

Differential Calculus and **Sage**

(revised edition)

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with extra material added by David Joyner
and updates by Charles Ross

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Added 2007: For further information on William granville, please see the Wikipedia article at http://en.wikipedia.org/wiki/William_Anthony_Granville, which has a short biography and links for further information.

Granville's book "Elements of the Differential and Integral Calculus" fell into the public domain and then much of it (but not all, at the time of this writing) was scanned into http://en.wikisource.org/wiki/Elements_of_the_Differential_and_Integral_Calculus primarily by P. J. Hall. This wikisource document uses MathML and L^AT_EX and some Greek letter fonts. The current L^AT_EX document is due to David Joyner, who is responsible for the formatting, editing for readability, the correction of any typos in the scanned version, and any extra material added (for example, the hyperlinked cross references, and the **Sage** material). Please email corrections to wdjoyner@gmail.com. In particular, the existence of this document owes itself primarily to three great open source projects: T_EX/L^AT_EX Wikipedia, and **Sage**. More information on **Sage** can be found at the **Sage** website (<http://www.sagemath.org>). Some material from Sean Mauch's public domain text on Applied Mathematics, <http://www.its.caltech.edu/~sean/book.html> was also included.

Though the original text of Granville is public domain, the extra material added in this version is licensed under the GNU Free Documentation License (please see <http://www.gnu.org/copyleft/fdl.html>), as is most of Wikipedia.

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Chapter 0

Preface



Figure 1: Sir Isaac Newton.

That teachers and students of the Calculus have shown such a generous appreciation of Granville's "Elements of the Differential and Integral Calculus" has been very gratifying to the author. In the last few years considerable progress has been made in the teaching of the elements of the Calculus, and in this revised edition of Granville's "Calculus" the latest and best methods are exhibited,—methods that have stood the test of actual classroom work. Those features of the first edition which contributed so much to its usefulness and popularity have been retained. The introductory matter has been cut down somewhat in order to get down to the real business of the Calculus sooner. As this is designed essentially for a drill book, the pedagogic principle that each result should be made intuitively as well as analytically evident to the student has been kept constantly in mind. The object is not to teach the student to rely on his intuition, but, in some cases, to use this faculty in advance of analytical investigation. Graphical illustration has been drawn on very liberally.

This Calculus is based on the method of limits and is divided into two main parts,—Differential Calculus and Integral Calculus. As special features, atten-

tion may be called to the effort to make perfectly clear the nature and extent of each new theorem, the large number of carefully graded exercises, and the summarizing into working rules of the methods of solving problems. In the Integral Calculus the notion of integration over a plane area has been much enlarged upon, and integration as the limit of a summation is constantly emphasized. The existence of the limit e has been assumed and its approximate value calculated from its graph. A large number of new examples have been added, both with and without answers. At the end of almost every chapter will be found a collection of miscellaneous examples. Among the new topics added are approximate integration, trapezoidal rule, parabolic rule, orthogonal trajectories, centers of area and volume, pressure of liquids, work done, etc. Simple practical problems have been added throughout; problems that illustrate the theory and at the same time are of interest to the student. These problems do not presuppose an extended knowledge in any particular branch of science, but are based on knowledge that all students of the Calculus are supposed to have in common.

The author has tried to write a textbook that is thoroughly modern and teachable, and the capacity and needs of the student pursuing a first course in the Calculus have been kept constantly in mind. The book contains more material than is necessary for the usual course of one hundred lessons given in our colleges and engineering schools; but this gives teachers an opportunity to choose such subjects as best suit the needs of their classes. It is believed that the volume contains all topics from which a selection naturally would be made in preparing students either for elementary work in applied science or for more advanced work in pure mathematics.

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Figure 2: Gottfried Wilhelm Leibnitz.

Chapter 1

Collection of Formulas

Chapter 2

Variables and Functions

Chapter 3

Theory of Limits

Chapter 4

Differentiation

Chapter 5

Rules for Differentiating Standard Elementary Forms

Chapter 6

Simple Applications of the Derivative

Chapter 7

Successive Differentiation

Chapter 8

Maxima, Minima and Inflection Points

Chapter 9

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Change of Variable

Chapter 11

Curvature; Radius of Curvature

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Theorem of Mean Value; Indeterminant Forms

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Circle of Curvature; Center of Curvature

Chapter 14

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