Introduction to Digital Signals and Digital Communication

Notes

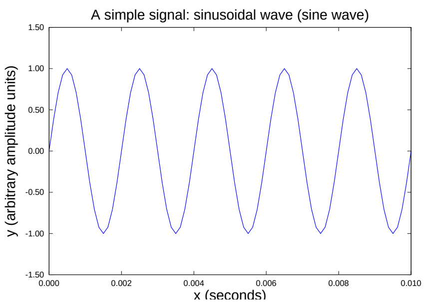
# Chapter Contents



Notes

# What is a Signal?

A simple example.

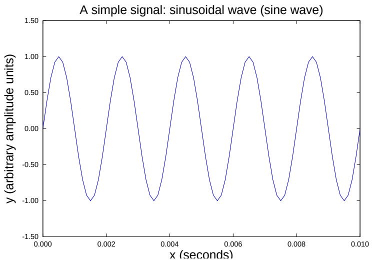



Notes

## What is a Signal?

**Can contain information for**

- *Communication*
- *Storage*
- *Calculation*




Notes

## Example of *What is a Signal?*

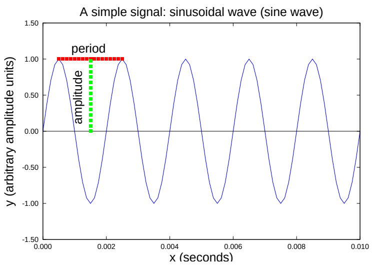
**Information is carried in the**

- *amplitude*, "$a$";
- *period*, "$T$";
- *frequency*, "$f = 1/T$";
- and *phase*, "$\phi$".

Equation for a sine wave:

$$y(x) = a \sin(2\pi f x + \phi)$$

where "$x$" is time in seconds for this example. Amplitude "$a = 1$" controls the height of the wave.



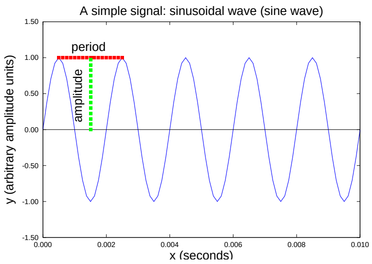

Notes

## Frequency and Period

Equation for a sine wave:

$$y(x) = a \sin(2\pi f x + \phi)$$

- $f$ is the frequency
- Measured in Hertz or Hz
- Here period, $T = 0.002$s
- $f = 1/T$ Hz, therefore $f = 1/0.002 = 500$Hz.




Notes

# Phase

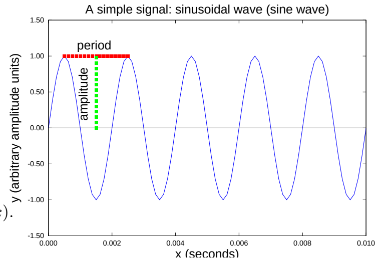
Equation for a sine wave:

$$y(x) = a\sin(2\pi fx + \phi)$$

- $\phi$ is the phase
- Here $\phi = 0$

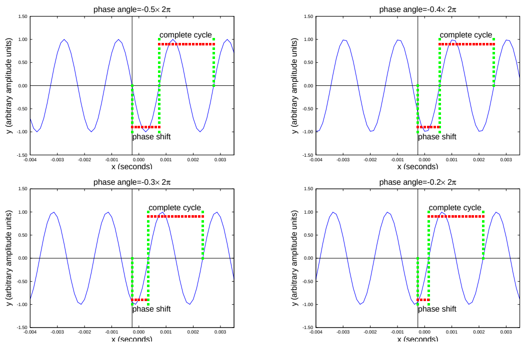
Therefore here,

$$y(x) = y(x, \phi = 0) = a\sin(2\pi fx).$$



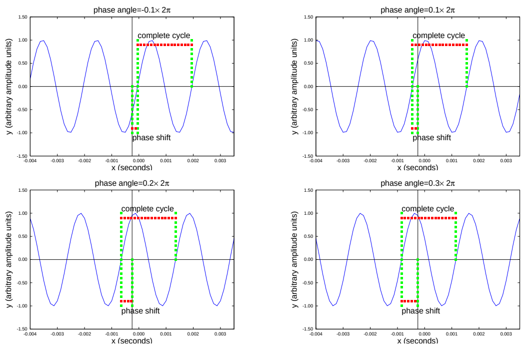

Notes

# Phase *examples*




Notes

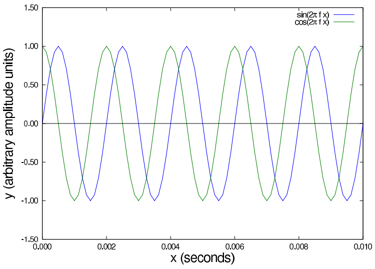
# Phase *examples cont'd*




Notes

## Cosine Vs Sine

Cosine and Sine functions are equivalent except for a phase shift ($1/4\times$period).



