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Game Theory-based and Heuristic Algorithms for Parking-lot Search

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Abstract— Increasing the population of cities has led to several problems in using the spatial sources of cities. One of these sources which imposes high expenses on city mates are car parks. To solve such problems many parking guidance systems have been developed, but unfortunately in most of them the efficiency has not been evaluated. In order to analyze efficiency of parking guidance systems, in this paper two models of parking selection systems are provided, using two concepts: game theory and priority heuristic. In the games theory model, drivers are considered as being rational entity that are seeking to maximize their payoffs. On the other hand, in the priority heuristic model, characteristics of drivers are taken into account for choosing a car park. We compared our model to the similar existing models based on three factors: the total number of drivers, the number of on-street car parks space, costs difference between private and on-street car parks and the influences of each factor on the efficiency of the parking guidance system. The results of comparison represent far higher efficiency compared to previous models.

Keywords— *Game Theory, Parking Search Process, Priority Heuristic*

I. INTRODUCTION

As predicted by researchers [1] it is expected that by 2050 about 70% of the world's population will be living in cities and urban areas. One side-effect of urban life is the increased number of vehicles. Vehicles are invented to make everyday life easier. However vehicles density in big cities has caused unpleasant problems including environmental issues, energy consumption, lack of parking space, traffic jams, air pollution. From among these problems, the parking space shortage is one of the most important ones, since cities' spatial resources are limited and the costs of building new car parks are astronomical. So drivers have to compete to find a parking space. People who lose this competition have to either wait for a long time (whose calculation is very difficult) or continue driving to find a new parking space [2]. This leads not only to drivers' time wasting but also to outbreak of traffic jams and overcrowded streets. This has made the effective management of urban resources and processes, a vital issue.

To cope with these phenomenon, in recent years many researches are carried out in relation to designing and development of parking guidance systems. Generally, parking

guidance systems are categorized into two groups: centralized and decentralized [3]. Centralized systems make use of wireless communication and sense technology for gathering and broadcast of information (including: the closest parking to the driver, the number of unoccupied spaces in the parking, the number of requests for the parking space, parking costs, the way to avoid traffic jams, time needed to arrive to the parking, the distance walk from parking to destination, etc.) [4], [5]. Decentralized systems also use the above-mentioned technologies to share the information [6], [7]. Recent advances in sensing technology and communication, which has led to the creation of smart cities, represents a promising way to reducing the time needed to find a parking space [8]. The newest researches in relation to parking guidance systems [9], [10] are trying to add a social layer to automobile systems. Users of such systems can utilize an application on their smart phone to share their knowledge of about parking spaces and/or even exchanging these spaces, so that in this way the overhead caused by searching parking spaces is reduced.

Unfortunately despite too many efforts done in this field, most researchers have done very little attention to evaluate their systems in real life. The efficiency of parking guidance systems depends on drivers' way of making decisions to choose a parking space. These systems are categorized into two major groups based on the type of decision-making. In the first class (most of the researches carried out are in this field) the parking guidance system only provides the drivers with information about the available parking resources and the coordinating costs, without any suggestions about which one to choose. So each driver based on the raw information obtained from such systems, decides which parking space suits him best. The second category is systems decide for the driver which parking space to choose using software which is installed on the car [11], [12].

In this paper it is tried to evaluate the efficiency of parking guidance systems by offering new solutions. Then the results are compared with other works. The results of the comparison showed higher efficiency in the present work in comparison to other ones. The decision making process for choosing the parking place has been done using two methods: games theory and priority heuristic. In the first, all drivers have exact knowledge of the parking space supply and costs and any

driver's level of information about the number of competitors is same. In this method, any driver is considered as a strategic, rational, and selfish agent who does any effort to minimize his parking costs based on information they have. But in priority heuristic, drivers instead of searching for the best parking space without analyzing all the information presented to them, looking to find a good enough alternative parking space. In fact, if a driver finds a relatively appropriate parking space, they will stop searching for the best parking space. In the presented models each driver can only choose one cheap on-street parking space or one expensive private parking space. Although private parking have unlimited parking spaces, on-street parking spaces are limited in number. Therefore drivers always have to compete for such parking spaces. Those drivers who choose to occupy an on-street parking space have to accept failure risk and consequently pay extra costs (caused by traffic, fuel consumption, stress, etc.) in order to get a space in a private parking.

