Stats 504 Assignment 4: New Taipei Housing Litigation

Introduction

The real estate business involves the buying and selling of houses, and there are a variety of factors that influence the price of a house. Due to the abstract nature of these factors, it can be tricky to accurately estimate the price of any house and this opens the door for malicious practices with respect to housing fraud. This report aims to convey the reasoning behind building a statistical model which can accurately predict the price of a house (in Taipei) given information regarding the house. The report (and appendix) is also meant to serve as an aid to Taiwanese law firms which are investigating cases of housing fraud, so as to enable them to gauge whether houses were truly sold for a reasonable price or not. The model was built while keeping in mind the tradeoff between an easily understandable model and a complex but accurate one. The nuances of this model are presented in the following sections. Following the analysis, an appendix with computer code is also made available so that the client may reproduce results, predict housing prices for specific suspect houses, as well as present for investigation by opposing counsel's statistical team.

Methods

The goal of the analysis is to build a reusable and understandable model that can predict house prices given information about the house, such as the date of the transaction, age of the house, distance to the nearest Mass Rapid Transit (MRT) station, number of convenience stores, and location of the house. The analysis and computer code presented in the appendix are intended to be used by the client to predict the prices of houses they suspect were bought/sold under fraudulent circumstances where the prices were unfairly misrepresented. Presenting the deviation between the actual price the house sold for, and the predicted price as per the model might be useful to present to a jury, as the client states they are a law firm investigating housing fraud in Taipei, Taiwan.

This goal is achieved by utilizing a statistical model known as Generalized Additive Models (GAM). GAMs are a sophisticated way of modeling the response variable (price of house) to depend on smooth polynomial curves with varying characteristics. These can capture non-linear relationships in the data very well. Another reason this method was chosen was because the client states that prediction is their primary goal and GAMs, due to their complex nature, can accurately model complex relationships in the data. However, this brings about a compromise on model interpretability. For this reason, a simple and interpretable linear model is also presented in the results section, to enable the client to understand the data at a superficial level before delving into the details of the GAM. This linear model is not meant to solve the primary goal of predicting house prices, but is rather just an addendum for convenience.

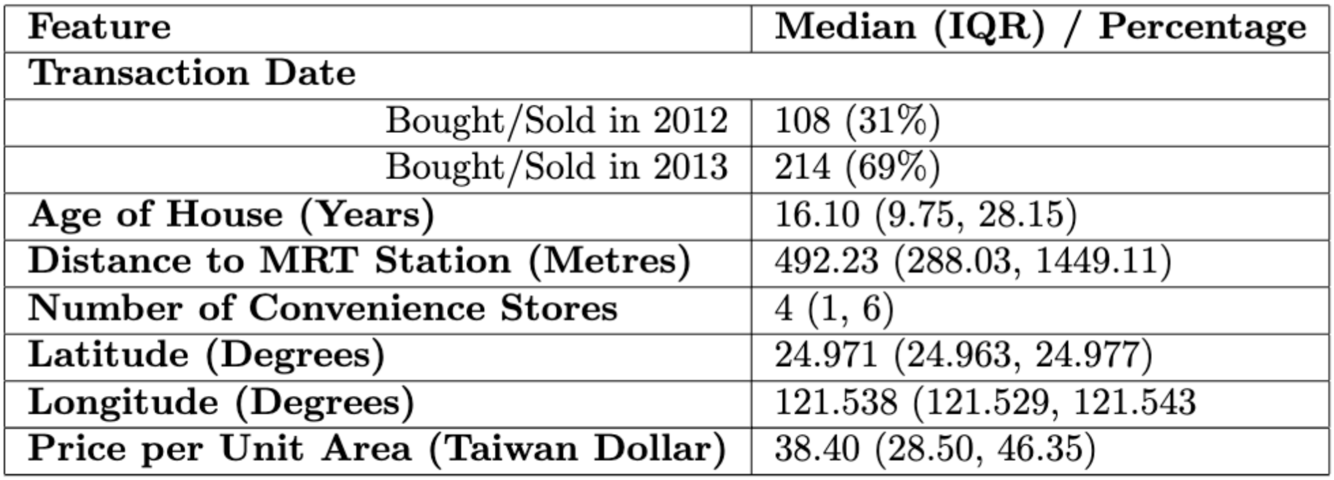
The appendix shows the various GAM models that were fitted, but the report only presents the best performing one. The best model was selected based on the Root Mean Score Error (RMSE) which is a metric that indicates how accurately the model predicts the house price. Akaike Information Criterion (AIC) was also used in model selection, as this is a single number score that can be used to determine which of multiple models is most likely to be the best model to fit the data.

