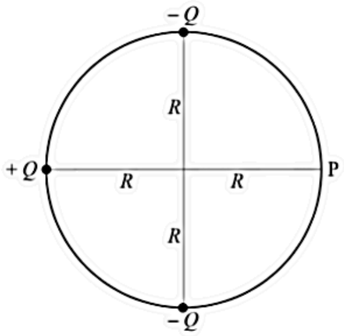
PHY103AA (2021 -22 IInd) End-sem exam

Q 1 (a): A vector $\vec{A}$ in cylindrical coordinate system is given as $\vec{A} = 2s\cos\phi\hat{s} + s\hat{\phi}$. Evaluate the closed loop line integral of $\vec{A}$ around the contour in the $z = 0$ plane bounded by $+x$ and $+y$ axes and the arc of the circle of radius 1 unit. **[3 marks]**

(b): For an arbitrary surface S with volume V, prove that $\int_V (\vec{\nabla}\phi)\, d\tau = \oint_S \phi\, d\vec{a}$. Here ϕ is a non-zero scalar. **[4 marks]**

(c): As shown in the figure, three charges are situated on the three points on the circumference of a circle of radius R.

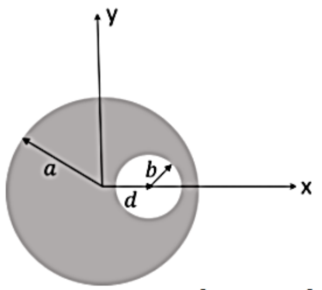


(i) How much work does it take to bring in another charge $+Q$ from far away and place it at the point P? **[2 marks]**
(ii) How much work does it take to assemble the whole configuration of four charges? **[2 marks]**

(d): If an electrostatic field (in spherical-polar coordinate system) due a charge configuration is given by the expression $\vec{E}(\vec{r}) = \frac{Ar\hat{r} + B\sin\theta\cos\phi\hat{\phi}}{r^2}$, where A and B are arbitrary constants, what is the corresponding charge density? **[2 marks]**

(e): The space between the plates of a parallel plate capacitor is filled with two slabs of linear dielectric material. One slab has a thickness a with dielectric constant 2.6 and the other slab has a thickness $2a$ with dielectric constant 1.5. The free charge density on the top plate is σ and on the bottom plate is $-\sigma$.
(i) Find the electric displacement $\vec{D}$ in each slab. **[2 marks]**
(ii) Find the electric field $\vec{E}$ in each slab. **[1 mark]**
(iii) Find the polarization $\vec{P}$ in each slab. **[1 mark]**
(iv) Find the potential difference between the plates. **[1 mark]**
(v) Find the location and amount of all the bound charges. **[2 marks]**

Q 2 (a): A cylindrical conductor of radius a, centered at the origin, has a hollow cylindrical bore of radius b parallel to, and centered a distance d from the origin ($d + b < a$), on the x-axis, as shown in the figure. The current density J_0 is uniform throughout the solid part (shaded portion in the figure) of the cylinder and is along the positive z-direction. Find the magnitude and the direction of the magnetic field everywhere inside the hollow (unshaded) cylindrical region. (Provide the final solution in Cartesian system.) **[8 marks + 2 marks]**

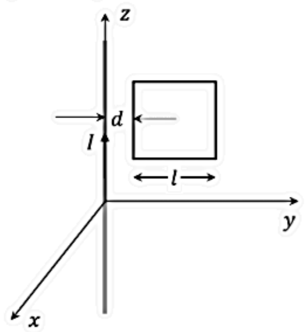


(b): Consider a system of two anti-parallel current carrying wires at $x = -d$ and $x = d$, with currents I and $-I$, respectively, along the positive z-direction.
(i) Calculate the vector potential everywhere when both wires have finite length $2L$ each, with midpoints at $z = 0$. **[8 marks]**
(ii) Using the expression in (i), calculate the vector potential everywhere for the same problem when $L \to \infty$. **[2 marks]**

