

Introduction to Digital Signal Processing

Digital Signal Processing

Notes

Contents

Digital Signal Processing
What is Digital Signal Processing?
Phase
Phasors and Complex Numbers

A Digital Signal Processing System

Notes

What is DSP?

Techniques include (e.g.)

- Filtering
- Frequency domain techniques (*i.e.* Fourier)
- Time domain techniques
- Random signals
- Predication and Estimation (e.g. time series estimation)

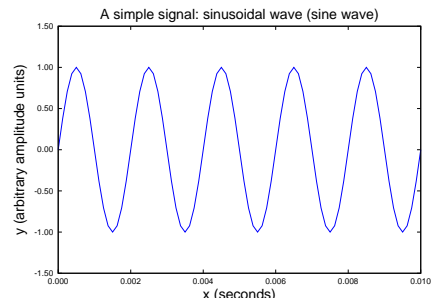
Example Applications

- Audio processing
- Communication systems
- Image processing
- Video processing
- Data compression
- Vehicle control
- Financial engineering

Notes

What is a Signal?

A simple example.

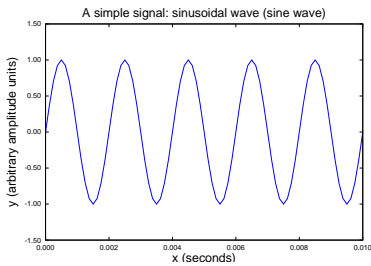


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What is a Signal?

Can contain information for

- Communication
- Storage
- Calculation



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Example of What is a Signal?

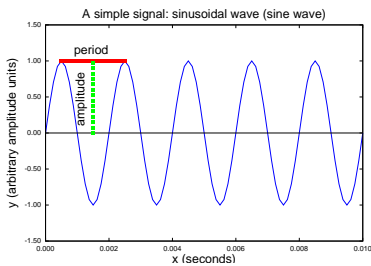
Information is carried in the

- amplitude, “ a ”;
- period, “ T ”;
- frequency, “ $f = 1/T$ ”;
- and phase, “ ϕ ”.

Equation for a sine wave:

$$y(x) = a \sin(2\pi f x + \phi)$$

where “ x ” is time in seconds for this example. Amplitude “ $a = 1$ ” controls the height of the wave.



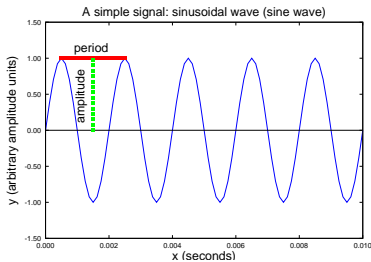
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Frequency and Period

Equation for a sine wave:

$y(x) = a \sin(2\pi f x + \phi)$

- f is the frequency
- Measured in Hertz or Hz
- Here period, $T = 0.002s$
- $f = 1/T$ Hz, therefore $f = 1/0.002 = 500Hz$.



Notes

Phase

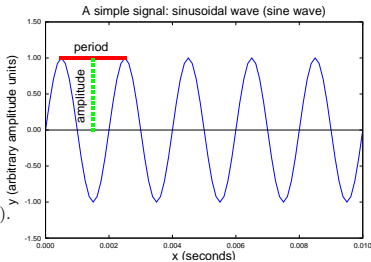
Equation for a sine wave:

$y(x) = a \sin(2\pi f x + \phi)$

- ϕ is the phase
- Here $\phi = 0$

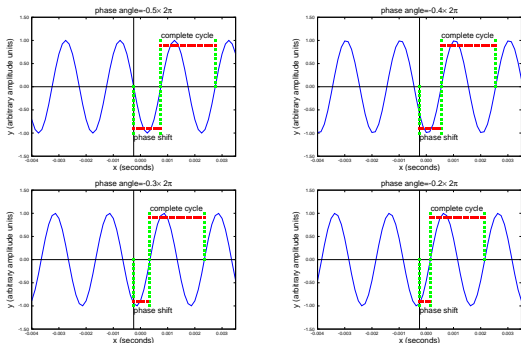
Therefore here,

$y(x) = y(x, \phi = 0) = a \sin(2\pi f x)$.



