Digital Modulation

Notes

Chapter Contents

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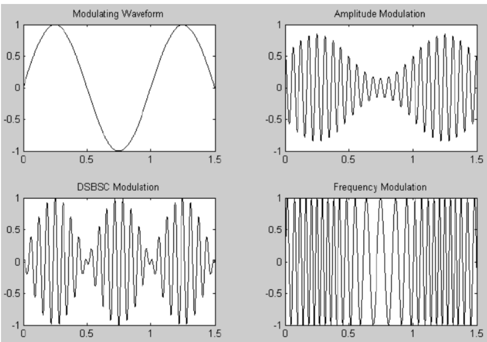
Waveform Types



Notes

Modulation

Notes

Amplitude Modulation:

$$x_m(t) = A(t)\cos(2\pi f_c t + \phi).$$

Phase Modulation (including FM):

$$x_m(t) = A\cos(2\pi f_c t + \phi(t)).$$

General Modulated Wave Formula:

$$x_m(t) = A(t)\cos(2\pi f_c t + \phi(t)),$$

QAM modulates both $A(t)$ and $\phi(t)$.

Digital Modulation

Notes

Famous digital modulation technique:

Morse Code

Morse code entered by **keying** a mechanical push button.

Digital modulation **over simplified** idea:

Standard (clocked) digital signal: $d(t) \in \{0, 1\}$.

Digital modulation signal: $m(t) = 2d(t) - 1$.

Then use $m(t)$ as an analogue modulating signal.

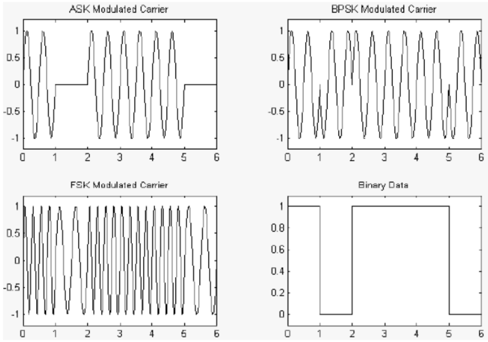
Digital Modulation: (Shift) Keying

Notes

Electronic digital modulation techniques include:

- Amplitude-Shift Keying (ASK)
- Frequency-Shift Keying (FSK)
- Phase-Shift Keying (PSK)
- (digital) Quadrature Amplitude Modulation (QAM)
- Binary Phase Shift Keying (BPSK)
    - *Similar to DSBSC or digital phase modulation.*

Digital Modulation: (Shift) Keying



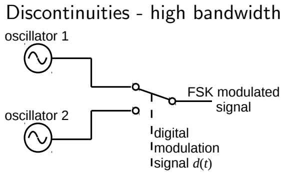
Notes

Frequency Shift Keying (FSK)

Varies analogue carrier frequency with digital modulating signal.
How to vary frequency?

BAD

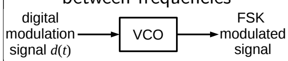
Discontinuities - high bandwidth



Only useful for low bite rate applications, e.g. storing data on audio cassette tapes.

GOOD

Continuous, smooth changes between frequencies

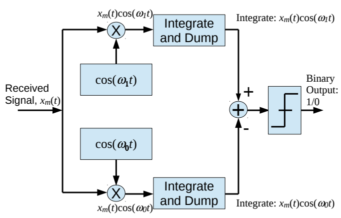


Notes

FSK

- Spacing of frequencies: $\Delta f = 2f_d$.
- Beneficial for receiver if $\Delta f \times T_b \in \mathbb{Z}$ (an integer)
  - Acts as modulation index.
  - T_b one bit period

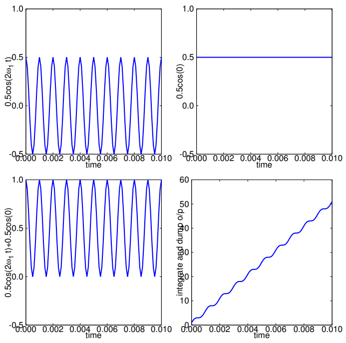
Receiver Circuit:



Integrate and Dump: effectively averages signal over one symbol duration.

