

# **New Social Collaborative Filtering Algorithms for Recommendation on Facebook**

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Except where otherwise indicated, this thesis is my own original work.

Joseph Christian G. Noel  
27 October 2011



To my family, who gave a 10 year old kid a 286 IBM PC running DOS back when the only use I could think for it was listing down all my favorite Dragon Ball Z characters on WordStar. The good old days when men were men and floppy disks were actually floppy. And for hooking me up to the internet 3 years later, when no one else I knew was on it, and all I did on it for months was to study deck strategies for Magic: The Gathering. I have my passion, my career, and my future only through the unbelievable foresight you guys had.



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# Abstract

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This thesis examines the problem of designing efficient, scalable, and accurate social *collaborative filtering* (CF) algorithms for personalized link recommendation on Facebook. Unlike standard CF algorithms using relatively simple user and item features (possibly just the user ID and link ID), link recommendation on social networks like Facebook poses the more complex problem of learning user preferences from a rich and complex set of user profile and interaction information. Most existing *social CF* (SCF) methods have extended traditional CF *matrix factorization* (MF) approaches, but have overlooked important aspects specific to the social setting; specifically, existing SCF MF methods (a) do not permit the use of item or link features in learning user similarity based on observed interactions, (b) do not permit directly modeling user-user information diffusion according to the social graph structure, and (c) cannot learn that two users may only have overlapping interests in specific areas. This thesis proposes a unified SCF optimization framework that addresses (a)–(c) and compares these novel algorithms with a variety of existing baselines. Evaluation is carried out via live user trials in a custom-developed Facebook App involving data collected over three months from over 100 App users and their nearly 30,000 friends. Not only do we show that our novel proposals to address (a)–(c) outperform existing approaches, but we also identify which offline ranking and classification evaluation metrics correlate most with human judgment of algorithm performance. Overall, this thesis represents a critical step forward in extending SCF recommendation algorithms to fully exploit the rich content and structure of social networks like Facebook.



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# Introduction

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Given the vast amount of content available on the Internet, finding information of personal interest (news, blogs, videos, movies, books, etc.) is often like finding a needle in a haystack. Recommender systems based on *collaborative filtering* (CF) aim to address this problem by leveraging the preferences of a user population under the assumption that similar users will have similar preferences. These principles underlie the recommendation algorithms powering websites like Amazon and Netflix.<sup>1</sup>

As the web has become more social with the emergence of Facebook, Twitter, LinkedIn, and most recently Google+, this adds myriad new dimensions to the recommendation problem by making available a rich labeled graph structure of social content from which user preferences can be learned and new recommendations can be made. In this socially connected setting, no longer are web users simply described by an IP address (with perhaps associated geographical information and browsing history), but rather they are described by a rich user profile (age, gender, location, educational and work history, preferences, etc.) and a rich history of user interactions with their friends (comments/posts, clicks of like, tagging in photos, mutual group memberships, etc.). This rich information poses both an amazing opportunity and a daunting challenge for machine learning methods applied to social recommendation — how do we fully exploit the social network content in recommendation algorithms?

## 1.1 Objectives

This thesis examines the problem of designing efficient, scalable, and accurate *social CF* (SCF) algorithms for *personalized link recommendation on Facebook* – quite simply the task of recommending personalized links to users that might interest them. We show an example link posted on Facebook in Figure 1.1 that we might wish to recommend to a subset of users; once link recommendations such as this one are made, we then need to gather feedback from users in order to (a) learn to make better recommendations in the future and (b) to evaluate the efficacy of our different recommendation approaches.

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<sup>1</sup>On Amazon, this is directly evident with statements displayed of the form “users who looked at item X ended up purchasing item Y 90% of the time”. While the exact inner workings of Netflix are not published, the best performing recommendation algorithm in the popular Netflix prize competition [Toscher and Jahrer 2009] used an ensemble of CF methods.



**Figure 1.1:** A link posted by the author that has been liked by three other users. The objective of this thesis is to build an automated link recommendation App for Facebook that learns user preferences via social collaborative filtering techniques and automatically recommends links (such as this one) to other users. We actively collect like/dislike feedback from the users to measure recommendation algorithm performance and to learn from for future recommendations.

Many existing SCF approaches that can be applied to recommendation tasks like this one extend *matrix factorization* (MF) techniques [Salakhutdinov and Mnih 2008] and have proved quite powerful in their ability to accurately model user preferences even when only a unique ID is available for both the user and item being recommended. The power of such methods stems from their ability to project users and items into latent vector spaces of reduced dimensionality where each is effectively grouped by similarity. Indeed, we will show in Chapter 4 that existing social extensions of MF are quite powerful and outperform a variety of other commonly used SCF approaches.

Given the strong performance of existing MF approaches to SCF, we aim to comparatively evaluate them and further improve on their performance when applied to link recommendation on Facebook. To do this, we first identify a number of major deficiencies of existing SCF MF methods that we make our objective to address in this thesis:

- (a) **Non-feature-based user similarity:** Existing SCF MF methods do not permit the use of item or link features in learning user similarity based on observed interactions. For example, the fact that two users have the same gender cannot be exploited by existing methods SCF MF methods that make use of user similarity.
- (b) **Model direct user-user information diffusion:** Existing SCF MF methods do not permit directly modeling user-user information diffusion according to the social graph structure. For example, if a certain user always likes content from another specific user, this simply cannot be learned by existing SCF MF methods.

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- (c) **Restricted common interests:** Existing SCF MF methods cannot learn from the fact that two users have common overlapping interests in specific areas. For example, a friend and their co-worker may both like the same links regarding technical content, but have differing interests when it comes to politically-oriented links — knowing this would allow one to recommend technical content posted by one user to the other, but existing SCF methods cannot explicitly encourage this.

This thesis addresses all of these problems with novel contributions in an efficient, scalable, and unified latent factorization component framework for SCF. We present results of our algorithms on live trials in a custom-developed Facebook App involving data collected over three months from over 100 App users and their nearly 30,000 friends. These results show that a number of extensions proposed to resolve (a)–(c) outperform all previously existing algorithms.

In addition, given that live online user evaluation trials are time-consuming, requiring many users and often an evaluation period of at least one month, we have one last important objective to address in this thesis:

- (d) **Identifying passive evaluation paradigms that correlate with actively elicited human judgments.** The benefits of doing this are many-fold. When designing new SCF algorithms, there are myriad design choices to be made, for which actual performance evaluation is the only way to validate the correct choice. Furthermore, simple parameter tuning is crucial for best performance and SCF algorithms are often highly sensitive to well-tuned parameters. Thus for the purpose of algorithm design and tuning, it is crucial to have methods and metrics that can be evaluated immediately on passive data (i.e., a passive data set of user likes) that are shown to correlate with human judgments in order to avoid the time-consuming process of evaluating the algorithms in live human trials.

## 1.2 Contributions

In the preceding section, we outlined three deficiencies of existing MF approaches for SCF. Now we discuss our specific contributions in this thesis to address these three deficiencies:

- (a) **User-feature social regularization:** One can encode prior knowledge into the learning process using a technique known as *regularization*. In the case of social MF, we often want to regularize the learned latent representations of users to enforce that users who interact heavily often have similar preferences, and hence similar latent representations.

Thus to address the deficiency noted in *non-feature-based user similarity*, we build on ideas used in Matchbox [Stern et al. 2009] to incorporate user features into the social regularization objective for SCF. There are two commonly used methods for social regularization in SCF — in Chapter 5 we extend both to handle user features and determine that the *spectral* regularization extension performs best.

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- (b) **Hybrid social collaborative filtering:** While MF methods prove to be excellent at projecting user and items into latent spaces, they suffer from the caveat that they cannot model joint features over user and items (they can only work with independent user features and independent item features). This is problematic when it comes to the issue of *modeling direct user-user information diffusion* — in short, the task of learning how often information flows from one specific user to another specific user.

The remedy for this turns out to be quite simple — we need only introduce an objective component in addition to the standard MF objective that serves as a simple linear regressor for such information diffusion observations. Because the resulting objective is a combination of latent MF and linear regression objectives, we refer to it simply as *hybrid SCF*. In Chapter 5, we evaluate this approach and show that it outperforms standard SCF.

- (c) **Copreference regularization:** Existing SCF methods that employ social regularization make a somewhat coarse assumption that if two users interact heavily (or even worse, are simply friends) that their latent representations must match as closely as possible. Considering that friends have different reasons for their friendships — co-workers, schoolmates, common hobby — it is reasonable to expect that two people (friends or not) may only share *restricted common interests*: co-workers may both enjoy technical content related to work, but differ otherwise; schoolmates may like to hear news about other schoolmates, but differ otherwise; people who share an interest in a common hobby are obviously interested in that hobby, but should not necessarily share common interests elsewhere.

To this end, we propose a finer-grained approach to regularizing users by restricting their latent user representation to be similar (or different) only in subspaces relevant to the items mutually liked/disliked (or disagreed upon — one user likes and the other dislikes). Because this method of regularization requires evidence of preferences between two users for the same item, we refer to it as regularizing based on *copreferences*. In Chapter 5, we evaluate this extension to standard SCF and show that it improves performance.

The previous contributions all relate to algorithmic and machine learning aspects of SCF algorithms. However, in a different dimension and as discussed in the previous section, we also have to know how to *evaluate* these algorithms both from active user feedback (ratings of new recommendations) and passive user content (simply a catalogue of previously rated links for a user). Thus as our final contribution, we perform the following extensive comparative evaluation:

- (d) **Comparative evaluation of active and passive metrics that align with user judgments:** In Chapter 3, we propose a number of training and testing regimes and a number of evaluation metrics for both ranking and classification paradigms. In both Chapters 4 and Chapters 5, we compare the performance of these metrics with the given algorithms and raw data in order to determine



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which regimes and metrics correlate closely with human judgment of performance in each setting.

## 1.3 Outline

The remaining chapters in this thesis are organized as follows:

- **Chapter 2:** We first define notation used throughout the thesis and then proceed to review both standard collaborative filtering approaches, specific MF approaches, and their social extensions.
- **Chapter 3:** We discuss the specific details of our Facebook link recommendation application and then our evaluation methodology for both offline and online (live user trial) experimentation. Our goal here is to evaluate a variety of performance objectives, both qualitative and quantitative, in order to evaluate the user experience with each recommendation algorithm and to determine which online evaluations correlate with which offline evaluations.
- **Chapter 4:** We empirically investigate existing SCF methods in our Facebook App and evaluation framework. Our objective here is to carry out a fair comparison and understand the settings in which each algorithm works — and most importantly for research progress — where these algorithms can be improved.
- **Chapter 5:** We begin by discussing novel algorithms that we propose along the lines of our contributions outlined in this Introduction. Then proceed to evaluate them in our Facebook App and evaluation framework to understand whether these improve over the baselines, as well as to understand if there are any obvious deficiencies in the new approaches.
- **Conclusions:** We summarize our conclusions from this work and outline directions for future research.

All combined, this thesis represents a critical step forward in SCF algorithms based on top-performing MF methods and their ability to fully exploit the breadth of information available on social networks to achieve state-of-the-art link recommendation.



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# Background

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In the following, we outline the high-level ideas behind *recommender systems* — systems whose task is to adapt to user preferences in order to recommend items that the user may like. Following a general discussion of techniques, we proceed to define mathematical notation used throughout the thesis along with a mathematically detailed discussion of various published techniques for recommender systems that we either compare to or extend in this thesis.

## 2.1 Definitions

There are two general approaches to recommender systems. The first is known as *content-based filtering* (CBF), which makes individual recommendations based on correlations between the item features of those items the user has explicitly liked and similar items that the system could potentially recommend; in practice CBF is simply the machine learning tasks of classification (will the user like a certain item?) or regression (how much will they like it?). The second approach is collaborative filtering (CF) [Resnick and Varian 1997], which is defined as the task of predicting whether a user will like (or dislike) an item by using that user’s preferences *as well as those of other users*. In general, CBF requires item features whereas CF requires multiple users in order to work.<sup>1</sup>

Our thesis work takes CF one step further than its traditional use [Resnick and Varian 1997] in that we assume we are recommending in the context of a social network. We loosely define *social CF* (SCF) as the task of CF augmented with additional social network information such as the following that are available on social networking sites such as Facebook:

- Expressive personal profile content: gender, age, places lived, schools attended; favorite books, movies, quotes; online photo albums (and associated comment text).
- Explicit friendship or trust relationships.

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<sup>1</sup>In this thesis we use item features (CBF) and user features in conjunction with CF. Since our ideas are mainly driven by CF extended to use item and user features, we generally refer to all of our newly proposed methods in this thesis as CF even when they are actually hybrid CF+CBF methods.

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- Content that users have personally posted (often text and links).
  - Content of interactions between users (often text and links).
  - Evidence of other interactions between users (being tagged in photos).
  - Publicly available preferences (likes/dislikes of posts and links).
  - Publicly available group memberships (often for hobbies, activities, social or political discussion).

We note that CF is possible in a social setting without taking advantage of the above social information, nonetheless we refer to any CF method that *can be applied* in a social setting *as* SCF.

## 2.2 Notation

Here we outline mathematical notation common to the SCF setting and models explored in this thesis:

- $N$  users. For methods that can exploit user features, we define an  $I$ -element user feature vector  $\mathbf{x} \in \mathbb{R}^I$  (alternately if a second user is needed,  $\mathbf{z} \in \mathbb{R}^I$ ). For methods that do not use user feature vectors, we simply assume  $\mathbf{x}$  is an index  $\mathbf{x} \in \{1 \dots N\}$  and that  $I = N$ .
- $M$  items. For methods that can exploit item features, we define a  $J$ -element feature vector  $\mathbf{y} \in \mathbb{R}^J$ . The feature vectors for users and items can consist of any real-valued features as well as  $\{0, 1\}$  features like user and item IDs. For methods that do not use item feature vectors, we simply assume  $\mathbf{y}$  is an index  $\mathbf{y} \in \{1 \dots M\}$  and that  $J = M$ .
- A (non-exhaustive) data set  $D$  of single user preferences of the form  $D = \{(\mathbf{x}, \mathbf{y}) \rightarrow R_{\mathbf{x}, \mathbf{y}}\}$  where the binary *response*  $R$  is represented by  $R_{\mathbf{x}, \mathbf{y}} \in \{0 \text{ (dislike)}, 1 \text{ (like)}\}$ .
- A (non-exhaustive) data set  $C$  of co-preferences (cases where *both* users  $\mathbf{x}$  and  $\mathbf{z}$  expressed a preference for  $\mathbf{y}$  – not necessarily in agreement) derived from  $D$  of the form  $C = \{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \rightarrow P_{\mathbf{x}, \mathbf{z}, \mathbf{y}}\}$  where co-preference class  $P_{\mathbf{x}, \mathbf{z}, \mathbf{y}} \in \{-1 \text{ (disagree)}, 1 \text{ (agree)}\}$ . Intuitively, if *both* user  $\mathbf{x}$  and  $\mathbf{z}$  liked or disliked item  $\mathbf{y}$  then we say they *agree*, otherwise if one liked the item and the other disliked it, we say they *disagree*.
- A similarity rating  $S_{\mathbf{x}, \mathbf{z}}$  between any users  $\mathbf{x}$  and  $\mathbf{z}$ . This is used to summarize all social interaction between user  $\mathbf{x}$  and user  $\mathbf{z}$  in the term  $S_{\mathbf{x}, \mathbf{z}} \in \mathbb{R}$ . A definition of  $S_{\mathbf{x}, \mathbf{z}} \in \mathbb{R}$  that has been useful is the following:

$$Int_{\mathbf{x}, \mathbf{z}} = \frac{\# \text{ interactions between } \mathbf{x} \text{ and } \mathbf{z}}{\text{average } \# \text{ interactions between all user pairs}} \quad (2.1)$$

$$S_{\mathbf{x}, \mathbf{z}} = \ln(Int_{\mathbf{x}, \mathbf{z}}) \quad (2.2)$$

The interactions between users that we include to define  $Int_{\mathbf{x},\mathbf{z}}$  are:

1. Being friends on Facebook
2. Posting an item (link, photo, video, photo, or message) on a user's wall.
3. Liking an item (link, photo, video, photo, or message) on a user's wall.
4. Commenting on an item (link, photo, video, photo, or message) on a user's wall.
5. Being tagged together in the same photo.
6. Being tagged together in the same video.
7. Two users tagging themselves as attending the same school.
8. Two users tagging themselves as attending the same class in school.
9. Two users tagging themselves as playing sports together.
10. Two users tagging themselves as working together for the same company.
11. Two users tagging themselves as working together on the same project for the same company.