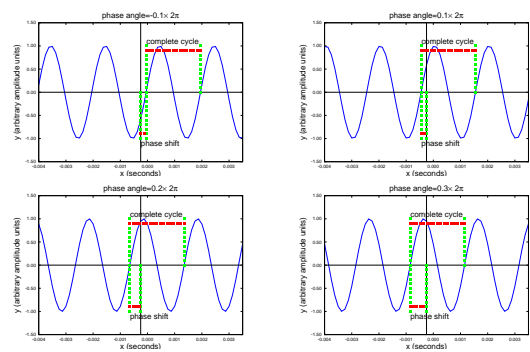
Notes

Phase examples



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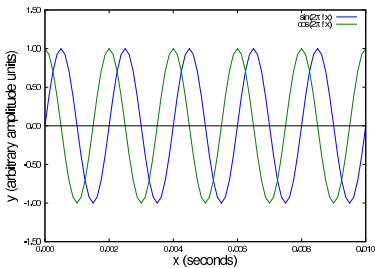
Phase examples cont'd



Notes

Cosine Vs Sine

Cosine and Sine functions are equivalent except for a phase shift ($1/4 \times \text{period}$).



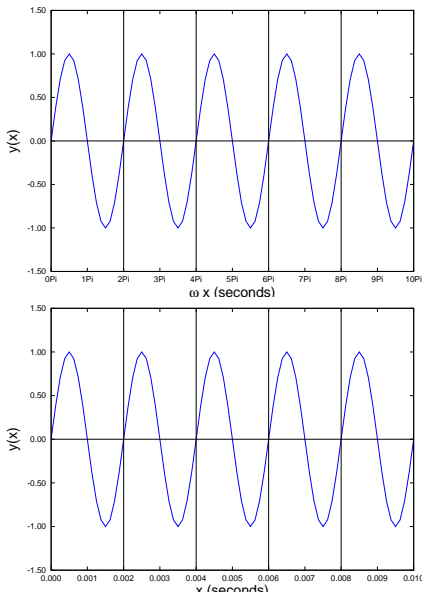
- $\cos(2\pi f x) = \sin(2\pi f x + \phi)$ where $\phi = \pi/2$.
- $\sin(2\pi f x) = \cos(2\pi f x + \phi)$ where $\phi = -\pi/2$.

Notes

Angular Frequency

- Frequency, $f = 1/T$
- Angular frequency, $\omega = 2\pi f$
- 1 period or cycle = 2π radians

$$y(x) = \sin(2\pi f x + \phi)$$
$$= \sin(\omega x + \phi)$$



Notes

Phasor Representation

A cosine (or sine) wave:

y(x) = a cos(ωx + φ)

can be represented as a phasor.
A phasor is a complex number:

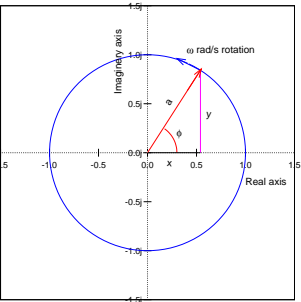
z = x + jy = a(cos(φ) + j sin(φ))

where x is known as the real part or Re(z) = x and y is known as the imaginary part or Im(z) = y.

x and y can be calculated with
x = a cos(φ) and y = a sin(φ).

Also remember j = √-1.

Argand or Phasor Diagram:



Notes

Complex Numbers

The square root of minus one is not defined so a symbol, j is used (sometimes i):

j = √-1.

Powers:

- j² = -1
- j³ = -j
- j⁻¹ = 1/j = -j

If z = x + jy (rectangular form) then alternative representations are:

- Polar form: z = a∠φ
- Exponential form: z = a exp(jφ)

where a = √(x² + y²) and φ = tan⁻¹(y/x).

Notes

Properties of Complex Numbers

If z = x + jy, z₁ = x₁ + jy₁ and z₂ = x₂ + jy₂ then

- Addition: z₁ + z₂ = x₁ + x₂ + j(y₁ + y₂)
 - Subtraction: z₁ - z₂ = x₁ - x₂ + j(y₁ - y₂)
 - Multiplication: z₁z₂ = a₁a₂∠(φ₁ + φ₂)
 - Division: z₁/z₂ = a₁/a₂∠(φ₁ - φ₂)
- Reciprocal: 1/z = 1/a∠(-φ)
 - Square root: √z = √a∠(φ/2)
 - Complex conjugate: z* = x - jy = a∠-φ

The polar form simplifies some operations such as multiplication and division of complex numbers.