A caveat that is worth mentioning is about the latitude and longitude variables present in the data. Since these variables are best treated together to denote the location of the house, it is not statistically sound to treat them as two separate variables. For this reason, they were combined together to build what is known as a tensor which is essentially a 2-Dimensional plane, as opposed to a 1-Dimensional smooth polynomial curve which is fitted for the other variables.

The primary limitations of using a GAM is that it can be hard to interpret the results from it. However, care is taken to ensure that the final model is explained with strong visuals depicting the influence of each variable on the price. Care was also taken to avoid overfitting, which is a phenomenon common to GAMs where the model is unable to generalize to new data provided to it. This was avoided by leaving out a small portion of the data when building the model, and then using the held out data to check the accuracy of the model.

Results

The client provided a URL to the data which was hosted on the internet at the UCI Data Repository website. This data represented houses in Taipei along with information about each house, including the price of the unit area in Taiwan Dollar. Each row of the data represents a single house and there are 414 such records. For each house, there was information about the transaction date, total age of the house, distance to the nearest MRT station, number of convenience stores within walking distance, and the latitude and longitude representing the location. The dataset was fairly clean, and did not require any additional processing in order to be reasonable. Table 1 provides a description of the summary statistics of the data.



Feature	**Median (IQR) / Percentage**
Transaction Date	
Bought/Sold in 2012	108 (31%)
Bought/Sold in 2013	214 (69%)
Age of House (Years)	16.10 (9.75, 28.15)
Distance to MRT Station (Metres)	492.23 (288.03, 1449.11)
Number of Convenience Stores	4 (1, 6)
Latitude (Degrees)	24.971 (24.963, 24.977)
Longitude (Degrees)	121.538 (121.529, 121.543
Price per Unit Area (Taiwan Dollar)	38.40 (28.50, 46.35)

Table 1: Baseline table indicating summary statistics of data relevant to the analysis

Prior to presenting the final GAM, results of a simple linear model are briefly presented for the purposes of interpretation. The linear model suggests that all the variables except longitude are statistically significant, and that as the age of the house and distance to MRT station increase, the unit price of the house goes down. Also, the price of the house increases if there are more convenience stores located close by.

