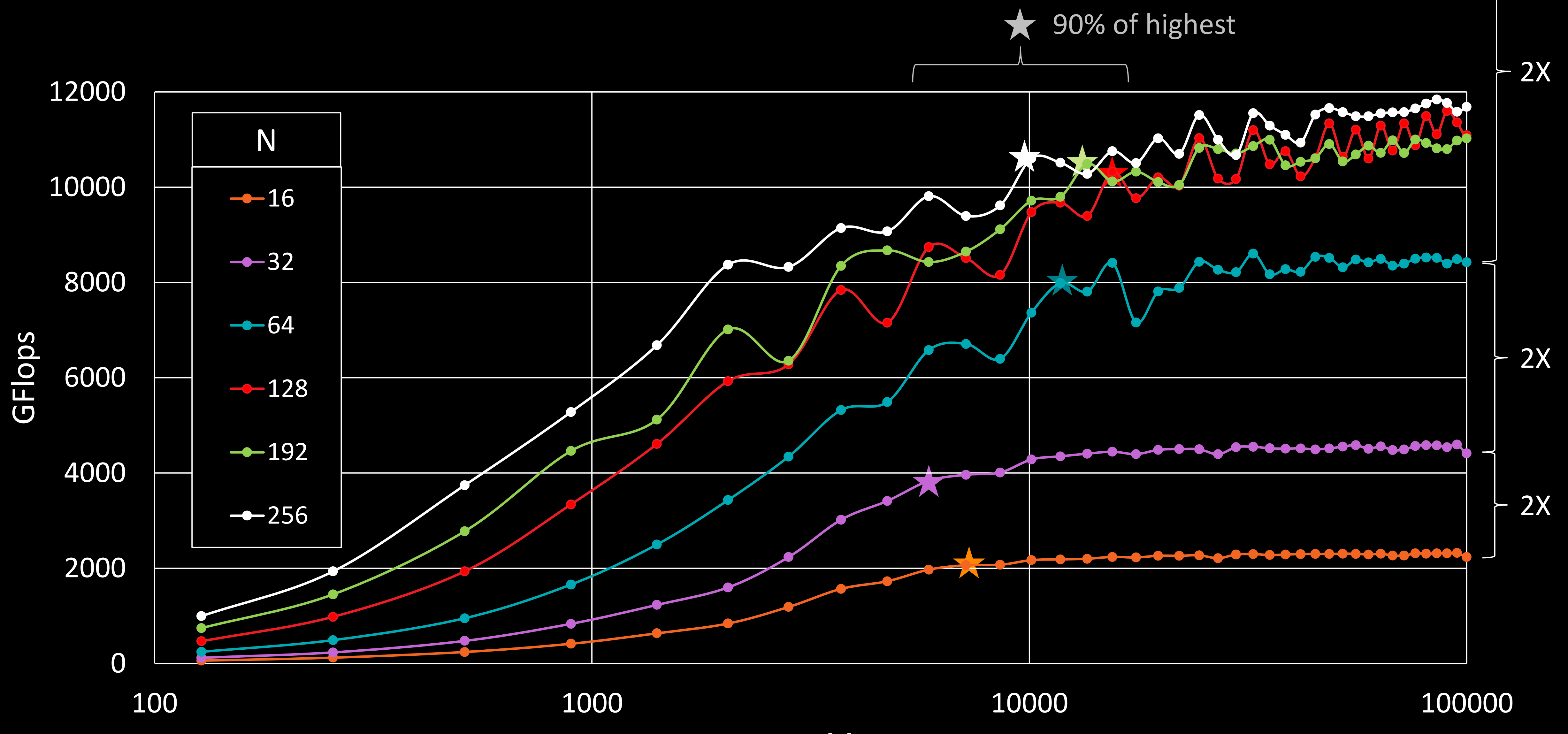
M \ N	16	32	64	112	176	256	352	464	592	736	896	1072	1264	1472	1696
16	29	62	118	205	314	448	596	735	890	1,024	1,131	1,204	1,401	1,502	1,562
32	60	116	238	406	615	880	1,137	1,424	1,722	2,006	2,271	2,304	2,628	2,877	2,953
64	121	236	452	774	1,168	1,635	2,030	2,402	2,785	3,173	3,539	3,625	4,206	4,506	4,723
112	206	409	790	1,270	1,831	2,365	2,758	3,275	3,590	3,989	4,353	4,398	5,002	5,487	5,422
176	316	633	1,201	1,848	2,404	3,178	3,457	4,012	4,506	4,695	5,479	5,397	6,223	5,894	6,204
256	456	878	1,575	2,342	3,065	4,021	4,271	4,847	5,052	5,810	5,780	5,968	6,579	6,898	6,811
352	604	1,186	2,141	2,971	3,566	4,566	4,720	5,458	5,924	6,001	6,555	6,683	7,363	7,298	7,224
464	755	1,447	2,489	3,340	4,113	5,100	5,480	5,877	5,810	6,906	7,229	7,075	7,789	7,932	7,927
592	947	1,711	2,892	3,749	4,510	5,561	6,022	5,942	6,393	7,288	7,512	7,550	7,734	8,013	8,120
736	1,073	2,018	3,415	4,254	4,966	6,241	6,099	6,967	7,254	7,666	7,913	8,111	8,476	8,693	8,424
896	1,228	2,225	3,755	4,450	5,394	6,442	6,447	6,794	6,950	7,527	7,974	8,695	8,841	8,860	9,428
1072	1,304	2,380	3,895	4,616	5,536	6,392	6,786	7,242	7,635	8,173	8,763	8,442	9,037	9,050	9,180
1264	1,427	2,716	4,490	5,222	6,367	7,212	7,401	7,782	7,755	8,474	9,031	9,012	9,125	9,373	9,506
1472	1,509	2,924	4,700	5,562	6,147	7,643	7,476	7,889	7,948	8,619	8,981	8,968	9,299	9,585	9,790
1696	1,608	3,128	4,896	5,594	6,562	7,673	7,503	8,015	8,196	8,409	9,484	9,100	9,521	9,792	9,641
1936	1,673	3,162	5,085	6,223	6,961	8,097	7,939	8,244	8,373	9,018	9,442	9,245	9,734	9,894	9,617
2192	1,666	3,291	5,152	6,069	7,078	8,025	7,870	8,285	8,692	8,773	9,876	9,419	9,929	9,894	9,946
2464	1,746	3,438	5,511	6,385	7,275	8,370	8,098	8,424	8,438	9,103	9,541	9,608	9,661	9,778	9,924
2752	1,805	3,501	5,703	6,353	7,109	8,311	8,062	8,530	8,825	9,046	9,846	9,765	10,032	10,185	10,053
3056	1,848	3,623	5,879	6,856	7,724	8,901	8,576	8,932	9,107	9,681	9,918	10,008	10,310	10,268	10,040
3376	1,836	3,570	5,910	6,907	7,934	8,733	8,671	8,510	8,930	9,739	10,468	9,701	10,125	10,145	10,398
3712	1,890	3,701	6,048	6,773	7,709	8,632	8,643	8,630	8,760	9,770	10,161	9,957	10,041	10,525	10,275
4064	1,948	3,824	6,511	7,413	8,252	9,440	8,982	9,380	9,553	10,282	10,555	10,289	11,005	10,828	10,748
4432	1,938	3,700	6,049	7,225	7,945	9,047	8,473	8,873	9,381	9,555	10,561	10,279	10,599	10,338	10,382
4816	1,966	3,819	6,334	7,244	7,879	9,041	8,608	9,070	9,868	9,534	10,245	9,994	11,052	10,466	10,211
5216	1,976	3,920	6,601	7,198	8,310	8,942	9,198	8,960	9,429	10,168	10,722	10,314	10,655	10,538	10,822
5632	2,034	3,996	6,180	7,169	7,800	9,051	8,528	9,295	9,440	9,765	10,740	10,183	10,998	10,577	10,521