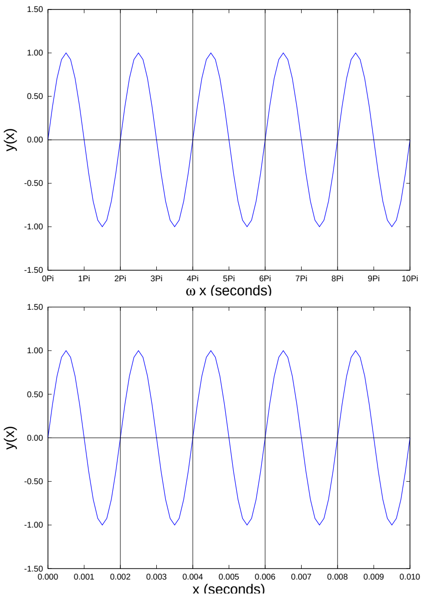

- $\cos(2\pi f x) = \sin(2\pi f x + \phi)$ where $\phi = \pi/2$.
- $\sin(2\pi f x) = \cos(2\pi f x + \phi)$ where $\phi = -\pi/2$.

Notes

## Angular Frequency

- Frequency, $f = 1/T$
- Angular frequency, $\omega = 2\pi f$
- 1 period or cycle $= 2\pi$ radians

$$y(x) = \sin(2\pi f x + \phi) \\ = \sin(\omega x + \phi)$$




Notes

## Complex Numbers

The square root of minus one is not defined so a symbol, $j$ is used (sometimes $i$):

$$j = \sqrt{-1}.$$

Powers:

- $j^2 = -1$
- $j^3 = -j$
- $j^{-1} = 1/j = -j$

If $z = x + jy$ (rectangular form) then alternative representations are:

- Polar form: $z = a\angle\phi$
- Exponential form: $z = a\exp(j\phi)$

where $a = \sqrt{x^2 + y^2}$ and $\phi = \tan^{-1}(y/x)$.

Notes

# Properties of Complex Numbers

If $z = x + jy$, $z_1 = x_1 + jy_1$ and $z_2 = x_2 + jy_2$ then

- Addition:
  $z_1 + z_2 = x_1 + x_2 + j(y_1 + y_2)$
- Subtraction:
  $z_1 - z_2 = x_1 - x_2 + j(y_1 - y_2)$
- Multiplication:
  $z_1 z_2 = a_1 a_2 \angle(\phi_1 + \phi_2)$
- Division:
  $z_1/z_2 = a_1/a_2 \angle(\phi_1 - \phi_2)$
- Reciprocal: $1/z = 1/a\angle(-\phi)$
- Square root: $\sqrt{z} = \sqrt{a}\angle(\phi/2)$
- Complex conjugate:
  $z^* = x - jy = a\angle - \phi$

The polar form simplifies some operations such as multiplication and division of complex numbers.

Notes

# Complex Exponentials, Sines and Cosines

Given

- $y_1(x) = b\exp(j\omega x) = b\cos(\omega x) + jb\sin(\omega x)$
- $y_2(x) = b\exp(-j\omega x) = b\cos(\omega x) + jb\sin(-\omega x)$
  as
  - $\cos(-\omega x) = \cos(\omega x)$ (even function)
  - $\sin(-\omega x) = -\sin(\omega x)$ (odd function)

Then

$$y_1(x) + y_2(x) = 2b\cos(\omega x).$$

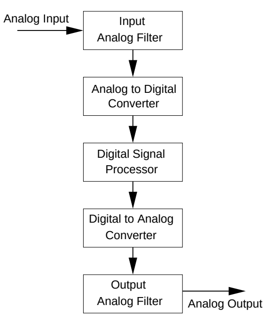
So that

$$b\cos(\omega x) = \frac{a}{2}\exp(j\omega x) + \frac{a}{2}\exp(-j\omega x).$$

A similar approach can be used to derive a sine function.

