A1.a. If $M2(i, j)$ = 1, it implies that there exists a path of length 2 from vertex i to vertex j in the graph G. In other words, there is a directed edge from i to some intermediate vertex k, and then another directed edge from k to j. If $M2(i, j)$ = 0, it means that there is no path of length 2 from vertex i to vertex j in the graph G. There are no intermediate vertices that connect i and j directly. b. If $M4$ is the product of $M2$ with itself, the entries of $M4$ signify the existence of paths of length 4 between vertices in the graph G. $M4(i, j)$ = 1 indicates that there exists a path of length 4 from vertex i to vertex j, whereas $M4(i, j)$ = 0 implies that no such path exists.Similarly, the entries of $M5$ = ($M4$) (M) represent the existence of paths of length 5 in the graph G. $M5(i, j)$ = 1 indicates the presence of a path of length 5 from vertex i to vertex j, and $M5(i, j)$ = 0 signifies the absence of such a path.In general, the matrix Mp represents the existence of paths of length p between vertices in the graph G. The entry $Mp(i, j)$ = 1 indicates the presence of a path of length p from vertex i to vertex j, and $Mp(i, j)$ = 0 implies the absence of such a path. c. If $M2(i, j)$ = k, it implies that the shortest path length from vertex i to vertex j is k in the weighted graph G. The min operation in the definition of $M2$ considers all possible intermediate vertices k' and calculates the sum of the weights of the edges from i to k' and from k' to j. $M2(i, j)$ gives the minimum of all such sums, representing the shortest path length.Therefore, if $M2(i, j)$ = k, it can be concluded that there exists a path from vertex i to vertex j of length k, and this path is the shortest among all paths from i to j in the graph G.

A3. To find the maximum bandwidth of a path between two switching centers a and b in a telephone network represented by a graph G, we can use a modified version of Dijkstra's algorithm. Here's the algorithm: 1. Create a priority queue Q to store vertices with their associated bandwidth values. Initialize all vertices with infinite bandwidth, except for vertex a, which is initialized with bandwidth 0. 2. While Q is not empty, do the following: a. Extract the vertex u with the minimum bandwidth value from Q. b. If u is b, stop the algorithm. The maximum bandwidth from a to b has been found. c. For each neighbor v of u, calculate the minimum bandwidth between u and v by taking the minimum of the bandwidth value of u and the bandwidth of the edge (u, v). Let this minimum bandwidth be new_bw. - If new_bw is greater than the current bandwidth value of v, update the bandwidth value of v to new_bw and add v to Q. 3. If the algorithm reaches this point, there is no path from a to b. Return an appropriate error message. Once the algorithm terminates, the maximum bandwidth of a path between a and b will be the bandwidth value associated with vertex b.The modified Dijkstra's algorithm ensures that at each step, we update the bandwidth value of a vertex with the minimum bandwidth possible for the path leading to it. By doing this, we effectively find the maximum bandwidth path from a to b. This algorithm assumes that the bandwidth values on the edges represent the capacity of the communication lines. If the bandwidth values represent some other metric, the algorithm may need to be adjusted accordingly.