Notes

Integration and Dump...



Notes

FSK

- Assume received signal should be "1"
- $\therefore$ output from upper mixer = +ve DC voltage (standard *coherent demodulation*)
  - Output of integrate-and-dump = +ve voltage
- Output from lower mixer is a difference frequency $2f_d$

$$\cos((\omega_1 - \omega_0)t) = \cos(2\pi(f_1 - f_0)t) = \cos(4\pi f_d t) = \cos(2\pi \times 2f_d \times t).$$

- If integration period contains whole number of cycles of $2f_d$ then
  - Output of integrate-and-dump = 0.
- Subtracting 0 (bottom) from +ve voltage (top) = +ve.
- Threshold forces +ve to 1 (or suitable voltage level).

If received signal should be 0, then top = 0 and bottom = -ve voltage,
$\therefore$ output threshold = 0.

Notes

Phase Shift Keying (PSK)

Information in phase, not amplitude:

$$x_m(t) = A\cos(2\pi f_c + \phi(t)).$$

Most basic PSK, **Binary PSK** (BPSK):

- Phase of 0 radians = binary 1
- Phase of π radians = binary 0

Can be encoded by:

$$\phi(t) = \begin{cases} 0 & \text{if} \quad d(t) = 0 \quad \Rightarrow \quad x_m(t) = A\cos(2\pi f_c) \\ \pi & \text{if} \quad d(t) = 1 \quad \Rightarrow \quad x_m(t) = A\cos(2\pi f_c + \pi) \end{cases}$$

But also $x_m(t) = A\cos(2\pi f_c + \pi) = -A\cos(2\pi f_c)$.
Identical to digital DSBSC.

Notes

Analogue QAM

- Analogue modulation technique:
  Quadrature Amplitude Modulation (QAM)
- Digital equivalent is important
- Review analogue QAM first.

QAM uses *compound angle formula*:

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

to obtain I/Q components.

Notes

QAM

Using the compound angular formula:

$$\begin{aligned} x_m(t) &= A(t)\cos(2\pi f_c t + \phi(t)) \\ &= A(t)[\cos(2\pi f_c t)\cos(\phi(t)) - \sin(2\pi f_c t)\sin(\phi(t))] \\ &= A(t)\cos(\phi(t))\cos(2\pi f_c t) - A(t)\sin(\phi(t))\sin(2\pi f_c t) \\ &= I(t)\cos(2\pi f_c t) - Q(t)\sin(2\pi f_c t) \end{aligned}$$

where

- $I(t) = A(t)\cos(\phi(t))$,
- $Q(t) = A(t)\sin(\phi(t))$.

Notes

QAM

QAM modulated wave:

$$x_m(t) = I(t)\cos(2\pi f_c t) - Q(t)\sin(2\pi f_c t)$$

with

- Amplitude Modulation:

$$A(t) = \sqrt{I(t)^2 + Q(t)^2}$$

- Phase Modulation:

$$\phi(t) = \tan^{-1}\left(\frac{Q(t)}{I(t)}\right).$$

Notes

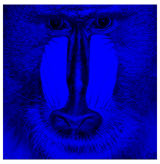
QAM: Subcarrier Chrominance Encoding

Computer Image: often RGB

Red

Green

Blue

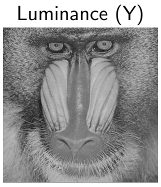


Notes

QAM: Subcarrier Chrominance Encoding

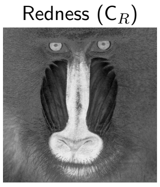
Colour Television: Luminance + Chrominance

Brightness
Luminance (Y)

