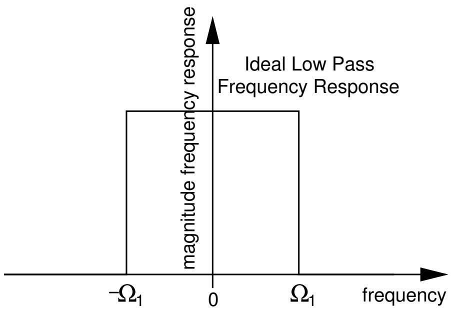
Solutions for: Non-Recursive Digital Filters Tutorial

1. What is the frequency domain representation for an ideal low pass filter with cut-off frequency Ω_1? Write the equation and sketch the function.
Solution The ideal low pass frequency response is given by:

$$H(\Omega) = \begin{cases} 1 & \text{for} \quad -\Omega_1 \leq \Omega \leq \Omega_1, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

This equation states that any frequency Ω between $-\Omega_1$ and $+\Omega_1$ can pass without attenuation ($H(\Omega) = 1$). All other frequencies are stopped altogether.



2. What is the time domain impulse response for an ideal low pass frequency response filter with $\Omega_1 = \pi/2$? (Calculate and sketch the result).
Solution The time domain representation of (1) is given by

$$\begin{aligned}
h[n] =& \frac{1}{2\pi}\int_{-\pi}^{\pi} H(\Omega)\exp(j\Omega\pi)\mathrm{d}\Omega = \frac{1}{2\pi}\left(\int_{-\pi}^{-\Omega_1} 0\mathrm{d}\Omega + \int_{-\Omega_1}^{\Omega_1} 1\times\exp(j\Omega\pi)\mathrm{d}\Omega + \int_{\Omega_1}^{\pi} 0\mathrm{d}\Omega\right) \\
=& \frac{1}{2\pi}\left([0]_{-\pi}^{-\Omega_1} + \left[\frac{\exp(j\Omega n)}{jn}\right]_{-\Omega_1}^{\Omega_1} + [0]_{\Omega_1}^{\pi}\right) = \frac{1}{2\pi}\left[\frac{\exp(j\Omega n)}{jn}\right]_{-\Omega_1}^{\Omega_1} \\
=& \frac{1}{2\pi jn}\left(\exp(j\Omega_1 n) - \exp(-j\Omega_1 n)\right) = \frac{1}{2\pi jn}2j\sin(\Omega_1 n) = \frac{1}{\pi n}\sin(\Omega_1 n) \\
=& \frac{\Omega_1}{\pi}\mathrm{sinc}(n\Omega_1)
\end{aligned}$$

where $\mathrm{sinc}(n\Omega_1) = \frac{\sin(n\Omega_1)}{n\Omega_1}$ is known as the sinc function.
Therefore if $\Omega_1 = \pi/2$ then:

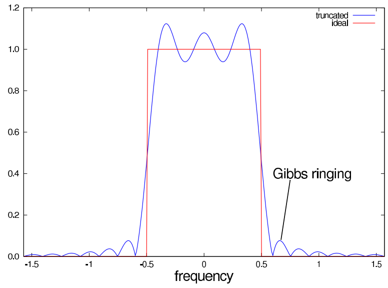
$$\begin{aligned}
h[n] =& \frac{\pi}{2\pi}\mathrm{sinc}\left(n\frac{\pi}{2}\right) = \frac{1}{2}\mathrm{sinc}\left(\frac{n\pi}{2}\right) \\
=& \frac{2}{n\pi}\frac{1}{2}\sin\left(\frac{n\pi}{2}\right) \\
=& \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right).
\end{aligned}$$

$h[n]$ is sampled at discrete values of n.

3. What is the period for a time domain sinc function?
Solution The period is ∞ as the time domain sinc function continues forever.