Notes

Phasor Representation

Euler's identity: $\exp(j\phi) = \cos(\phi) + j \sin(\phi)$

Therefore

- $\cos(\phi) = \text{Re}(\exp(j\phi)) \longrightarrow$ or the real part, x
- $\sin(\phi) = \text{Im}(\exp(j\phi)) \longrightarrow$ or the imaginary part, y

Recall the cosine wave: $y(x) = \cos(\omega x + \phi)$

which can be written as:

$$y(x) = \text{Re}(a \exp(j(\omega x + \phi))) = \text{Re}(a \exp(j\omega x) \exp(j\phi))$$
$$= \text{Re}(A \exp(j\omega x))$$

where A is the phasor representation of $y(x)$ given by

$$A = a \exp(j\phi) = a\angle(\phi).$$

Notes

Complex Exponentials,
Sines and Cosines

Given

- $y_1(x) = b \exp(j\omega x) = b \cos(\omega x) + j b \sin(\omega x)$
- $y_2(x) = b \exp(-j\omega x) = b \cos(\omega x) + j b \sin(-\omega x)$

as

- $\cos(-\omega x) = \cos(\omega x)$ (even function)
- $\sin(-\omega x) = -\sin(\omega x)$ (odd function)

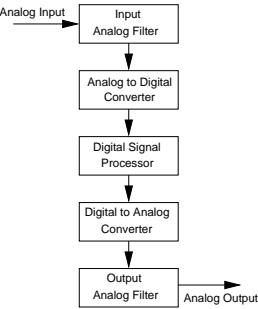
Then $y_1(x) + y_2(x) = 2b \cos(\omega x).$

So that $b \cos(\omega x) = \frac{a}{2} \exp(j\omega x) + \frac{a}{2} \exp(-j\omega x).$

A similar approach can be used to derive a sine function.

Notes

A Typical Digital Signal
Processing System



- Input Analog Filter
(antialiasing):
Limits frequency range
- Analog to Digital Converter
Converts signal to digital samples
- Digital Signal Processor
Storage, Communication and or Calculations
- Digital to Analog Converter
Convert to continuous signal
- Output Analog Filter
Removes sharp transitions

Notes

Analog to Digital Converter

- Real world is typically *analog* (continuous)
- Digital signal approximates analog signal with discrete quantised samples
- ADC converts an analog signal to a digital signal
- Signal is digitised in two ways:
 - Signal is sampled at a sampling rate or frequency: Information is collected about the signal at regular intervals.
 - The continuous or analog signal is then quantised: *i.e.* put into digital form, where only a finite set of numbers are represented.

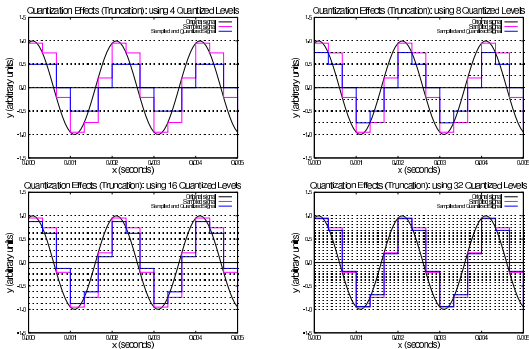
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Quantisation using Truncation

- Signal can be quantised using *e.g.* truncation where numbers following specified position are removed.
- Examples:
 - 5.7 truncated to integer is 5
 - 5.11 truncated to 1 decimal place is 5.1
- Negative numbers are truncated in the same way (note different to the common *floor* function in matlab), *e.g.*
 - -5.78 truncated to integer is -5
 - -5.135 truncated to 2 decimal places is -5.13

Notes

Truncation Quantisation *examples*



- Errors can be seen between the sampled and the sampled and quantized signals.

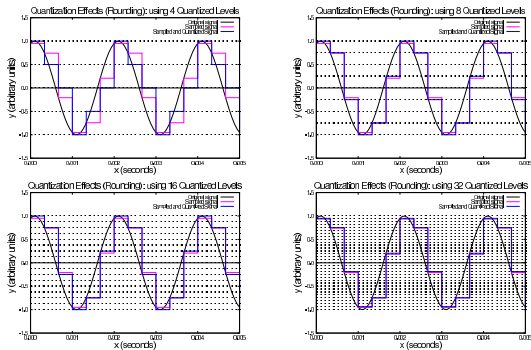
Notes

Quantisation using Rounding

- Rounding can be a quantization method associated with smaller errors, e.g.
 - 5.7 rounded to nearest integer is 6
 - 5.11 rounded to 1 decimal place is 5.1
 - -5.78 rounded to nearest integer is -6
 - -5.135 rounded to 2 decimal places is -5.14

Notes

Rounding Quantisation examples

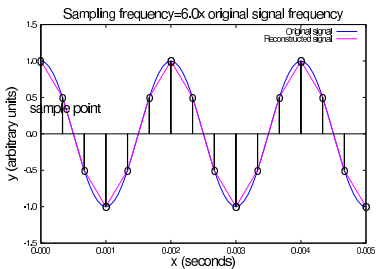


- Errors can be seen between the sampled and the sampled and quantized signals.