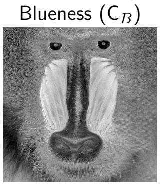


Chrominance (Colour Information)

Redness (C_R)

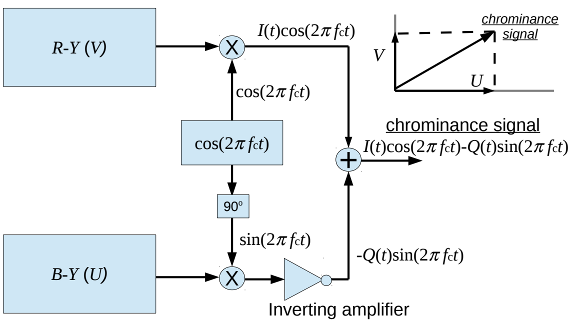


Blueness (C_B)



Notes

QAM: Subcarrier Chrominance Encoding



Notes

Digital QAM

Simultaneously carries 2 digital signals.
Digitally amplitude modulating 2 quadrature carriers:

- $\cos(2\pi f_c t)$
- $\sin(2\pi f_c t)$

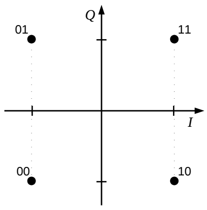
Possible states (4-QAM):

State Table

Data	I	Q
11	+1	+1
10	+1	-1
01	-1	+1
00	-1	-1

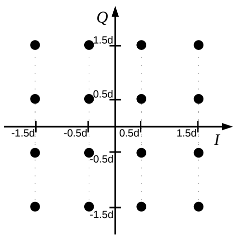
Can be drawn as constellation diagram...

Constellation Diagram (4QAM)



Notes

16QAM

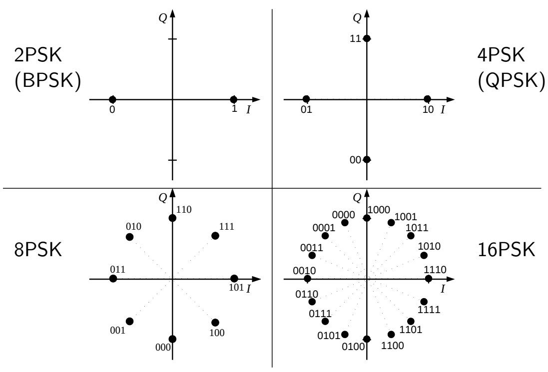


- Two bits (4 levels) to modulate I
  - Equivalent to 2bit D/A converter
- Two bits (4 levels) to modulate Q
  - Equivalent to 2bit D/A converter

Notes

M-ary PSK Schemes

To convey N bits, then $M = 2^N$:

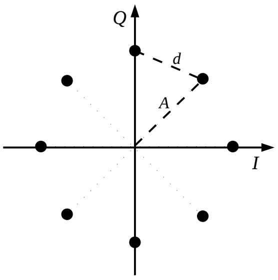


Notes

M-ary PSK

Variables in M-ary PSK scheme:

- A = amplitude or signal strength
- d = distance between states (state spacing)



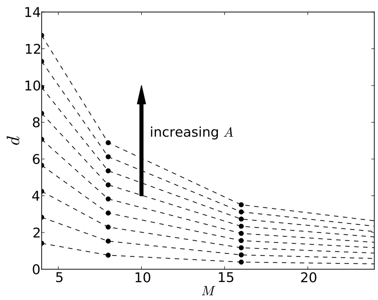
Distance between state points:

$$d = 2A\sin\left(\frac{\pi}{M}\right)$$

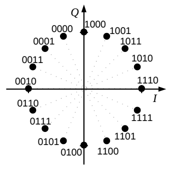
Notes

M-ary PSK Variables

As M increases it becomes more difficult for a receiver to distinguish (decide) between states:

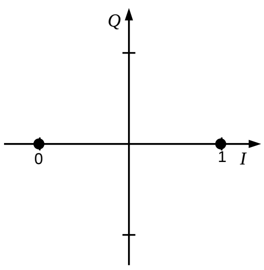


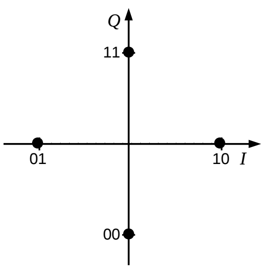
- Greater A (signal strength) can help to separate state points,
- but not useful as $M \geq 16$ because of small d.



