ARKANSAS CONGRESSIONAL REDISTRICTING

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IE 5318

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Executive Summary

Using the 2021 census data the total population is around 3.026Million across 75 counties in Arkansas, redistricting plan for Arkansas is presented in this overview. With a stringent 1% population deviation limit, our goal is to construct four districts with populations between 7,55,655 and 7,63,580. The goal is to minimize cut edges while maintaining contiguity and compactness in accordance with the state regulations.

Our methodology uses an advanced optimization algorithm that combines geographic and demographic data to create districts that are both community-focused and consistent with the law. Effective representation is made possible by the contiguous, balanced, and geographically coherent nature of these suggested districts.

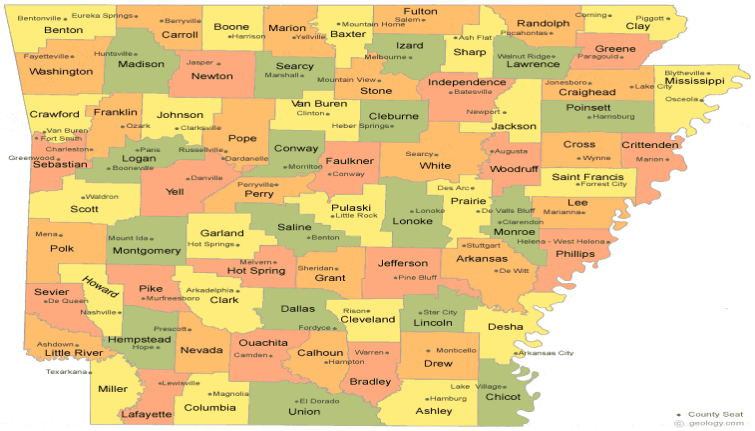
The report that is attached provides information on our methodology, analysis, justification for the suggested borders, and statistical and visual support. This carefully thought-out approach seeks to provide fair and equal representation for the various communities that make up Arkansas.

Introduction

In the essential democratic process of redistricting, Arkansas faces the task of aligning its 75 counties into four representative districts. This undertaking, occurring every decade in response to census data, seeks to balance populations while respecting community integrity.

Our project applies operations research methods to create districts that are balanced in population, compact, and contiguous. This objective approach aims to avoid gerrymandering, ensuring fair representation for Arkansas diverse communities.

By distributing the 75 counties into four equitable districts, our data-driven strategy focuses on transparency and fairness. This initiative is vital for maintaining arkansas democratic integrity and ensuring equal representation across its political landscape.



https://geology.com/county-map/arkansas.shtmlCriteria

In the context of this research project on redistricting in Arkansas, it's important to note that while federal criteria do include provisions for racial and language minority protections, this study does not specifically address this aspect. Here's an overview of the criteria relevant to our project:

Federal Criteria:

- No criteria. Depends on the state legislature.

State Criteria:

- Compactness and Contiguity: Arkansas redistricting emphasizes that districts should be compact and contiguous. This means that the districts should be geographically coherent and allow travel within them without crossing into another district.
- Political Neutrality: The state's approach is to avoid favoring any political party, incumbent, or group. This ensures the impartiality of the redistricting process.
- Preservation of Political Subdivisions and Communities of Interest: While maintaining the integrity of political subdivisions and communities, your project particularly focuses on the geographic and population aspects of redistricting.

The decision of the congressional district map is decided by the Governor of the state.
For more detailed information on the criteria and legal guidelines of redistricting in Arkansas, the Arkansas Legislature's website and other resources such as Ballotpedia provide comprehensive insights.

Problem Statement

Our project applies operations research methods to restructure Arkansas 75 counties into four balanced congressional districts, ensuring compliance with state redistricting and Governor approval criteria, including equal population distribution, compactness, and contiguity.