In addition, we can define  $S_{\mathbf{x},\mathbf{z}}^+$ , a *non-negative* variant of  $S_{\mathbf{x},\mathbf{z}}$ :

$$S_{\mathbf{x},\mathbf{z}}^+ = \ln(1 + Int_{\mathbf{x},\mathbf{z}}) \quad (2.3)$$

Having now defined all notation, we proceed to a discussion of CF algorithms compared to or extended in this thesis.

## 2.3 Content-based Filtering (CBF) Algorithms

As noted previously, CBF methods can be viewed as classification or regression approaches. Since our objective here is to classify whether a user likes an item or not (i.e., a classification objective), we focus on classification CBF approaches in this thesis. For an initial evaluation, perhaps the most well-known and generally top-performing classifier is the support vector machine, hence it is the CBF approach we choose to compare to in this work.

### 2.3.1 Support Vector Machines

A *support vector machine* (SVM) [Cortes and Vapnik 1995] is a type of supervised learning algorithm for classification based on finding optimal separating hyperplanes in a possibly high-dimensional feature space. During training, an SVM builds a model by constructing a set of hyperplanes that separates one class of data from another class with the maximum margin possible. Data are classified by finding out on which side of a hyperplane they fall on.

For the experiments in this thesis, the SVM uses a fixed-length feature vector  $\mathbf{f} \in \mathbb{R}^F$  derived from the  $(\mathbf{x}, \mathbf{y}) \in D$ , denoted as  $\mathbf{f}_{\mathbf{x},\mathbf{y}}$ . In general,  $\mathbf{f}_{\mathbf{x},\mathbf{y}}$  may include features that

are non-zero only for specific items and/or users, e.g., a  $\{0, 1\}$  indicator feature that user  $\mathbf{x}$  and user  $\mathbf{z}$  have both liked item  $\mathbf{y}$ . Specific features used in the SVM for the Facebook link recommendation task are defined in Section 3.2.

The SVM implementation used for this thesis is *LibSVM* [Chang and Lin 2001], which provides a regression score on the classification (i.e., the non-thresholded learned linear function) which can be used for ranking the results.

## 2.4 Collaborative Filtering (CF) Algorithms

### 2.4.1 $k$ -Nearest Neighbor

One of the most common forms of CF is the nearest neighbor approach [Bell and Koren 2007]. The  $k$ -nearest neighbor algorithm is a method of classification or regression that is based on finding the  $k$ -closest training data neighbors in the feature space nearest to a target point and combining the information from these neighbors — perhaps in a weighted manner — to determine the classification or regression value for the target point.

There are two main variants of nearest neighbors for collaborative recommendation, *user-based* and *item-based* — both methods generally assume that no user or item features are provided, so here  $\mathbf{x}$  and  $\mathbf{y}$  are simply respective user and item indices. Given a user  $\mathbf{x}$  and an item  $\mathbf{y}$ , let  $N(\mathbf{x} : \mathbf{y})$  be the set of *user* nearest neighbors of  $\mathbf{x}$  that have also given a rating for  $\mathbf{y}$ , let  $N(\mathbf{y} : \mathbf{x})$  be the set of *item* nearest neighbors of  $\mathbf{y}$  that have also been rated by  $\mathbf{x}$ , let  $S_{\mathbf{x}, \mathbf{z}}$  some measure of similarity rating between users  $\mathbf{x}$  and  $\mathbf{z}$  (as defined previously), and let  $S_{\mathbf{y}, \mathbf{y}'}$  be some measure of similarity rating for items  $\mathbf{y}$  and  $\mathbf{y}'$ . Following [Bell and Koren 2007], the predicted rating  $\hat{R}_{\mathbf{x}, \mathbf{y}} \in [0, 1]$  that the user  $\mathbf{x}$  gives item  $\mathbf{y}$  can then be calculated in one of two ways:

- **User-based similarity:**

$$\hat{R}_{\mathbf{x}, \mathbf{y}} = \frac{\sum_{\mathbf{z} \in N(\mathbf{x} : \mathbf{y})} S_{\mathbf{x}, \mathbf{z}} R_{\mathbf{z}, \mathbf{y}}}{\sum_{\mathbf{z} \in N(\mathbf{x} : \mathbf{y})} S_{\mathbf{x}, \mathbf{z}}}$$

- **Item-based similarity:**

$$\hat{R}_{\mathbf{x}, \mathbf{y}} = \frac{\sum_{\mathbf{y}' \in N(\mathbf{y} : \mathbf{x})} S_{\mathbf{y}, \mathbf{y}'} R_{\mathbf{x}, \mathbf{y}'}}{\sum_{\mathbf{y}' \in N(\mathbf{y} : \mathbf{x})} S_{\mathbf{y}, \mathbf{y}'}}$$

The question of which approach to use depends on the dataset. When the number of items is far fewer than the number of users, it has been found that the item-based approach usually provides better predictions as well as being more efficient in computations [Bell and Koren 2007].

### 2.4.2 Matrix Factorization (MF) Models

As done in standard CF methods, we assume that a matrix  $U$  allows us to project users  $\mathbf{x}$  (and  $\mathbf{z}$ ) into a latent space of dimensionality  $K$ ; likewise we assume that a matrix  $V$

allows us to project items  $\mathbf{y}$  into a latent space also of dimensionality  $K$ . Formally we define  $U$  and  $V$  as follows:

$$U = \begin{bmatrix} U_{1,1} & \dots & U_{1,I} \\ \vdots & U_{k,i} & \vdots \\ U_{K,1} & \dots & U_{K,I} \end{bmatrix} \quad V = \begin{bmatrix} V_{1,1} & \dots & V_{1,J} \\ \vdots & V_{k,j} & \vdots \\ V_{K,1} & \dots & V_{K,J} \end{bmatrix}$$

If we *do not* have user and item features then we simply use  $\mathbf{x}$  and  $\mathbf{y}$  as indices to pick out the respective rows and columns of  $U$  and  $V$  so that  $U_{\mathbf{x}}^T V_{\mathbf{y}}$  acts as a measure of affinity between user  $\mathbf{x}$  and item  $\mathbf{y}$ . If we *do* have user and item features, we can respectively represent the latent projections of user and item as  $(U\mathbf{x})_{1\dots K}$  and  $(V\mathbf{y})_{1\dots K}$  and hence use  $\langle U\mathbf{x}, V\mathbf{y} \rangle = \mathbf{x}^T U^T V \mathbf{y}$  as a measure of affinity between user  $\mathbf{x}$  and item  $\mathbf{y}$ . Either way, using  $U\mathbf{x}$  and  $V\mathbf{y}$  with features or  $U_{\mathbf{x}}$  and  $V_{\mathbf{y}}$  with no features, we see that the basic idea behind matrix factorization techniques is to project the user  $\mathbf{x}$  and item  $\mathbf{y}$  into some  $K$ -dimensional space where the dot product in this space indicates the relative affinity of  $\mathbf{x}$  for  $\mathbf{y}$ . Because this latent space is low-dimensional, i.e.,  $K \ll I$  and  $K \ll J$ , similar users and similar items will tend to be projected “nearby” in this  $K$ -dimensional space.

But there is still the question as to how we learn the matrix factorization components  $U$  and  $V$  in order to optimally carry out this user and item projection. The answer is simple: we need only define the objective we wish to maximize as a function of  $U$  and  $V$  and then use gradient descent to optimize them. Formally, we can optimize the following objectives based on whether or not we have user or item features:

- **Without item and user features [Salakhutdinov and Mnih 2008]:**

$$\sum_{(\mathbf{x}, \mathbf{y}) \in D} \frac{1}{2} (R_{\mathbf{x}, \mathbf{y}} - U_{\mathbf{x}}^T V_{\mathbf{y}})^2 \quad (2.4)$$

- **With item and user features (Matchbox) [Stern et al. 2009]:**

$$\sum_{(\mathbf{x}, \mathbf{y}) \in D} \frac{1}{2} (R_{\mathbf{x}, \mathbf{y}} - \mathbf{x}^T U^T V \mathbf{y})^2 \quad (2.5)$$

Taking gradients w.r.t.  $U$  and  $V$  here (holding one constant while taking the derivative w.r.t. the other), we can easily define an alternating gradient descent approach to approximately optimize these objectives and hence determine good projections  $U$  and  $V$  that minimize the reconstruction error of the observed responses  $R_{\mathbf{x}, \mathbf{y}}$ .

These are well-known MF approaches to CF, however in the context of social networks, we’ll need to adapt them to this richer setting to obtain SCF approaches. We discuss these extensions next.

### 2.4.3 Social Collaborative Filtering

There are essentially two general classes of MF methods applied to SCF that we discuss below. All of the social MF methods defined to date *do not* make use of user or item features and hence  $\mathbf{x}$  and  $\mathbf{z}$  below should be treated as user indices as defined previously for the non-feature case.

The first class of social MF methods can be termed as *social regularization* approaches in that they somehow constrain the latent projection represented by  $U$ .

There are two social regularization methods that directly constrain  $U$  for user  $\mathbf{x}$  and  $\mathbf{z}$  based on evidence  $S_{\mathbf{x},\mathbf{z}}$  of interaction between  $\mathbf{x}$  and  $\mathbf{z}$ . We call these methods:

- **Social regularization** [Yang et al. 2011; Cui et al. 2011]:

$$\sum_{\mathbf{z} \in \text{friends}(\mathbf{x})} \sum \frac{1}{2} (S_{\mathbf{x},\mathbf{z}} - \langle U_{\mathbf{x}}, U_{\mathbf{z}} \rangle)^2$$

- **Social spectral regularization** [Ma et al. 2011; Li and Yeung 2009]:

$$\sum_i \sum_{\mathbf{z} \in \text{friends}(\mathbf{x})} \frac{1}{2} S_{\mathbf{x},\mathbf{z}}^+ \|U_{\mathbf{x}} - U_{\mathbf{z}}\|_2^2$$

We refer to the latter as *spectral* regularization methods since they are identical to the objectives used in spectral clustering [Ng et al. 2001].

The *SoRec* system [Ma et al. 2008] proposes a slight twist on social spectral regularization in that it learns a third  $N \times N$  (n.b.,  $I = N$ ) *interactions matrix*  $Z$ , and uses  $U_{\mathbf{z}}^T Z_{\mathbf{z}}$  to predict user-user interaction preferences in the same way that standard CF uses  $V$  in  $U_{\mathbf{x}}^T V_{\mathbf{y}}$  to predict user-item ratings. *SoRec* also uses a sigmoidal transform  $\sigma(o) = \frac{1}{1+e^{-o}}$  on the predictions:

- **SoRec regularization** [Ma et al. 2008]:

$$\sum_{\mathbf{z}} \sum_{\mathbf{z} \in \text{friends}(\mathbf{x})} \frac{1}{2} (S_{\mathbf{x},\mathbf{z}} - \sigma(\langle U_{\mathbf{x}}, Z_{\mathbf{z}} \rangle))^2$$

The second class of SCF MF approaches represented by the single exemplar of the *Social Trust Ensemble* can be termed as a *weighted average* approach since this approach simply composes a prediction for item  $\mathbf{y}$  from a weighted average of a user  $\mathbf{x}$ 's predictions *as well as* their friends ( $\mathbf{z}$ ) predictions (as evidenced by the additional  $\sum_{\mathbf{z}}$  in the objective below):

- **Social Trust Ensemble** [Ma et al. 2009] (Non-spectral):

$$\sum_{(\mathbf{x},\mathbf{y}) \in D} \frac{1}{2} (R_{\mathbf{x},\mathbf{y}} - \sigma(U_{\mathbf{x}}^T V_{\mathbf{y}} + \sum_k U_{\mathbf{x}}^T V_{\mathbf{z}}))^2$$



---

As for the MF CF methods, all MF SCF methods can be optimized by alternating gradient descent on the respective matrix parameterizations.

#### 2.4.4 Tensor Factorization Methods

On a final note, we observe that *Tensor factorization* (TF) methods can be used to learn latent models of interaction of 2 dimensions and higher. A 2-dimensional TF method is simply standard MF. An example of a 3-dimensional TF method is given by [Rendle et al. 2009], where recommendation of user-specific tags for an item are modeled with tags, user, and items each in one dimension. To date, TF methods have not been used for social recommendation, however, we will draw on the idea of using additional (more than 2) dimensions of latent learning in some of our novel SCF approaches in Chapter 5.

## 2.5 Summary

In this chapter we have seen some of the different existing methods for CF and SCF. Each of these methods has its own weaknesses, some of which we detailed in Chapter 1. Next, we discuss how we evaluate these existing CF and SCF algorithms on the task of link recommendation on Facebook; following this, we proceed in subsequent chapters to evaluate these algorithms as well as propose novel algorithms that extend the background work covered here.



# Evaluation of Social Recommendation Systems

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In this chapter we first discuss our Facebook Link Recommendation (LinkR) application and then proceed to discuss how it can be evaluated using general principles of evaluation used in the machine learning and information retrieval fields.