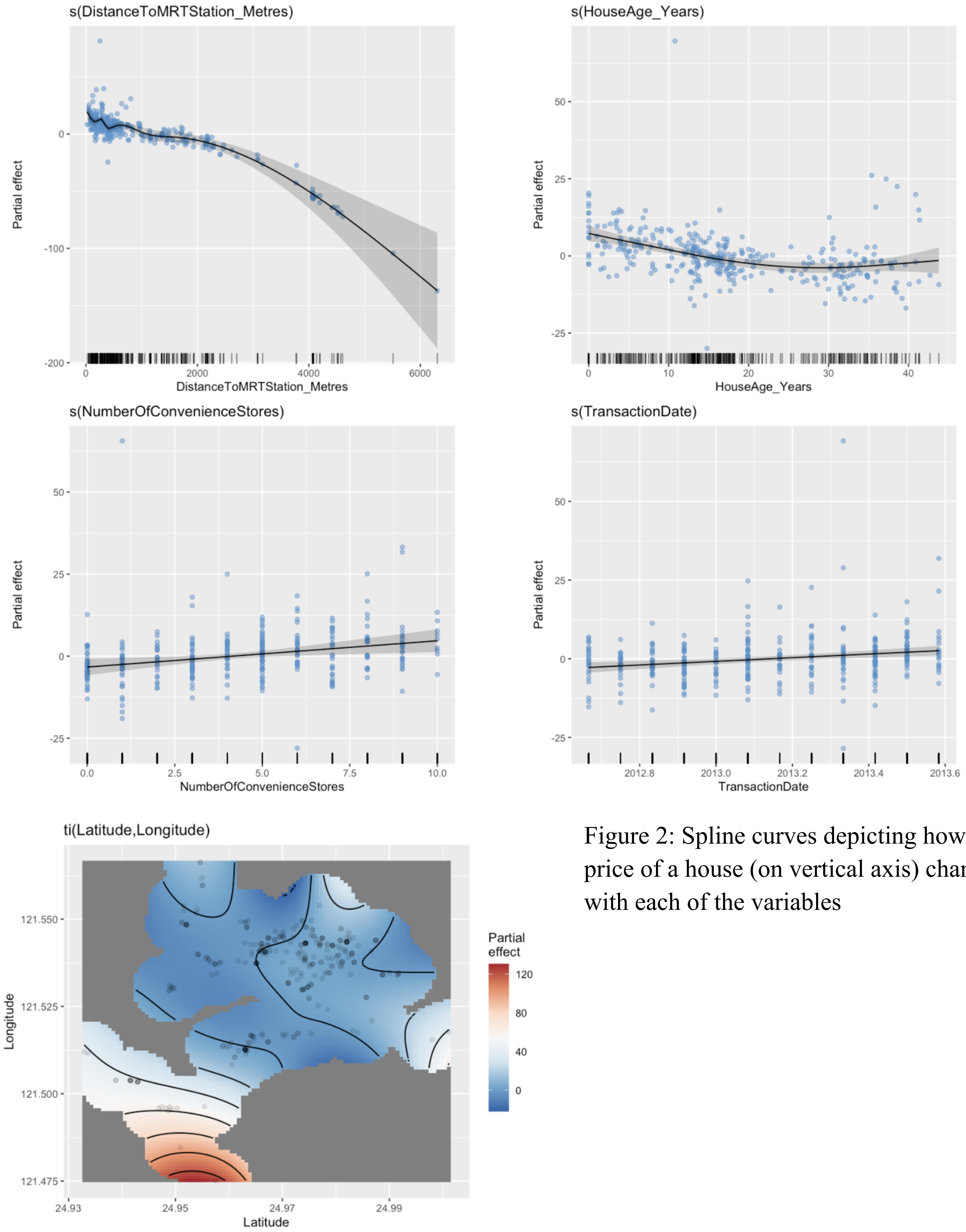


Figure 2: Spline curves depicting how the price of a house (on vertical axis) changes with each of the variables

However, as the visuals in Figure 1 show, not all variables depict a linear relationship so a GAM is required to improve on prediction accuracy. Six different models were fitted to the data, with different types of smoothing curves and combinations variables. These models were then evaluated based on their AIC and RMSE scores which are described in the methods section. The best performing model had an AIC score of 2383.8 on the training data, and an RMSE of 6.63. This model also used a tensor to treat the latitude and longitude as one variable instead as separate ones.

The model results of the GAM are not easily interpreted, which is why they are only presented in the appendix. The report displays visuals of each of the smooth curves, and how they influence the unit price of a house above in Figure 2. The partial effect of each variable is isolated by holding all other variables constant at their median values.

As an example interpretation, there is a slight U-shaped curve in house age. This is intuitive as antique houses and brand new houses are likely to have higher prices compared to slightly older houses (with all other variables held constant at median values). Similarly, the number of convenience stores has a seemingly linear relationship which is also intuitive as the house is likely to be more expensive if it has good accessibility to convenience stores.

The final panel in Figure 2 represents the effect of the location of the house in terms of latitude and longitude. The red and white areas of the image are areas where the unit house price is likely to be higher, while the blue areas are expected to be cheaper.

Conclusion

This report presents the results of an analysis performed on housing prices of the real estate market in Taiwan, with the objective of trying to accurately predict the price based on various characteristics of the house. The most optimal GAM model is presented, and the appendix contains computer code that can be utilized to plug in values to obtain predicted prices for houses that the client is interested in investigating as part of their lawsuit. The report also presents interpretability for each of the variables on how they affect the price, such as the age of the house having a nonlinear relationship where both very new and very old houses are more expensive than middle aged houses. It must be noted that GAMs are prone to overfitting, and there is a possibility that the data used in this analysis is not representative enough to build a good enough model which could potentially be a limitation. Furthermore, as house prices are a vast and abstract subject, it cannot be guaranteed that the data contains all the variables that are truly important in determining the price of a house. However, it is expected that this report will serve as a good aid for the law firm to make their case to the judge and jury.

