

Assignment 2 Question 4

First Induction Proof

Base Case

The tree is empty and therefore the flatten function will return an empty list

Induction Hypothesis

Let T has k nodes then all k nodes will be in $\text{flatten}(T)$

Induction Step

Let T be a non-empty tree with $k+1$ nodes with root N , left subtree L , and right subtree R . N is first added to the list. Flatten is then called on the left subtree and right subtree recursively adding the root of the tree to the list on all levels until the left and right subtrees of the node are empty. So given that all k nodes were in the list, then flatten will also be called on the $k+1$ node and be added to the list since it is a left/right subtree of one of the previous k nodes.

Also all the nodes have at most one parent so the flatten function will only be called once on every node and since a node is added to the list only when flatten is called on it then the list and the tree will have the exact same number of elements. All elements of the tree are in the list and they are both the same size which means that an element x is in the binary tree T iff x is in $\text{flatten}(T)$. QED