M \ N	1936	2192	2464	2752	3056	3376	3712	4064	4432	4816	5216	5632
16	1,625	1,646	1,750	1,754	1,823	1,833	1,887	1,937	1,890	1,976	1,977	2,036
32	3,120	3,171	3,365	3,452	3,617	3,619	3,654	3,800	3,710	3,848	3,896	3,992
64	4,959	4,936	5,146	5,483	5,846	5,644	5,979	6,404	5,869	6,251	6,449	6,510
112	6,033	5,625	5,791	6,362	6,879	6,652	6,891	7,436	7,058	7,203	7,016	7,458
176	6,908	6,755	7,237	7,001	7,332	7,864	7,630	8,156	7,868	7,704	8,343	8,023
256	7,593	7,095	7,458	7,936	8,597	8,012	8,558	9,211	8,357	8,761	8,387	8,835
352	7,899	7,811	7,934	7,886	8,573	8,636	8,741	9,013	8,483	8,534	9,188	8,854
464	8,217	8,232	8,403	8,562	8,950	8,526	8,708	9,437	8,882	9,007	8,905	9,378
592	8,403	8,677	8,430	8,848	9,118	8,947	8,898	9,573	9,414	9,861	9,461	9,707
736	9,010	8,796	9,114	9,079	9,678	9,715	9,801	10,293	9,534	9,505	10,182	9,853
896	9,395	9,785	9,516	9,655	9,758	10,327	10,173	10,338	10,442	10,216	10,456	10,786
1072	9,259	9,446	9,620	9,836	10,038	9,745	10,054	10,334	10,281	10,016	10,399	10,338
1264	9,739	9,929	9,614	10,071	10,300	10,133	10,140	11,079	10,544	11,055	10,580	11,158
1472	9,854	9,897	9,804	10,213	10,185	10,124	10,565	10,824	10,323	10,434	10,564	10,728
1696	9,605	9,943	9,926	10,056	10,052	10,381	10,304	10,753	10,358	10,234	10,841	10,593
1936	10,001	10,076	10,313	10,328	10,756	10,301	10,183	10,874	10,575	10,411	10,514	11,049
2192	10,081	9,841	10,083	10,276	10,515	10,485	10,543	10,970	10,806	10,726	10,663	10,651
2464	10,247	10,056	10,339	10,607	10,[illegible]77	10,396	10,549	11,076	11,013	11,005	11,025	11,065
2752	10,301	10,260	10,617	10,726	10,[illegible]85	10,647	11,294	11,298	10,608	10,669	10,732	10,868
3056	10,734	10,520	10,516	10,508	11,[illegible]74	10,751	11,497	11,552	10,864	11,024	11,207	11,393
3376	10,296	10,481	10,394	10,669	10,[illegible]22	10,744	10,801	10,989	11,199	10,775	11,037	11,290
3712	10,136	10,441	10,462	11,227	11,[illegible]72	10,682	10,935	11,159	11,420	11,065	11,343	11,700
4064	10,806	10,915	11,081	11,257	11,[illegible]83	10,935	11,228	11,521	11,210	11,548	11,328	11,685
4432	10,563	10,819	11,061	10,643	10,[illegible]56	11,197	11,554	11,277	11,100	11,480	11,371	11,310
4816	10,420	10,701	11,044	10,694	11,[illegible]23	10,786	11,199	11,605	11,480	11,416	11,375	11,396
5216	10,464	10,578	11,008	10,755	11,[illegible]46	10,974	11,427	11,338	11,310	11,311	11,364	11,433
5632	10,906	10,524	10,999	10,797	11,[illegible]50	11,160	11,654	11,602	11,166	11,244	11,355	11,513



Conclusion 2: Performance depends on size, not aspect ratio (unless skinny).

LARGE & SKINNY



K = 3104

LARGE & SKINNY - 90% OF HIGHEST

MINIMUM PROBLEM SIZE FOR HIGH PERFORMANCE

AMD

N	TFlops	90%	M	MT	Flops/Byte	# WG	WG/CU	CU Occ
16	2.3	2.0	7168	64x32	10.6	112	1.75	4
32	4.3	3.8	5888	32x32	8	184	1.3	8
64	8.7	8.0	11776	128x32	12.8	184	1.3	4
128	11.5	10.3	15488	64x128	21.3	242	3.8	3
192	11.0	10.5	13568	64x64	16	636	5.6	4
256	11.8	10.6	10112	128x64	21.3	316	4.9	3

"skinny"

Not "skinny"

Large Tile

Overfill CUs

Conclusion 3: To achieve peak performance, need sufficient work to fill GPU with large tiles.

Conclusion 4: Performance of small or skinny sizes bottle-necked by L2 bandwidth.

Vega10 @ 27.3 Flops / byte =

(13200 Gflop/sec) / (483 Gbyte/sec)

L2 is 2X faster

SMALL MXN LARGE K

N

M	64	128	192	256	320	384	448	512
64	4483	6171	6703	7411	6678	7831	7693	7805
128	6126	9009	8219	10818	7979	9139	9941	11316
192	6657	8971	8077	10675	7318	8529	9542	8735
256	7398	10930	8519	11310	9395	11177	10142	11595
320	7081	9745	8275	10822	8587	9384	9595	10888
384	7799	11516	8702	11409	9370	11237	10132	11568
448	7782	11611	8830	10867	9842	11788	10341	10897
512	8235	11500	8902	11599	9837	11675	10190	11630

Conclusion 5: For peak, MxN has to be at least 128x256, and M*N*K has to be sufficiently large (~3000^3).