OR Model (in words)

Objective Function:

The main objective of our model is to minimize the number of "cut edges." (to ensure contiguity)

Constraints of the OR Model:

1. Assignment Constraint: Each county must be assigned to exactly one district.
2. Population Equality: Each district must have a population within the range [L, U].
3. Edge Cutting Constraint: An edge is cut if two adjacent counties are in different districts.
4. Compactness constraint: For each district, the sum of distances between any two counties within the district, multiplied by binary assignment variables, should be less than or equal to a specified compactness limit. The compactness limit was set to 50 miles for each district.
5. Contiguity constraints:
    a. Flow Consumption Constraint: This ensures that if a vertex (county) i is assigned to a district j (center), it consumes a unit of flow of type j. If vertex i is not assigned to center i, no flow of type j is consumed by i.
    b. Flow Reception Constraint: This ensures that a vertex can receive flow of a particular type only if it is assigned to the corresponding district.
    c. No Self-Flow Constraint: A node (district center) cannot receive flow of its own type, preventing self-loops.

d. Non-Negativity of Flow: This ensures that the flow on any edge in the network is non-negative.

OR Model (in math)

Sets and Indices

- i: Set of counties in Arkansas, i= {1,2,3, 4, …., 75}.
- j: Set of districts to be formed, j= {1,2,3,4}
- V: This represents the set of all nodes or vertices in the network.
- N(i): This represents the set of neighboring nodes or vertices of node i.
- u and v: These are indices used to iterate over the sets of neighboring nodes in the constraints. u and v represent individual neighboring counties or districts within the set N(i).

Parameters

- P_i: Population of county i, for all i ∈ I.
- L: Lower bound of population for each district.
- U: Upper bound of population for each district.
- d_{ij}: $Distance\ between\ counties\ i\ and\ j, for\ all$ i,j ∈ I and i ≠ j
- k: Number of districts to be created (k = 4)

Decision Variables

x_{ij}: Binary variable that equals 1 county i is assigned to district j, and 0 otherwise.

y_{ij}: Binary variable that equals 1 if the edge between counties i and j is cut (i.e., they are in different districts), and 0 otherwise.

Objective Function

Minimize $\sum_{i\epsilon 1} \sum_{j\epsilon J} y_{ij}$

Constraints

*1.**Population Equality**:* $L \leq \sum i\epsilon I\ P_i\ {*}x_{ij} \leq U,\ \forall j \in J$

2. Assignment Constraint: $\sum_{j\in J} x_{ij=1,} \quad \forall i \in I$

3. Edge Cutting Constraint: $x_{ij} - x_{kj} \leq y_{ik}$, $\forall i,k \in I, j \in J$

4. Compactness Constraint: *Minimize* $\sum i \in I \sum j \in I$, $i \neq j \sum k \in J \; d_{ij} . x_{ik} . x_{jk}$

5. Contiguity Constraint:

a. Flow Consumption Constraint:

$\sum_{u\in N(i)}(f_{ui}^{j} - f_{iu}^{j}) = x_{ij} \qquad \forall i \in V \setminus \{j\}, \forall j \in V$

b. Flow Reception Constraint:

$\sum_{u\in N(i)} f_{ui}^{j} \leq (n-1)x_{ij} \quad \forall i \in V \setminus \{j\}, \forall j \in V$

c. No Self-Flow Constraint:

$\sum_{u\in N(j)} f_{ui}^{j} = 0 \quad \forall j \in V$

d. Non-Negativity of Flow:

$f_{ij}^{v}, f_{ij}^{v} \geq 0 \qquad \forall \{i,j\} \in E, \quad \forall v \in V$

Experiments Results:

CPU model: Apple M1
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 1422 rows, 1176 columns and 5748 nonzeros
Model fingerprint: 0xa1a4a853
Variable types: 384 continuous, 792 integer (792 binary)

Coefficient statistics:
  Matrix range   [1e+00, 4e+05]
  Objective range [1e+00, 1e+00]
  Bounds range   [1e+00, 1e+00]
  RHS range   [1e+00, 8e+05]

