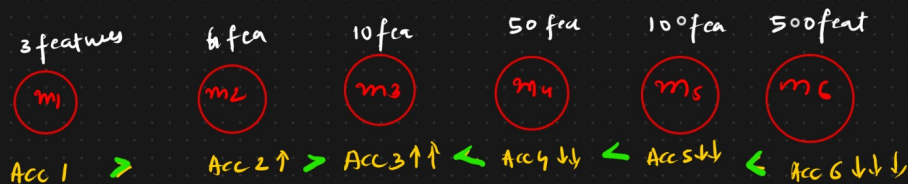


- Principal component Analysis (PCA) [dimensionality reduction]

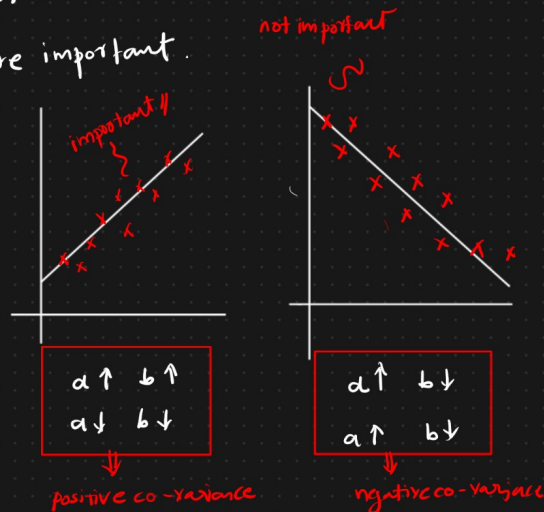
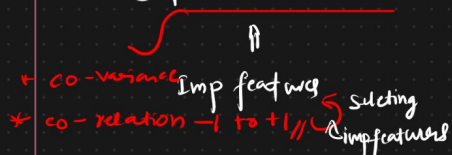
① curse of dimensionality.



- \* if you are feeding model more features or dimensions more than the model will perform bad n t knowing which is more important.

- two diff ways to remove the wire

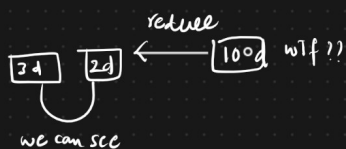
① feature selection    ② Dimension reduction (pot)



## ② Feature Selection vs Feature Extraction

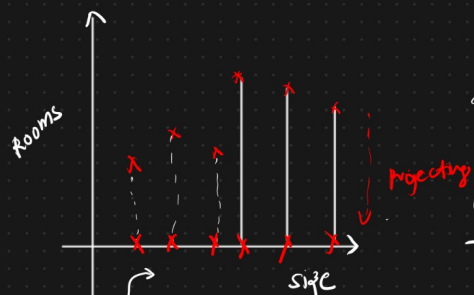
Why dimensionality reduction??

- \* prevent overfit
- \* improve the acc' ↑
- \* visualize the data



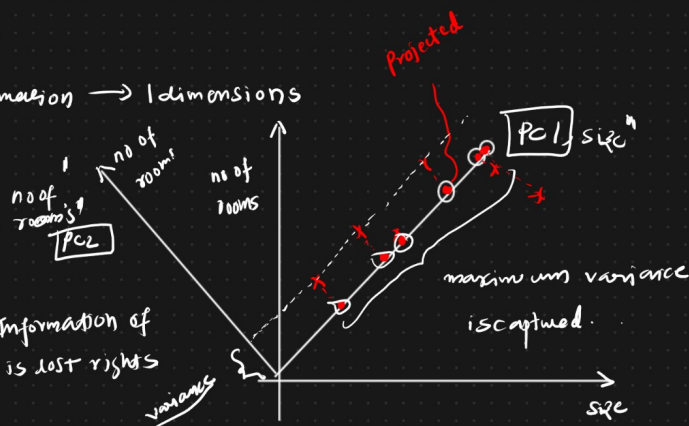
## ② PCA geometric Intuition

Size of house	No of room	Price
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PC A

2 dimension  $\rightarrow$  1 dimensions



\* So, here after projecting the datapoints the data is getting lost

\* So, using Eigen decomposition we create pc1  
after projecting it will deose into  $S_0$