The rest of the paper is organized as follows. Section II is dedicated to discuss some related works. Section III, has formulized the strategic game of choosing a parking space. Then the game will be analyzed using a PoA metric to investigate the efficiency of equilibrium strategies. Section IV includes introduction, formulization and analysis of priority heuristic to decide on either a private or on-street parking space. The comparison of the designed game and previous models is presented in section V. The results of the comparison indicate a higher efficiency of the present model. Finally the paper ends with a conclusion of the main findings in section VI.

II. RELATED WORK

In relation to smart parking guidance systems, the author of paper [13] have designed a parking guidance system based on message board. They could design a framework for implementing parking guidance information system and the necessary entities and processes for such systems were described. In [14] a conceptual architecture for smart parking guidance systems (IPA) is presented in order to offer parking management solutions. In this research, availability of parking spaces is analyzed using sensor network and drivers can reserve a parking space using IPA. Researchers [15] have extended the concept of reservation and designed a smart parking guidance system based on wireless networks. They applied a queuing model in order to resolve the optimal allocation problem in their work. In [16] the usefulness of reservation systems have been reviewed. The researchers in this paper have suggested that reservation based parking guidance systems could contribute to simplification of parking-searching operation and reduce the traffic jams.

In [11] researchers have used the games theory to design a parking guidance system. The main focus of researchers is on the influence of the amount of information on drivers' decisions. They concluded that in uncooperative contexts where any player is only seeking to maximize their benefit, the spread of information has a mutual effect on the efficiency of parking searching process; on the one hand contributes to an easier way of finding a parking space, and on the other hand it sharpening the competition. So researchers [17] attempted to

design a cooperative game to analyze the efficiency of decentralized parking search systems. They have changed the game by manipulating payoff matrix so that a driver's failing costs also influences a winner driver's cost. Due to this, drivers try to cooperate with each other to decrease their own costs as far as possible. The researchers claim that in this method the competing players' final costs are reduced and therefore the efficiency is increased. Researchers in [18] have defined a condition of the game in which drivers utilize other driver's information (in other locations) to find a space as close as possible (in this method the distance from the selected parking space to the driver's destination is neglected). Researchers have offered parking space assigning algorithms to find out or approximate the NE states and then compared their work with greedy Naïve algorithm. Authors of [19] have utilized fixed-distance heuristic method (that is an instance of sequential search problems) to find a parking space. In the presented method all parking spaces are neglected until reaching the first closest spaces to the driver's destination. They have reviewed their method in two cases. In the first case drivers are aware weather there is parking space closer to their destination or not, and in the second case this awareness is lacking.

In this research, we analyzed parking space selection issue from strategic and psychological points of view. In the strategic method, we manipulated costs function and used potential function to find out the optimum number of drivers in the competition. Since the number of competitors is selected in a smart way and optimally, the number of those who fail is decreased. The effect of this optimality on total efficiency of system is significant and considerable. In the second method, by including drivers' personal characteristics in the type of decision making and by using costs ratio function, we managed to design a model which is much more efficient in comparison to similar models.

III. MODELING THE PARKING SPACE SELECTION GAME

In the present paper, players are the drivers (D) that are searching for a parking space in the center of a big city. The car parks are divided into the two categories of private and on-street parking spaces. Parking cars in street-sides usually causes car density and traffic jam. Therefore on-street parking spaces (P_{os}) are restricted. As a result, drivers who intend to travel to such places should decide either to head for an expensive private parking which has unlimited spaces or to compete on an inexpensive on-street parking space with other drivers. The game is designed in a way that for a driver who is trying to get a parking space, it doesn't matter where to park in a street. In fact the whole street is considered as a single parking, and its value is the same for all drivers. This is also true about private parking and it doesn't matter for the driver which private parking in the area to choose. So, for their decision-making any driver is faced to two choices of private and on-street parking instead of several separate complexes. Another assumption which is taken into account is that the drivers are completely aware of the capacity of parking, costs of each of them, and the possibility of getting an unoccupied space. Since all drivers make their decisions at the same time none of them has information about the exact percentage of whether the parking space is occupied or not (so they don't know whether a parking is full or not). Also any driver

occupies the selected space for a long time. People whose houses are located in these areas of the city, leave their cars in the car park from morning to the end of the working hour.