K = 1,048,576

FUTURE WORK

- ds_read2
- Buffer loads, rather than flat loads, would use fewer vgprs for some important cases.
- Image loads could handle out of bounds better than current vector-shifting.
- Increase sampling using Monte Carlo style search, rather than brute force.
- Support higher LocalSplitU by not requiring all threads to write.
- Support more tile/unroll combination by not requiring all threads to participate in global reads.
- Don’t use LDS at all? Thumbs down.
- Is dpp useable? focus on 2x2 threads; low priority, only eliminates ~8 vgprs.
- Greedy A,B w/ 1wg/CU; thin rows, store A or B in registers; requires broadcast
- Greedy A,B w/ 1wg/CU; store A or B in LDS and keep there while iterating over tile assignments

BACKUP SLIDES

IN CASE THE VARSITY SLIDES JUST WEREN'T ENOUGH

TERMS

- Compute-Unit (CU)
- Local Data Share (LDS)
- Vector General-Purpose Register (VGPR)
- Instruction-Level Parallelism (ILP)
  - issue high latency memory operation
  - instructions independent of memory op
  - wait for memory op
  - instructions dependent on memory op

SUPPORTED PROBLEM TYPES

- Example:

$$C_{ijk} = \alpha \sum_{lmn} A_{inlkm} * B_{mjlkn} + \beta C_{ijk}$$

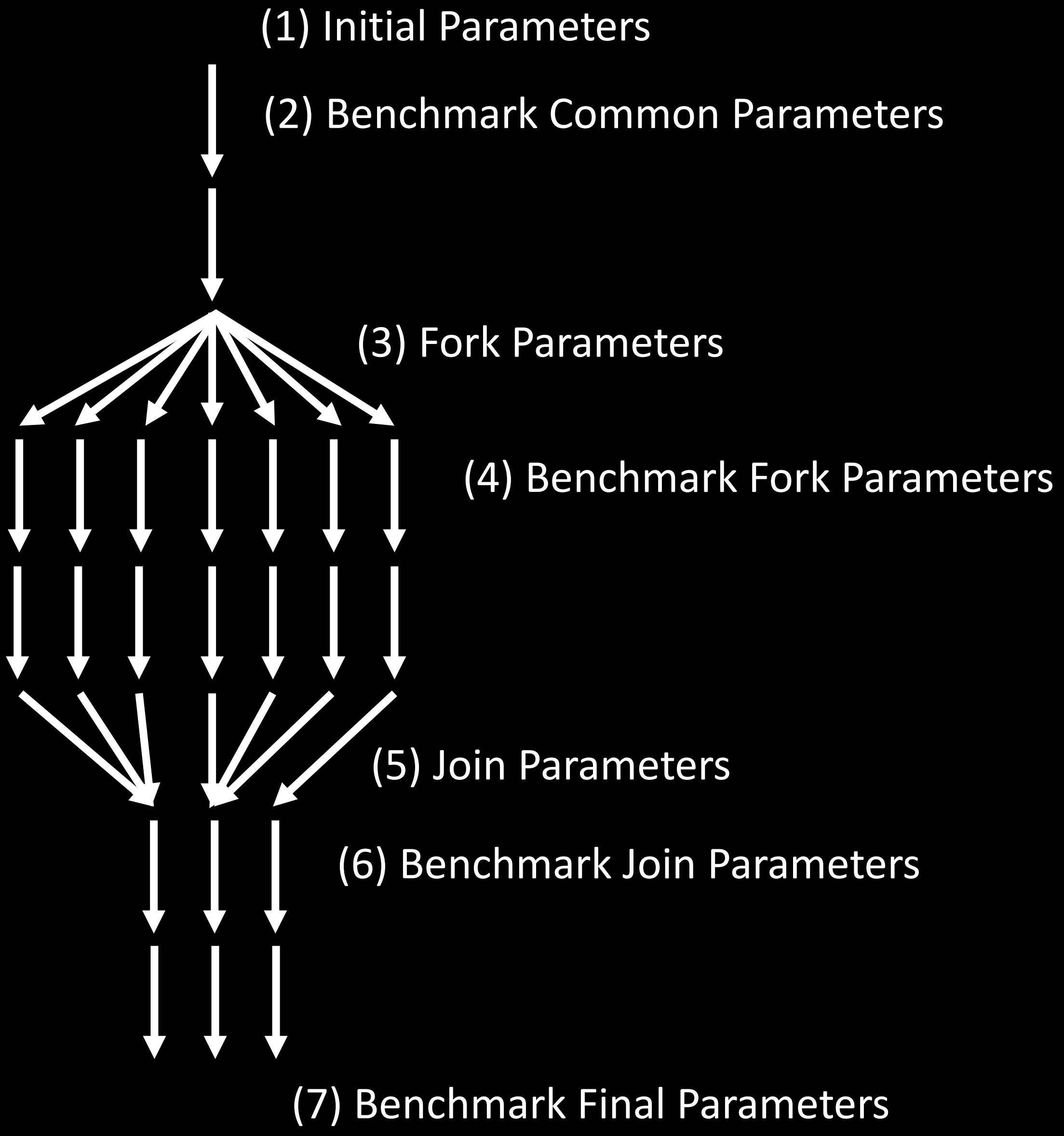
- Tensor indices are ordered shortest to largest stride, i.e., the zeroth index/dimension has a stride of 1 (typically).
- C indices are labeled alphabetically starting with “i”.
- Summation indices are labeled alphabetically picking up where C indices left off.
- A, B indices are then labeled according to which summation or C index they correspond to.
- Tensile kernel will employ:
  – Enough work-groups to cover all the dimensions of C
  – Enough nested loops to carry out the multi-dimensional summation.
- A kernel for a given problem type will give correct answer for any problem sizes.