In [47]:

```
library(readxl)
library(mgcv)
library(ggplot2)
library(GGally)
library(gratia)
library(Metrics)
```

```
Registered S3 method overwritten by 'GGally':
  method from
  +.gg   ggplot2
```

Code for Prediction [remove # and run this code]

In [185…

```
data = read_excel("Real estate valuation data set.xlsx")
data = subset(data, select = -c(1) )
colnames(data) = c("TransactionDate",
                   "HouseAge_Years",
                   "DistanceToMRTStation_Metres",
                   "NumberOfConvenienceStores",
                   "Latitude",
                   "Longitude",
                   "Price")
gam_mod_final <- gam(Price ~ s(TransactionDate, bs="cr") +
                s(HouseAge_Years, bs="cr") +
                s(DistanceToMRTStation_Metres, bs="cr") +
                s(NumberOfConvenienceStores, bs="cr") +
                ti(Latitude, Longitude, bs="cr"), data=data)
```

In [ ]:

```
# test = read_excel("<Enter your file name here>.xlsx")
# unit_house_price = predict(gam_mod_final, data=test)
# data.frame(unit_house_price)
```

In [ ]:

Read in data

In [ ]:

In [52]:

```
data = read_excel("Real estate valuation data set.xlsx")
data = subset(data, select = -c(1) )
```

In [135…

```
dim(data)
```

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In [55]:

```
colnames(data) = c("transaction_date", "age", "dist_to_mrt", "num_conv_stores", "lat", "long", "pric
```

In [56]:

```
head(data)
```

A tibble: 6 × 7

transaction_date	age	dist_to_mrt	num_conv_stores	lat	long	price
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
2012.917	32.0	84.87882	10	24.98298	121.5402	37.9

transaction_date	age	dist_to_mrt	num_conv_stores	lat	long	price
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
2012.917	19.5	306.59470	9	24.98034	121.5395	42.2
2013.583	13.3	561.98450	5	24.98746	121.5439	47.3
2013.500	13.3	561.98450	5	24.98746	121.5439	54.8
2012.833	5.0	390.56840	5	24.97937	121.5425	43.1
2012.667	7.1	2175.03000	3	24.96305	121.5125	32.1

Exploratory Data Analysis

In [156…

```
dim(train)
```

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In [153…

```
sum(train$transaction_date < 2013)
```

108

In [163…

```
quantile(train$lat)
```

0%: 24.93293 **25%:** 24.96305 **50%:** 24.9711 **75%:** 24.97744 **100%:** 25.00115