The presented game can be formulized as an example of *resource selection games*. In this game type N players compete for a limited number of public resources [20]. We defined the one-shot parking space selection game as follows:

Definition III.1:

- $D = \{1, \dots, D\}$, $D > 1$ is the set of drivers who searching for the parking space.
- P_{os} , $|P_{os}| \geq 1$ is the set of on-street spots.
- P_{priv} , $|P_{priv}| \geq D$ is the set of spots in parking lot.
- $A_i = \{os, priv\}$ is the action set for each driver $i \in D$, containing the actions “on-street” (os) and “private parking” ($priv$).
- $CF_{os}(\cdot)$ is the cost functions of action os .
- $CF_{priv}(\cdot)$ is the cost functions of action $priv$.

We have considered parking space selection game as a larger family of *congestion games*. Since its payers’ payoffs is a non-descending function of the number of players competing on the capacity of car parks. In fact, it is not important which driver chooses which space, what matters is how many drivers intend to head to which car park type. Drivers who intend to get an on-street parking space have to compete with others because of the limited number of them. Those who win the competition pay C_{os} costs per time unit, but those who fail it have to pay extra cost arisen from wasting time, fuel consumption (C_{exc}) and head for a private car park. If $N \geq P_{os}$ number of drivers participates in the competition from the D number drivers in the game, P_{os}/N percent of them will succeed and as a result $L_{os} = 1 - P_{os}/N$ percent of them will lose. According to the aforementioned discussions we can suggest claim 1.

Claim 1: The expected cost for a driver that plays action os , $CF_{os}: A_1 \times \dots \times A_D \rightarrow \mathbb{R}$ is a function of n drivers competing in the competition and equals:

$$CF_{os}(n) = L_{os} \cdot (C_{exc} + C_{priv}) + (1 - |L_{os}|) \cdot C_{os} \quad (1)$$

$$= C_{os} + L_{os} \cdot (C_{exc} + C_{priv} - C_{os})$$

On the other hand, the costs for those drivers who avoid participating in the competition and directly go to private car parks equals:

$$CF_{priv} = C_{priv} \quad (2)$$

A. Analysis for parking space selection game

In the present paper, to investigate and review the efficiency of game’s equilibrium strategies, theoretic concepts and findings [11], [21] are utilized. In the parking space selection game one action profile is a vector $a = (a_i, a_{-i}) \in \times_{n=1}^D A_n$ in which a_{-i} is indicative of the action of all drivers except for driver i in profile a . As the players of these games are rational all of them are willing to minimize their costs. So to find the game’s Nash equilibrium, cases must be reviewed in which players get the most benefit by changing strategy.

Therefore, researchers [11] have claimed game’s pure NE may happen in either of these cases:

- When $D \leq m_{NE}$, a single pure strategy $m(a') = D$.
- When $D > m_{NE}$ and m_{NE} is not an integer, $\binom{D}{\lfloor m_{NE} \rfloor}$ pure NE with $m(a') = \lfloor m_{NE} \rfloor$.
- When $D > m_{NE}$ and m_{NE} is an integer, $\binom{D}{m_{NE}}$ pure NE with $m(a') = m_{NE} - 1$.

In which m_{NE} is the optimum number of drivers who have to compete in the game.

They also used (3), (4) and (5) respectively to calculate social costs, optimum social costs, and evaluating the efficiency of the their method.

$$C_{Soc}(m_{NE}) = \begin{cases} \sigma + m_{NE} \cdot (C_{os} - C_{priv}), & \text{if } m_{NE} \leq P_{os} \\ \sigma + P_{os} \cdot \tau + m_{NE} \cdot C_{exc}, & \text{if } m_{NE} > P_{os} \end{cases} \quad (3)$$

$$C_{opt} = \varepsilon \cdot C_{os} + \zeta \cdot C_{priv} \quad (4)$$

PoA

$$= \begin{cases} \frac{\varepsilon \cdot \tau + D \cdot (C_{exc} + C_{priv})}{\varepsilon \cdot C_{os} + \zeta \cdot C_{priv}}, & \text{if } m_{NE} \geq D \\ \frac{\sigma + P_{os} \cdot \tau + \lfloor m_{NE} \rfloor \cdot C_{exc}}{\varepsilon \cdot C_{os} + \zeta \cdot C_{priv}}, & \text{if } m_{NE} < D \end{cases} \quad (5)$$

Where $\sigma = D \cdot C_{priv}$, $\tau = (C_{os} - C_{priv} - C_{exc})$,

$\varepsilon = \min(D, P_{os})$, $\zeta = \max(0, D - P_{os})$. In (5) the most important factor in the efficiency of designed game is the value of m_{NE} . Since most competitors have to return to a private car park after paying extra costs. Therefore such drivers have the most negative effects on social costs and system’s efficiency.