## Eigen decomposition on matrix.

1) transform

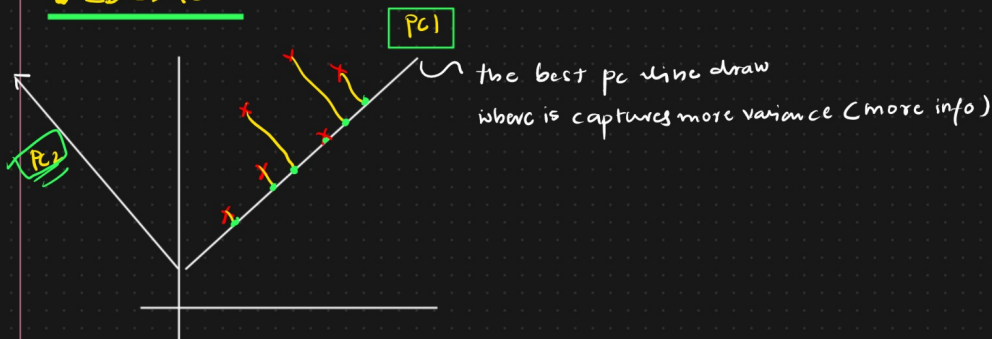
PC1  $\rightarrow$  captures the maximum variance.

PC2  $\rightarrow$  captures the second maximum variance.

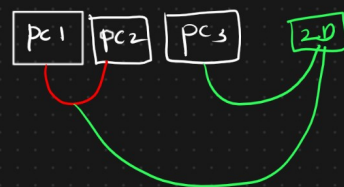
\* we create  $pc_2$  adjacent to the  $pc_1$  and this will cover second maximum variance

- 2 Dimensions  $\Rightarrow pc_1, pc_2, \text{var}(pc_1) > \text{var}(pc_2)$
- 3 Dimensions  $\Rightarrow \text{var}(pc_1) > \text{var}(pc_2) > \text{var}(pc_3)$

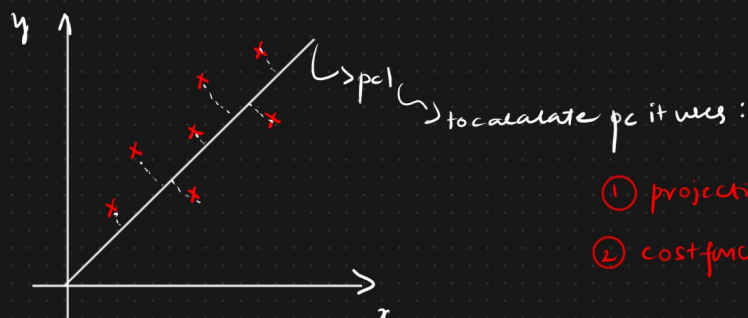
• 2D  $\Rightarrow$  1D



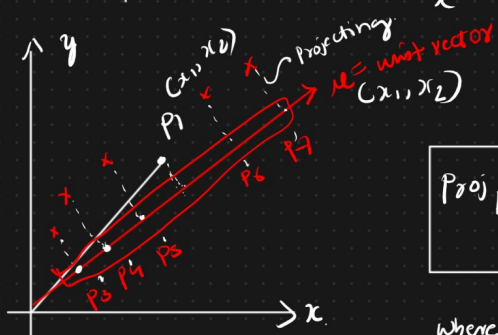
3D  $\Rightarrow$  2D



## Maths Intuition behind PCA Algorithm



- ① projection
- ② cost function (variance)



$$\text{Proj}_{P_1} u = \frac{P_1 \cdot u}{\|u\|}$$

where unit vector magnitude is "1"

$$\text{Proj}_{P_1} u = P_1 \cdot u \quad \|u\| = 1$$

$p_0, p_1, p_2, p_3, p_4$

Scalars values

- (tell's us the only distance)
- this makes it easier to calculate the variance

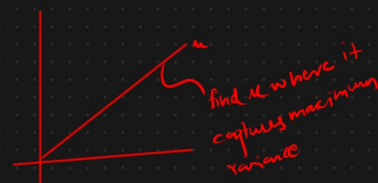
$x_0', x_1', x_2', x_3', x_4'$

cost function

$$\text{Variance} = \frac{(x_i - \bar{x})^2}{N}$$

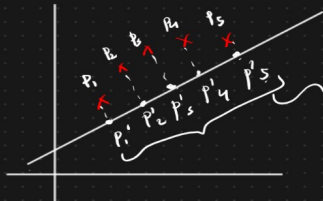
here our main goal is to find a best unit vector which captures maximum variance

about unit vector = magnitude of 1  
 $\hat{u}$  denoted by vector  $\|u\| = 1$   
 let say we have  $\vec{a} = (3, 4)$   
 magnitude  $\|\vec{a}\| = \sqrt{3^2 + 4^2} = 5$   
 to convert vector to unit vector we divide vectors with their magnitude.  
 $\sqrt{\frac{3^2}{5} + \frac{4^2}{5}} = 1$