In [136…

```
summary(train)
```

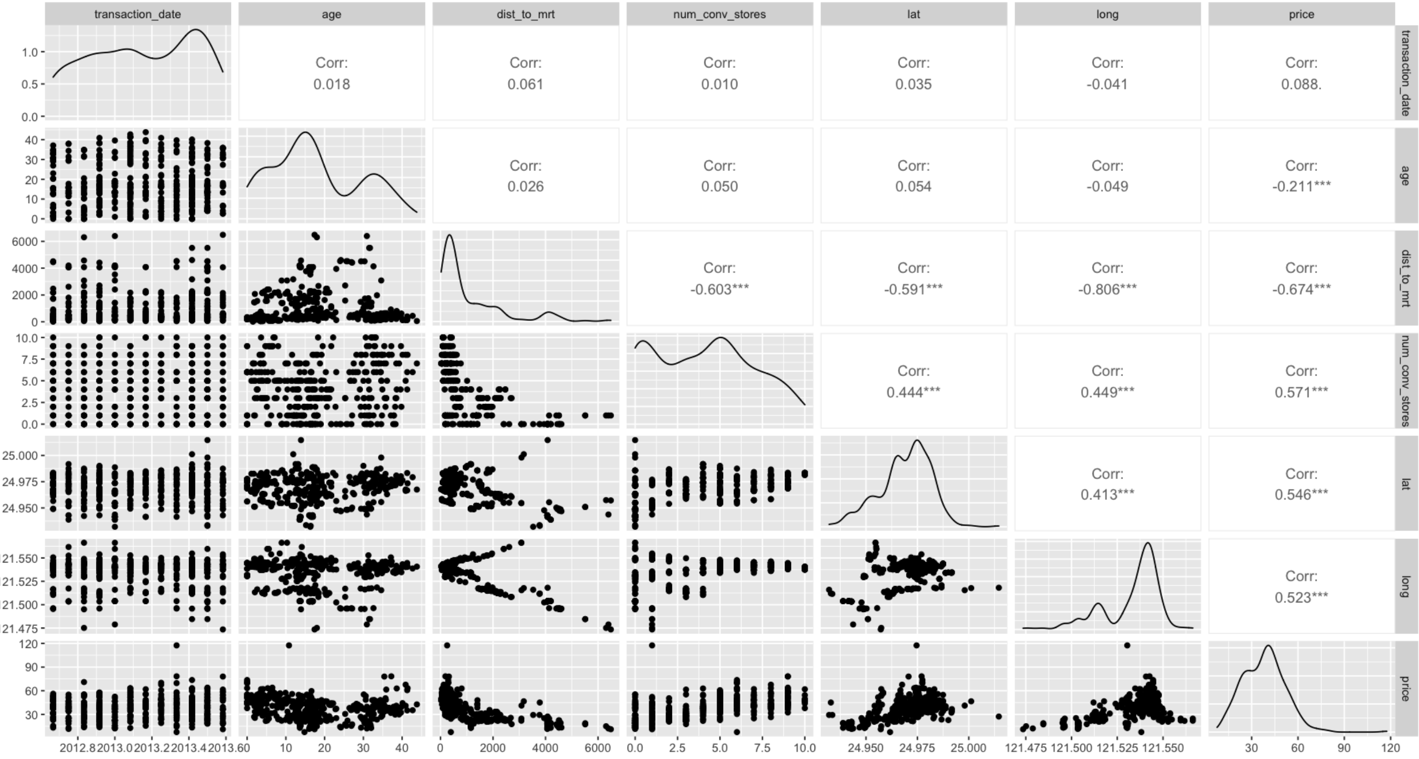
```
transaction_date      age          dist_to_mrt      num_conv_stores
Min.   :2013     Min.   : 0.00   Min.   :  23.38   Min.   : 0.000
1st Qu.:2013     1st Qu.: 9.75   1st Qu.: 288.03   1st Qu.: 1.000
Median :2013     Median :16.10   Median : 492.23   Median : 4.000
Mean   :2013     Mean   :17.79   Mean   :1029.68   Mean   : 4.146
3rd Qu.:2013     3rd Qu.:28.15   3rd Qu.:1449.11   3rd Qu.: 6.000
Max.   :2014     Max.   :43.80   Max.   :6306.15   Max.   :10.000
     lat             long           price
Min.   :24.93   Min.   :121.5   Min.   :  7.60
1st Qu.:24.96   1st Qu.:121.5   1st Qu.: 28.50
Median :24.97   Median :121.5   Median : 38.40
Mean   :24.97   Mean   :121.5   Mean   : 38.00
3rd Qu.:24.98   3rd Qu.:121.5   3rd Qu.: 46.35
Max.   :25.00   Max.   :121.6   Max.   :117.50
```

In [59]:

```
options(repr.plot.width=15, repr.plot.height=8)
```

In [60]:

```
ggpairs(data)
```



Simple Linear Model

In [130…

```
summary(lm(price ~ ., data=data))
```

```
Call:
lm(formula = price ~ ., data = data)

Residuals:
    Min      1Q  Median      3Q     Max 
-35.667  -5.412  -0.967   4.217  75.190 

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)    
(Intercept)      -1.444e+04  6.775e+03  -2.132  0.03364 *  
transaction_date  5.149e+00  1.557e+00   3.307  0.00103 ** 
age              -2.697e-01  3.853e-02  -7.000 1.06e-11 ***
dist_to_mrt      -4.488e-03  7.180e-04  -6.250 1.04e-09 ***
num_conv_stores   1.133e+00  1.882e-01   6.023 3.83e-09 ***
lat               2.255e+02  4.457e+01   5.059 6.38e-07 ***
long             -1.243e+01  4.858e+01  -0.256  0.79820    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 8.858 on 407 degrees of freedom
Multiple R-squared:  0.5824,	Adjusted R-squared:  0.5762 
F-statistic:  94.6 on 6 and 407 DF,  p-value: < 2.2e-16
```