Notes

# A Typical Digital Signal Processing System




- Input Analog Filter (antialiasing):
  *Limits frequency range*
- Analog to Digital Converter
  *Converts signal to digital samples*
- Digital Signal Processor
  *Storage, Communication and or Calculations*
- Digital to Analog Converter
  *Convert to continuous signal*
- Output Analog Filter
  *Removes sharp transitions*

Notes

# Analog to Digital Converter

- Real world is typically *analog* (continuous)
- Digital signal approximates analog signal with discrete quantised samples
- ADC converts an analog signal to a digital signal
- Signal is digitised in two ways:
  - Signal is sampled at a sampling rate or frequency: Information is collected about the signal at regular intervals.
  - The continuous or analog signal is then quantised: *i.e.* put into digital form, where only a finite set of numbers are represented.

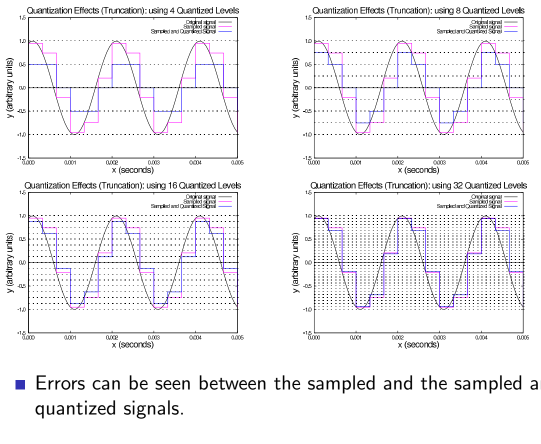
Notes

# Quantisation using Truncation

- Signal can be quantised using *e.g.* truncation where numbers following specified position are removed.
- Examples:
  - 5.7 truncated to integer is 5
  - 5.11 truncated to 1 decimal place is 5.1
- Negative numbers are truncated in the same way (note <u>different</u> to the common *floor* function in matlab), *e.g.*
  - -5.78 truncated to integer is -5
  - -5.135 truncated to 2 decimal places is -5.13

Notes

# Truncation Quantisation *examples*




- Errors can be seen between the sampled and the sampled and quantized signals.

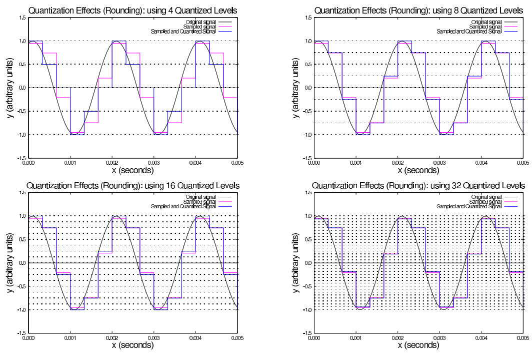
Notes

## Quantisation using Rounding

- Rounding can be a quantization method associated with smaller errors, *e.g.*
  - 5.7 rounded to nearest integer is 6
  - 5.11 rounded to 1 decimal place is 5.1
  - -5.78 rounded to nearest integer is -6
  - -5.135 rounded to 2 decimal places is -5.14

Notes

## Rounding Quantisation *examples*

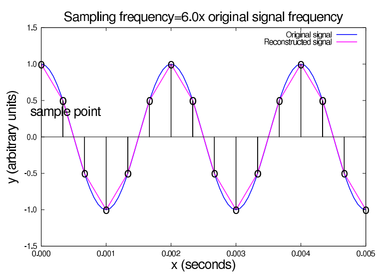



- Errors can be seen between the sampled and the sampled and quantized signals.

Notes

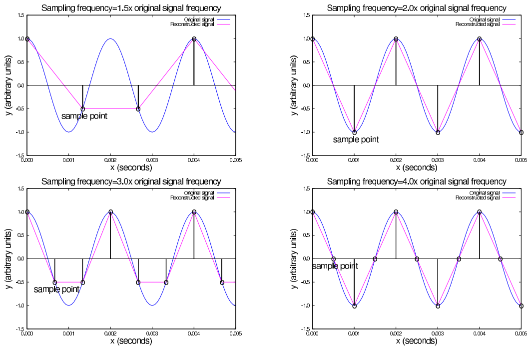
## Sampling

- Sampling also affects the quality of the digitised signal.
- Higher sampling rate reduces error and enables better representation of the original analog signal in digital form.