GENERAL KERNEL-LEVEL STRATEGY

OTHER

- High CU-Occupancy and prefetching to prevent latencies from being bottleneck
- High GPU-Occupancy to create enough parallel work to fill all CUs
- Eliminate divergence from summation loop

AUTO-TUNING

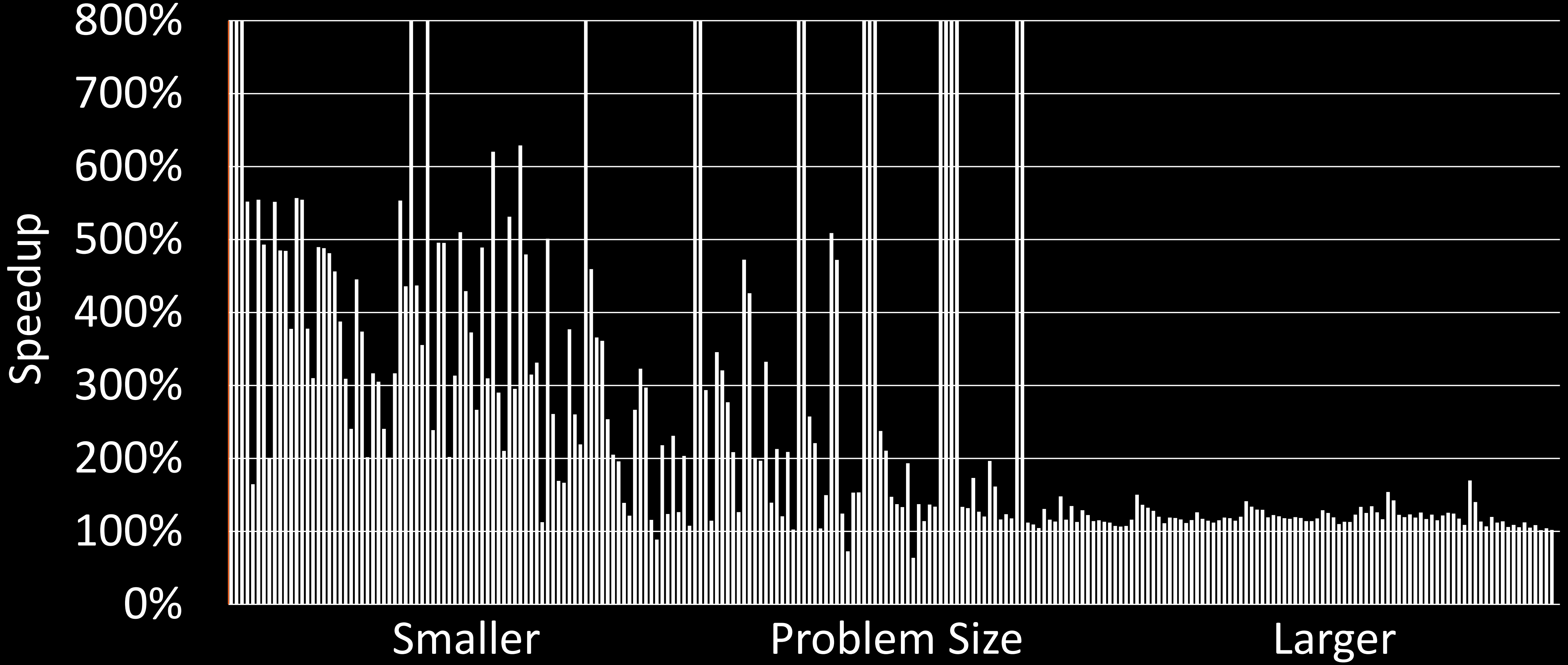


- Steps 1-6 create a set of candidates which will be benchmarked against all problem sizes.
- User can limit final kernels to keep backend library size manageable. Otherwise each problem size could have own custom kernel.
- User can create multiple sets of candidates and benchmark each against a set of problem sizes.
  – small/skinny tiles for small/skinny problem sizes
  – large tiles for large problem sizes

EXPERIMENT 2 - DEEPBENCH

- Set of thousands of kernels.
- Benchmark kernels against ~250 problem sizes of DeepBench.
- Analyze speedup of kernel tuned for exact problem size vs “fastest” 128x128 tile kernel.

