Department of Physics
IIT Kanpur, 2021-22 Ist Semester

PHY103AA (Physics - II) **Mid-sem exam** **14-01-2022**

CT Question id: 3037

Question 1 [A]: For a cylinder of radius $3\ \mathrm{m}$ and height $5\ \mathrm{m}$, calculate the area of the curved surface between $\phi = \frac{\pi}{6}$ and $\phi = \pi$. **[2 marks]**

[B]: For an arbitrary surface S with volume V, prove that $\int_V (\vec{\nabla} \times \vec{A})\ \mathrm{d}\tau = \oint_S (\hat{n} \times \vec{A})\ \mathrm{d}a$. Here $\hat{n}$ represents the direction of the area element $\mathrm{d}a$. **[3 marks]**

[C]: For a given function $f(\theta) = \cos\theta$, calculate the gradient of the function in Spherical-polar and Cartesian coordinates to evaluate $\hat{\theta} \cdot \hat{x}$, $\hat{\theta} \cdot \hat{y}$ and $\hat{\theta} \cdot \hat{z}$. **[3 marks]**

[D]: Among the given four forces in Cartesian and Spherical-polar coordinate system,
(i) $\vec{F_1} = K \exp\left(\frac{-r^2}{R^2}\right)\hat{r}$
(ii) $\vec{F_2} = K\left(x^3\hat{y} - y^3\hat{z}\right)$
(iii) $\vec{F_3} = K\left(x^3\hat{x} + y^3\hat{y}\right)$
(iv) $\vec{F_4} = K\left(\frac{\hat{\phi}}{r}\right)$
show, using explicit calculations, which force(s) represent(s) conservative vector field. **[2 marks]**

CT Question id: 3040

Question 2 [A]: The time-averaged electric potential of a neutral hydrogen atom is given by $V(\vec{r}) = q\frac{e^{-\lambda r}}{r}(1 + \frac{\lambda r}{2})$, where q is the electronic charge and λ^{-1} is half the Bohr radius. Find out the charge density $\rho(r)$ that will give rise to this potential? **[3 Marks]**

[B]: Calculate the energy per unit length stored in a cylinder with radius R and uniform volume charge density ρ. [HINT: *Use the electric field approach. Calculate the energy relative to the configuration where all the charge is initially distributed uniformly over a hollow cylinder with larger radius $B(> R)$, since the field outside radius B is the same in both configurations. Don't forget to include the field inside the cylinder.*] **[4 marks]**

[C]: The given vector function represents a possible electrostatic field:
$E_x = 6xy, E_y = 3x^2 - 3y^2, E_z = 0$.
(*For convenience, a multiplicative factor with units of $\mathrm{V/m^3}$ is ignored which is necessary to make the units correct.*) Calculate the line integral of $\vec{E}$ from the point $(0, 0, 0)$ to the point $(x_1, y_1, 0)$ along the path
(i) that runs from $(0, 0, 0)$ to $(x_1, 0, 0)$ and thence to $(x_1, y_1, 0)$,
(ii) that runs from $(0, 0, 0)$ to $(0, y_1, 0)$ and thence to $(x_1, y_1, 0)$.
Also find the potential function $V(x, y, z)$. **[3 marks]**

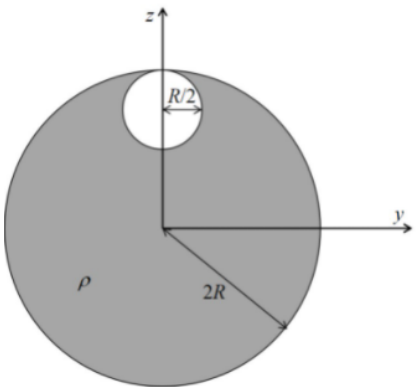
CT Question id: 3041

Question 3 [A]: Consider a semiconductor pn junction diode at equilibrium with depletion region in the range $-d_p < x < d_n$. If the charge distribution in the depletion region is given by

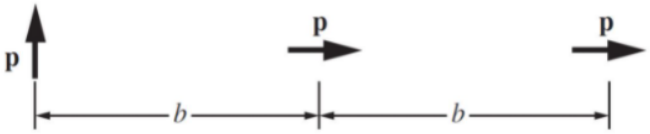
$$\begin{aligned} \rho &= -eN_a; && d_p < x < 0 \\ &= +eN_d; && 0 < x < d_n \\ &= 0; && \text{otherwise} \end{aligned}$$

Calculate and plot the electric field and the electric potential in the depletion region. **[4 marks]**

[B]: Consider a sphere of radius $2R$ and uniform volume charge density ρ. If a smaller sphere of radius $R/4$ is carved out this sphere (see figure), find the potential at a far away point $\vec{r}$. **[3 marks]**



[C]: Consider the arrangement of three dipoles shown in the figure. What are the magnitude and direction of the force on the central dipole caused by the field of the other two dipoles? **[3 marks]**



CT Question id: 3042

Question 4 [A]: It is given that the electric breakdown strength of air is $3\ \mathrm{V}/\mu\mathrm{m}$ and that of glass is $30\ \mathrm{V}/\mu\mathrm{m}$. Consider a parallel plate capacitor with the plate separation of $1\ \mathrm{cm}$, where glass is filled in half, i.e., $0.5\ \mathrm{cm}$ of the space and air in the remaining half. Calculate the maximum permissible potential that can be applied on the capacitor plates. **[3 marks]**

[B]: Consider a concentric spherical conductor having inner solid sphere of radius a and charge Q, and an outer shell of inner radius b and outer radius c. If the entire region between the solid sphere and outer shell is filled with a dielectric material of dielectric constant $K = \alpha r$ (K increases linearly with radial distance), find the energy confined in the system. **[3 marks]**

[C]: In the previous problem, if the outer shell is at zero potential and the dielectric constant K of the medium increases linearly with radial distance, i.e. $K = \alpha r$, find
(i) all bound charge densities.
(ii) potential difference between the inner and outer shell. **[4 marks]**