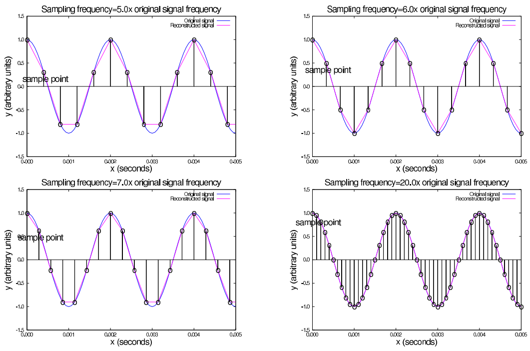

Notes

# **Sampling** *examples*




Notes

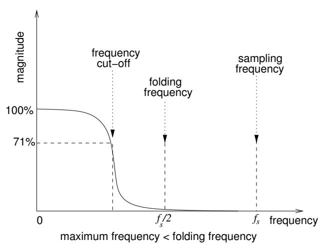
# **Sampling** *examples cont'd*




Notes

# **Input Analog Filter: Antialiasing Filter**

- Analog to Digital Converter (ADC) requires signal below a particular frequency (Nyquist Frequency)
- ∴ Limit frequency range to below Nyquist frequency ($f_s/2$) before Analog to Digital Conversion.



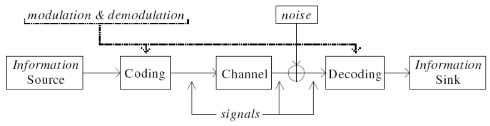

- Otherwise next stage produces frequency errors (*i.e.* aliasing)
- Sampling produces copies of signal at multiples of sampling frequency
- Aliasing occurs when copies of signal overlap each other

Notes

## Basic Communication System

Notes

How to perform electronic communication?




- *Coding can be analogue or digital*
- **Coding prepares the signal for transmission or storage**.

## Coding

Notes

Analogue coding can include:

- Modulation / Demodulation e.g.
  - Amplitude Modulation (AM),
  - Frequency Modulation (FM)

Digital coding can include:

- Cryptography
- Compression
- Channel coding:
  - Error correction coding (adding redundancy)

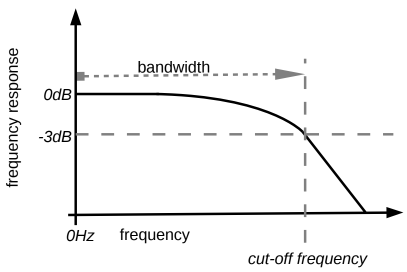
## Frequency Response

Notes

Bandwidth =

- range of frequencies with response above -3dB.

Low Pass Filter or Baseband System

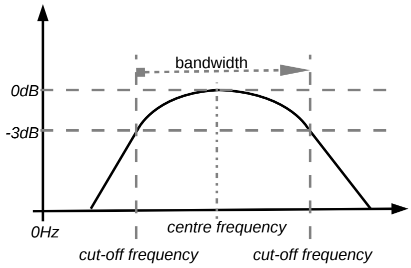


## Frequency Response

Bandwidth =

- range of frequencies with response above -3dB.

Bandpass Filter or Bandpass System




Notes

## Audio

| | Frequency Range *min* | *max* | Dynamic Range |
|---|---|---|---|
| **Human Hearing** | 20Hz | 20kHz | ~140dB |
| **CD** | 2Hz | 22.05kHz | 96dB |
| **Telephone** | 300Hz | 3.4kHz | <48dB |
| **Speech** | 300Hz | 8kHz | <70dB |
| **Woofer** | <10kHz | | >96dB |
| **Tweeter** | >2kHz | | |

Notes

Digital Dynamic Range (DNR) affected by **Quantisation Noise**:
e.g. CD records with $Q = 16$ bit signal:

$$\text{DNR} = 20 \times \log_{10}(2^Q) = 96.3\text{dB}$$

Analogue DNR dependent on minimum and maximum measurable signal values.
e.g. if $V_{\min} = 100\text{mV}$ and $V_{\max} = 4.5\text{V}$ then:

$$\text{DNR} = 20 \times \log_{10}\left(\frac{V_{\max}}{V_{\min}}\right) = 33.1\text{dB}$$

Notes

## Summary

- Description of a sinusoidal signal
- Cosine and Sine functions
- Complex numbers and alternative representations
- A typical digital signal processing system
- A typical digital communication system

Notes