SPEEDUP OF DEEPBENCH SIZES



100% means equal performance

HGEMM - TFLOPS

											896	1072	1264	1472	1696	1936	2192	2464	2752	3056	3376	3712	4064	4432	4816	5216	5632	6064	6512	6976	7456	7952	8464	8992	9536	10096	10672	11264	11872	12496	13136	13792	14464	15152	15856	16576	17312	18064	18832	19616	20416	21232	22064
16	0.0	0.0	0.0	0.0	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.5	0.5	0.6	0.7	0.8	0.9	1.1	1.2	1.3	1.5	1.6	1.7	1.9	2.1	2.2	2.4	2.6	2.8	3.0	3.2	3.4	2.4	2.6	2.7	2.9	3.0	3.2	3.4	3.5	3.7	3.9	4.1	4.1	4.2	3.4	3.6	3.7	3.8	4.0	4.1	4.2	4.2
32	0.0	0.0	0.1	0.1	0.2	0.2	0.3	0.4	0.5	0.6	0.8	0.9	1.1	1.3	1.5	1.7	1.9	2.1	2.4	2.6	2.9	3.2	3.5	3.8	4.1	4.5	4.8	5.1	5.5	5.9	6.3	6.7	4.8	5.1	5.5	5.8	6.1	6.4	6.8	7.1	7.4	7.8	8.1	8.3	8.4	6.9	6.4	7.4	7.6	7.9	8.2	8.4	8.4
64	0.0	0.1	0.1	0.2	0.3	0.4	0.6	0.8	1.0	1.3	1.6	1.8	2.2	2.5	2.9	3.4	3.8	4.2	4.8	5.3	5.8	6.4	7.0	7.6	8.3	9.0	9.6	10.4	11.1	11.8	12.5	13.5	9.6	10.1	10.8	11.3	12.0	12.8	13.4	14.1	14.7	15.4	16.3	16.4	16.8	13.6	14.1	14.7	15.3	15.9	16.2	16.6	16.9
112	0.0	0.1	0.2	0.3	0.5	0.8	1.1	1.4	1.8	2.3	2.7	3.3	3.8	4.4	5.2	5.9	6.6	7.5	8.4	9.2	10.1	11.2	12.2	8.8	9.6	10.4	11.2	12.1	12.9	13.9	14.8	15.8	12.3	13.0	13.7	14.6	15.4	16.2	17.1	14.1	14.8	15.6	16.3	17.1	17.9	15.4	16.1	16.7	17.4	18.1	18.9	16.7	17.3
176	0.1	0.2	0.3	0.5	0.8	1.2	1.7	2.2	2.8	3.6	4.3	5.1	6.0	7.0	8.1	9.2	10.3	11.6	8.7	9.6	10.5	11.6	12.7	13.8	15.0	16.2	12.8	13.7	14.7	15.7	16.9	18.0	15.0	16.0	16.9	17.9	18.9	16.4	17.3	18.1	19.1	17.0	17.9	18.7	19.6	17.8	18.6	19.3	20.2	18.6	19.3	20.1	18.7
256	0.1	0.2	0.4	0.8	1.2	1.8	2.4	3.2	4.2	5.1	6.2	7.5	8.7	10.3	11.6	13.3	10.0	11.2	12.5	13.8	15.2	17.0	18.3	14.6	15.8	17.1	18.5	19.9	16.7	18.0	19.2	20.5	17.9	19.0	20.2	21.3	19.2	20.2	21.3	19.5	20.5	21.5	20.0	20.9	21.8	20.5	21.3	22.3	21.0	21.9	22.8	21.7	22.5
352	0.2	0.3	0.6	1.1	1.7	2.4	3.4	4.5	5.7	7.0	8.5	10.2	12.1	9.3	10.6	12.1	13.6	15.4	12.5	13.9	15.3	16.8	18.4	15.8	17.1	18.6	16.4	17.7	18.9	17.2	18.4	19.6	18.2	19.1	20.5	19.1	20.2	19.1	20.2	19.2	20.2	19.4	20.3	19.6	20.5	19.9	20.8	20.2	21.0	20.5	21.3	20.8	20.4
464	0.2	0.4	0.8	1.4	2.2	3.2	4.5	5.9	7.4	9.2	11.3	8.9	10.4	12.1	14.0	15.9	13.1	14.7	16.4	18.2	15.8	17.3	19.0	17.0	18.4	17.0	18.3	19.7	18.4	19.7	18.6	19.8	19.0	20.1	19.3	20.5	19.8	20.8	20.2	19.8	20.8	20.3	19.9	20.9	20.5	20.2	21.1	20.9	20.6	20.5	21.3	21.1	21.0
592	0.3	0.5	1.0	1.8	2.8	4.2	5.7	7.4	9.5	11.7	9.4	11.2	13.2	15.4	12.9	14.7	16.7	14.8	16.4	18.2	16.6	18.2	19.9	18.4	20.0	18.8	20.3	19.3	20.7	19.9	19.3	20.6	20.0	19.6	20.7	20.4	20.1	21.1	20.9	20.6	20.5	21.5	21.3	21.2	21.1	21.1	21.0	20.9	21.0	21.0	21.9	21.9	21.9
736	0.3	0.6	1.3	2.2	3.5	5.1	7.0	9.2	11.8	9.6	11.6	14.0	16.5	13.9	16.0	18.3	16.3	18.3	16.8	18.6	20.5	19.2	20.9	19.8	19.1	20.6	20.0	21.5	20.9	20.5	21.9	21.5	21.2	21.0	22.3	22.1	21.9	21.8	21.8	21.7	21.7	21.8	21.8	21.9	22.0	22.1	22.2	22.3	22.4	22.6	22.7	22.2	22.4
896	0.4	0.8	1.6	2.7	4.3	6.2	8.6	11.2	9.5	11.7	14.3	17.1	14.5	17.0	19.5	17.5	19.8	18.2	20.4	19.2	21.2	20.3	22.2	21.4	20.8	22.5	22.1	21.7	21.4	23.0	22.7	22.6	22.5	22.4	22.5	22.5	22.6	22.7	22.8	22.9	23.1	23.3	23.4	23.6	23.0	23.2	23.5	23.7	23.2	23.5	23.8	23.4	23.7
1072	0.5	0.9	1.9	3.2	5.1	7.4	10.1	8.8	11.2	14.0	16.9	14.8	17.4	15.9	18.3	17.2	19.4	18.5	20.7	20.0	19.5	21.4	21.0	20.8	20.6	22.3	22.2	22.1	22.1	22.2	22.3	22.4	22.6	22.7	23.0	23.2	22.5	22.7	23.0	22.5	22.8	23.1	22.7	23.1	23.4	23.1	22.9	23.2	23.0	23.4	23.3	23.1	23.5
1264	0.5	1.1	2.2	3.9	6.0	8.7	12.1	10.4	13.2	16.4	14.5	17.4	16.1	18.8	17.7	20.2	19.4	19.0	21.2	20.8	20.6	20.5	22.4	22.3	22.4	22.4	22.6	22.7	23.0	22.0	22.3	22.6	22.9	22.3	22.7	23.1	22.6	23.1	22.7	23.2	22.9	23.3	23.2	23.0	23.5	23.3	23.2	23.2	23.2	23.6	23.6	23.6	23.7
