

# Electrostatics Module

Based on Hermes2D (<http://hpfem.org/hermes>)

## 1 Module Description

The electrostatics module calculates the distribution of the electric potential  $\varphi$  induced by stationary electric charges. For charged objects one can specify either their voltage or the surface charge density. Various objects and/or subdomains can have different values of the relative electric permittivity  $\epsilon$ .

The example below shows a 2D model of an oscilloscope that consists of two electrodes (in the left part of the image) and a metallic screen (on the right). The voltage on the upper and lower electrode is 2000 V and 0 V, respectively, The screen has zero surface charge density  $\sigma$  and electric permittivity  $\epsilon = 2$ . Electric permittivity of the surrounding air is  $\epsilon = 1$ .

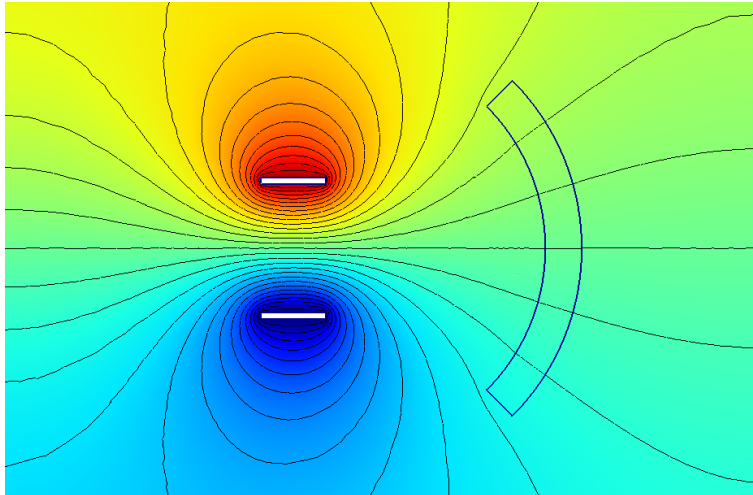


Figure 1: Electric field of an oscilloscope.

## 2 Underlying Equations

The equation for the electric potential  $\varphi$  is

$$-\text{div}(\epsilon \nabla \varphi) = \varrho$$

where  $\varrho$  is electric charge density. Once the electric potential  $\varphi$  is calculated, the electric field vector  $E$  can be obtained as its negative gradient,

$$E = -\nabla \varphi.$$

### 3 Boundary Conditions

The following two types of boundary conditions are typical for electrostatics calculations:

- *Fixed voltage:*  $\varphi = \varphi^*$  where  $\varphi^*$  is a constant. Various objects in the arrangement can have different voltages. Grounded objects usually have  $\varphi = 0$  V (such as the lower electrode in the above example).
- *Surface charge density:*  $\sigma = \sigma^*$  where  $\sigma^*$  is a constant. Various objects in the arrangement can have different values of  $\sigma$ . Objects that do not carry any surface charges have  $\sigma = 0$  C/m<sup>2</sup> (such as the screen in the above example).