Train Test Split

In [94]:

```
set.seed(1029)
sample <- sample(c(TRUE, FALSE), nrow(data), replace=TRUE, prob=c(0.8,0.2))
train  <- data[sample, ]
test   <- data[!sample, ]
```

Various Generalized Additive Models (Comparison based on AIC and RMSE)

In [96]:

```
gam_mod1 <- gam(price ~ s(transaction_date) +
                s(age) +
                s(dist_to_mrt) +
                s(num_conv_stores) +
                ti(lat, long), data=train)
```

In [97]:

```
gam_mod2 <- gam(price ~ s(transaction_date, bs="cr") +
                s(age, bs="cr") +
                s(dist_to_mrt, bs="cr") +
                s(num_conv_stores, bs="cr") +
                ti(lat, long, bs="cr"), data=train)
```

In [98]:

```
gam_mod3 <- gam(price ~ s(transaction_date, bs="cr") +
                s(dist_to_mrt, bs="cr") +
                s(num_conv_stores, bs="cr") +
                ti(lat, long, bs="cr"), data=train)
```

In [99]:

```
gam_mod4 <- gam(price ~ s(transaction_date, bs="bs") +
                s(dist_to_mrt, bs="bs") +
                s(num_conv_stores, bs="bs") +
                ti(lat, long, bs="bs"), data=train)
```

In [108…

```
gam_mod5 <- gam(price ~ transaction_date +
                s(age, bs="cr") +
                s(dist_to_mrt, bs="cr") +
                s(num_conv_stores, bs="cr") +
                ti(lat, long, bs="cr"), data=train)
```

In [109…

```
gam_mod6 <- gam(price ~ transaction_date +
                s(age, bs="bs") +
                s(dist_to_mrt, bs="bs") +
                s(num_conv_stores, bs="bs") +
                ti(lat, long, bs="bs"), data=train)
```

In [100…

```
AIC(gam_mod1)
```

```
2396.22825450415
```

In [101…

```
AIC(gam_mod2)
```

```
2383.87284822368
```

In [102…

```
AIC(gam_mod3)
```

```
2431.33703612261
```

In [103…

```
AIC(gam_mod4)
```

```
2448.4556477285
```

In [110…

```
AIC(gam_mod5)
```

```
2383.8728356601
```

In [111…

```
AIC(gam_mod6)
```

```
2395.98870087607
```

In [104…

```
rmse(test$price, predict(gam_mod1, test))
```

```
6.63125299168043
```