1472	0.6	1.3	2.6	4.4	7.1	10.2	9.2	12.1	15.4	14.0	17.0	15.9	18.8	17.9	17.6	20.0	19.7	19.5	21.7	21.7	21.7	21.8	22.0	22.2	22.5	22.8	23.1	22.2	22.6	23.1	22.5	22.9	22.5	23.0	23.5	23.2	23.0	23.5	23.4	23.3	23.2	23.7	23.7	23.7	23.7	23.7	23.8	23.4	23.5	23.6	23.7	23.9	23.6
1696	0.7	1.5	2.9	5.1	8.1	11.7	10.7	13.9	12.9	16.1	19.5	18.3	17.7	17.5	20.2	20.0	20.0	20.2	20.4	20.7	21.0	21.4	21.8	22.3	21.5	22.0	22.6	22.0	22.6	22.3	22.8	22.6	22.4	23.1	23.0	22.9	22.9	22.9	23.0	23.0	23.1	23.2	23.4	23.1	23.2	23.4	23.2	23.4	23.3	23.5	23.4	23.3	23.5
1936	0.8	1.7	3.4	5.9	9.2	13.2	12.1	15.9	14.7	18.3	17.5	17.2	20.2	19.9	20.0	20.2	20.5	20.8	21.2	21.7	20.7	21.3	21.9	22.6	22.0	22.7	22.4	22.1	22.8	22.7	22.6	22.6	22.6	22.7	22.8	22.9	23.0	23.2	22.9	23.1	23.3	23.1	23.3	23.2	23.1	23.4	23.3	23.3	23.3	23.3	23.3	23.3	23.4
2192	0.9	1.9	3.8	6.8	10.4	9.9	13.7	13.0	16.6	16.2	19.7	19.4	19.4	19.6	20.0	20.4	21.0	21.5	20.6	21.2	22.0	22.7	22.2	21.9	22.6	22.4	22.3	22.3	23.1	22.4	22.5	22.7	22.9	23.1	22.7	23.0	23.2	23.0	23.4	23.2	23.2	23.1	23.5	23.1	23.5	23.2	23.3	23.3	23.4	23.3	23.4	23.5	23.4
2464	1.1	2.1	4.3	7.5	11.6	11.2	15.3	14.6	14.7	18.3	18.2	18.5	19.0	19.5	20.1	20.8	21.6	20.7	21.5	22.3	21.9	22.7	22.5	22.4	22.3	23.2	23.3	22.6	22.8	23.0	22.7	22.9	23.2	23.0	23.3	23.2	23.1	23.5	23.5	23.5	23.6	23.6	23.7	23.5	23.6	23.5	23.6	23.5	23.5	23.7	23.6	23.6	23.6
2752	1.2	2.4	4.8	8.4	8.7	12.5	12.5	16.4	16.4	16.8	20.4	20.7	21.2	21.8	20.4	21.3	20.5	21.5	22.5	22.1	22.0	23.0	22.9	23.0	23.1	23.2	23.5	23.0	23.3	23.6	23.4	23.2	23.1	23.6	23.5	23.6	23.6	23.7	23.8	23.9	23.7	23.9	24.0	24.0	23.9	23.8	23.8	23.8	23.9	23.9	24.0	24.0	23.9
3056	1.3	2.6	5.3	9.3	9.6	13.8	13.9	18.2	18.3	18.5	19.2	19.9	20.8	21.6	20.7	21.7	21.3	22.4	22.1	23.2	23.2	23.2	23.4	22.7	23.0	23.4	23.7	23.5	23.3	23.7	23.7	23.6	23.7	23.7	23.8	23.6	23.8	23.9	23.8	23.7	23.7	23.9	23.9	24.0	24.0	23.8	23.9	23.8	23.9	23.9	24.1	24.0	23.9
3376	1.5	2.9	5.8	10.2	10.5	15.2	15.3	15.8	16.6	20.5	21.2	19.5	20.6	21.7	21.0	20.7	21.9	21.9	22.0	23.2	22.4	22.7	23.0	23.4	23.1	23.6	23.4	23.3	23.4	23.4	23.5	23.2	23.4	23.6	23.5	23.7	23.6	23.9	23.6	23.7	23.7	23.8	23.9	23.8	24.0	23.9	23.8	23.8	23.9	23.9	23.9	24.0	23.9
3712	1.6	3.2	6.4	11.2	11.6	16.9	16.8	17.4	18.2	19.2	20.3	21.4	20.5	21.8	21.4	21.3	22.7	22.7	23.0	23.2	22.7	23.1	23.6	23.4	23.3	23.2	23.8	23.4	23.5	23.7	23.5	23.7	23.6	23.9	23.9	23.9	23.9	24.0	23.9	24.0	23.9	24.0	24.0	24.0	24.0	24.0	24.1	24.0	24.1	24.1	24.1	24.1	24.1
4064	1.8	3.5	7.0	12.1	12.6	18.4	18.5	19.0	19.9	20.9	22.1	21.0	22.4	22.0	21.8	21.9	22.2	22.5	22.9	23.4	23.0	23.6	23.4	23.4	23.5	23.6	23.7	23.5	23.7	23.6	23.5	23.5	23.6	23.6	23.8	23.9	23.8	24.0	23.9	23.9	23.9	24.0	24.0	23.9	24.1	23.9	24.0	23.8	24.0	23.9	24.0	23.9	23.8
4432	1.9	3.8	7.5	8.8	13.7	14.5	15.8	17.0	18.4	19.8	21.4	20.8	22.3	22.2	22.3	22.5	21.8	22.4	23.0	22.7	23.4	23.4	23.4	23.0	23.2	23.5	23.3	23.7	23.6	23.6	23.7	23.5	23.6	23.5	23.7	23.7	23.7	23.7	23.8	23.7	23.8	23.7	23.9	23.8	23.9	23.9	23.8	23.8	23.7	23.8	23.9	23.8	23.7
4816	2.1	4.1	8.3	9.6	14.9	15.8	17.1	18.4	20.0	19.1	20.8	20.6	22.4	22.5	21.5	22.0	22.6	22.3	23.1	23.0	23.1	23.3	23.5	23.2	23.1	23.5	23.4	23.5	23.6	23.7	23.6	23.5	23.7	23.7	23.8	23.8	23.7	23.8	23.8	23.7	23.7	23.8	23.8	23.9	23.9	23.9	23.9	23.7	23.9	23.8	23.8	23.7	23.7
5216	2.2	4.5	9.0	10.4	16.1	17.1	18.5	17.0	18.8	20.7	22.5	22.3	22.4	22.8	22.0	22.7	22.4	23.2	23.2	23.3	23.6	23.2	23.6	23.5	23.4	23.5	23.6	23.8	23.6	23.9	23.8	23.5	23.6	23.7	23.9	23.8	24.0	24.0	23.8	23.8	23.9	23.9	24.0	24.0	24.0	24.0	23.8	23.8	23.8	23.7	23.8	23.8	23.7