## 3.1 Facebook

Facebook is a social networking service that is currently the largest in the world. As of July 2011 it had more than 750 million active users. Users in Facebook create a profile and establish “friend” connections between users to establish their social network. Each user has a “Wall” where they and their friends can make posts to. These posts can be links, photos, status updates, etc. Items that have been posted by a user can be “liked”, shared, or commented upon by other users. An example of a link post on a Wall that had been liked by others was provided previously in Figure 1.1.

This thesis seeks to find out how best to recommend links to individual users such that there is a high likelihood that they will “like” their recommended links. We do this by creating a Facebook application (i.e., ‘Facebook “App”’) that recommends links to users everyday, where the users may give their feedback on the links indicating whether they *liked* it or *disliked* it. We discuss this application in detail next.

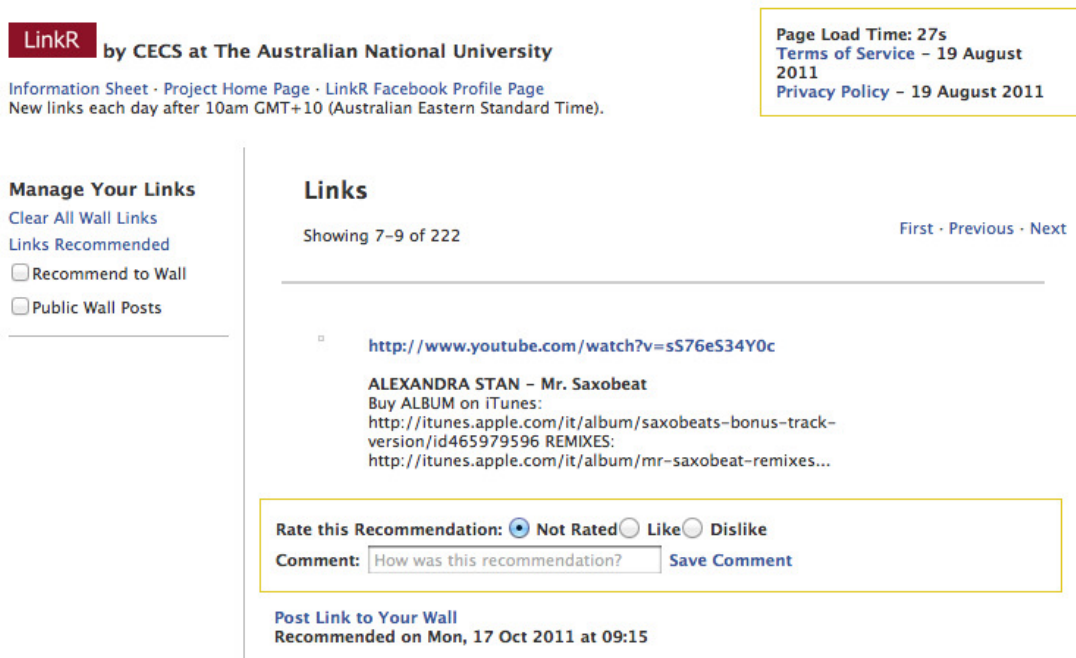
### 3.1.1 LinkR

Facebook allows applications to be developed that can be installed by their users. As part of this thesis project, the LinkR Facebook application was developed.<sup>1</sup> The functionalities of the LinkR application are as follows:

1. Collect data that have been shared by users and their friends on Facebook.
2. Recommend (three) links to the users daily.

---

<sup>1</sup>The main developer of the LinkR Facebook App is Khoi-Nguyen Tran, a PhD student at the Australian National University. Khoi-Nguyen wrote the user interface and database crawling code for LinkR. All of the learning and recommendation algorithms used by LinkR were written solely by the author for the purpose of this thesis.



**Figure 3.1:** The Facebook LinkR App showing one of the recommendations as it appears to users of the system. Users have the option of liking or disliking a recommendation as well as providing explicit feedback commentary.

3. Collect feedback from the users on whether they liked or disliked the recommendations.

Figure 3.1 shows the Facebook LinkR App as it appears to users.

## 3.2 Dataset

Using the LinkR Facebook App developed for this project, we were able to gather data on 34,245 users and 407,887 links.<sup>2</sup>

### 3.2.1 User Data

Data that are collected and used for the user features are as follows:

- Gender: male or female
- Birthday: year
- $location_{id}$ : an integer ID corresponding to the user's specific present location (city and country)
- $hometown_{id}$ : an integer ID corresponding to the user's specific home town (city and country)

<sup>2</sup>As of October 18, 2011, 12:15am.

- $F_{\mathbf{x},\mathbf{z}} \in \{0,1\}$ : indicator of whether users  $\mathbf{x}$  and  $\mathbf{z}$  are friends.
- $Int_{\mathbf{x},\mathbf{z}} \in \mathbb{N}$ : interactions on Facebook between users  $\mathbf{x}$  and  $\mathbf{z}$  as defined in Section 2.2.

### 3.2.2 Link Data

Data that are used for the link features are:

- $id$  of the user who posted the link.
- $id$  of the user on whose wall the link was posted.
- Text description of the link from the user who posted it.
- Text link summary from the metatags on the target link webpage.
- Number of times the link has been liked.
- Number of times the link has been shared.
- Number of comments posted on the link.
- $F'_{\mathbf{x},\mathbf{y}} \in \{0,1\}$ : indicator of whether user  $\mathbf{x}$  has liked item  $\mathbf{y}$ .

Additionally, links that have been recommended by the LinkR application have the following extra features:

- $id$ 's of users who have clicked on the link url.
- Optional “Like” or “Dislike” rating of the LinkR user on the link.

### 3.2.3 Implicit Dislikes

Outside of the “Dislike” ratings that we are able to get from the LinkR data, there is no other functionality within Facebook itself that allows users to explicitly define which link they do not like. Therefore, we need some way to infer disliked links during training. During training we consider links that were posted by the user’s friends and which they have not likes as an evidence that they dislike a link. This is a major assumption since users may have simply not seen the link, yet they may have actually liked it *if* they had seen it. Nevertheless, we find both in our offline and online evaluations that this assumption allows us to augment our training data in practice and does help performance despite these caveats.

## 3.3 Evaluation Metrics

We define *true positives* (TP) to be the count of relevant items that were returned by the algorithm, *false positives* (FP) to be the count of non-relevant items that were returned by the algorithm, *true negatives* (TN) to be the count of non-relevant items

that weren't returned by the algorithm, and *false negatives* (FN) to be the non-relevant items that were returned by the algorithm.

*Precision* is a measure of what fraction of items returned by the algorithm were actually relevant.

$$Precision = \frac{TP}{TP + FP}$$

For some problems, results are returned as a ranked list. The position of an item in the list must also be evaluated, not just whether the item is in the returned list or not. A metric that does this is *average precision* (AP), which computes the precision at every position in a ranked sequence of documents.  $k$  is the rank in a sequence of retrieved documents,  $n$  is the number of retrieved documents, and  $P(k)$  is the precision at cut-off  $k$  in the list.  $rel(k)$  is an indicator function equalling 1 if the item at position  $k$  is a relevant document, and 0 otherwise. The average precision is then calculated as follows:

$$AveP = \frac{\sum_{k=1}^n (P(k) \times rel(k))}{\text{number of relevant problems}}$$

The main metric we use in this thesis is the *mean average precision* (MAP). Since we make a recommendation for each user, these recommendations can be viewed as a separate problem per user, and evaluate the AP for each one. Getting the mean of APs across all users gives us an effective ranking metric for the entire recommendation system:

$$MAP = \frac{\sum_{u=1}^U AveP(u)}{\text{number of users}}$$

## 3.4 Training and Testing Issues

### 3.4.1 Training Data

Because of the sheer size of the Facebook data, it was impractical to run training and recommendations over the entire dataset. To keep the runtime of our experiments within reason, we used only the most recent four weeks of data for training the recommenders. This also helps alleviate some temporal aspects of the user's changing preferences, i.e., what the user liked last year may not be the same as what he or she likes this year. We also distinguish between the three types of link like/dislike data we can get from the dataset:

- **ACTIVE:** The explicit "Like" and "Dislike" rating that a LinkR user gives on links recommended by the LinkR application. In addition to this, a click by a user on a recommended link also counts as a like by that user on that particular link. This data is only available for LinkR users as it is specific to the LinkR App.

- 
- PASSIVE: The list of likes by users on links in the Facebook data and the inferred dislikes detailed above. This data can be collected from all users (App users and non-App users).
  - UNION: Combination of the ACTIVE and PASSIVE data.

### 3.4.2 Live Online Recommendation Trials

For the recommendations made to the LinkR application users, we select only links posted in the most recent two weeks that the user has not liked. We use only the links from the last two weeks since an informal user study has indicated a preference for recent links. Furthermore, older links have a greater chance of being outdated and are also likely to represent broken links that are not working anymore. We have settled on recommending three links per day to the LinkR users and according to the survey done at the end of the first trial, three links per day seems to be the generally preferred number of daily recommendations.

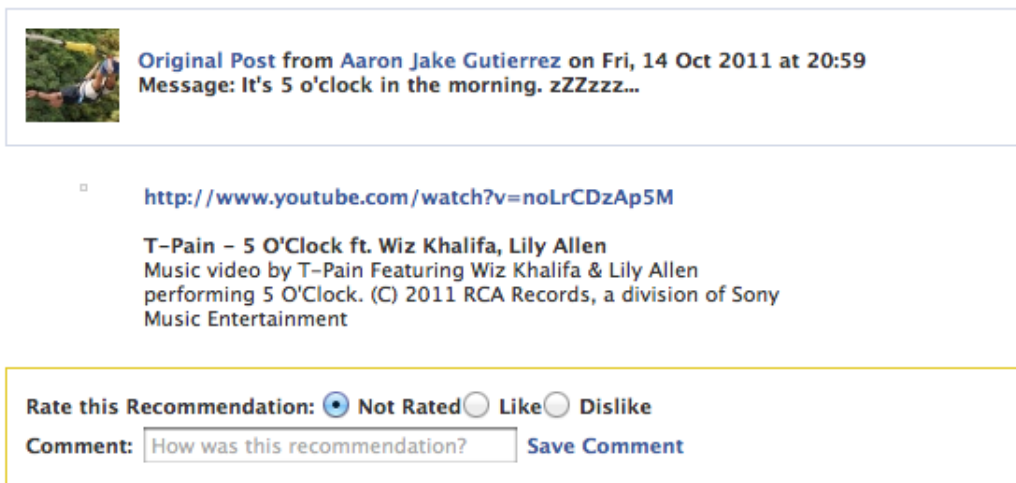
For the live trials, Facebook users who installed the LinkR application were *randomly assigned one of four algorithms in each of the two trials*. Users were not informed which algorithm was assigned to them to remove any bias. We distinguish our recommended links into two major classes, links that were posted by the LinkR user's friends and links that were posted by users other than the LinkR user's friends. The LinkR users were encouraged to rate the links that were recommended to them, and even provide feedback comments on the specific links. In turn these ratings became part of the training data for the recommendation algorithms, and thus were used to improve the performance of the algorithms over time. Based on the user feedback, we filtered out non-English links and links without any descriptions from the recommendations to prevent user annoyance.

At the end of the first trial, we conducted a user survey with the LinkR users to find out how satisfied they were with the recommendations they were getting.

### 3.4.3 Test Data

Similar to our selection for training data, the test data used for our passive experiment also uses only the most recent 4 weeks of data. We distinguish the test data into the following classes:

- FB-USER-PASSIVE: The PASSIVE like/dislike data for all Facebook users in the dataset.
- APP-USER-PASSIVE: The PASSIVE like/dislike data for only the LinkR application users.
- APP-USER-ACTIVE-FRIENDS: The ACTIVE like/dislike data for the LinkR users, but only for friend recommended links.
- APP-USER-ACTIVE-NON-FRIENDS: The ACTIVE like/dislike data for the LinkR users, but only for non-friend recommended links.



**Figure 3.2:** Screenshot of a recommendation made by the LinkR application with the rating and feedback options. In contrast to Figure 3.1, which showed a recommendation from a non-friend, this example shows a recommendation from a friend, where we are then able to also provide the friend’s name and comment text along with the link recommendation.

- APP-USER-ACTIVE-ALL: The entire active like/dislike data for the LinkR users.

During PASSIVE experiments, we simply selected which combination of training data and testing data to use. This helped us to determine which training-test data combination best reflected the results of the live trials as we discuss in the next chapter. In cases where training and testing data overlap, i.e., training on PASSIVE and testing on APP-USER-PASSIVE, we sample a random 20% subset of the training data per user for testing. These links are then removed from the training data to ensure that there are no common links between the training data and the test data. When there is no train/test overlap, i.e., training only on PASSIVE and testing only on ACTIVE, then we use the full respective datasets for training and testing.



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# Comparison of Existing Recommender Systems

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In this chapter we discuss the first set of four SCF algorithms that was implemented for the LinkR application and then show how each algorithm performed during the live user trial, how satisfied the users were with links being recommended to them through LinkR, and the results of offline passive experiments with the algorithms.

## 4.1 Objective components

Here we present full derivations for the components of the CF MF methods which we first described in Section 2.4.2. For the SCF method we use here, we extend the Social regularization technique also described in Section 2.4.2 to exploit user features as in Matchbox.

We take a composable approach to collaborative filtering (CF) systems where a (social) CF minimization objective  $Obj$  is composed of sums of one or more objective components:

$$Obj = \sum_i \lambda_i Obj_i \quad (4.1)$$

Because each objective may be weighted differently, a weighting term  $\lambda_i \in \mathbb{R}$  is included for each component and should be optimized via cross-validation.

Most target predictions are binary classification-based ( $\{0,1\}$ ), therefore in the objectives a sigmoidal transform

$$\sigma(o) = \frac{1}{1 + e^{-o}} \quad (4.2)$$

of regressor outputs  $o \in \mathbb{R}$  is used to squash it to the range  $[0, 1]$ . In places where the  $\sigma$  transform may be optionally included, this is written as  $[\sigma]$ .

#### 4.1.1 Matchbox Matrix Factorization ( $Obj_{pmcf}$ )

The basic objective function we use for our MF models is the Matchbox [Stern et al. 2009] model. Matchbox extends the matrix factorization model [Salakhutdinov and Mnih 2008] for CF by using the user features and item features in learning the latent spaces for these users and items. The Matchbox MF objective component is:

$$\sum_{(\mathbf{x}, \mathbf{y}) \in D} \frac{1}{2} (R_{\mathbf{x}, \mathbf{y}} - [\sigma] \mathbf{x}^T U^T V \mathbf{y})^2 \quad (4.3)$$

#### 4.1.2 L2 $U$ Regularization ( $Obj_{ru}$ )

To help in generalization, it is important to regularize the free parameters  $U$  and  $V$  to prevent overfitting in the presence of sparse data. This can be done with the  $L_2$  regularizer that models a prior of 0 on the parameters. The regularization component for  $U$  is

$$\frac{1}{2} \|U\|_{\text{Fro}}^2 = \frac{1}{2} \text{tr}(U^T U) \quad (4.4)$$

#### 4.1.3 L2 $V$ Regularization ( $Obj_{rv}$ )

We also regularize  $V$  as done with  $U$  above. The regularization component for  $V$  is

$$\frac{1}{2} \|V\|_{\text{Fro}}^2 = \frac{1}{2} \text{tr}(V^T V) \quad (4.5)$$

#### 4.1.4 Social Regularization ( $Obj_{rs}$ )

The social aspect of SCF is implemented as a regularizer on the user matrix  $U$ . What this objective component does is constrain users with a high similarity rating to have the same values in the latent feature space. This models the assumption that users who are similar socially should also have similar preferences for items.