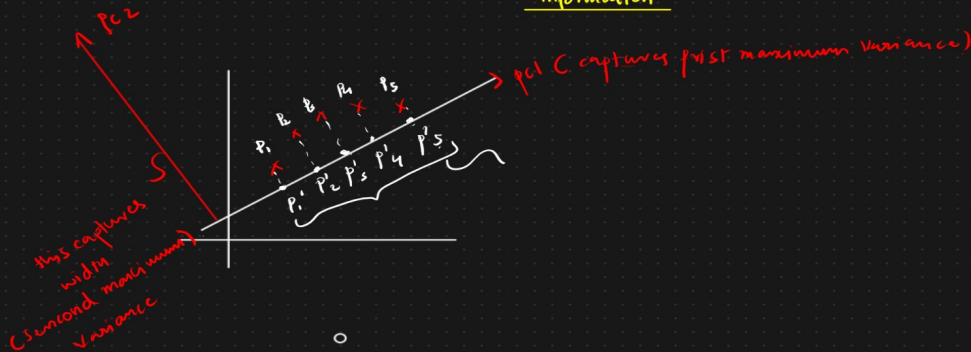


- Why do we need pc 2 or pc 3 ?? We have pc 3 right ??

let's see

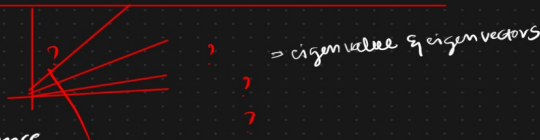


here as you can see we are capturing the only length and we are missing out the width which is nothing but loss of information



## Eigen Decomposition on Covariance Matrix

in order to find the best pc the captures the most variance we use this



- Eigen vector and eigen values [linear transformation]

[Eigen decomposition of covariance matrix]

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \text{eigen value} \times \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

matrix vector

$$\begin{bmatrix} \end{bmatrix} * \begin{bmatrix} v \end{bmatrix} = \lambda * v$$

eigen value

$$A * v = \lambda * v \Rightarrow \text{eigen vect } v \rightarrow \text{maximum magnitude}$$

(mathematically that'll capture maximum variance proven)

vector bases //

Vectors

1) vector multiplication . =

2) Scalar addition . = 2 .



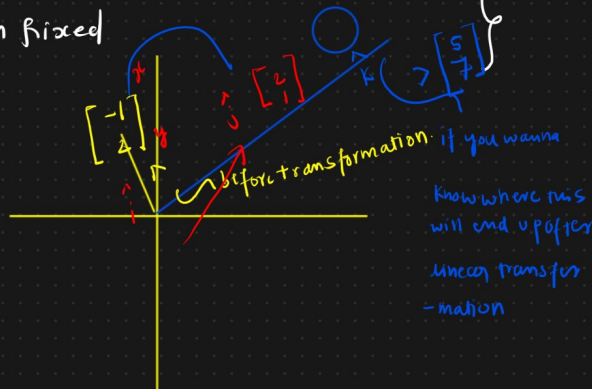


## linear transformation

function takes vector input and gives a vector output

When the lines after transformation is straight & not bent. } grids are spaced parallel and linearly  
and the origin remain fixed

numerically ??



$$= 5 \begin{bmatrix} -1 \\ 2 \end{bmatrix} + 7 \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{pmatrix} x(\hat{i}) + y(\hat{j}) \end{pmatrix}$$

## Steps to calculate eigen vector and eigen values

$$\text{cov}(x, y) = \frac{\sum_{i=1}^{n-1} (x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

$$A = \begin{matrix} & \begin{matrix} x & y \end{matrix} \\ \begin{matrix} x \\ y \end{matrix} & \begin{bmatrix} \text{var}(x) & \text{cov}(x, y) \\ \text{cov}(y, x) & \text{var}(y) \end{bmatrix} \end{matrix}$$

Annotations:  $\frac{(x - \bar{x})^2}{n-1}$  points to  $\text{var}(x)$ ;  $\frac{(y - \bar{y})^2}{n-1}$  points to  $\text{var}(y)$ .

$$A \cdot v = \lambda \cdot v$$

$$\lambda_1, \lambda_2$$

magnitude of pc1

eigen values //

= magnitude of pc2