5632	2.4	4.8	9.7	11.2	12.8	18.5	16.4	18.3	20.3	20.0	22.0	22.2	22.6	23.1	22.6	22.4	22.4	23.3	23.5	23.7	23.4	23.8	23.7	23.3	23.4	23.6	23.9	23.8	23.7	24.1	23.8	24.0	23.9	24.0	24.0	24.0	24.1	24.2	24.1	24.1	24.0	24.1	24.1	24.1	24.0	24.0	24.0	24.0	24.0	24.0	24.0	23.8	23.8
6064	2.6	5.2	10.4	12.1	13.7	19.9	17.6	19.7	19.3	21.5	21.7	22.1	22.7	22.2	22.1	22.1	22.3	22.6	23.0	23.5	23.4	23.4	23.5	23.7	23.5	23.8	23.8	23.5	23.9	23.7	23.7	23.6	23.6	23.7	23.8	23.9	23.9	24.1	23.9	24.0	24.0	23.9	24.0	23.8	23.8	23.8	23.8	23.8	23.7	23.7	23.7	23.7	23.7
6512	2.8	5.5	11.1	12.9	14.7	16.8	18.9	18.4	20.7	20.9	21.4	22.1	23.0	22.6	22.6	22.8	23.1	22.8	23.3	23.3	23.4	23.5	23.7	23.6	23.6	23.6	23.7	23.9	23.8	24.0	23.8	23.9	24.0	23.9	24.1	24.1	23.9	24.0	24.0	23.9	23.9	23.8	23.8	23.9	23.9	24.0	23.8	23.8	23.9	23.7	23.9	23.8	23.7
6976	3.0	6.0	11.8	13.9	15.8	18.0	17.2	19.7	19.9	20.5	23.0	22.2	21.9	23.1	22.3	22.6	22.4	23.0	23.6	23.7	23.4	23.7	23.6	23.6	23.7	23.9	24.1	23.7	24.0	23.8	23.7	23.7	23.7	23.9	24.0	24.0	23.8	24.1	24.0	24.0	23.8	23.9	23.9	23.8	23.8	23.8	23.7	23.7	23.6	23.6	23.7	23.7	23.6
7456	3.1	6.3	12.6	14.8	16.9	19.3	18.4	18.6	19.3	21.9	22.7	22.3	22.3	22.5	22.8	22.6	22.5	23.2	23.4	23.6	23.5	23.5	23.5	23.7	23.5	23.8	23.8	23.6	23.8	23.7	23.7	23.5	23.7	23.8	23.8	23.9	23.8	24.0	23.8	23.7	23.7	23.8	23.8	23.6	23.7	23.7	23.6	23.5	23.5	23.5	23.7	23.6	23.5
7952	3.4	6.7	13.3	15.8	18.0	20.5	19.6	19.8	20.5	21.5	22.6	22.4	22.6	23.0	22.6	22.6	22.7	22.9	23.2	23.7	23.2	23.7	23.5	23.5	23.5	23.6	24.0	23.6	23.9	23.7	23.6	23.5	23.7	23.7	23.8	23.9	23.7	23.9	23.7	23.6	23.6	23.7	23.7	23.6	23.6	23.7	23.6	23.5	23.5	23.5	23.5	23.6	23.5
8464	2.4	4.8	9.5	12.2	15.1	17.9	18.1	18.9	20.0	21.2	22.5	22.6	22.9	22.6	22.5	22.6	22.9	23.2	23.2	23.7	23.4	23.6	23.6	23.6	23.7	23.6	23.9	23.7	24.0	23.7	23.7	23.7	23.6	23.7	23.9	23.8	24.0	23.8	23.7	23.7	23.6	23.6	23.7	23.5	23.7	23.5	23.5	23.5	23.4	23.5	23.5	23.6	23.4
8992	2.6	5.1	10.2	13.0	16.0	19.0	19.3	20.1	19.6	21.0	22.5	22.8	22.3	23.0	23.1	22.6	23.0	23.0	23.6	23.7	23.6	23.9	23.6	23.5	23.7	23.7	24.0	23.7	23.9	23.9	23.8	23.7	23.7	23.8	23.8	23.8	23.7	24.0	23.8	23.7	23.7	23.8	23.7	23.5	23.7	23.5	23.5	23.5	23.5	23.4	23.5	23.6	23.4
9536	2.7	5.5	10.8	13.7	16.9	20.1	20.4	19.3	20.7	22.2	22.5	22.9	22.7	23.5	22.9	22.7	22.7	23.3	23.5	23.8	23.4	23.9	23.8	23.7	23.7	23.8	24.0	23.8	24.0	24.0	23.8	23.8	23.9	23.7	23.8	23.9	23.7	23.9	23.8	23.7	23.7	23.7	23.7	23.6	23.8	23.6	23.6	23.5	23.6	23.5	23.6	23.6	23.5
10096	2.9	5.7	11.3	14.5	17.9	21.3	19.1	20.4	20.3	22.0	22.5	23.2	23.1	23.2	22.9	22.9	23.0	23.2	23.6	23.6	23.7	23.9	23.9	23.7	23.8	23.8	24.0	23.9	24.1	24.1	23.9	23.9	23.8	23.9	24.0	23.8	23.9	23.9	23.7	23.8	23.7	23.9	23.8	23.7	23.8	23.7	23.8	23.7	23.6	23.6	23.7	23.7	23.5
10672	3.0	6.1	12.0	15.3	18.9	19.1	20.2	19.7	20.1	21.9	22.6	22.5	22.6	23.0	22.9	23.0	23.2	23.2	23.6	23.7	23.6	24.0	23.8	23.7	23.7	24.0	24.1	23.9	23.9	23.9	23.8	23.7	23.8	23.7	23.8	23.9	23.6	23.9	23.7	23.7	23.7	23.8	23.7	23.5	23.6	23.5	23.5	23.5	23.5	23.5	23.6	23.7	23.5
11264	3.2	6.4	12.8	16.1	16.4	20.2	19.1	20.8	21.1	21.8	22.7	22.7	23.0	23.6	22.9	23.1	23.0	23.5	23.7	23.9	23.9	24.0	23.9	23.7	23.8	24.0	24.1	24.1	24.0	24.1	23.8	23.8	23.8	23.8	23.9	23.8	23.8	23.8	23.9	23.6	23.8	23.7	23.8	23.6	23.6	23.6	23.6	23.5	23.6	23.6	23.6	23.6	23.5
11872	3.4	6.8	13.4	17.1	17.3	21.2	20.1	20.2	20.9	21.7	22.8	23.0	22.7	23.4	23.0	22.8	23.3	23.5	23.8	23.8	23.6	23.8	23.9	23.8	23.7	23.8	24.2	23.9	23.9	24.0	23.8	23.8	23.7	23.8	23.8	23.7	23.7	24.0	23.6	23.7	23.6	23.8	23.7	23.6	23.7	23.6	23.7	23.6	23.6	23.7	23.6	23.7	23.4
12496	3.6	7.1	14.1	14.1	18.1	19.5	19.2	19.7	20.6	21.7	22.9	22.5	23.2	23.3	23.0	23.0	23.2	23.5	23.9	23.7	23.7	24.0	23.9	23.7	23.7	23.9	24.1	23.9	23.9	24.0	23.7	23.7	23.7	23.7	23.8	23.7	23.8	23.7	23.6	23.8	23.8	23.7	23.7	23.6	23.8	23.7	23.5	23.5	23.6	23.6	23.6	23.6	23.5