This method is an extension of existing SCF techniques [Yang et al. 2011; Cui et al. 2011] described in Section 2.4.2 that constrain the latent space to enforce users to have similar preferences latent representations when they interact heavily. Like Matchbox which extends regular matrix factorization methods by making use of user and link features, our extension to the Social Regularization method incorporates user features to learn similarities between users in the latent space.

$$\begin{aligned}
& \sum_{\mathbf{x}} \sum_{\mathbf{z} \in \text{friends}(\mathbf{x})} \frac{1}{2} (S_{\mathbf{x},\mathbf{z}} - \langle U\mathbf{x}, U\mathbf{z} \rangle)^2 \\
&= \sum_{\mathbf{x}} \sum_{\mathbf{z} \in \text{friends}(\mathbf{x})} \frac{1}{2} (S_{\mathbf{x},\mathbf{z}} - \mathbf{x}^T U^T U \mathbf{z})^2
\end{aligned} \tag{4.6}$$

#### 4.1.5 Derivatives

We seek to optimize sums of the above objectives and will use gradient descent for this purpose.

For the overall objective, the partial derivative w.r.t. parameters  $\mathbf{a}$  are as follows:

$$\begin{aligned}
\frac{\partial}{\partial \mathbf{a}} \text{Obj} &= \frac{\partial}{\partial \mathbf{a}} \sum_i \lambda_i \text{Obj}_i \\
&= \sum_i \lambda_i \frac{\partial}{\partial \mathbf{a}} \text{Obj}_i
\end{aligned}$$

Previously we noted that that we may want to transform some of the regressor outputs  $o[\cdot]$  using  $\sigma(o[\cdot])$ . This is convenient for our partial derivatives as

$$\frac{\partial}{\partial \mathbf{a}} \sigma(o[\cdot]) = \sigma(o[\cdot])(1 - \sigma(o[\cdot])) \frac{\partial}{\partial \mathbf{a}} o[\cdot]. \tag{4.7}$$

Hence anytime a  $[\sigma(o[\cdot])]$  is optionally introduced in place of  $o[\cdot]$ , we simply insert  $[\sigma(o[\cdot])(1 - \sigma(o[\cdot]))]$  in the corresponding derivatives below.<sup>1</sup>

Before we proceed to our objective gradients, we define abbreviations for two useful vectors:

$$\begin{aligned}
\mathbf{s} &= U\mathbf{x} & \mathbf{s}_k &= (U\mathbf{x})_k; \ k = 1 \dots K \\
\mathbf{t} &= V\mathbf{y} & \mathbf{t}_k &= (V\mathbf{y})_k; \ k = 1 \dots K
\end{aligned}$$

Please see The Matrix Cookbook [Petersen and Pedersen 2008] for more details on the matrix derivative identities used in the following calculations.

Now we proceed to derivatives for the previously defined primary objective components:

- **Matchbox Matrix Factorization:** Here we define alternating partial derivatives between  $U$  and  $V$ , holding one constant and taking the derivative w.r.t. the

<sup>1</sup>We note that our experiments using the sigmoidal transform in objectives with  $[0, 1]$  predictions do not generally demonstrate a clear advantage vs. the omission of this transform as originally written (although they do not demonstrate a clear disadvantage either).

other:<sup>2</sup>

$$\begin{aligned}
\frac{\partial}{\partial U} Obj_{pmcf} &= \frac{\partial}{\partial U} \sum_{(\mathbf{x}, \mathbf{y}) \in D} \frac{1}{2} \left( \underbrace{(R_{\mathbf{x}, \mathbf{y}} - [\sigma] \overbrace{\mathbf{x}^T U^T V \mathbf{y}}^{o_{\mathbf{x}, \mathbf{y}}})}_{\delta_{\mathbf{x}, \mathbf{y}}} \right)^2 \\
&= \sum_{(\mathbf{x}, \mathbf{y}) \in D} \delta_{\mathbf{x}, \mathbf{y}} \frac{\partial}{\partial U} - [\sigma] \mathbf{x}^T U^T \mathbf{t} \\
&= - \sum_{(\mathbf{x}, \mathbf{y}) \in D} \delta_{\mathbf{x}, \mathbf{y}} [\sigma(o_{\mathbf{x}, \mathbf{y}})(1 - \sigma(o_{\mathbf{x}, \mathbf{y}}))] \mathbf{t} \mathbf{x}^T \\
\frac{\partial}{\partial V} Obj_{pmcf} &= \frac{\partial}{\partial V} \sum_{(\mathbf{x}, \mathbf{y}) \in D} \frac{1}{2} \left( \underbrace{(R_{\mathbf{x}, \mathbf{y}} - [\sigma] \overbrace{\mathbf{x}^T U^T V \mathbf{y}}^{o_{\mathbf{x}, \mathbf{y}}})}_{\delta_{\mathbf{x}, \mathbf{y}}} \right)^2 \\
&= \sum_{(\mathbf{x}, \mathbf{y}) \in D} \delta_{\mathbf{x}, \mathbf{y}} \frac{\partial}{\partial V} - [\sigma] \mathbf{s}^T V \mathbf{y} \\
&= - \sum_{(\mathbf{x}, \mathbf{y}) \in D} \delta_{\mathbf{x}, \mathbf{y}} [\sigma(o_{\mathbf{x}, \mathbf{y}})(1 - \sigma(o_{\mathbf{x}, \mathbf{y}}))] \mathbf{s} \mathbf{y}^T
\end{aligned}$$

For the regularization objective components, the derivatives are:

- $L_2$   $U$  regularization:

$$\begin{aligned}
\frac{\partial}{\partial U} Obj_{ru} &= \frac{\partial}{\partial U} \frac{1}{2} \text{tr}(U^T U) \\
&= U
\end{aligned}$$

- $L_2$   $V$  regularization:

$$\begin{aligned}
\frac{\partial}{\partial V} Obj_{rv} &= \frac{\partial}{\partial V} \frac{1}{2} \text{tr}(V^T V) \\
&= V
\end{aligned}$$

- Social regularization:

$$\begin{aligned}
\frac{\partial}{\partial U} Obj_{rs} &= \frac{\partial}{\partial U} \sum_{\mathbf{x}} \sum_{\mathbf{z} \in \text{friends}(\mathbf{x})} \frac{1}{2} \left( \underbrace{S_{\mathbf{x}, \mathbf{z}} - \mathbf{x}^T U^T U \mathbf{z}}_{\delta_{\mathbf{x}, \mathbf{y}}} \right)^2 \\
&= \sum_{\mathbf{x}} \sum_{\mathbf{z} \in \text{friends}(\mathbf{x})} \delta_{\mathbf{x}, \mathbf{y}} \frac{\partial}{\partial U} - \mathbf{x}^T U^T U \mathbf{z} \\
&= - \sum_{\mathbf{x}} \sum_{\mathbf{z} \in \text{friends}(\mathbf{x})} \delta_{\mathbf{x}, \mathbf{y}} U (\mathbf{x} \mathbf{z}^T + \mathbf{z} \mathbf{x}^T)
\end{aligned}$$

<sup>2</sup>We will use this method of alternation for all objective components that involve bilinear terms.

---

Algorithm	Users
Social Matchbox	26
Matchbox	26
SVM	28
Nearest Neighbor	28

**Table 4.1:** Number of Users Assigned per Algorithm.

Hence, for any choice of primary objective and one or more regularizers, we simply add the derivatives for  $U$  and/or  $V$  according to (4.7).

## 4.2 Algorithms

The CF and SCF algorithms used for the first user trial were:

1.  **$k$ -Nearest Neighbor (KNN):** We use the user-based approach as described in Section 2.4.1.
2. **Support Vector Machines (SVM):** We use the the SVM implementation described in Section 2.3.1 using the features described in Section 3.2.
3. **Matchbox (Mbox):** Matchbox MF + L2  $U$  Regularization + L2  $V$  Regularization
4. **Social Matchbox (Soc. Mbox):** Matchbox MF + Social Regularization + L2 Regularization

Social Matchbox uses the Social Regularization method to incorporate the social information of the FB data. SVM incorporates social information in the  $\mathbf{f}_{\mathbf{x},\mathbf{y}}$  features that it uses. Matchbox and Nearest Neighbors do not make use of any social information.

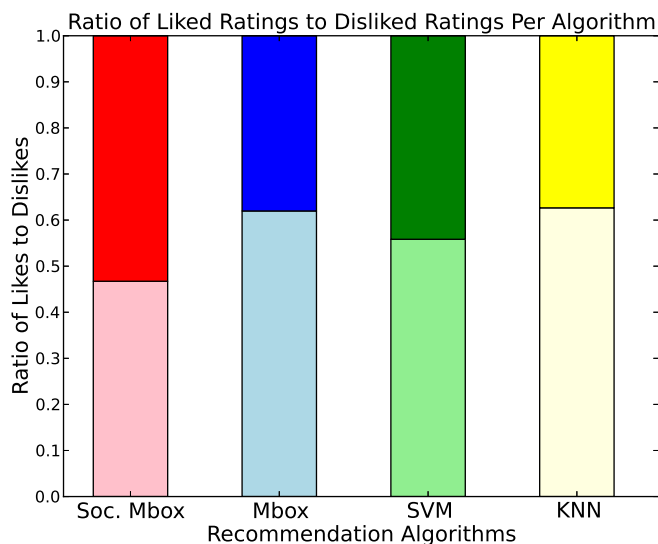
## 4.3 Online Results

The first live user trial was run from August 25 to October 13. The algorithms were randomly distributed among the 106 users who installed the LinkR application. The distribution of the algorithms to the users are show in Table 4.1

Each user was recommended three links everyday and they were able to rate the links on whether they ‘Liked’ or ‘Disliked’ it. Results shown in Figure 4.1 are the percentage of Like ratings and the percentage of Dislike ratings per algorithm stacked on top of each other with the Like ratings on top.

As shown in Figure 4.1, Social Matchbox was the best performing algorithm in the first trial and in fact was the only algorithm to get receive more like ratings than dislike ratings. This would suggest that using social information does indeed provide useful information that resulted in better link recommendations from LinkR.

We also look at the algorithms with the results split between friend links and non-friend links recommendations. Again, the results shown in Figure 4.2 are the percentage



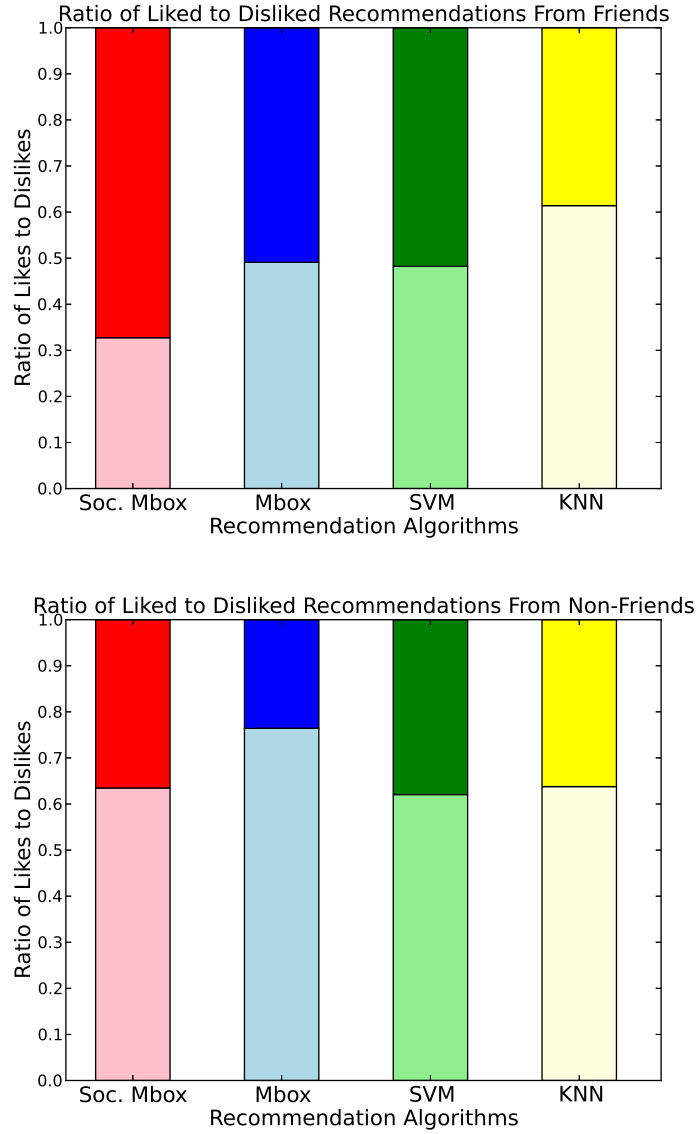
**Figure 4.1:** Results of the online live trials. The percentage of Liked ratings are stacked on top of the percentage of Disliked ratings per algorithm. Social Matchbox was found to be the best performing of the four algorithms evaluated in the first trial.

of Like ratings and the percentage of Dislike ratings per algorithm stacked on top of each other with the Like ratings on top. As shown in Figure 4.2, all four algorithms experienced a significant performance drop in the ratio of Likes to Dislikes when it came to recommending non-friend links. This suggests that aside from Liking or Disliking a link solely from the quality of the link being recommended, users are also more likely to Like a link simply because a friend had posted it and more likely to Dislike it because it was posted by a stranger.