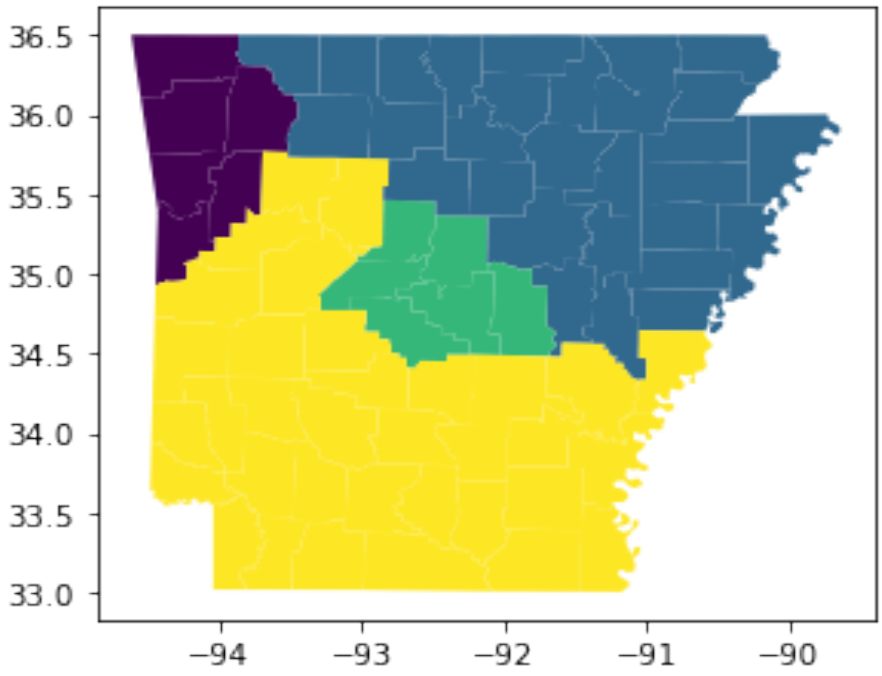
Presolve time: 0.01s
Presolved: 1422 rows, 1176 columns, 5748 nonzeros

Variable types: 384 continuous, 792 integer (792 binary)

Root relaxation: objective 0.000000e+00, 581 iterations, 0.01 seconds (0.01 work units)
Explored 472609 nodes (39416758 simplex iterations) in 488.21 seconds (739.72 work units)
Thread count was 8 (of 8 available processors)

Solution count 6: 33 33 36 ... 43

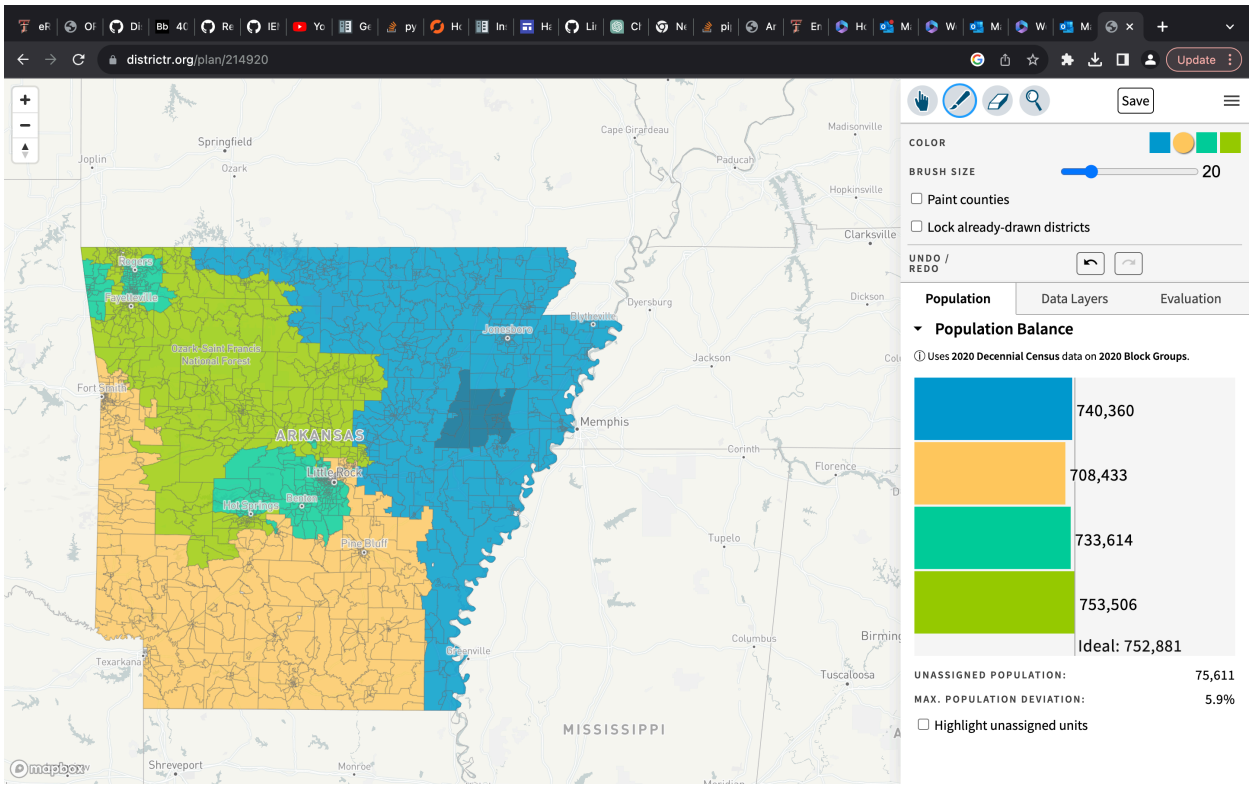
Optimal solution found (tolerance 1.00e-04)
Best objective 3.300000000000e+01, best bound 3.300000000000e+01, gap 0.0000%