Claim 2: The most appropriate value for m_{NE} is:

$$m_{NE} = P_{os} \cdot \frac{C_{exc} + C_{priv} - C_{os}}{C_{exc} + C_{os}} \quad (6)$$

Proof: We considered the parking space selection game as *symmetric games* (in which all drivers have the access to action set) to find the appropriate value for m_{NE} . Action set of this game is composed of two possible action os and $priv$. In [22] it has been shown that any symmetric congestion game including two strategies will have an equilibrium in pure strategies. From the game’s symmetric it can be shown that all 2^D action profiles can be mapped in $D + 1$ meta-profile. Any meta-profile $a(m)$ indicates the number of drivers that have decided on competing for on-street parking spaces. The expected costs for these m drivers and $D - m$ drivers decided to directly drive to a private parking, is a function of meta-profile $a(m)$ instead of the real profile. As mentioned, parking space selection game is categorized as congestion games. Hence, this game accepts an exact potential function $\Phi(\cdot)$ [23]. The game’s potential function is effectively a function of m and it can be written:

$$\Phi(a) \sim \Phi(m) = \sum_{j \in p} \sum_{k=0}^{n_j(a)} CF_j(k) \quad (7)$$

In which $n_j(a)$ is the number of drivers that have used source j under action profile a , $p = P_{os} + P_{priv}$. As a result for $m \leq P_{os}$

$$\Phi(m) = \lambda + \sum_{k=1}^m C_{os} = \sigma + m \cdot (C_{os} - C_{priv}) \quad (8)$$

Where $\lambda = (D - m) \cdot C_{priv}$. In other words, if the number of competitors is less than the on-street parking capacity all competitors can be placed in the car park and none has to pay extra costs. Those drivers that have not participated in the competition will pay private parking cost. But for $m > P_{os}$

$$\Phi(m) = \lambda + \sum_{k=1}^m C_{os} + L_{os} \cdot (C_{exc} + C_{priv} - C_{os}) \quad (9)$$

In the congestion games pure NE strategies will be equal to the potential function's local minimum. So for $m \leq P_{os}$ $\partial\Phi(m)/\partial m$ is smaller than zero and as a result minimum is obtained at m . But for $m > P_{os}$, demanding $\partial\Phi(m)/\partial m = 0$; so $\partial\Phi(m)/\partial m = P_{os} \cdot \frac{C_{exc} + C_{priv} - C_{os}}{C_{exc} + C_{os}}$ is the value of (6). As a result, according to the calculations done it seems that optimum condition happens that from D drivers, m_{NE} participate in the competition.

IV. PRIORITY HEURISTIC MODELING

The designed model in the previous section had considered players rational aspect only and was completely neglectful of the drivers' psychological and unique traits. In fact the software installed on the car made all the decisions and the driver (with whatever characteristics) had to follow it and not interfere with it. But in this section it is tried to include the drivers' behavioral and psychological features in their decision making. In fact, any driver provides a profile of personal features (such as sensitivity to prices, distance from car park to the destination, etc.) to the software and then the software offers a suggestion based on their priorities. In order to do this a concept named priority heuristic came to help. Researchers [24] have tried to answer these questions: What do people pay attention to when making risk-based decisions? What information-search methods do they use in this process? According to their findings in conditions in which people have loss instead of gain, priority heuristic defines the decision making process like this (to clarify on the subject an example is provided in Table I. Where in case decision A the numerical outcome -300 is loss with probability 0.05% and 0 is loss with probability 0.95% and in case decision B is made there is 5 is loss with probability 1).

1. Compare the minimum losses of the decisions A and B (0 and 5). If the difference between them is equal or greater 10% of the maximum loss, choose the option with the smaller value between minimum losses (In this example, 10% of maximum loss is 20 and greater than 5-0, so move to Step 2).

2. Otherwise, compare the probabilities of the minimum losses of the decisions A and B (0.95 and 1). If the two probabilities differ by more than 1/10, choose the option with higher probability between minimum losses. (In this example, 1-0.95 is smaller than 1/10, so move to Step 3).

3. Otherwise, compare the maximum losses of the decisions A and B (200 and 5) and choose the option with lower maximum loss (So, decision B is chosen).