## 4.4 Survey Results

Near the end of the first trial, the LinkR users were invited to answer a survey regarding their experiences with the recommendations they were getting. They were asked a number of questions, with the following pertaining to the quality of the recommendations:

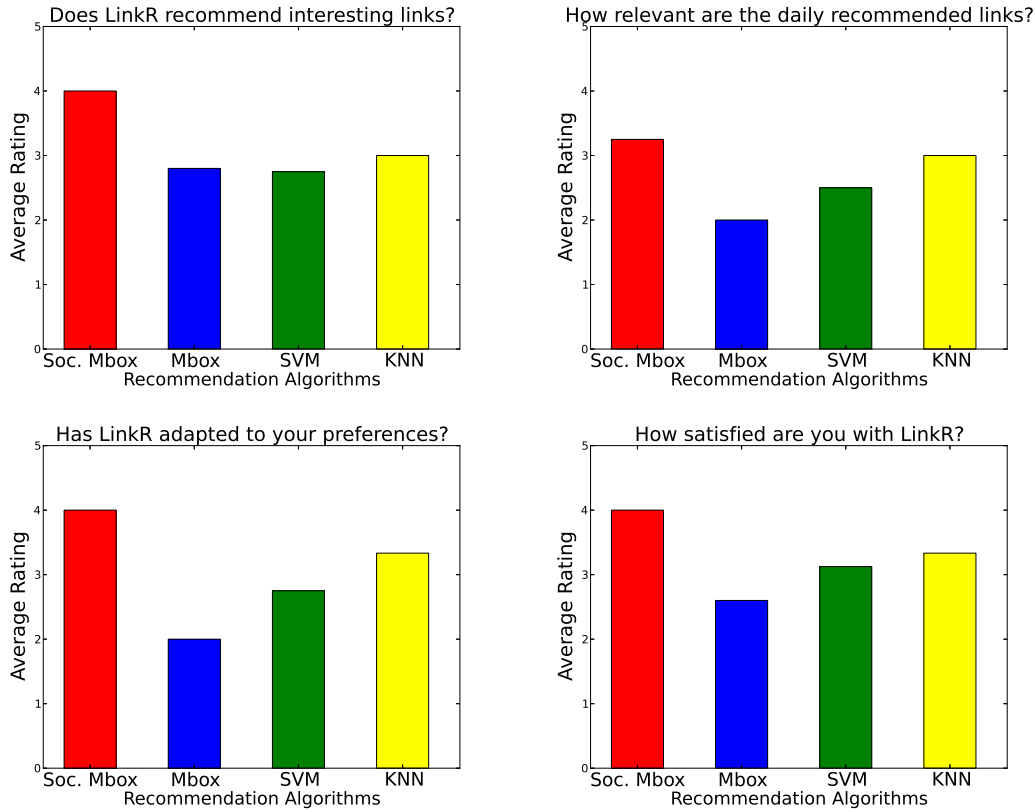
- Do you find that ANU LinkR recommends interesting links that you may not have otherwise seen?
- Do you feel that ANU LinkR has adapted to your preferences since you first started using it?
- How relevant are the daily recommended links?
- Overall, how satisfied are you with LinkR?



**Figure 4.2:** Results of the online live trials, split between friends and non-friends. The percentage of Liked ratings are stacked on top of the percentage of Disliked ratings per algorithm. There is a significant drop in performance between recommending friend links and recommending non-friend links.

They gave their answers to each question as an integer rating with range  $[1 - 5]$ , with a higher value being better. Results are shown in Figure 4.3. Their answers were grouped together according to the recommendation algorithm that was assigned to them, and the averages per algorithm are below.

As shown in Figure 4.3, Social Matchbox achieved higher survey scores than the other recommendation algorithms, in all four questions. The results of the survey reflected the results in the online live trial and confirms that Social Matchbox was the



**Figure 4.3:** Results of the user survey after the first trial. The survey answers from the users reflect the online results that Social Matchbox was the best recommendation algorithm in this trial.

best recommendation algorithm in the first trial.

## 4.5 Offline Results

The goal of the offline experiments was to see how which training and testing subset best correlates with user preferences. The algorithms were evaluated using the mean average precision metric discussed in Section 3.3. The offline experiments were also used to tune the parameters of the different algorithms.

- The results of training on the PASSIVE data are shown in Figure 4.4. The results show that training on PASSIVE data does not correlate with the results of the live user trials. Also, compared to training on the ACTIVE and UNION subsets, training on PASSIVE data gave worse results on the MAP metric.
- The results of training on the ACTIVE data are shown in Figure 4.5. When training on ACTIVE, the algorithms were tested only on links and users that were also in the ACTIVE data as well. Training on ACTIVE data gave much better MAP results than training on PASSIVE data. The main advantage of



ACTIVE data over PASSIVE data is the explicit dislikes that only the ACTIVE data has. Even though the ACTIVE data is far smaller than the PASSIVE data, this inclusion of explicit dislikes allowed the SCF algorithms to perform better in the offline experiments.

However, one weakness of ACTIVE is the small amount of data in it, only the explicit likes and dislikes of only the LinkR application users. ACTIVE data can't provide information on LinkR users that have just installed the application, old LinkR users that haven't rated any links, and other non-LinkR users on Facebook.

- The results of training on UNION data are shown in Figure 4.6. UNION combines the advantage of PASSIVE with having a larger dataset and the advantage of ACTIVE. When testing on the APP-USER-ACTIVE-ALL dataset, the results correlate with the results of the live user trial and the user surveys.

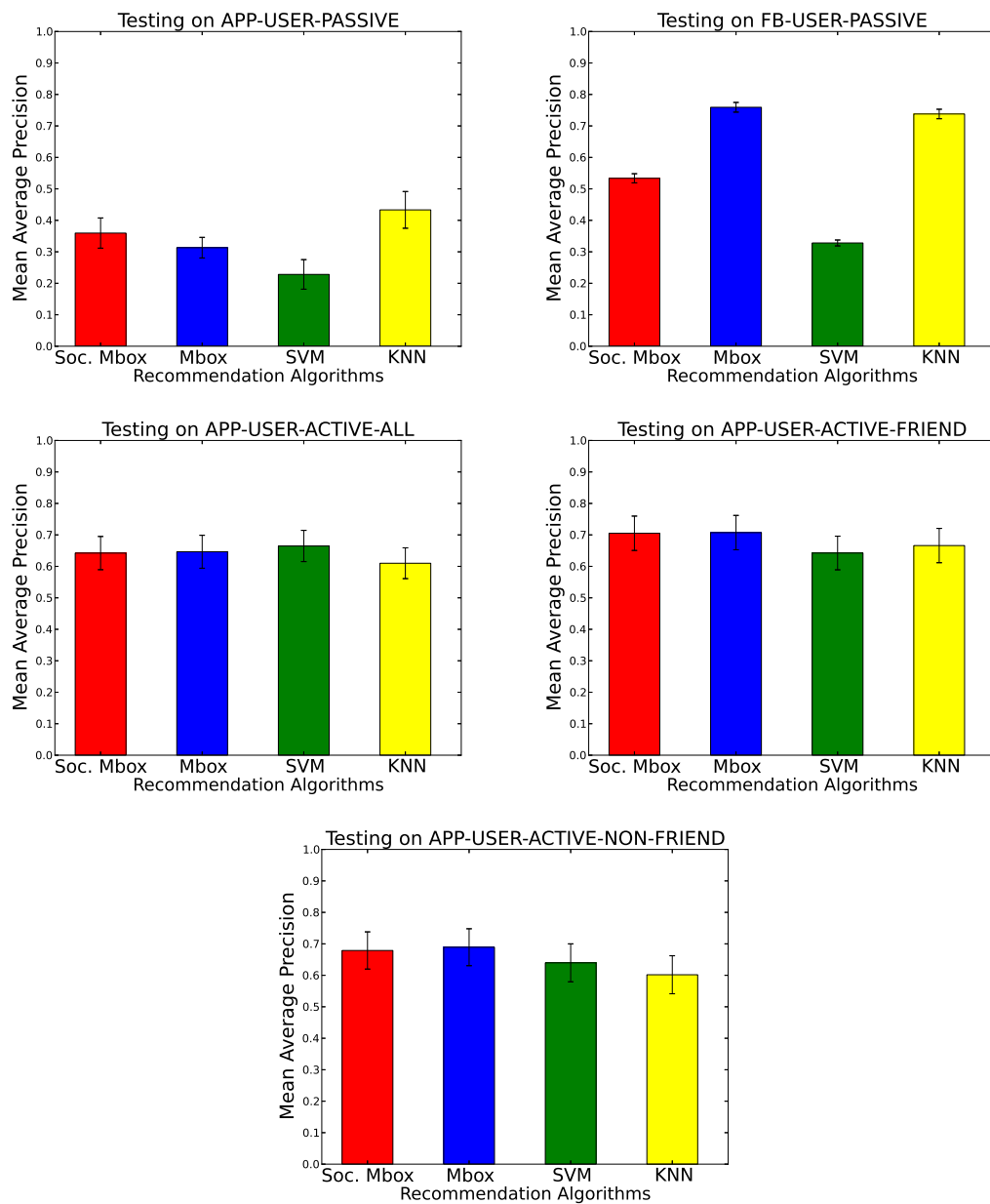
To give the best recommendations for all users, *the SCF algorithms are best trained on the larger size of the PASSIVE data with the explicit likes and dislikes of the ACTIVE data*. Hence, for the second user trial in the next chapter we keep on training the UNION dataset.

## 4.6 Summary

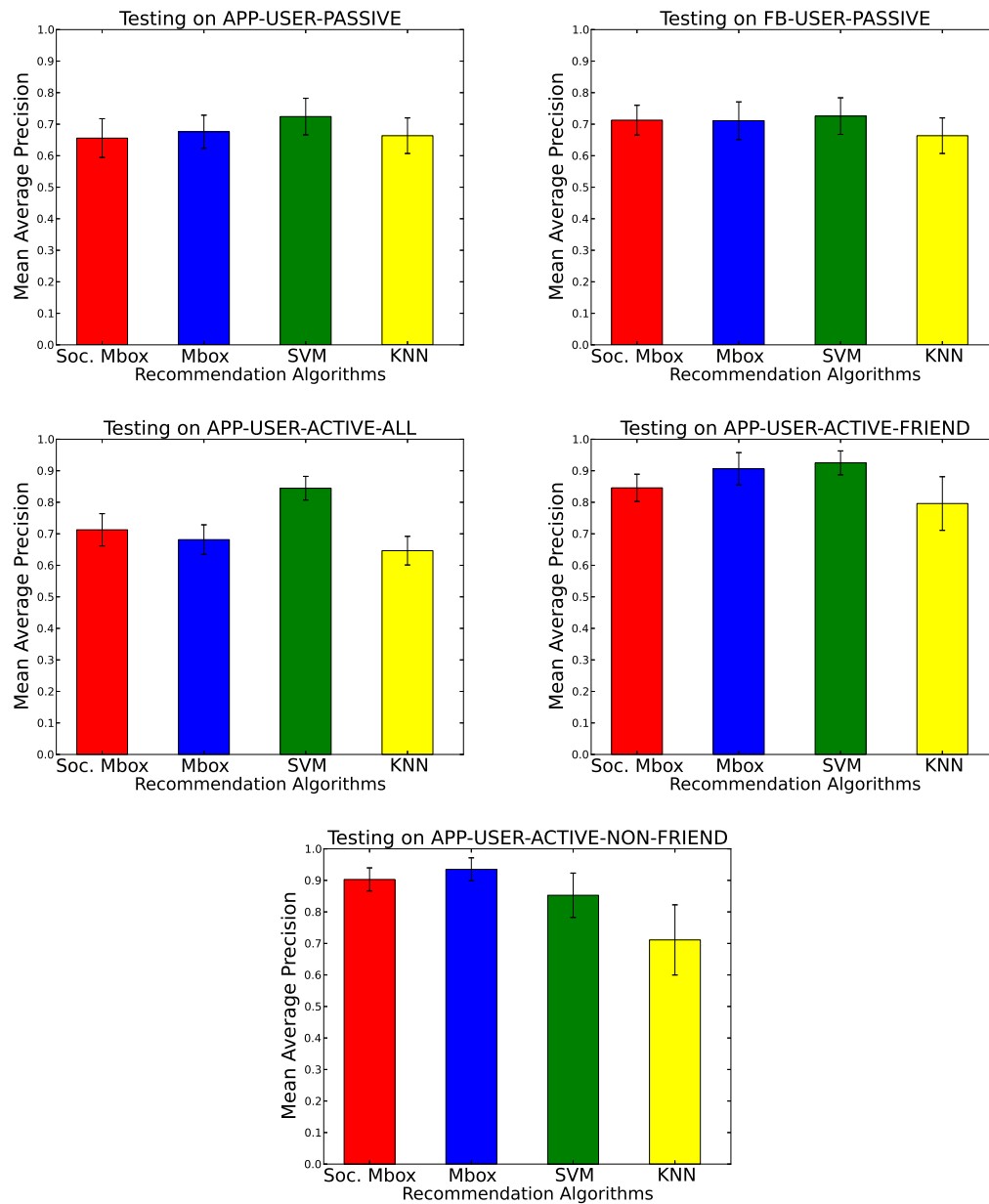
At the end of the first trial, we have observed the following:

- Social Matchbox was the best performing algorithm in the live user trial in terms of percentage of likes.
- Social Matchbox received the highest user evaluation scores in the user survey at the end of the user trial.
- Of the various combinations of training and testing data in the offline passive experiment, we found that training on the UNION subset and testing on the APP-USER-ACTIVE-ALL subset best correlated with the results of the live user trial and the user survey. Training on the UNION dataset had advantages compared to training on the other data subsets, namely that it had the large amount of information of the PASSIVE data and the explicit dislikes information of the ACTIVE data.

