

- Symmetry $\sigma : \Theta \rightarrow \Theta$ is a measurable function with a measurable inverse satisfying

$$\Pi_k F_k(\theta) \propto \Pi_k F_k(\sigma(\theta))$$

- Local symmetry σ satisfies

$$F_k(\theta) \propto F_k(\sigma(\theta))$$

- Class of transformations of model parameters $T \subset \{f : \Theta \rightarrow \Theta\}$. Subset of local symmetries contained in T $S_T \subset T$ defined by

$$S_T = \{\sigma \in T \mid \sigma \text{ is a local symmetry}\} = \bigcap_k \{\sigma \in T \mid F_k(\theta) \propto F_k(\sigma(\theta))\}$$

- Scaling symmetry is a symmetry σ which multiplies θ pointwise by a vector

$$v = (r_1, \dots, r_N) = (e^{d_1}, \dots, e^{d_N})$$

where $r_n \in \mathbb{R}_+$ and $d_n = \log r_n$.

Case of $r_n \in \mathbb{R}_-$ is covered by a combination of scaling symmetry and sign-flip symmetry.

Let $\Sigma(v)$ be the diagonal matrix with v on the diagonal. Then $\sigma(\theta) = \Sigma(v)\theta$.

Let matrix C be the matrix of constraints on d_n . The scaling symmetries of the model are the vectors in the null space of C , $\mathcal{N}(C)$.

Let $d = (d_1, \dots, d_N)$. Let $v(d) = (e^{d_1}, \dots, e^{d_N})$. Then the scaling symmetries of the model are $\sigma_d(\theta) = \Sigma(v(d))\theta$, $d \in \mathcal{N}(C)$.

Then $\{\Sigma(v(d))\theta : d \in \mathcal{N}(C)\} \subset \Theta$ is an equivalence class for the model (θ, F) .

Definition 1?

Let Ξ be a σ -algebra on Θ . A symmetry breaker $\phi : \Xi \rightarrow \Theta$ is a deterministic function that satisfies

1. $\phi(\emptyset)$ is undefined?
2. for $\mathcal{T} \in \Xi$, $\phi(\mathcal{T}) = \theta$ where $\theta \in \mathcal{T}$

Definition 2?

A symmetry breaker $\phi : \Theta \rightarrow \Theta$ is a deterministic measurable function that satisfies

Equivalence class

- symmetry detection: canonical surjection?
- symmetry breaking: section?