13136	3.7	7.4	14.8	14.9	19.0	20.4	20.2	20.7	20.5	21.7	23.1	22.8	22.9	23.2	23.1	23.3	23.2	23.6	23.7	23.7	23.7	23.9	23.9	23.8	23.7	24.0	24.0	24.0	23.9	23.9	23.7	23.6	23.6	23.7	23.8	23.6	23.7	23.9	23.6	23.7	23.7	23.5	23.7	23.7	23.5	23.7	23.6	23.5	23.5	23.5	23.6	23.6	23.4
13792	3.9	7.8	15.5	15.6	17.0	21.5	19.4	20.3	21.5	21.8	23.2	23.1	23.3	23.7	23.2	23.1	23.1	23.6	23.9	23.9	23.8	24.0	24.0	23.7	23.7	23.9	24.0	23.9	23.8	24.0	23.8	23.7	23.6	23.9	23.7	23.9	23.8	23.8	23.8	23.8	23.8	23.7	23.7	23.6	23.7	23.7	23.7	23.6	23.6	23.7	23.7	23.6	23.6
14464	4.1	8.2	16.2	16.4	17.8	20.0	20.3	19.9	21.3	21.8	23.4	22.7	23.1	23.7	23.4	23.3	23.4	23.7	24.1	23.9	23.9	24.0	24.0	23.8	23.8	24.0	24.2	24.0	23.8	23.9	23.8	23.7	23.7	23.7	23.8	23.7	23.7	23.9	23.7	23.7	23.6	23.7	23.6	23.6	23.7	23.7	23.6	23.6	23.6	23.6	23.7	23.7	23.4
15152	4.1	8.2	16.6	17.1	18.7	20.9	19.6	20.9	21.2	21.9	23.6	23.1	23.0	23.7	23.1	23.2	23.1	23.5	24.0	23.9	23.8	24.0	23.9	23.8	23.9	24.0	24.1	23.8	23.8	23.7	23.5	23.6	23.6	23.5	23.6	23.5	23.6	23.8	23.5	23.6	23.5	23.6	23.6	23.4	23.6	23.6	23.5	23.4	23.4	23.5	23.6	23.5	23.3
15856	4.2	8.4	16.7	17.9	19.5	21.8	20.5	20.5	21.1	22.0	23.0	23.3	23.5	23.7	23.2	23.1	23.5	23.6	23.9	24.0	24.0	24.0	24.0	23.9	23.9	24.0	24.1	23.8	23.9	23.8	23.7	23.6	23.7	23.7	23.8	23.6	23.8	23.6	23.9	23.7	23.8	23.7	23.8	23.7	23.7	23.7	23.6	23.6	23.5	23.6	23.7	23.7	23.5
16576	3.4	6.8	13.6	15.4	17.8	20.5	19.9	20.2	21.0	22.1	23.2	23.1	23.3	23.7	23.4	23.4	23.2	23.5	23.9	23.8	23.9	24.0	23.9	23.8	23.9	23.9	24.1	23.8	24.0	23.9	23.8	23.7	23.5	23.6	23.6	23.7	23.6	23.7	23.7	23.7	23.6	23.7	23.7	23.6	23.7	23.5	23.5	23.5	23.4	23.5	23.5	23.6	23.5
17312	3.6	7.1	14.2	16.0	18.6	21.3	20.7	21.1	21.0	22.2	23.5	22.9	23.2	23.8	23.2	23.3	23.2	23.6	23.8	23.9	23.8	24.1	24.0	23.7	23.8	23.8	24.1	23.8	23.8	23.7	23.6	23.6	23.5	23.5	23.6	23.8	23.5	23.7	23.7	23.5	23.6	23.6	23.5	23.6	23.6	23.5	23.5	23.5	23.4	23.4	23.5	23.5	23.5
18064	3.7	7.4	14.8	16.7	19.4	22.3	20.2	20.9	21.0	22.3	23.7	23.2	23.2	23.4	23.4	23.3	23.3	23.6	23.9	23.8	23.8	24.0	23.9	23.8	23.7	23.8	24.0	23.7	23.8	23.7	23.5	23.6	23.4	23.6	23.5	23.7	23.5	23.7	23.5	23.6	23.5	23.6	23.6	23.4	23.5	23.5	23.5	23.4	23.4	23.5	23.5	23.5	23.5
18832	3.8	7.7	15.3	17.3	20.1	21.1	21.0	20.7	21.0	22.4	23.3	23.0	23.2	23.5	23.3	23.3	23.4	23.5	23.9	23.9	23.9	24.1	23.9	23.7	23.9	23.8	24.0	23.7	23.9	23.6	23.5	23.5	23.4	23.4	23.6	23.6	23.5	23.7	23.5	23.5	23.5	23.6	23.6	23.4	23.6	23.5	23.4	23.4	23.4	23.4	23.5	23.5	23.4
19616	4.0	7.9	15.8	18.0	18.6	21.9	20.5	20.5	21.9	22.6	23.5	23.3	23.6	23.6	23.5	23.3	23.3	23.7	23.9	23.9	23.9	24.1	23.9	23.7	23.8	23.7	24.1	23.7	23.7	23.6	23.6	23.5	23.5	23.5	23.6	23.6	23.5	23.8	23.7	23.6	23.6	23.6	23.7	23.5	23.6	23.5	23.4	23.5	23.4	23.5	23.5	23.4	23.5
20416	4.1	8.1	16.3	18.7	19.3	22.8	21.3	21.3	21.9	22.7	23.8	23.2	23.6	23.7	23.4	23.3	23.4	23.7	24.0	24.1	23.9	24.1	24.1	23.8	23.8	23.8	24.1	23.8	23.9	23.7	23.6	23.5	23.5	23.6	23.6	23.8	23.7	23.8	23.6	23.6	23.6	23.7	23.6	23.5	23.7	23.5	23.5	23.4	23.5	23.5	23.5	23.6	23.4
21232	4.2	8.4	16.6	16.6	20.1	21.7	20.8	21.1	21.9	22.2	23.4	23.1	23.6	23.9	23.3	23.3	23.6	23.6	24.1	24.0	24.0	24.1	23.9	23.8	23.7	23.8	23.8	23.8	23.8	23.6	23.6	23.6	23.6	23.6	23.8	23.7	23.7	23.7	23.7	23.6	23.6	23.6	23.7	23.5	23.7	23.6	23.5	23.6	23.5	23.5	23.5	23.5	23.5
22064	4.2	8.4	16.9	17.3	18.7	22.5	20.4	21.0	21.9	22.4	23.7	23.4	23.7	23.7	23.6	23.4	23.4	23.6	23.9	23.9	23.9	24.1	23.8	23.7	23.6	23.7	23.9	23.7	23.7	23.5	23.5	23.5	23.4	23.4	23.5	23.5	23.5	23.7	23.5	23.5	23.5	23.4	23.6	23.7	23.5	23.4	23.6	23.6	23.5	23.5	23.5	23.5	23.4