Notes

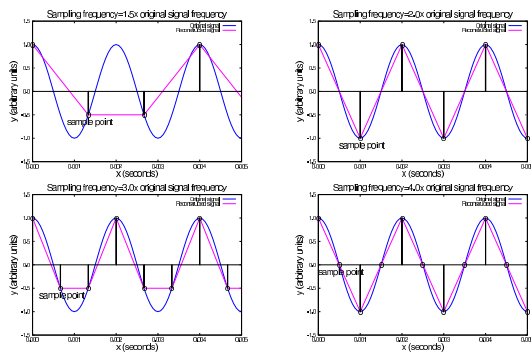
Sampling

- Sampling also affects the quality of the digitised signal.
- Higher sampling rate reduces error and enables better representation of the original analog signal in digital form.



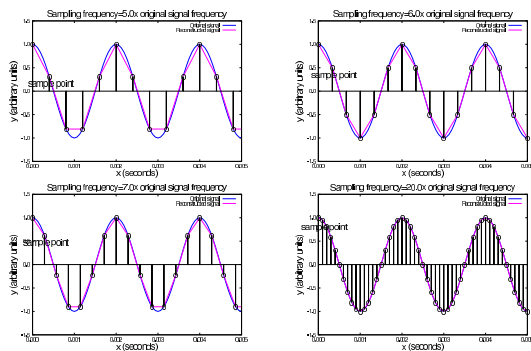
Notes

Sampling examples



Notes

Sampling examples cont'd

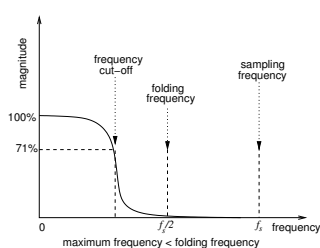


Notes

Input Analog Filter: Antialiasing Filter

- Analog to Digital Converter (ADC) requires signal below a particular frequency (Nyquist Frequency)
- \therefore Limit frequency range to below Nyquist frequency ($f_s/2$) before Analog to Digital Conversion.

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- Otherwise next stage produces frequency errors (*i.e.* aliasing)
- Sampling produces copies of signal at multiples of sampling frequency
- Aliasing occurs when copies of signal overlap each other

Digital Signal Processor

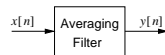
- After digitisation (with the ADC) digital signal processing may then be performed on the digitised signal.
- Simple example
 - Averaging filter:

$$y[n] = \frac{x[n] + x[n-1] + \dots + x[n-k+1]}{k}$$

for window width $k = 3$

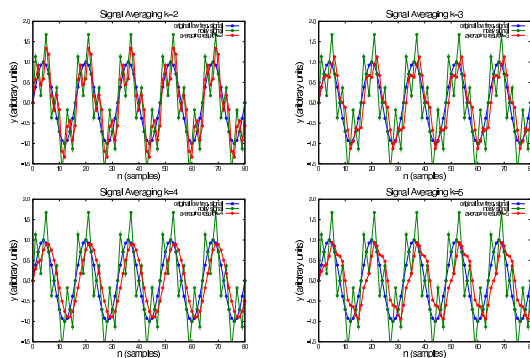
$$y[n] = \frac{x[n] + x[n-1] + x[n-2]}{3}$$

where $x[n]$ is an input value at sample time n and $y[n]$ is an output at sample time n



Notes

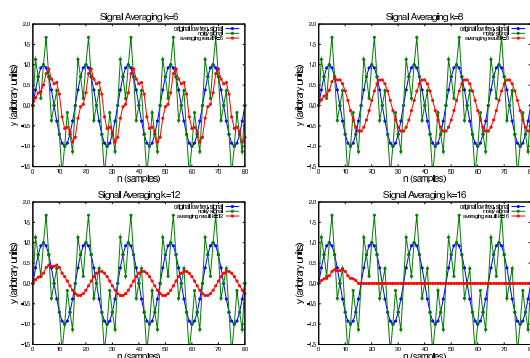
Averaging Filter Examples



Window width k controls the response of the filter. If k is too low, there is little benefit on output signal.

Notes

Averaging Filter Examples cont'd



Window width k controls the response of the filter. If k is too high, the filter removes all of the output signal.

Notes

Summary

- Definition of digital signal processing
- Description of phase
- Cosine and Sine functions
- Complex numbers and alternative representations
- A typical digital signal processing system

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