So, Parking-space-selection gamble is modeled like the Table II (invoking the notation in Section III). In which for decision *on-street* the output $-C_{os}$ with probability $\min(1, \frac{P_{os}}{D})$ and output $-(C_{priv} + C_{exc})$ with probability $1 - \min(1, \frac{P_{os}}{D})$ will happen. For decision *private* which has only one state, output $-C_{priv}$ will happen with probability 1. On the basis of this, steps based on priority heuristic for parking space selection game include:

- *Step one:* if $|C_{os} - C_{priv}| > thr_c \cdot (C_{priv} + C_{exc})$ drivers will decide to choose on-street parking. Otherwise step two is considered.
- *Step two:* If $mid = \frac{C_{priv} - C_{os}}{C_{priv} + C_{exc}} > thr_c$ again drivers decide to go to on-street car parks. Otherwise step three is considered.
- *Step three:* If $1 - \min(1, \frac{P_{os}}{D}) < thr_p$, Maximum costs will be compared and since $(C_{priv} + C_{exc}) > C_{priv}$, drivers have to decide on private parking.

Researchers [24] have offered value $1/10$ for both thresholds thr_c and thr_p , but since drivers' economic status, trip purpose (work, entertainment), and characteristics differ, in designing parking space selection gamble two different values are considered for these thresholds.

A. Parking space selection gamble analysis

To analyze parking space selection gamble findings by other researchers [12] are used. They have modeled drivers' heterogeneity using two probability density function PDF_{thr_c} and PDF_{thr_p} . To do this, they supposed that majority of drivers won't accept risk of difference in very high or low prices. So, most of them prefer to compete when differences in price are reasonable (like *mid* condition).

On this basis, by applying $\int_0^{c_{max}} PDF_{thr_c}(x) = 1$ and $PDF_{thr_c}(0) = PDF_{thr_c}(c_{max}) = 0$ they resulted that

$$PDF_{thr_c}(x) = \begin{cases} \frac{6x}{c_{max}^2} \cdot (1 - \frac{x}{c_{max}}), & \text{if } x \in [0, c_{max}] \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

In which c_{max} shows drivers' sensitivity to the difference in costs of the two types of parking. After calculating probability density function, they have calculated cumulative distribution function CDF_{thr_c} and CDF_{thr_p} for each and concluded that the number of drivers decided to participate in the competition will be:

$$D_{os}^{PH} = D \cdot CDF_{thr_c}(mid) \quad (11)$$

So, the number of derivers that not tending to participate in the competition and directly go to private parking will be equal to:

$$D_{priv}^{PH} = D \cdot (1 - CDF_{thr_c}(mid)) \quad (12)$$

Claim 3: in parking space selection gamble, drivers' total costs equal to:

$$C_T^{PH} = \begin{cases} D_{os}^{PH} \cdot (C_{os} - C_{priv}) + \sigma, & \text{if } D_{os}^{PH} \leq P_{os} \\ D_{os}^{PH} \cdot C_{exc} + \sigma + P_{os} \cdot \tau, & \text{if } D_{os}^{PH} > P_{os} \end{cases} \quad (13)$$

TABLE I. PARAMETERS FOR MAKING A CHOICE BETWEEN A OR B

Decision	Values
A	-200 with p = 0.05 0 with p = 0.95
B	-5 with p = 1.00

Proof: Here drivers need to be divided into three groups. The first groups includes people that participate in the competition and win (P_{os}), the second includes people who participate in the competition but lose ($D_{os}^{PH} - P_{os}$), and third group includes people who directly go to a private car park without participating in the competition (D_{priv}^{PH}). If the number of competitors is fewer than the on-street parking spaces, all competitors are placed in them and no one pays any extra cost. Those drivers not competing in the competition will only pay private parking cost. But if competitors outnumber the on-street parking spaces, the first group pay C_{os} per time unit; and the second group have to pay $C_{exc} + C_{priv}$; and finally the third group pay C_{priv} per time unit.

For the purpose of optimizing the designed model's performance, we used a metric called costs ratio $r_c = \frac{(C_{exc} + C_{priv})}{C_{priv}}$. This norm reflects the costs from drivers' risk-taking in the competition over costs caused by not participating in the competition (and directly going to a private parking). Based on any driver's sensitivity to differences in costs we concluded that if we limit the number of drivers who participate in the competition, according to (14) the efficiency of the parking space selection gamble increases significantly.

$$thr_{cmp}^{PH} = \frac{(C_{exc} + C_{priv})}{C_{priv}} \cdot \frac{P_{os}}{c_{max}} \quad (14)$$

In other words, we changed the gamble so that: parking guidance system determines the number of drivers competing, using priority heuristic before reaching thr_{cmp}^{PH} threshold. When the number of competitor drivers reach the thr_{cmp}^{PH} threshold the system besides what the priority heuristic suggest only allows the participation of thr_{cmp}^{PH} drivers in the competition.