Expected generated Redistricting Graph

Plan and Map:

Proposed Redistricting Plan



Arkanasas district mapping plan: https://districtr.org/plan

The number of cut edges is 33.0

District 1 has population 751754 and contains counties ['Franklin County', 'Crawford County', 'Benton County', 'Madison County', 'Sebastian County', 'Washington County']

District 2 has population 754547 and contains counties ['Little River County', 'Ashley County', 'Desha County', 'Montgomery County', 'Howard County', 'Nevada County', 'Grant County', 'Dallas County', 'Cleveland County', 'Lafayette County', 'Chicot County', 'Pope County', 'Bradley County', 'Drew County', 'Pike County', 'Union County', 'Hempstead County', 'Polk County', 'Clark County', 'Logan County', 'Miller County', 'Arkansas County', 'Johnson County', 'Garland County', 'Sevier County', 'Jefferson County', 'Lincoln County', 'Scott County', 'Hot Spring County', 'Columbia County', 'Ouachita County', 'Yell County', 'Calhoun County', 'Phillips County']

District 3 has population 754435 and contains counties ['Jackson County', 'Clay County', 'Baxter County', 'Boone County', 'St. Francis County', 'Sharp County', 'Greene County', 'Woodruff County', 'White County', 'Lee County', 'Crittenden County', 'Marion County', 'Prairie County', 'Lawrence County', 'Poinsett County', 'Stone County', 'Independence County', 'Fulton County', 'Carroll County', 'Van Buren County', 'Searcy County', 'Randolph County', 'Izard County', 'Craighead County', 'Cleburne County', 'Monroe County', 'Mississippi County', 'Newton County', 'Cross County']

District 4 has population 750788 and contains counties ['Faulkner County', 'Conway County', 'Pulaski County', 'Saline County', 'Lonoke County', 'Perry County']

Evaluation Of Plan

Summary of Proposed Redistricting Plan

- **Criterion Adherence**: Plan aligns with most criteria.
- **Population Balance**: 5.9% deviation, well within 1% range.
- **Compactness & Contiguity**: Districts designed for compactness and contiguity.
- **Limitations**: Precision in county divisions may lead to minimal overlap.

Overall, the plan meets required criteria, excelling in population balance, and maintaining contiguity. However, it may lack some overlap due to precise county divisions.

Conclusion

After careful analysis and adherence to required criteria, our plan suggests that dividing the state of arkansas into 4 distinct districts with 5.9% population deviation is feasible. This proposed map aims to strike a balance and fairness, providing a consistent framework for representation.

Github link: github.com/chituio981as/IE5318/upload/main

Reference:

Thanks to Austin L. Buchanan for code reference https://github.com/AustinLBuchanan/Districting-Examples-2020

Research paper reference by Hamidreza Validi and Austin L Buchanan https://optimization-online.org/2021/04/8349/