Notes

BPSK and QPSK





BPSK QPSK

QPSK uses two orthogonal modulated carriers (cos and sin).
Therefore using available bandwidth more efficiently.

Notes

Bit-Rate versus Bandwidth

In general:

channel bandwidth to transmit digital signal	$\propto$	number of symbols sent per second (Baud rate)

Simple binary system (such as BPSK):

$$\text{Baud rate} = \text{Bit rate}.$$

M-ary schemes need less channel bandwidth than binary schemes for same bit rate.

Notes

Relative Bandwidth

$$\begin{aligned} B_{\text{rel}} &= \frac{\text{Baud Rate}}{\text{Bit Rate}} = \frac{\text{symbols/sec}}{\text{bits/sec}} \\ &= \frac{1}{\text{bits/symbol}} = \frac{1}{N} \\ &= \frac{1}{\log_2(M)}. \end{aligned}$$

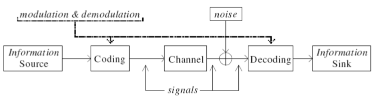
Scheme	B_{rel}
BPSK	1
QPSK	$\frac{1}{2}$
8-PSK	$\frac{1}{3}$
16-PSK	$\frac{1}{4}$
16-QAM	$\frac{1}{4}$

- M number of states in the scheme.

Low B_{rel} *is (usually)* **Good** *- but may become e.g. too sensitive to noise.*

Notes

Noise Sensitivity



- All **communication systems** affected by **noise**
- Need to minimise **Bit-Error-Ratio (BER)**:

$$\text{BER} = \frac{\text{Number of Bits in Error}}{\text{Number of Bits Sent}}.$$

- Need to move states as far apart as possible: **increase** d
- Could increase transmitter power (**increase** A). However
  - Increases Costs
  - High Power Amplifier Non-linearities
- Alternatively reducing B_{rel} using higher order modulation schemes.

Notes

Root Mean Square (RMS)

RMS amplitude of a carrier can be found by:

- Finding A^2 for each state
  - **Square**
- Finding Average A^2 for all states
  - **Mean**
- Taking the square root
  - **Root**

$$A_{\text{rms}} = \sqrt{\frac{1}{M} \sum_{i \text{ in all states}} A_i^2}.$$

16QAM has RMS amplitude:

$$A_{\text{rms}} = d\sqrt{\frac{5}{2}}.$$

Notes

Noise Sensitivity

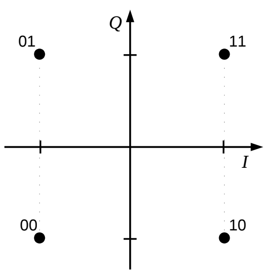
- Decreasing channel bandwidth reduces noise because

$$\text{noise power}, kTB \propto \text{channel bandwidth}, B$$
$$B \propto B_{\text{rel}} = \frac{\text{Baud rate}}{\text{Bit rate}}$$
$$\therefore \text{noise power} \propto B_{\text{rel}}$$
$$\therefore \text{noise RMS} \propto \sqrt{B_{\text{rel}}}$$

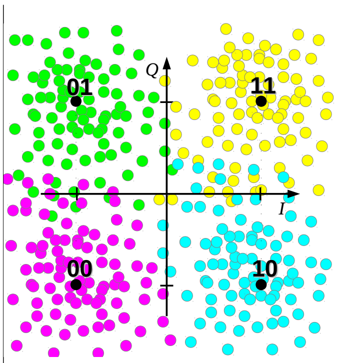
- *Higher order systems are more sensitive to noise* (for fixed transmitter power) because
  - states are closer together.
  - If A_{rms} is constant then d must get smaller.
  - Leads to more sensitivity to noise...

