Tutorial Questions:
Recursive Digital Filters

1. What is the condition on α for stability?
2. What procedure would you use to find $h(t)$ from $H(s)$?
3. Use the structure of the AR part of the filter to explain why its impulse response never quite dies away and thus why digital filters with feedback (and potentially feed-forward as well) are generally known as Infinite Impulse Response (IIR) filters.
4. A first order filter has a transfer function:

$$H(z) = \frac{a_0 + a_1 z^{-1}}{1 + b_1 z^{-1}}$$

   Determine the zeros and poles for this filter.

5. Explain why the set of values $z = e^{j\Omega}$ is the unit circle.
6. A first-order digital filter has impulse response:

$$h[n] = \frac{a_1}{b_1}\delta[n] + \left(a_0 - \frac{a_1}{b_1}\right)(-b_1)^n.$$

   Explain why this must be an IIR filter rather than an FIR filter.

7. Find the z-plane poles and zeros of the transfer function of the biquadratic filter with

$$H(z) = \frac{a_0 z^2 + a_1 z + a_2}{z^2 + b_1 z + b_2}.$$

8. Find the previous question filter's impulse response $h[n]$ by using the partial fraction method. Use it to explain why this filter must be IIR rather than FIR.
9. Draw a block diagram which represents the difference equation $y[n] = a_0 x[n] + a_1 x[n-1] + a_2 x[n-2] - b_1 y[n-1] - b_2 y[n-2]$ and explain the similarities and differences between it and the diagram of the biquadratic filter.
10. Find the frequency-domain response of this filter from the difference equation by assuming its input is a complex sinusoid of frequency f and its output is the same frequency but multiplied by complex $H(f)$. Compare the result with the z-domain transfer-function H(z).
11. Determine (i) a cascade and (ii) a parallel realisation for the following transfer function using only first-order structures:

$$H(z) = \frac{z(z-1)}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{8}\right)}.$$

   Sketch the block diagrams which result and compare them with the block diagram you would get from implementing the above as a single biquadratic structure.

12. Follow the steps below to design an IIR digital low-pass filter $G(z)$ with sampling frequency 44.1kHz and 3dB frequency of 11.025kHz using a second-order Butterworth filter analogue prototype:

$$G(s) = \frac{1}{\left(\frac{s}{\alpha}\right)^2 + \sqrt{2}\left(\frac{s}{\alpha}\right) + 1}.$$

   where α is the analogue 3dB angular frequency.

   (a) Find the value of α given the above sampling and cut-off frequency.
   (b) Find the z-domain transfer function of the digital filter.
   (c) Find the poles and zeros of $G(z)$
   (d) Find a difference equation that can approximate the transfer function then draw a direct form 1 realisation for the difference equation.