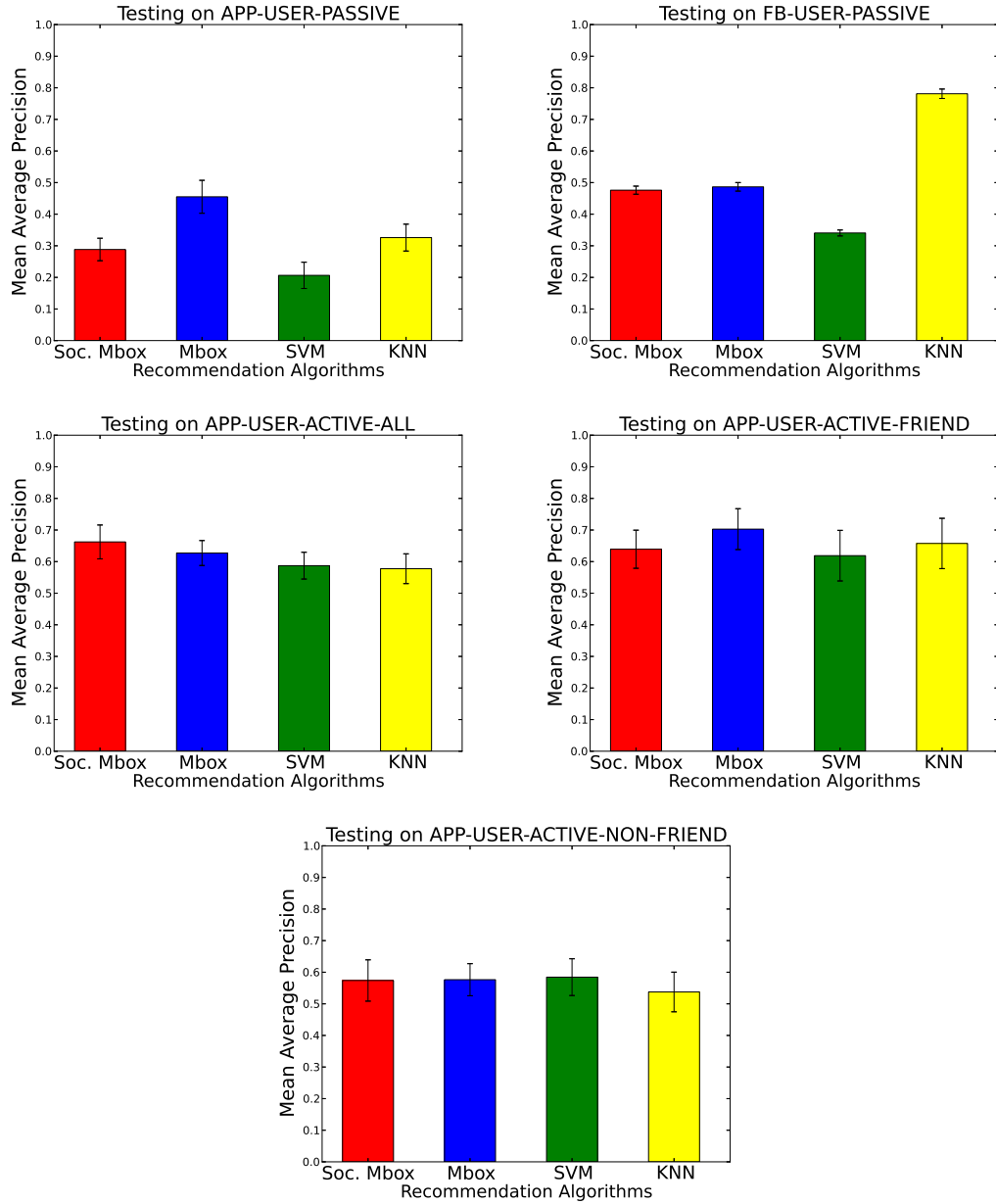
In the next chapter, we discuss new techniques for incorporating social information and show how they improve on Social Matchbox.



**Figure 4.4:** Results of training on PASSIVE data.



**Figure 4.5:** Results of training on ACTIVE data.



**Figure 4.6:** Results of training on UNION data. When testing on APP-USER-ACTIVE-ALL, we find that Social Matchbox was again the best recommendation algorithm. Training on UNION and testing on APP-USER-ACTIVE-ALL is the training/test data combination that is most similar to the online setup.

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# New Algorithms for Social Recommendation

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After studying the results of the first trial, we made use of what we learned to design new algorithms to improve upon deficiencies of existing recommendation algorithms. Our goal in designing these algorithms was to address deficiencies in current SCF methods that was previously discussed at-length in Section 1.1:

- Non-feature-based user similarity
- Modeling direct user-user information diffusion
- Restricted common interests

We address these deficiencies by modifying the optimization objectives that were discussed in the previous chapter, creating new objective functions. We discuss these new objectives in the following sections.

## 5.1 New Objective Components

As in Chapter 4, we assume that the CF optimization objective decomposes into a sum of objective components as follows:

$$Obj = \sum_i \lambda_i Obj_i \quad (5.1)$$

### 5.1.1 Hybrid Objective ( $Obj_{phy}$ )

As specified in Chapter 1, one weakness of MF methods is that they cannot model joint features over user and items, and hence the cannot model direct user-user information diffusion. Information diffusion models the unidirectional flow of links from one user to another (i.e., one user likes/shares what another user posts). We believe that this information will be useful for SCF, and is lacking in current SCF methods.

We fix this by introducing another objective component in addition to the standard MF objective, and this component serves as a simple linear regressor for such information diffusion observations. The resulting hybrid objective component becomes a combination of latent MF and linear regression objectives.

We make use of the  $\mathbf{f}_{\mathbf{x},\mathbf{y}}$  features detailed in Section 3.2 to make the linear regressor.  $\mathbf{f}_{\mathbf{x},\mathbf{y}}$  models user-user information diffusion because it is a joint feature between users and the links. It allows us to learn information diffusion models whether certain users always likes content from another specific user.

Using  $\langle \cdot, \cdot \rangle$  to denote an inner product, we define a weight vector  $\mathbf{w} \in \mathbb{R}^F$  such that  $\langle \mathbf{w}, \mathbf{f}_{\mathbf{x},\mathbf{y}} \rangle = \mathbf{w}^T \mathbf{f}_{\mathbf{x},\mathbf{y}}$  is the prediction of the system. The objective of the linear regression component is therefore

$$\sum_{(\mathbf{x},\mathbf{y}) \in D} \frac{1}{2} (R_{\mathbf{x},\mathbf{y}} - [\sigma] \mathbf{w}^T \mathbf{f}_{\mathbf{x},\mathbf{y}})^2$$

We combine the output of the linear regression objective with the Matchbox output  $[\sigma] \mathbf{x}^T U^T V y$ , to get a hybrid objective component. The full objective function for this hybrid model is

$$\sum_{(\mathbf{x},\mathbf{y}) \in D} \frac{1}{2} (R_{\mathbf{x},\mathbf{y}} - [\sigma] \mathbf{w}^T \mathbf{f}_{\mathbf{x},\mathbf{y}} - [\sigma] \mathbf{x}^T U^T V y)^2 \quad (5.2)$$

### 5.1.2 L2 w Regularization ( $Obj_{rw}$ )

In the same manner as  $U$  and  $V$ , it is important to regularize the free parameter  $\mathbf{w}$  to prevent overfitting in the presence of sparse data. This can again be done with the  $L_2$  regularizer that models a prior of 0 on the parameters. The objective component for the L2 regularizer for  $\mathbf{w}$  is:

$$\frac{1}{2} \|\mathbf{w}\|_2^2 = \frac{1}{2} \mathbf{w}^T \mathbf{w} \quad (5.3)$$

### 5.1.3 Social Spectral Regularization ( $Obj_{rss}$ )

As we did with the Social Regularization method in Section 4.1.4, we build on ideas used in Matchbox [Stern et al. 2009] to extend social spectral regularization [Ma et al. 2011; Li and Yeung 2009] by incorporating user features into the objective. The objective function for our extension to social spectral regularization is:

$$\begin{aligned}
& \sum_{\mathbf{x}} \sum_{\mathbf{z} \in \text{friends}(\mathbf{x})} \frac{1}{2} S_{\mathbf{x}, \mathbf{z}}^+ \|U\mathbf{x} - U\mathbf{z}\|_2^2 \\
&= \sum_{\mathbf{x}} \sum_{\mathbf{z} \in \text{friends}(\mathbf{x})} \frac{1}{2} S_{\mathbf{x}, \mathbf{z}}^+ \|U(\mathbf{x} - \mathbf{z})\|_2^2 \\
&= \sum_{\mathbf{x}} \sum_{\mathbf{z} \in \text{friends}(\mathbf{x})} \frac{1}{2} S_{\mathbf{x}, \mathbf{z}}^+ (\mathbf{x} - \mathbf{z})^T U^T U (\mathbf{x} - \mathbf{z}) \tag{5.4}
\end{aligned}$$

#### 5.1.4 Social Co-preference Regularization ( $Obj_{rsc}$ )

A crucial aspect missing from other SCF methods is that while two users may not be globally similar or opposite in their preferences, there may be sub-areas of their interests which can be correlated to each other. For example, two friends may have similar interests concerning music, but different interests concerning politics. The social co-preference regularizers aim to learn such selective co-preferences. The motivation is to constrain users  $\mathbf{x}$  and  $\mathbf{z}$  who have similar or opposing preferences to be similar or opposite in the same latent latent space relevant to item  $\mathbf{y}$ .

We use  $\langle \cdot, \cdot \rangle_\bullet$  to denote a re-weighted inner product. The purpose of this inner product is to tailor the latent space similarities or dissimilarities between users to specific sets of items. This fixes the issue detailed in the previous paragraph by allowing users  $\mathbf{x}$  and  $\mathbf{z}$  to be similar or opposite in the same latent latent space relevant only to item  $\mathbf{y}$ .

The objective component for social co-preference regularization along with its expanded form is

$$\begin{aligned}
& \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \frac{1}{2} (P_{\mathbf{x}, \mathbf{z}, \mathbf{y}} - \langle U\mathbf{x}, U\mathbf{z} \rangle_{V\mathbf{y}})^2 \\
&= \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \frac{1}{2} (P_{\mathbf{x}, \mathbf{z}, \mathbf{y}} - \mathbf{x}^T U^T \text{diag}(V\mathbf{y}) U \mathbf{z})^2 \tag{5.5}
\end{aligned}$$

#### 5.1.5 Social Co-preference Spectral Regularization ( $Obj_{rscs}$ )

This is the same as the social co-preference regularization above, except that it uses the spectral regularizer format for learning the co-preferences.

We use  $\|\cdot\|_{2,\bullet}$  to denote a re-weighted  $L_2$  norm. The reweighing of this norm serves the same purpose as the re-weighted inner product in Section 5.1.4, it tailors the similarities or dissimilarities between users to specific sets of items. This allows users  $\mathbf{x}$  and  $\mathbf{z}$  to be similar or opposite in the same latent latent space relevant only to item  $\mathbf{y}$ .

The objective component for social co-preference spectral regularization along with its expanded form is

$$\begin{aligned}
& \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \frac{1}{2} P_{\mathbf{x}, \mathbf{z}, \mathbf{y}} \|U\mathbf{x} - U\mathbf{z}\|_{2, V\mathbf{y}}^2 \\
&= \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \frac{1}{2} P_{\mathbf{x}, \mathbf{z}, \mathbf{y}} \|U(\mathbf{x} - \mathbf{z})\|_{2, V\mathbf{y}}^2 \\
&= \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \frac{1}{2} P_{\mathbf{x}, \mathbf{z}, \mathbf{y}} (\mathbf{x} - \mathbf{z})^T U^T \text{diag}(V\mathbf{y}) U (\mathbf{x} - \mathbf{z}) \quad (5.6)
\end{aligned}$$

### 5.1.6 Derivatives

As before, we seek to optimize sums of the above objectives and will use gradient descent for this purpose. Please see The Matrix Cookbook [Petersen and Pedersen 2008] for more details on the matrix derivative identities used in the following calculations. We again use the following useful abbreviations:

$$\begin{aligned}
\mathbf{s} &= U\mathbf{x} & \mathbf{s}_k &= (U\mathbf{x})_k; \quad k = 1 \dots K \\
\mathbf{t} &= V\mathbf{y} & \mathbf{t}_k &= (V\mathbf{y})_k; \quad k = 1 \dots K
\end{aligned}$$

The derivatives for the hybrid objective functions as well as the new social regularizers are:

- **Hybrid:**

$$\begin{aligned}
\frac{\partial}{\partial \mathbf{w}} \text{Obj}_{phy} &= \frac{\partial}{\partial \mathbf{w}} \sum_{(\mathbf{x}, \mathbf{y}) \in D} \frac{1}{2} \left( \underbrace{R_{\mathbf{x}, \mathbf{y}} - [\sigma] \overbrace{\mathbf{w}^T \mathbf{f}_{\mathbf{x}, \mathbf{y}}}^{o_{\mathbf{x}, \mathbf{y}}^1} - [\sigma] \mathbf{x}^T U^T V \mathbf{y}}_{\delta_{\mathbf{x}, \mathbf{y}}} \right)^2 \\
&= \sum_{(\mathbf{x}, \mathbf{y}) \in D} \delta_{\mathbf{x}, \mathbf{y}} \frac{\partial}{\partial \mathbf{w}} - [\sigma] \mathbf{w}^T \mathbf{f}_{\mathbf{x}, \mathbf{y}} \\
&= - \sum_{(\mathbf{x}, \mathbf{y}) \in D} \delta_{\mathbf{x}, \mathbf{y}} [\sigma (o_{\mathbf{x}, \mathbf{y}}^1) (1 - \sigma(o_{\mathbf{x}, \mathbf{y}}^1))] \mathbf{f}_{\mathbf{x}, \mathbf{y}}
\end{aligned}$$



$$\begin{aligned}
\frac{\partial}{\partial U} Obj_{phy} &= \frac{\partial}{\partial U} \sum_{(\mathbf{x}, \mathbf{y}) \in D} \frac{1}{2} \left( \underbrace{R_{\mathbf{x}, \mathbf{y}} - [\sigma] \mathbf{w}^T \mathbf{f}_{\mathbf{x}, \mathbf{y}} - [\sigma] \overbrace{\mathbf{x}^T U^T V \mathbf{y}}^{o_{\mathbf{x}, \mathbf{y}}^2}}_{\delta_{\mathbf{x}, \mathbf{y}}} \right)^2 \\
&= \sum_{(\mathbf{x}, \mathbf{y}) \in D} \delta_{\mathbf{x}, \mathbf{y}} \frac{\partial}{\partial U} - [\sigma] \mathbf{x}^T U^T V \mathbf{y} \\
&= - \sum_{(\mathbf{x}, \mathbf{y}) \in D} \delta_{\mathbf{x}, \mathbf{y}} [\sigma(o_{\mathbf{x}, \mathbf{y}}^2)(1 - \sigma(o_{\mathbf{x}, \mathbf{y}}^2))] \mathbf{t} \mathbf{x}^T \\
\frac{\partial}{\partial V} Obj_{phy} &= \frac{\partial}{\partial V} \sum_{(\mathbf{x}, \mathbf{y}) \in D} \frac{1}{2} \left( \underbrace{R_{\mathbf{x}, \mathbf{y}} - [\sigma] \mathbf{w}^T \mathbf{f}_{\mathbf{x}, \mathbf{y}} - [\sigma] \overbrace{\mathbf{x}^T U^T V \mathbf{y}}^{o_{\mathbf{x}, \mathbf{y}}^2}}_{\delta_{\mathbf{x}, \mathbf{y}}} \right)^2 \\
&= \sum_{(\mathbf{x}, \mathbf{y}) \in D} \delta_{\mathbf{x}, \mathbf{y}} \frac{\partial}{\partial V} - [\sigma] \mathbf{x}^T U^T V \mathbf{y} \\
&= - \sum_{(\mathbf{x}, \mathbf{y}) \in D} \delta_{\mathbf{x}, \mathbf{y}} [\sigma(o_{\mathbf{x}, \mathbf{y}}^2)(1 - \sigma(o_{\mathbf{x}, \mathbf{y}}^2))] \mathbf{s} \mathbf{y}^T
\end{aligned}$$

- $L_2$  w regularization:

$$\begin{aligned}
\frac{\partial}{\partial \mathbf{w}} Obj_{rw} &= \frac{\partial}{\partial \mathbf{w}} \frac{1}{2} \mathbf{w}^T \mathbf{w} \\
&= \mathbf{w}
\end{aligned}$$