Notes

Noisy Constellation Example



Transmitted or Theoretical 4QAM



Received / Noisy 4QAM

Notes

Signal Power

To compare modulation schemes...

- Assume transmitter power for modulation scheme selected so that d is fixed multiple of noise RMS.
- Does not guarantee exactly same Bit-Error-Ratio between modulation schemes
- Leads to Signal Power for approximate equivalence:

$$P \propto B_{\text{rel}} \times \left(\frac{A}{d}\right)^2.$$

- B_{rel} depends on
  - number of states, M
- $\left(\frac{A}{d}\right)^2$ depends on
  - Type of keying, *and*
  - Number of states.

Notes

Relative Signal Power

Transmitter power P required to obtain same level of Symbol Errors (not BER) using

$$P \propto B_{\text{rel}} \times \left(\frac{A}{d}\right)^2$$

Scheme	B_{rel}	$(A/d)^2$	$(A/d)^2 B_{\text{rel}}$	dB relative to BPSK
BPSK	1	0.25	0.250	0dB
4PSK	1/2	0.5	0.250	0dB
8PSK	1/3	1.7071	0.569	+3.57dB
16PSK	1/4	6.5685	1.642	+8.17dB
16QAM	1/4	2.5	0.625	+3.98dB

- Higher order schemes increase transmitter power need
- QAM needs less power than PSK for same channel bandwidth

Notes

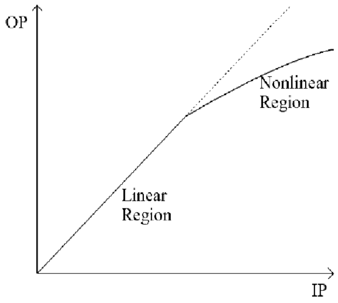
High Power Amplifier Nonlinearities

- All devices slightly nonlinear
- Very high power devices can be very nonlinear
  - e.g. satellite broadcasting transmitters using TWT or SSPAs
- Nonlinear effects include:
  - Generation of harmonics
  - Distortion components with same frequency band as input signal
    - Cannot be removed by filtering
- Examples:
  - Instantaneous amplitude amplifier output not a constant multiple of input amplitude
    - **AM/AM conversion distortion**.
  - Amplifier phase shift not constant
    - **AM/PM conversion distortion**

Notes

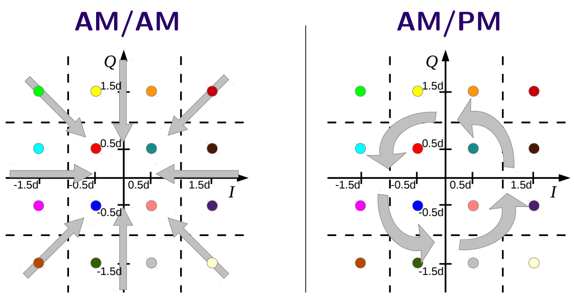
AM/AM Conversion Distortion

Characterized by nonlinearities in the input signal power versus output signal power.



Notes

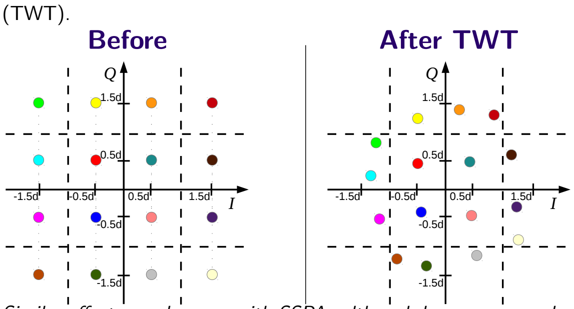
AM/AM versus AM/PM on QAM



Notes

Combined Affect on QAM

Characterized by nonlinearities in the phase.
This might be seen after simulating a Travelling Wave Tube (TWT).