In [105…
```
rmse(test$price, predict(gam_mod2, test))
```

6.63351143591346

In [106…
```
rmse(test$price, predict(gam_mod3, test))
```

8.45638741256293

In [107…
```
rmse(test$price, predict(gam_mod4, test))
```

9.92676030404349

In [112…
```
rmse(test$price, predict(gam_mod5, test))
```

6.63352175171342

In [113…
```
rmse(test$price, predict(gam_mod6, test))
```

9.64911308238745

Choose `gam_mod2` as final model based on AIC and RMSE

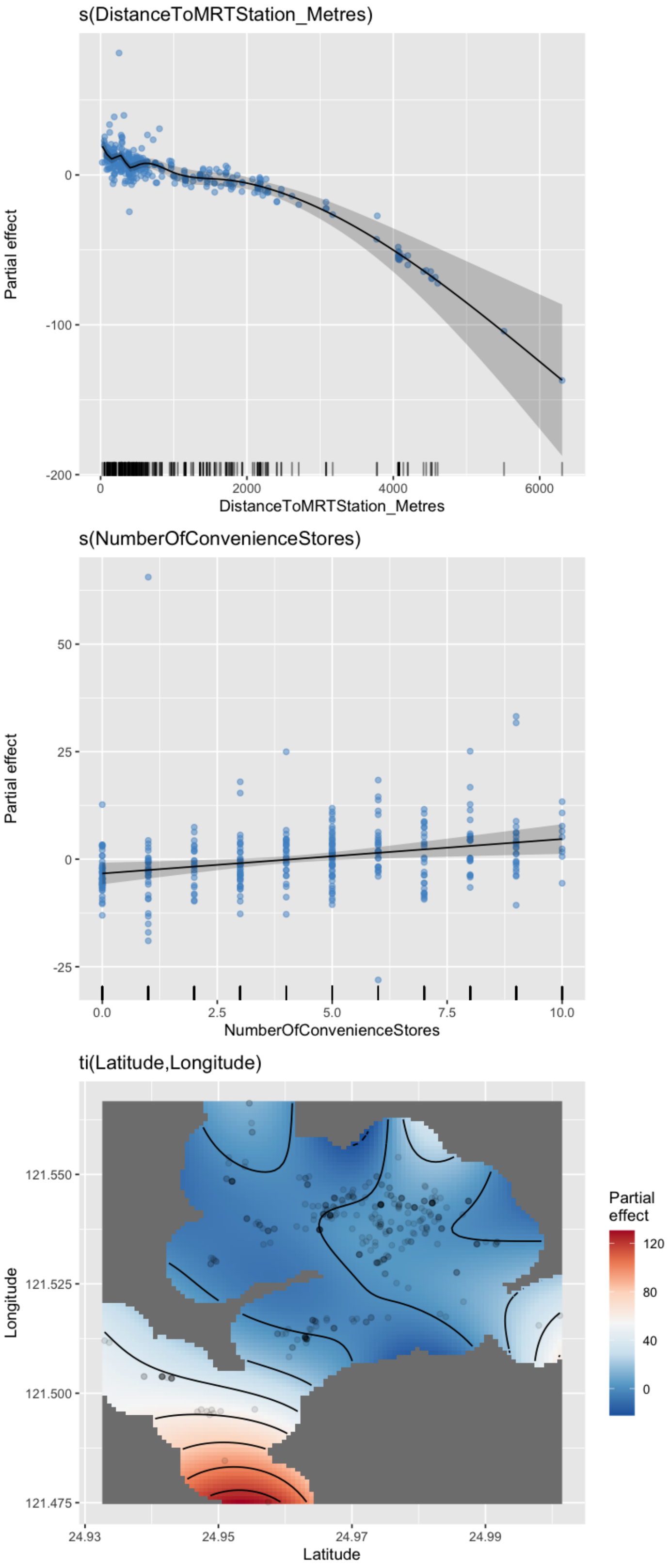
In [114…
```
summary(gam_mod2)
```

```
Family: gaussian
Link function: identity

Formula:
price ~ s(transaction_date, bs = "cr") + s(age, bs = "cr") +
    s(dist_to_mrt, bs = "cr") + s(num_conv_stores, bs = "cr") +
    ti(lat, long, bs = "cr")

Parametric coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   32.534      1.087   29.93   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Approximate significance of smooth terms:
                      edf Ref.df      F  p-value
s(transaction_date)  1.000  1.000 14.335 0.000184 ***
s(age)               2.596  3.228 16.908  < 2e-16 ***
s(dist_to_mrt)       8.756  8.968 20.035  < 2e-16 ***
s(num_conv_stores)   1.000  1.000  7.513 0.006473 **
ti(lat,long)        10.221 11.434  6.162  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

R-sq.(adj) =  0.692   Deviance explained = 71.3%
GCV = 62.308  Scale est. = 57.831    n = 342
```

In [179…
```
options(repr.plot.width=12, repr.plot.height=15)
```

Visualize the GAM

In [180…
```
gratia:::draw.gam(gam_mod_final, scales="free", residuals=TRUE, nrow=3)
```


s(DistanceToMRTStation_Metres)
Partial effect
0
-100
-200
0
2000
4000
6000
DistanceToMRTStation_Metres
s(HouseAge_Years)
Partial effect
50
25
0
-25
0
10
20
30
40
HouseAge_Years
s(NumberOfConvenienceStores)
Partial effect
50
25
0
-25
0.0
2.5
5.0
7.5
10.0
NumberOfConvenienceStores
s(TransactionDate)
Partial effect
50
25
0
-25
2012.8
2013.0
2013.2
2013.4
2013.6
TransactionDate
ti(Latitude,Longitude)
Longitude
121.550
121.525
121.500
121.475
24.93
24.95
24.97
24.99
Latitude
Partial effect
120
80
40
0