$$(a \wedge b)\Big|_{((a\ (p \wedge q))(b\ \ a))}$$

$$\equiv ((p \wedge q) \wedge a)$$

$$\cancel{(p \wedge \neg p)\Big|_{((p\ t)(p\ nil))}}$$

Conjecture
Theorem
Axiom
Corollary
Lemma

Sat.

$$(a \wedge b)\Big|_{((a\ t)(b\ t))}$$

Falsifiable

$$(a \wedge b)\Big|_{((a\ nil)(b\ t))}$$

Instantiation

$$(p \oplus q) \wedge \neg(p \oplus q)$$

$$\equiv \{\ p \wedge \neg p \equiv nil \Big|_{((p\ (p \oplus q)))}$$

nil

Conjecture

$$(p \oplus q) \wedge \neg(p \oplus q) \equiv nil$$

$$\boxed{(\text{listp } z) \Rightarrow (\text{equal } (\text{len } (\text{cons } x\ z)) \ (\text{len } (\text{cons } y\ z)))}$$

$P \Rightarrow Q$

Proof

C.I. (listp z)

Prove $\Rightarrow$

Assume P

Prove Q

Use $\Rightarrow$

Show p is true

conclude Q

Modus Ponens

```
(defunc alen (x)
  :input-contract t
  :output-contract (natp (alen x))
  (if (atom x)
      0
      (+ 1 (alen (rest x)))))
```

]] or } or whatever ≡ magic parens

conjecture 1:

(alen x) = (alen (list x))

0 ... 1

Counter: x = 4

Conjecture 2

(alen (cons x z)) = (alen (cons y z))

Lemma 1

(alen (cons x z)) ≡ (+ 1 (alen z))

Lemma 1

LHS

(alen (cons x z))

$\equiv$ { Def alen $|_{((x\ (cons\ x\ z)))}$ }

(if (atom (cons x z))
    0
    (+ 1 (alen (rest (cons x z)))))

$\equiv$ { Def atom $|_{((x\ (cons\ x\ z)))}$ }

(if (not (consp (cons x z)))
    0
    (+ 1 (alen (rest (cons x z)))))

$\equiv$ { Def not $|_{((a\ (consp\ (cons\ x\ z))))}$ }

(if (if (consp (cons x z)) nil t) 0 (+1 (alen (rest (cons x z)))))

$\equiv$ { if axiom, consp axiom }

(if nil 0 (+1 (alen (rest (cons x z)))))

$\equiv$ { if axiom }

(+1 (alen (rest (cons x z))))

$\equiv$ { first-rest axioms }

$\boxed{\text{(+1 (alen z))}}$ $\Leftarrow$ RHS

$\boxed{\text{LHS = RHS Q.E.D.}}$

RHS (alen (cons y z))

$\equiv$ {

LHS of conjecture

(alen (cons x z))

$\equiv$ { Lemma 1 $|_{((x\ x))}$ }

(+1 (alen z))

RHS

(alen (cons y z))

$\equiv$ { Lemma 1 $|_{((x\ y))}$ }

(+1 (alen z))

LHS = RHS Q.E.D.

Reflexivity

$$X \equiv X$$

Symmetry of Equality

$$(a = b) \Rightarrow (b = a)$$

Transitivity of Equality

$$(a = b) \land (b = c) \Rightarrow (a = c)$$

if axioms

$$(x = nil) \Rightarrow (if\ x\ y\ z) \equiv z$$

$$(x \neq nil) \Rightarrow (if\ x\ y\ z) \equiv y$$

t

consp axiom

$$(consp\ (cons\ x\ y)) \equiv t$$

first-rest axioms

$$(first\ (cons\ x\ y)) \equiv x$$

$$(rest\ (cons\ x\ y)) \equiv y$$

```
(defunc f (x1 x2 ... xn)
  :input-contract (and (R1 x1) (R2 x2) ... (Rn xn))
  :output-contract (R (f x1 x2 ... xn))
  <body>)
```

$IC \Rightarrow OC$ contract theorem eg $(\text{and } (R_1\ x_1) \ldots (R_n\ x_n)) \Rightarrow (R\ (f\ x_1 \ldots x_n))$

$IC \Rightarrow ((f\ x_1\ x_2 \ldots x_n) = \langle body \rangle)$

Equality Axiom Schema for Functions

$x_1 = y_1 \wedge x_2 = y_2 \ldots x_n = y_n \Rightarrow$

$(f\ x_1 \ldots x_n) = (f\ y_1 \ldots y_n)$

(equal l1 l2) $\equiv$ (l1 = l2) $\Leftarrow$ Shortcut!

$$(\text{listp } x) \wedge (\text{listp } y) \wedge (\text{listp } z) \cancel{\Rightarrow} \wedge$$

Exportation

$$(\text{endp } x) \Rightarrow (\text{app } (\text{app } x\ y)\ z) = (\text{app } x\ \overbrace{(\text{app } y\ z)}^{\text{"y"}}))$$

C1. (listp x)
C2. (listp y)
C3. (listp z)
C4 (endp x)

General context

C5. (x = nil) {C1, C4, Def listp}

① LHS = x
RHS = x
LHS = RHS

② LHS = RHS
x = x

③ LHS = RHS

(app (app x y) z)

≡ {Def of app, if axiom, C4}

(app $\boxed{y}$ z)

≡ {if axiom | ((x (endp x)) (y (app y z)) (z (cons (first x) (app (rest x) (app y z)))), C4}

(if (endp x)
    (app y z)
    (cons (first x) (app (rest x) (app y z))))

≡ {Def app | ((y (app y z)))}

(app x (app y z))