Similar effects can be seen with SSPAs although less pronounced especially for phase.

Notes

SNR for Digital Communication

Analogue communications use Signal to Noise Ratio (SNR)

$$\text{SNR} = \frac{\text{signal power}}{\text{noise power}}$$

Digital systems use:

$$\frac{\text{bit energy}}{\text{noise power spectral density}} = \frac{\text{amount of power for each bit}}{\text{amount of noise across bandwidth}}$$

$$= \frac{\text{signal power} \times \text{bit time}}{\frac{\text{noise power}}{\text{bandwidth}}}$$

Notes

SNR for Digital Communication

Analogue communications use Signal to Noise Ratio (SNR)

$$\text{SNR} = \frac{\text{signal power}}{\text{noise power}}$$

Digital systems use:

$$\frac{\text{bit energy}}{\text{noise power spectral density}} = \frac{\text{signal power} \times \text{bit time}}{\frac{\text{noise power}}{\text{bandwidth}}}$$

$$= \frac{\text{signal power} \times \frac{1}{\text{bit rate}}}{\frac{\text{noise power}}{\text{bandwidth}}}$$

$$= \frac{\text{signal power}}{\text{noise power}} \times \frac{\text{bandwidth}}{\text{bit rate}}$$

Notes

SNR for Digital Communication

Analogue communications use Signal to Noise Ratio (SNR)

$$\text{SNR} = \frac{S}{N}$$

Digital systems use:

$$\frac{E_b}{N_0} = \frac{S \times T_b}{\frac{N}{W}}$$

$$= \frac{S \times \frac{1}{R}}{\frac{N}{W}}$$

$$= \frac{S}{N} \times \left(\frac{W}{R}\right).$$

Notes

SNR for Digital Communication

$$\frac{E_b}{N_0} = \frac{S}{N} \times \left(\frac{W}{R}\right).$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

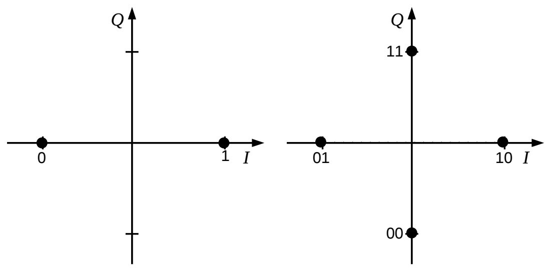
$$\frac{\text{bit energy}}{\text{noise power spectral density}} = \text{SNR} \times \frac{\text{bandwidth}}{\text{bit rate}}$$

SNR is normalized by: bandwidth to bit rate ratio.
Also dimensionless.

Useful for comparing performance of different digital communication processes.

Notes

BPSK and QPSK



BPSK QPSK

Same Bit Error Probability.

Notes

BPSK and QPSK

BPSK	QPSK
$\times 1$ BPSK signal	$\times 2$ orthogonal BPSK signals
$\times 1$ A amplitude signal	$\times 2$ $A/\sqrt{2}$ signal amplitudes
S Average Power	$S/2$ Average Power
R Bit Rate	$R/2$ Bit Rate
$\frac{E_b}{N_0} = \frac{S}{N}\left(\frac{W}{R}\right)$	$\frac{E_b}{N_0} = \frac{S/2}{N}\left(\frac{W}{R/2}\right) = \frac{S}{N}\left(\frac{W}{R}\right)$

Same Bit Error Probability.

Notes

Summary

- Modulation is a way of preparing a signal for transmission
- It needs to make efficient use of available bandwidth
- Overcome problems with noise and other sources of distortion

Notes

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