- Social spectral regularization:

$$\begin{aligned}
\frac{\partial}{\partial U} Obj_{rss} &= \frac{\partial}{\partial U} \sum_{\mathbf{x}} \sum_{\mathbf{z} \in \text{friends}(\mathbf{x})} \frac{1}{2} S_{\mathbf{x}, \mathbf{z}}^+ (\mathbf{x} - \mathbf{z})^T U^T U (\mathbf{x} - \mathbf{z}) \\
&= \sum_{\mathbf{x}} \sum_{\mathbf{z} \in \text{friends}(\mathbf{x})} \frac{1}{2} S_{\mathbf{x}, \mathbf{z}}^+ U ((\mathbf{x} - \mathbf{z})(\mathbf{x} - \mathbf{z})^T + (\mathbf{x} - \mathbf{z})(\mathbf{x} - \mathbf{z})^T) \\
&= \sum_{\mathbf{x}} \sum_{\mathbf{z} \in \text{friends}(\mathbf{x})} S_{\mathbf{x}, \mathbf{z}}^+ U (\mathbf{x} - \mathbf{z})(\mathbf{x} - \mathbf{z})^T
\end{aligned}$$

Before we proceed to the final derivatives, we define one additional vector abbreviation:

$$\mathbf{r} = U \mathbf{z} \quad \mathbf{r}_k = (U \mathbf{z})_k; \quad k = 1 \dots K.$$

• **Social co-preference regularization:**

$$\begin{aligned}
\frac{\partial}{\partial U} Obj_{rsc} &= \frac{\partial}{\partial U} \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \frac{1}{2} \left( \underbrace{P_{\mathbf{x}, \mathbf{z}, \mathbf{y}} - \mathbf{x}^T U^T \text{diag}(\mathbf{V} \mathbf{y}) U \mathbf{z}}_{\delta_{\mathbf{x}, \mathbf{z}, \mathbf{y}}} \right)^2 \\
&= \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \delta_{\mathbf{x}, \mathbf{z}, \mathbf{y}} \frac{\partial}{\partial U} - \mathbf{x}^T U^T \text{diag}(\mathbf{V} \mathbf{y}) U \mathbf{z} \\
&= - \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \delta_{\mathbf{x}, \mathbf{z}, \mathbf{y}} (\text{diag}(\mathbf{V} \mathbf{y})^T U \mathbf{x} \mathbf{z}^T + \text{diag}(\mathbf{V} \mathbf{y}) U \mathbf{z} \mathbf{x}^T) \\
&= - \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \delta_{\mathbf{x}, \mathbf{z}, \mathbf{y}} \text{diag}(\mathbf{V} \mathbf{y}) U (\mathbf{x} \mathbf{z}^T + \mathbf{z} \mathbf{x}^T)
\end{aligned}$$

In the following,  $\circ$  is the Hadamard elementwise product:

$$\begin{aligned}
\frac{\partial}{\partial V} Obj_{rsc} &= \frac{\partial}{\partial V} \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \frac{1}{2} (P_{\mathbf{x}, \mathbf{z}, \mathbf{y}} - \mathbf{x}^T U^T \text{diag}(\mathbf{V} \mathbf{y}) U \mathbf{z})^2 \\
&= \frac{\partial}{\partial V} \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \frac{1}{2} \left( \underbrace{P_{\mathbf{x}, \mathbf{z}, \mathbf{y}} - (\widehat{U \mathbf{x}}^{\mathbf{s}} \circ \widehat{U \mathbf{z}}^{\mathbf{r}})^T \mathbf{V} \mathbf{y}}_{\delta_{\mathbf{x}, \mathbf{z}, \mathbf{y}}} \right)^2 \\
&= \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \delta_{\mathbf{x}, \mathbf{z}, \mathbf{y}} \frac{\partial}{\partial V} - (\mathbf{s} \circ \mathbf{r})^T \mathbf{V} \mathbf{y} \\
&= - \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \delta_{\mathbf{x}, \mathbf{z}, \mathbf{y}} (\mathbf{s} \circ \mathbf{r}) \mathbf{y}^T
\end{aligned}$$

- **Social co-preference spectral regularization:**

$$\begin{aligned}
\frac{\partial}{\partial U} Obj_{rscs} &= \frac{\partial}{\partial U} \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \frac{1}{2} P_{\mathbf{x}, \mathbf{z}, \mathbf{y}} (\mathbf{x} - \mathbf{z})^T U^T \text{diag}(V \mathbf{y}) U (\mathbf{x} - \mathbf{z}) \\
&= \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \frac{1}{2} P_{\mathbf{x}, \mathbf{z}, \mathbf{y}} (\text{diag}(V \mathbf{y})^T U (\mathbf{x} - \mathbf{z}) (\mathbf{x} - \mathbf{z})^T \\
&\quad + \text{diag}(V \mathbf{y}) U (\mathbf{x} - \mathbf{z}) (\mathbf{x} - \mathbf{z})^T) \\
&= \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} P_{\mathbf{x}, \mathbf{z}, \mathbf{y}} \text{diag}(V \mathbf{y}) U (\mathbf{x} - \mathbf{z}) (\mathbf{x} - \mathbf{z})^T \\
\frac{\partial}{\partial V} Obj_{rscs} &= \frac{\partial}{\partial V} \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \frac{1}{2} P_{\mathbf{x}, \mathbf{z}, \mathbf{y}} (\mathbf{x} - \mathbf{z})^T U^T \text{diag}(V \mathbf{y}) U (\mathbf{x} - \mathbf{z}) \\
&= \frac{\partial}{\partial V} \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} \frac{1}{2} P_{\mathbf{x}, \mathbf{z}, \mathbf{y}} (U (\mathbf{x} - \mathbf{z}) \circ U (\mathbf{x} - \mathbf{z}))^T V \mathbf{y} \\
&= \frac{1}{2} \sum_{(\mathbf{x}, \mathbf{z}, \mathbf{y}) \in C} P_{\mathbf{x}, \mathbf{z}, \mathbf{y}} (U (\mathbf{x} - \mathbf{z}) \circ U (\mathbf{x} - \mathbf{z})) \mathbf{y}^T
\end{aligned}$$

Hence, for any choice of primary objective and one or more regularizers, we simply add the derivatives for each of  $\mathbf{w}$ ,  $U$ , and  $V$  according to (4.7).

## 5.2 Second Trial

For the second online trial, we chose four algorithms again to randomly split between the LinkR application users. Social Matchbox was included again as a baseline since it was the best performing algorithm in the first trial. The distribution count of the algorithms to the users is shown in Table 5.1

The four SCF algorithms are:

- **Social Matchbox (Soc. Mbox)** : Matchbox MF + Social Regularization + L2  $U$  Regularization + L2  $V$  Regularization
- **Spectral Matchbox (Spec. Mbox)**: Matchbox MF + Social Spectral Regularization + L2  $U$  Regularization + L2  $V$  Regularization
- **Social Hybrid (Soc. Hybrid)**: Hybrid + Social Regularization + L2  $U$  Regularization + L2  $V$  Regularization + L2  $\mathbf{w}$  Regularization
- **Spectral Co-preference (Spec. CP)**: Matchbox MF + Social Co-preference Spectral Regularization + L2  $U$  Regularization + L2  $V$  Regularization

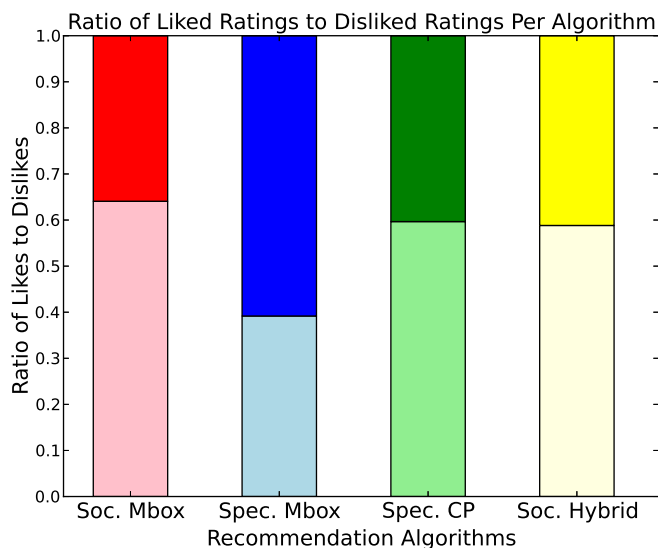
### 5.2.1 Online Results

The online experiments were switched to the new algorithms on October 13, 2011. For the online results reported here, since the second live trial is still currently ongoing,

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Algorithm	Users
Social Matchbox	26
Spectral Matchbox	25
Spectral Co-preference	27
Social Hybrid	25

**Table 5.1:** Number of Users Assigned per Algorithm.

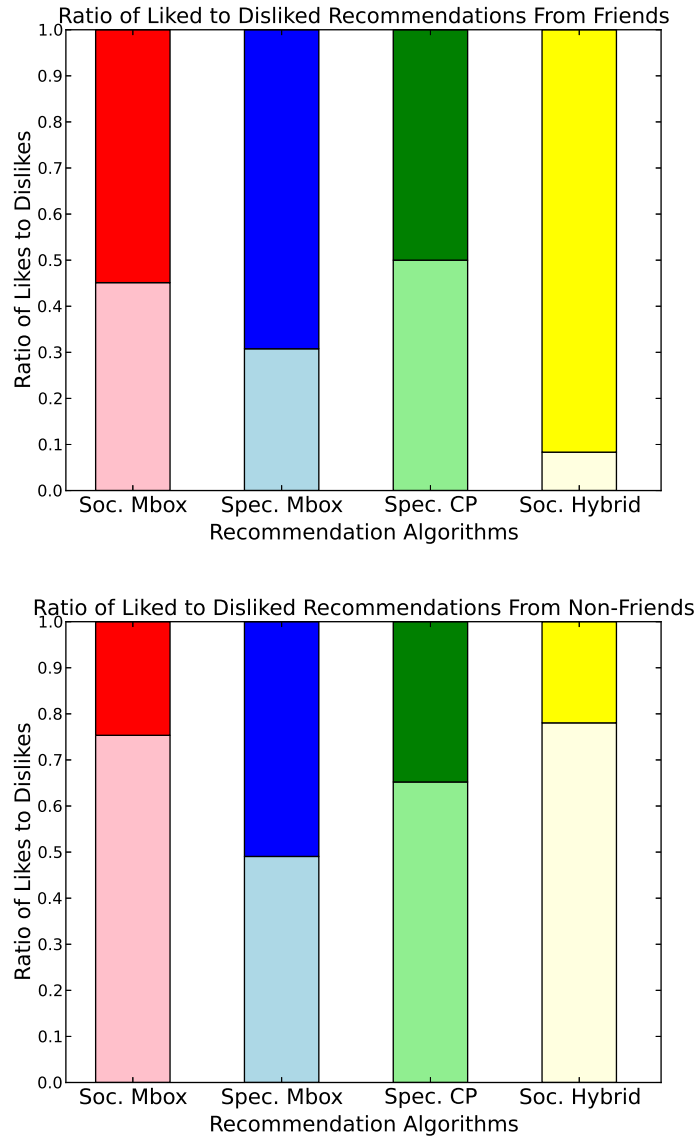


**Figure 5.1:** Results of online live trials. The percentage of Liked ratings are stacked on top of the percentage of Disliked ratings per algorithm. Spectral Matchbox achieved the highest ratio of likes to dislikes among the four algorithms. Spectral social regularization in general appears to be a better way to socially regularize compared to social regularization.

we took a snapshot of the data as it was on October 22, 2011. The algorithms were randomly distributed among the 103 users who still had the LinkR application installed. The distribution of the algorithms to the users are show in Table 5.1

Results shown in Figure 5.1 are the percentage of Like ratings and the percentage of Dislike ratings per algorithm stacked on top of each other with the Like ratings on top.

First thing we note in Figure 5.1 is the decrease in performance for Social Matchbox, and in fact for all SCF algorithms in general. Except for Spectral Matchbox, they all received more Dislike ratings than Like ratings. What we noticed is that of the recommendations being made in the week that we switched over to the new algorithms, the majority of the links were about Steve Jobs, who had died the week previously. We believe that the redundancy and lack of variety of the links being recommended caused an increase in the Dislike ratings being given by users on the recommended links. Taking out the skewed results that follows an unusual event such as this, the relative algorithm performance was better.



**Figure 5.2:** Results of the online live trials, split between friends and non-friends. As in the first trial, there is a significant drop in performance between recommending friend links and recommending non-friend links.

We again split the results again between friend link recommendations and non-friend link recommendations, with the results shown in Figure 5.2 being the percentage of Like ratings and the percentage of Dislike ratings per algorithm stacked on top of each other with the Like ratings on top. As shown Figure 5.2, all four algorithms experienced significant performance drop in the number of likes when it came to recommending non-friend links. This reflects the results of the first trial.