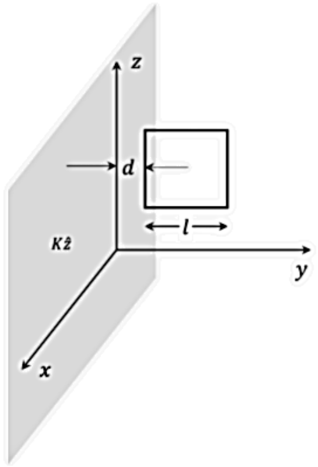
Q 3 (a): A coil of wire with 3000 turns and radius 4 cm, is placed in stable equilibrium (horizontally) on a table. A block of magnetic material of size 1 cm^3 (each side 1 cm long), relative permeability 2000 and zero `frozen-in' magnetization is placed on the table at a distance 30 cm from the coil. The current through the coil is 1 A. Find the net magnetic moment induced in the block. **[5 marks]**

(b): Consider a magnetized material in the shape of a right circular cylinder of length L and radius a with the axis along z-direction. The cylinder has a permanent `frozen-in' magnetization M_0, uniform through-out its volume and parallel to its axis. Determine the magnetic field $\mathbf{B}$ and auxiliary field $\mathbf{H}$ at all points *only* on the axis of the cylinder, both inside and outside. For $L/a = 5$, plot the scaled magnetic field $\mathbf{B}/\mu_0 M_0$ as a function of z/L . **[12 marks + 3 marks]**

Q 4 (a): **(i)** An infinitely long wire is carrying current I along the $+z$ direction. A square loop of side l is initially located on the YZ plane at a distance d from the z axis (see figure below). Calculate the induced electromotive force generated in the loop when it moves along the positive y and z directions, respectively, with a constant velocity v. **[3 marks]**



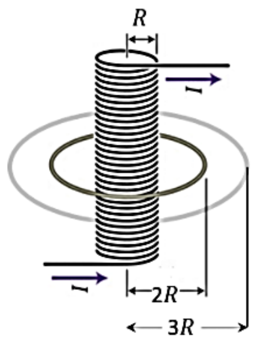
(ii) Repeat the calculations in (i) for an infinite uniform surface current $\mathbf{K} = K\hat{z}$ flowing over the XZ plane (see figure below). K is a positive constant. **[2 marks]**



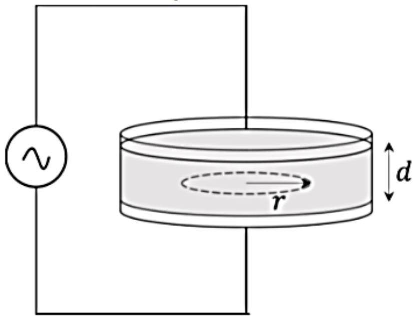
(b): An infinitely long solenoid of radius R and n turns per unit length is carrying a very slowly varying current $I = I_0 e^{-\alpha t}$ along the $\hat{\phi}$ direction. α is a positive constant.

(i) Calculate the electric field inside the solenoid at a distance r from the axis. **[2 marks]**

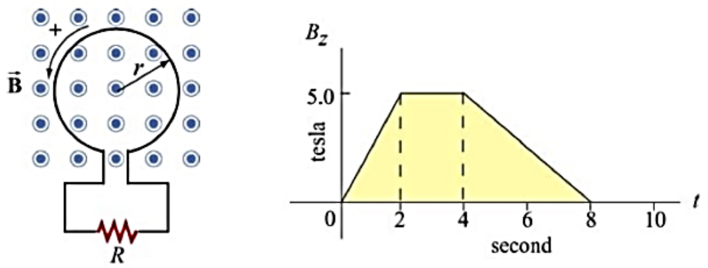
(ii) Two loops of radius $2R$ and $3R$ are placed over the solenoid symmetrically as shown in the figure. Determine the direction of induced current in the loops. Calculate the ratio I_1/I_2, where I_1 and I_2 are the induced currents in the first (radius $2R$) and the second loop (radius $3R$), respectively. Resistivity and cross-section of both the loop wires are ρ and A, respectively. **[3 marks]**



(c): A thin parallel-plate capacitor of plate separation d is filled with a medium of conductivity σ and dielectric constant ϵ_r. The plates of the capacitor are circular with radius $R \gg d$. A time-varying voltage $V = V_0 \sin(\omega t)$ is applied to the capacitor, as shown in the figure. Assuming that the electric field between the plates is homogeneous, find the auxiliary field H at a distance r from the axis ($r \ll R$). **[4 marks]**



(d): A uniform magnetic field $\mathbf{B}$ is perpendicular to a one-turn circular loop of wire of negligible resistance, as shown in the figure below. The field changes with time as shown in the right panel (the z direction is out of the plane of the page). The loop is of radius $r = 0.5$ m and is connected in series with a resistance $R = 20\Omega$.



