TODO

STAT 548 Qualifying Paper

Kenny Chiu

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Abstract. TODO

1 Introduction

2 Notation

Throughout this report, we closely follow the notation used by Forastiere et al. (2021). Let $G = (\mathcal{N}, \mathcal{E})$ be an undirected network where $\mathcal{N}$ is a set of N units (nodes) and $\mathcal{E}$ is a set of edges (i, j). A partition $(i, \mathcal{N}_i, \mathcal{N}_{-i})$ of $\mathcal{N}$ describes unit i's neighbourhood $\mathcal{N}_i$ (the set of N_i units connected to unit i) and the set $\mathcal{N}_{-i}$ of all other units that are not i and are not in $\mathcal{N}_i$. Let $Z_i \in \{0, 1\}$ be the treatment assigned to unit i and $Y_i \in \mathcal{Y}$ the observed outcome of unit i. Denote the treatment and outcome vector for the population $\mathcal{N}$ as $\mathbf{Z}$ and $\mathbf{Y}$, respectively, and the corresponding vectors for partition $(i, \mathcal{N}_i, \mathcal{N}_{-i})$ as $(Z_i, \mathbf{Z}_{\mathcal{N}_i}, \mathbf{Z}_{\mathcal{N}_{-i}})$ and $(Y_i, \mathbf{Y}_{\mathcal{N}_i}, \mathbf{Y}_{\mathcal{N}_{-i}})$. Let $G_i = g_i(\mathbf{Z}_{\mathcal{N}_i}) \in \mathcal{G}_i$ be some known and well-specified summary g_i of the treatments in unit i's neighbourhood, and denote the vector of neighbourhood treatments for the population as $\mathbf{G}$. Let $\mathbf{X}_i \in \mathcal{X}$ be a vector of covariates for unit i that partitions into individual-level characteristics $\mathbf{X}_i^{\text{ind}} \in \mathcal{X}^{\text{ind}}$ and neighbourhood-level/aggregated individidual-level characteristics $\mathbf{X}_i^{\text{neigh}} \in \mathcal{X}^{\text{neigh}}$. TODO$V_g, v_g$

3 Proposed methodology in context of the literature

TODOIn this section, we explain the method proposed by Forastiere et al. (2021) and discuss its relevance in the context of the related literature. We also discuss its advantages over other methods used in similar contexts.

3.1 Setting, objective and method

Forastiere et al. (2021) examine the problem of performing causal inference of treatment effects from observational network data under interference. The setting is challenging for causal inference because

1. the assignment mechanism of treatments is unknown with observational data and so estimated effects may be non-causal in the presence of unmeasured confounders, and
2. naive inference methods that ignore interference may produce biased estimates.

In the randomized study literature, these issues can generally be dealt with by designing the study in such a way that the influence of confounders is minimized and that inference methods that account for interference can be used (e.g., Saveski et al., 2017; Doudchenko et al., 2020; Jagadeesan et al., 2020; Imai et al., 2021). In the observational setting, these considerations need to be addressed by the inference method in the analysis phase. Forastiere et al. consider a setting involving a binary treatment (e.g., intervention and control) and where the interference on a unit is limited to that of its immediate neighbouring units. They propose a method to estimate the causal treatment and spillover (interference) effects that yield unbiased estimates under certain assumptions.

Under the potential outcome framework, the general procedure for estimating effects is to match units into sets based on covariate values, compute the effect contrast within each matched set, and estimate the effect by the (weighted) average of the contrasts across sets. However, matching may be difficult when the space of possible covariate values is large or if there are many covariates. Forastiere et al. (2021) address this problem in their proposed method by instead matching on a defined joint propensity score $\psi(z; g; x)$ that factorizes into a neighbourhood propensity score $\lambda(g; z; \mathbf{x}^g)$ (probability of being exposed to neighbourhood treatment g given individual treatment z and relevant covariates $\mathbf{x}^g$) and an individual propensity score $\phi(z; \mathbf{x}^z)$ (probability of being assigned treatment z given relevant covariates $\mathbf{x}^z$). Note that $\mathbf{X}^g$ and $\mathbf{X}^z$ do not necessarily correspond to $\mathbf{X}^{\text{neigh}}$ and $\mathbf{X}^{\text{ind}}$, respectively, and as they may not be disjoint. The steps for their propensity-based method are as follows:

1. **Subclassify units**.
   (a) Fit a logistic regression model on the individual treatments Z_i given covariates $\mathbf{X}_i^z$, and use the model to predict the individual propensity scores $\phi(1; \mathbf{X}_i^z)$.
   (b) Partition the units into J subclasses B_j, $j \in \{1, \ldots, J\}$, based on similar estimated individual propensity scores $\hat{\phi}(1; \mathbf{X}_i^z)$ and such that each subclass is approximately balanced in the number of treated and untreated units.

