nclab
Online STEM Laboratory

Electrostatics Module

Based on Hermes2D (`http://hpfem.org/hermes`)

1 Module Description

The electrostatics module calculates the distribution of the electric potential φ induced by stationary electric charges. For charged objects one can specify either their voltage or the surface charge density. Various objects and/or subdomains can have different values of the relative electric permittivity ϵ.

The example below shows a 2D model of an oscilloscope that consists of two electrodes (in the left part of the image) and a metallic screen (on the right). The voltage on the upper and lower electrode is 2000 V and 0 V, respectively, The screen has zero surface charge density σ and electric permittivity $\epsilon = 2$. Electric permittivity of the surrounding air is $\epsilon = 1$.

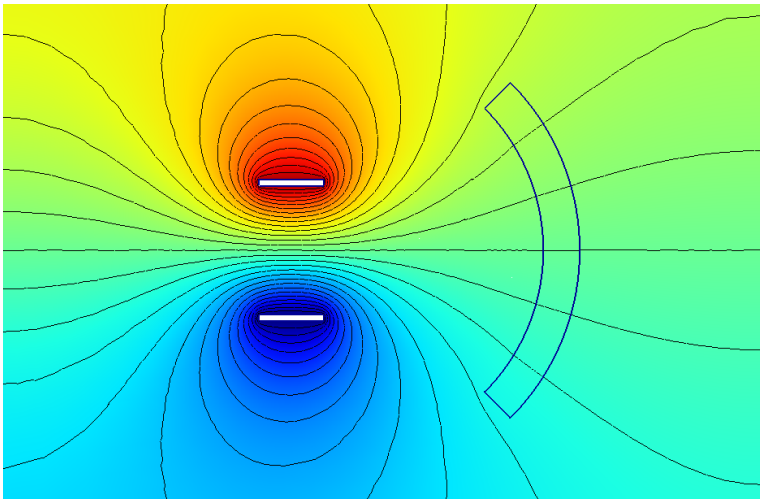

Figure 1: Electric field of an oscilloscope.

2 Underlying Equations

The equation for the electric potential φ is

$$-\mathrm{div}(\epsilon \nabla \varphi) = \varrho$$

where ϱ is electric charge density. Once the electric potential φ is calculated, the electric field vector E can be obtained as its negative gradient,

$$E = -\nabla \varphi.$$

3 Boundary Conditions

The following two types of boundary conditions are typical for electrostatics calculations:

- *Fixed voltage*: $\varphi = \varphi^*$ where φ^* is a constant. Various objects in the arrangement can have different voltages. Grounded objects usually have $\varphi = 0$ V (such as the lower electrode in the above example).
- *Surface charge density*: $\sigma = \sigma^*$ where σ^* is a constant. Various objects in the arrangement can have different values of σ. Objects that do not carry any surface charges have $\sigma = 0$ C/m^2 (such as the screen in the above example).