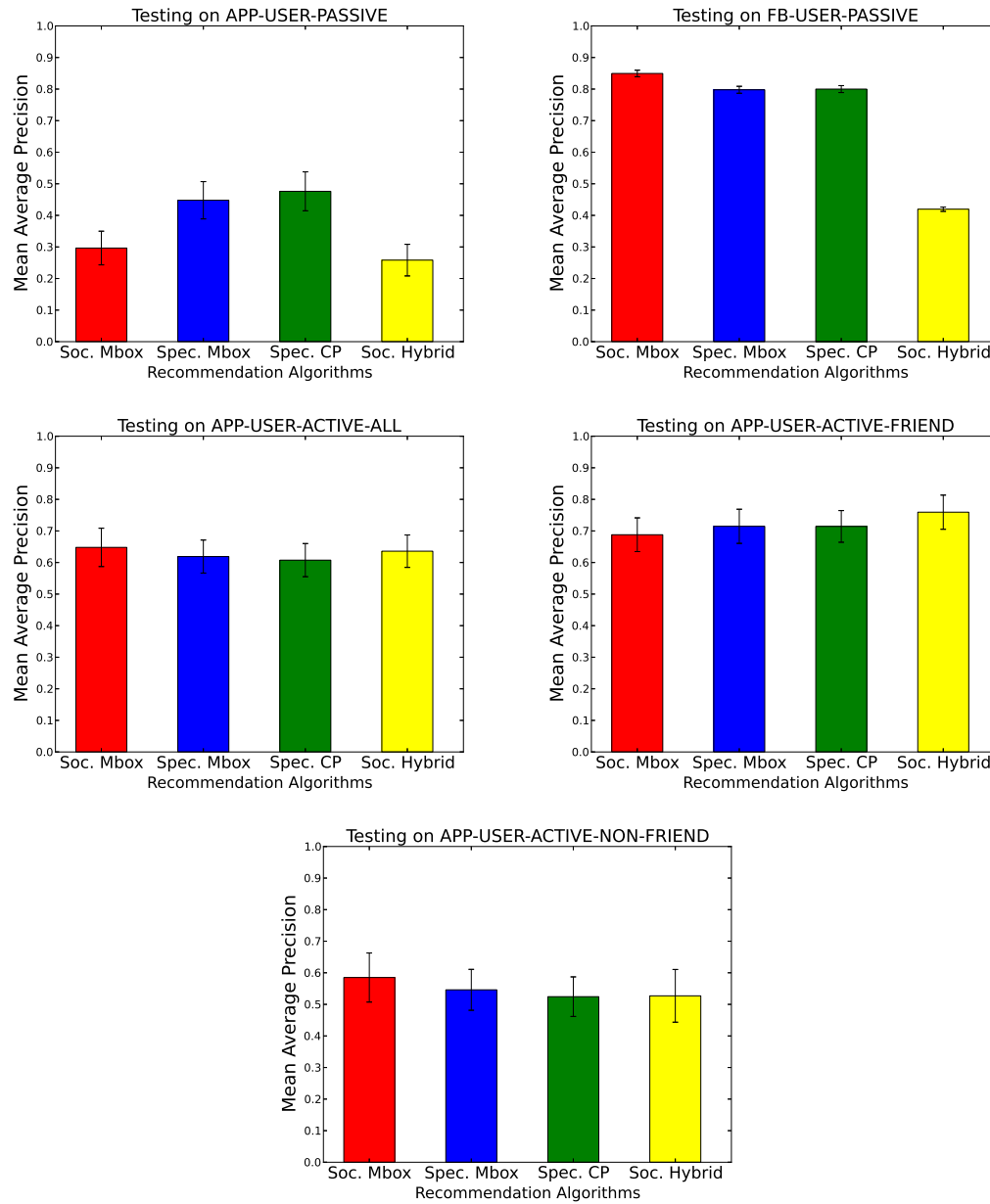
We note the following observations from the results shown in Figures 5.1 and 5.2:

- 
- Compared to the other algorithms, Spectral Matchbox achieved the best ratio of likes to dislikes as seen, as seen in Figure 5.1. Combined with the results for Spectral Co-preference, Spectral social regularization in general appears to be a better way to socially regularize compared to social regularization. This comparison holds even when the results are split between friend links recommendations and non-friend links recommendations, as seen in Figure 5.2.
  - When looking at just the friend link recommendations in Figure 5.2, Social Hybrid was the best performing algorithm. This result comes from the user-user information diffusion among its friends that Social Hybrid learns, which could not be learned by the other SCF algorithms. Learning information diffusion thus helps when it comes to building better SCF algorithms.
  - Spectral Co-preference didn't do well on friend link recommendations, however it did better on the non-friend link recommendations. When it comes to recommending friend links, friend interaction information coming through social regularizer seems more important than implicit co-likes information provided by the co-preference regularizer. When there is no social interaction information such as with non-friend links, co-preference methods with its implicit co-likes information appear much better than just vanilla collaborative filtering at projecting users into subspaces of common interest.

### 5.3 Offline Results

Aside from the online user trial, we also evaluate the algorithms using the mean average precision metric discussed in Section 3.3. We note the following observations from the results shown in Figures 5.3, 5.4, and 5.5:

- Most results are not statistically significant.
- In Figure 5.5, when training UNION and testing on APP-USER-ACTIVE-FRIEND, Social Hybrid gets the score on the MAP metric. This reflects the results of the live user trial in Section 5.2.1.
- In Figure 5.4, when training on ACTIVE and testing on APP-USER-ACTIVE-NON-FRIEND, Spectral Co-preference shows slightly better (though not statistically significant) performance than when testing on APP-USER-ACTIVE-FRIEND. This also reflects the results of the live user trial in Section 5.2.1. The explicit dislikes of the Active data allow the co-preference regularizer to learn the Dislike data better.
- The better performance of the social spectral regularization methods in the live user trials do not show in the offline experiments. This may be caused by the difference in the dataset size between the live user trials and the offline experiments, which affected optimal parameter settings for the offline experiments. Further tuning of the parameters may cause the social spectral regularization methods to perform better

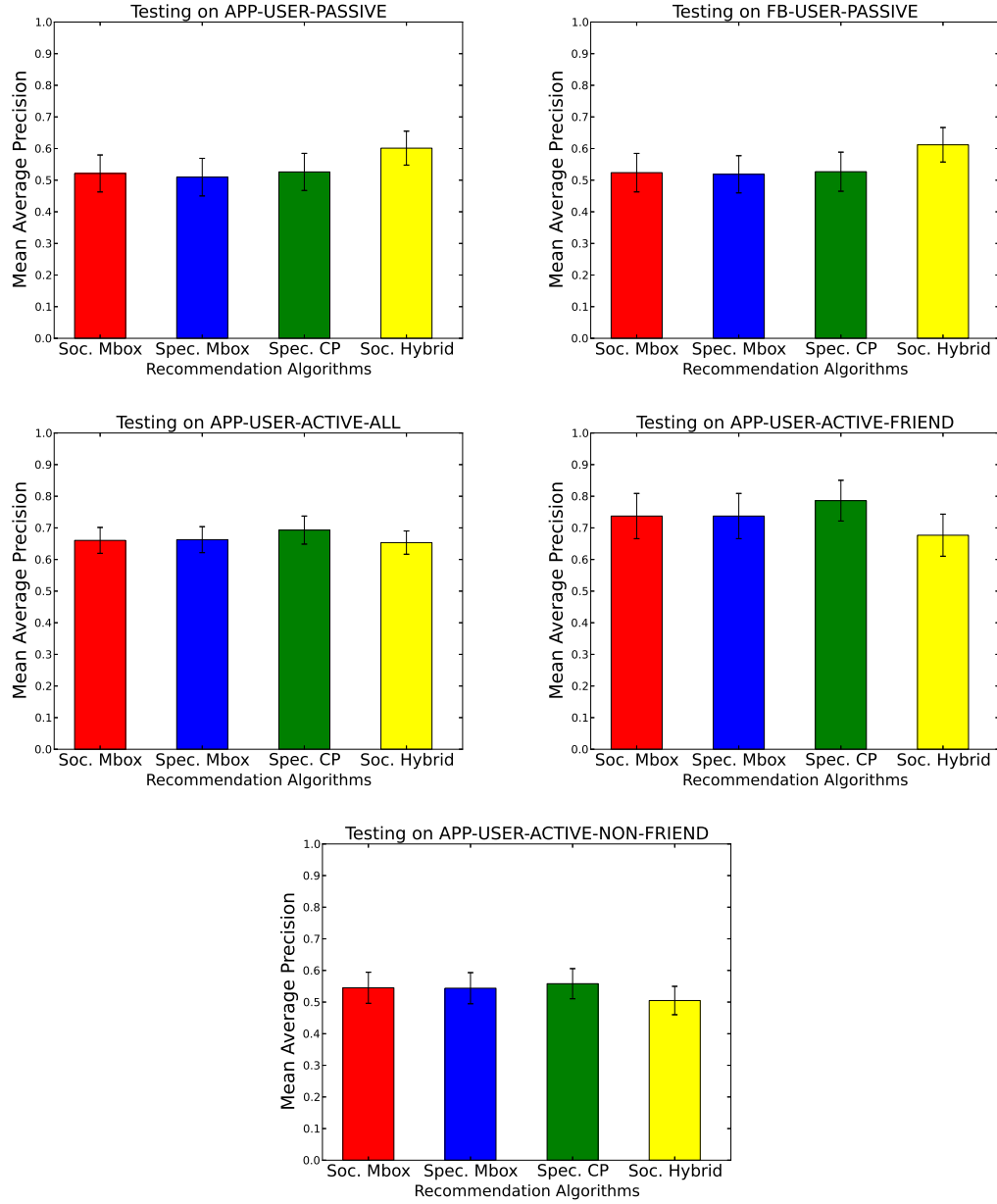


**Figure 5.3:** Results of training on Passive data

- Another reason may be that perhaps the MAP metric isn't the best metric that correlates with human performance evaluation, and some other metric that we could have used instead may better reflect the results of live user trials.

## 5.4 Summary

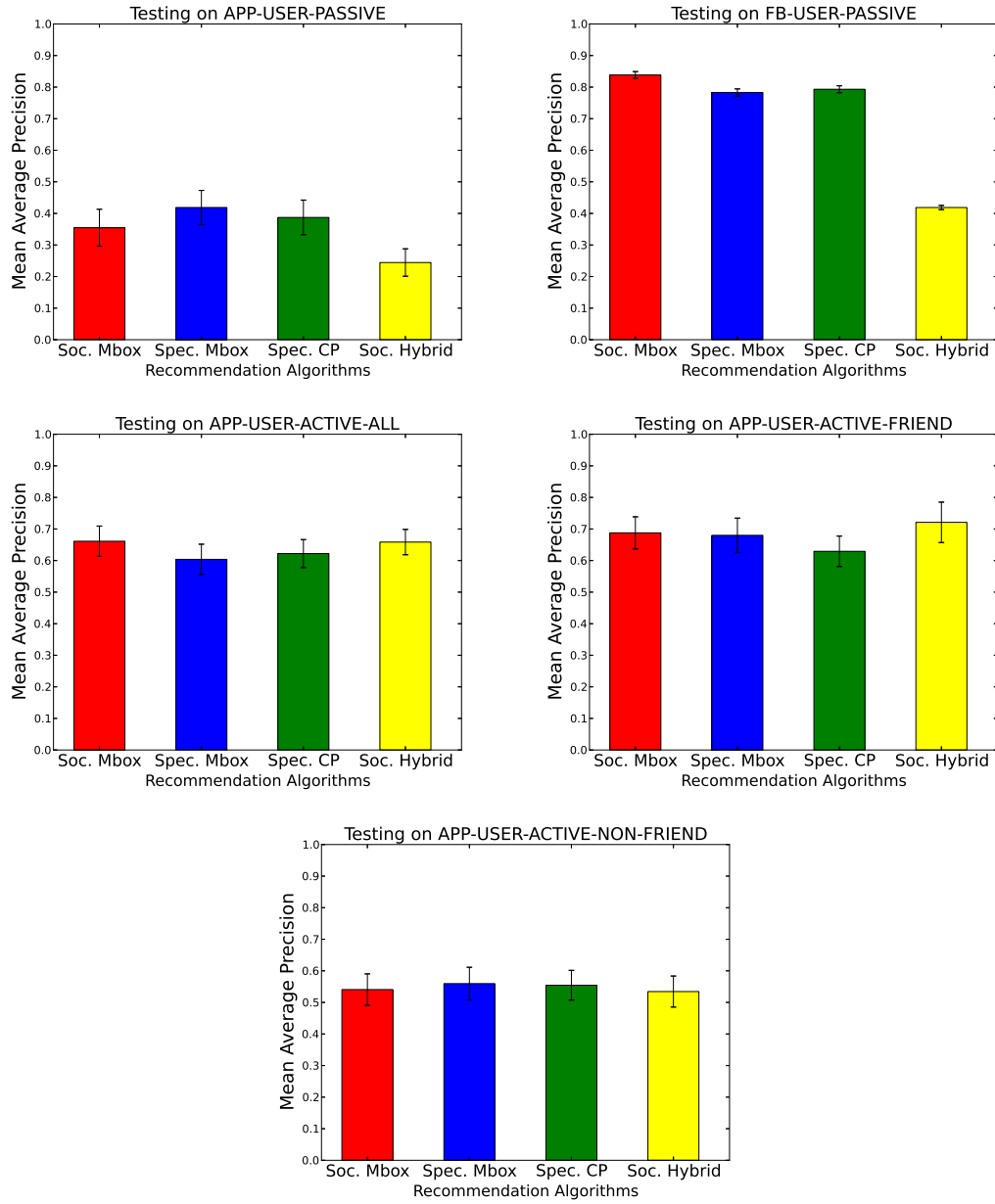
We summarize the observations made during the second trial:



**Figure 5.4:** Results of training on Active data

- The social spectral regularization methods generally performed better in the live user trials, even when the results were split between friend link recommendations and non-friend link recommendations.
- Learning information diffusion models helps in SCF, as evidenced by the strong performance of Social Hybrid when recommending friend links.
- When there is no social interaction information, learning implicit co-likes information is better than using plain CF methods.





**Figure 5.5:** Results of training on Union data

- The better performance of the social spectral regularization methods in the live trials were not reflected in the offline experiments. Perhaps there is a better metric than MAP that correlates with human preferences.



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# Conclusion

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## 6.1 Summary

We summarize the results and contributions of this paper as follows:

- We outlined three main deficiencies in current SCF techniques and discuss how we solve them:
  - (a) **Non-feature-based user similarity:** We extended existing social regularization and social spectral regularization methods to incorporate user features to learn user-user similarities in the latent space.
  - (b) **Model direct user-user information diffusion:** We defined a new hybrid SCF method which combines a linear CBF function with a CF MF method to incorporate user-user information diffusion into the model.
  - (c) **Restricted common interests:** We defined a new social co-preference method that learns user like or dislike similarities only for specific areas.
- Our extension to social regularization is a very effective method for SCF, and beat all other algorithms in the first trial. This was reflected in the live user trial, user survey, and offline passive experiments. However, social regularization can still be improved to address other deficiencies outlines in this thesis.
- Learning user-user information diffusion models can result in better SCF algorithms. Our Social Hybrid algorithm which uses this information outperformed all other SCF methods when recommending friend links.
- Friend interaction information coming from social regularization is more useful than the implicit co-likes information of co-preference regularization. However when there is no social interaction information available, learning this implicit co-likes information is better than plain CF methods.

## 6.2 Future Work

- We used only a subset of the possible feature data in Facebook and in the links themselves. Extending the recommenders to handle more of the rich information

that is available may result in better performance for the SCF algorithms. One feature that would have been useful is including a *genre* feature in the links (blog, news, video, etc.) to fine-grain which types of links that users prefers to receive. This would have prevented a number of dislikes that from users that do not listen to music much and kept getting recommended music videos from YouTube. A specific genre like “music video” in the links would have allowed the SCF algorithms to learn these types of user preferences.

- Related to having more features like the genre of the links, enforcing diversity in the recommended links would prevent redundant links about the same topic being recommended again and again. This is especially useful when an unusual event happens like the death of Steve Jobs and the ensuing massive amount of links about it being posted on Facebook by other users.
- Another future direction this work can go to is to incorporate active learning in the algorithms. This would ensure that the SCF algorithm do not exploit the learned preferences too much and even discover better link preferences that are available.
- Probably the biggest assumption we have made in our implementations is how we inferred the implicit dislikes of users in the Facebook data. A better and method of inferring implicit dislikes will give a big boost to the SCF algorithms. As evidenced by the results of training on ACTIVE and UNION data, having a more accurate list of likes and dislikes greatly improves the performance of the SCF algorithms.
- Finally, there may be a better metric than MAP for offline testing that more accurately correlates with live human preference. What that metric is is still an open question.

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