2. **Estimate potential outcomes**. Let $B_j^g = V_g \cap B_j$. For each subclass B_j:
   (a) Fit some regression model on the neighbourhood treatments G_i given the individual treatments Z_i and covariates $\mathbf{X}_i^g$, and use the model to predict the neighbourhood propensity scores $\lambda(g; z; \mathbf{X}_i^g)$.
   (b) Fit some regression model on the potential outcomes $Y_i(z, g)$ given the estimated neighbourhood propensity scores $\hat{\lambda}(g; z; \mathbf{X}_i^g)$.
   (c) Estimate the dose-response function by averaging over the estimated potential outcomes for a particular level of the joint treatment, i.e.,

$$\hat{\mu}_j(z, g; V_g) = \frac{\sum_{i \in B_j^g} \hat{Y}_i(z, g)}{|B_j^g|} .$$

3. **Estimate the average dose-response function** (ADRF) for a particular level of the joint treatment by taking the weighted average of the estimated dose-response functions over the subclasses, i.e.,

$$\hat{\mu}(z, g; V_g) = \sum_{j=1}^{J} \hat{\mu}_j(z, g; V_g) \left(\frac{|B_j^g|}{v_g} \right) .$$

4. **Estimate** the treatment effects $\tau(g)$, overall treatment effect τ, spillover effects $\delta(g; z)$, and overall spillover effects $\Delta(z)$ by

$$\hat{\tau}(g) = \hat{\mu}(1, g; V_g) - \hat{\mu}(0, g; V_g) , \qquad \hat{\tau} = \sum_{g \in \mathcal{G}} \hat{\tau}(g) \mathbb{P}(G_i = g) ,$$

$$\hat{\delta}(g; z) = \hat{\mu}(z, g; V_g) - \hat{\mu}(z, 0; V_g) , \qquad \hat{\Delta}(z) = \sum_{g \in \mathcal{G}} \hat{\delta}(g; z) \mathbb{P}(G_i = g) .$$

Forastiere et al. (2021) show that their estimators for the treatment and spillover effects are unbiased under three assumptions, the first two of which form the Stable Unit Treatment on Neighbourhood Value Assumption (SUTNVA, a generalization of SUTVA that relaxes the no interference assumption to allow interference of immediate neighbours) and the third being an unconfoundedness assumption that says the treatment assignment mechanism is conditionally independent of the outcomes for the given set of covariates.

3.2 Comparison to the literature

There appears to be a limited number of works in the literature that examine the similar problem of causal inference in observational data under general forms of interference. As noted by Forastiere et al., Liu et al. (2016) proposed inverse probability-weighted (IPW) and Hájek-type estimators for the causal treatment effect, and van der Laan (2014) proposed a novel targeted maximum likelihood estimator for the effect that was later further developed by Sofrygin and van der Laan (2017) and Ogburn et al. (2017). In IPW, effects are estimated by weighted averages of observed outcomes where one component of the weight is defined with respect to a hypothetical allocation strategy (an assumed distribution over the neighbouring treatments). The Bernoulli allocation strategy (Tchetgen & VanderWeele, 2012) is generally used, which assumes that each unit in the neighbourhood is assigned the treatment independently with probability α and that the unit assignment is independent of its neighbours' assignment. Homophily—the tendency for units with similar characteristics to form ties—is disregarded when the Bernoulli allocation strategy is used. As noted by Forastiere et al. (2021), one strength of their proposed method is that it does not rule out potential homophily and is able to do so by directly defining its estimators in terms of the observed neighbourhood treatments. TODO: TMLE? TODO: strengths–observed neighbourhood treatment vs hypothetical; weaker unconfoundedness assumption?

Jackson et al. (2020) use propensity-based estimators similar to Forastiere et al. (2021) but take homophily into account by modeling correlated treatment assignments as an incomplete information game. The recent work by Sánchez-Becerra (2021) questioned the justification of the unconfoundedness assumption with respect to a constructed statistic (e.g., the joint propensity score) and proposed a model-based estimator that is obtained by optimizing a loss function. TODO

More commonly, related works in the literature examine the inference problem under the assumption of partial interference. Under this assumption, individuals can be partitioned into groups where it is assumed that there are no spillover effects between groups. The focus on partial interference settings seems to be primarily due to momentum of earlier works (e.g., Sobel, 2006; Hudgens & Halloran, 2008) that looked at causal inference in randomized studies with interference, in which group-randomization tends to be more practical. Examples of recent work that assume partial interference include the work by Liu et al. (2019), Barkley et al. (2020), and Qu et al. (2021). It is notable that these works all propose IPW estimators despite exploring slightly difference contexts of the interference problem. Estimators based on IPW appear to be another idea in the literature that persisted since its original introduction by Tchetgen and VanderWeele (2012) for grouped observational data.

Several other works in the literature consider inference in observational studies with interference under specific contexts. For example, Toulis et al. (2018) explore the problem of treatment entanglement where treatment assignments are assumed to satisfy certain restrictions. Zigler and Papadogeorgou (2021) focus on the problem of bipartite causal inference with interference where treatments are applied to one unit and the outcome is measured on another.

4 Potential bias of naive estimator when unconfoundedness holds

TODOIn this section, we provide an example that illustrates the relevance of Theorem 2.A, Corollary 2 and Corollary 3 in the paper by Forastiere et al. (2021).

5 Reproducing results of simulation study

TODOIn this section, we aim to (partially) reproduce the results of the simulation study conducted by Forastiere et al. (2021). Note that we work with only the subset of the data that they use that is publicly accessible. The subset contains data for 6504 students (TODO: schools?) as opposed to the 16,410 students from 29 schools considered in their study.

6 Extending the simulation study

TODOIn this section, we consider a small extension to the simulation study.

7 Critical appraisal and concluding remarks

TODOWe conclude this report with a critical appraisal of the method proposed by Forastiere et al. (2021).
TODOcontributions, limitations
TODOunconfoundedness assumption? Sánchez-Becerra (2021)

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