(i) Calculate and plot the electromotive force induced in the circuit as a function of time. Label the axes quantitatively (numbers and units). **[2 marks]**
(ii) Calculate and plot the current I through the resistor R. Label the axes quantitatively (numbers and units). **[2 marks]**
(iii) Calculate and plot the power dissipated in the resistor as a function of time. Label the axes quantitatively (numbers and units). **[2 marks]**

Q 5: Sunlight with the magnetic field $\vec{B} = 10^{-6}T\,(1.5\hat{x} + B_y\hat{y})\cos(\omega t - k_x x + k_y y)$, shines on an arctic polar ice lying in the x-z plane, where $k_x = 4\pi/a$, $k_y = 3\pi/a$ and $a = 1500$ nm. The ice covers an area of $100,000$ square kilometers. Assume polar ice to be a perfect reflector.

(a) Find B_y and angular frequency of the incident sunlight. **[2 marks + 2 marks]**

(b) Find incident and reflected electric field vectors. **[2 marks + 2 marks]**

(c) Find the total electric field above the surface of the polar ice. Comment on the nature of the electromagnetic wave above the surface of the polar ice. **[2 marks + 1 mark]**

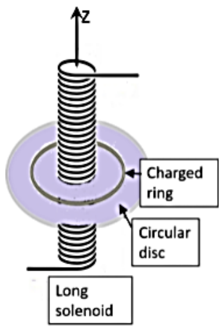
(d) Find the Poynting vector and the momentum densities corresponding to the incident and reflected sunlight. **[2 marks + 2 marks]**

(e) Calculate the pressure exerted by the sunlight on the surface of the polar ice. What fraction of atmospheric pressure does this amount to? **[2 marks + 1 marks]**

(f) Suppose the patch of polar ice melts as a result of global warming, exposing the fully absorbing rocks underneath. How much extra energy per second is absorbed by the earth due to the melting of ice? **[2 marks]**

Proper units should be written in each case.

Q 6 (a): Consider a thin long current carrying solenoid with axis along z-direction. The magnetic flux through the solenoid is Φ_0. A nonconducting circular disc is located in the xy plane and is free to rotate about the z-axis. A thin circular ring of uniformly distributed total charge Q is embedded in the disc, as shown in the figure. Now suppose that the current source is disconnected so that the magnetic field reduces to zero.



(i) Calculate the angular momentum imparted to the charged ring when the magnetic field reduces to zero. **[4 marks]**
(ii) One may wonder, where is the angular momentum coming from! You have to be careful here! Note that the magnetic field outside the solenoid is zero but the vector potential is not! Take the following expression for field angular momentum under the Coulomb gauge (No need to prove the expression!).
$L_{em} = \epsilon_0 \int_V \vec{r} \times (\vec{E} \times \vec{B})\, d\tau = \int_V \vec{r} \times (\rho\vec{A})\, d\tau$; where ρ is volume charge density.
Use the expression to *explicitly* calculate the initial field angular momentum stored in the system. **[6 marks]**

(b): A plane polarized electromagnetic wave, with the electric field in the plane of incidence, is incident from a dielectric medium of refractive index n_1 at angle θ_I, at the interface with another dielectric medium of refractive index n_2 ($n_2 < n_1$). You may use Fresnel equation given in formula sheet.

(i) Find out the phase shift on reflection. **[8 marks]**
(ii) Using the expression in (i), find out the phase shift (in radian) on reflection for a dielectric-vacuum interface, when the incidence angle is 60° and the refractive index of the dielectric (incident medium) is $\frac{4}{3}$. **[2 marks]**