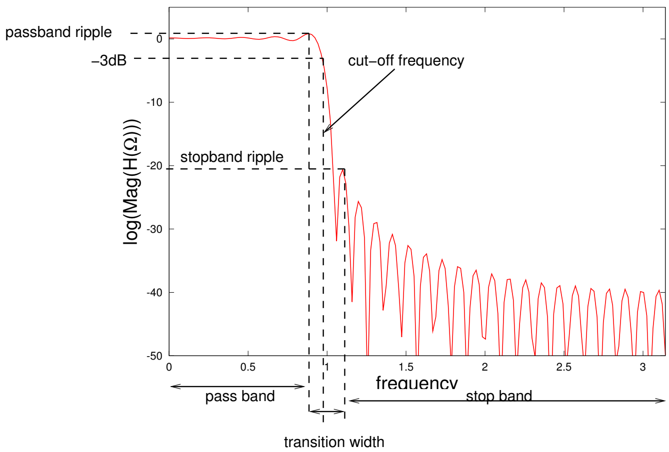
4. What is a window function used for in FIR filter design?
Solution A window function is used to shorten the sinc function to a finite length so that it can be implemented as a discrete number of filter coefficients.

5. What is Gibbs ringing? Explain in 3 sentences and draw a labelled sketch of Gibbs ringing.
Solution Gibbs ringing is an effect in the frequency domain which results from cutting the sinc function in the time domain to a finite length. The effect is ripples or bumps in the frequency domain surrounding the edges of the cut-off frequencies. The frequency domain function is therefore no longer ideal and the ripples mean that some frequencies are allowed to pass through the filter that would not have passed through an ideal filter.



6. Give the name and equation for a commonly used window function in FIR filter design.
   Solution Rectangle window: $w[n] = 1$ for $|n| \leq$ window width$/2$.

7. Sketch the frequency domain response of the window (above) which has been convolved (convolution in frequency domain) with an ideal low pass filter. Label the sketch with the following filter parameters: stop band, transition width, pass band, cut-off frequency, passband ripple, stopband ripple.
   Solution



8. How much attenuation (in decibels) should there be at the cut-off frequency (or half power point)?
   Solution -3dB.

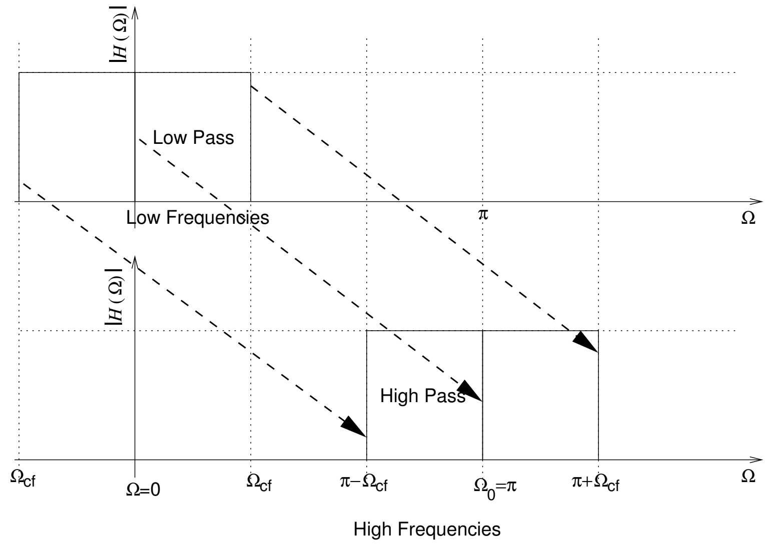
9. Give the equation for the time domain filter coefficients for a band pass FIR filter using an ideal low pass filter and window function $w[n]$.
   Solution

$$h[n] = \frac{\Omega_{\text{cf}}}{\pi} \text{sinc}(n\Omega_{\text{cf}}) \times w[n] \times \cos(n\Omega_0)$$

   where Ω_{cf} is the cut-off frequency, Ω_0 is the centre band frequency $w[n]$ is the window function.

10. How can the bandpass filter be made into a high pass filter? Sketch the transformation of the frequency responses.
    Solution A high pass filter can be achieved by

    - Shifting the impulse response to $\Omega_0 = \pi$,
    - Via multiplication by $\cos(n\Omega_0) = \cos(n\pi)$.
    - This is the limit of the unique part of the digital spectrum.


$|H(\Omega)|$
Low Pass
Low Frequencies
π
Ω
$|H(\Omega)|$
High Pass
Ω_{cf}
$\Omega=0$
Ω_{cf}
$\pi-\Omega_{cf}$
$\Omega_0=\pi$
$\pi+\Omega_{cf}$
Ω
High Frequencies