V. NUMERICAL RESULTS

In this section symbol-labeling is in a way that the strategic game suggested by us is called $\Gamma(D)$ and the game suggested in [11] is labeled $\Gamma(N)$. It should be noted that in $\Gamma(N)$ method, $m_{NE} = P_{os} \cdot (C_{priv} + C_{exc} - C_{os})/C_{exc}$ but in our model, m_{NE} equals to (6). Also, the parking space selection gamble using priority heuristic designed by us is named $PH(D)$ and the method suggested in [12] is called $PH(N)$. We used three factors of the total number of drivers, total number of on-street parking spaces, and cost difference between on-street and private parking, in comparing our model to the others. We also,

divide this section to two main parts: comparing strategic methods, and comparing priority heuristic methods. In the part, comparing strategic games, we analyzed the influence of the mentioned factors on PoA and social costs of $\Gamma(D)$ and $\Gamma(N)$. Also, in the other part $PH(D)$ is compared to $PH(N)$. The purpose of this comparison is to discover the influence of the mentioned factors on the number of drivers participating in the competition and total costs caused by them. It should be noted that in our model, the number of

TABLE II. PRIORITY HEURISTIC GAMBLE FOR PARKING SELECTION

Decision	Values
On-street	$-C_{os}$ with $p = \min(1, \frac{P_{os}}{D})$ $-(C_{priv} + C_{exc})$ with $p = 1 - \min(1, \frac{P_{os}}{D})$
Private	$-C_{priv}$ with $p = 1.00$

competitor drivers will be calculated with (14) in case of $D_{os}^{PH} > thr_{cmp}^{PH}$. But in $PH(N)$, competitor drivers will be calculated with (11) in any condition.

A. Comparing Strategic Methods $\Gamma(D)$ vs. $\Gamma(N)$

Fig. 1 shows the number of total drivers (a) and the number of on-street parking spaces (b) effect on PoA metric. As mentioned earlier, the efficiency of parking guidance systems is greatly dependent on the number of drivers who participate in the competition. In $\Gamma(N)$ method, with the increased total number of drivers, PoA increases as well. This trend continues until the total number of drivers equals m_{NE} . Then PoA decreases since after reaching this point no more drivers are not allowed in the competition. But in our suggested method, since m_{NE} has a completely optimum value, the value of PoA does not experience a great change with the increased number of drivers. Comparing $\Gamma(N)$ and $\Gamma(D)$ methods from the influence of the number of drivers on PoA point of view shows significant higher efficiency of $\Gamma(D)$ method to $\Gamma(N)$, since PoA value in the suggested method is always very close to optimum value (Fig. 1a).

On the other hand, if the total number of drivers is fixed and the number of on-street parking spaces are increasing in number, again in $\Gamma(N)$ the value of PoA increases until $D \geq m_{NE}$. When m_{NE} exceeds D , the PoA starts descending.

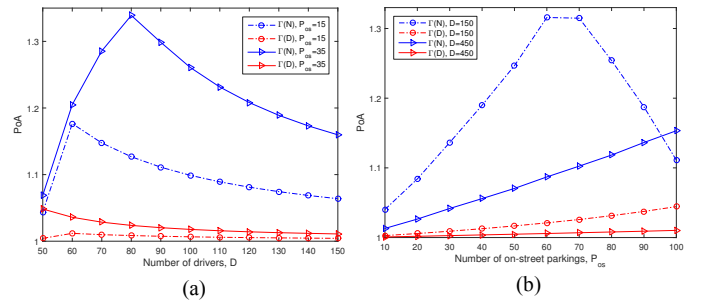


Fig. 1. PoA as a function of the number of total drivers (a) and number of on-street parkings (b), under fixed $C_{os} = 2$, $C_{priv} = 5$, $C_{exc} = 3$.

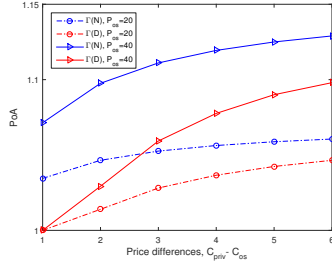


Fig. 2. PoA as a function of price differences between private and on-street parkings under fixed $D = 300$, $C_{os} = 1$, $C_{exc} = 4$ and variable private parking fee.

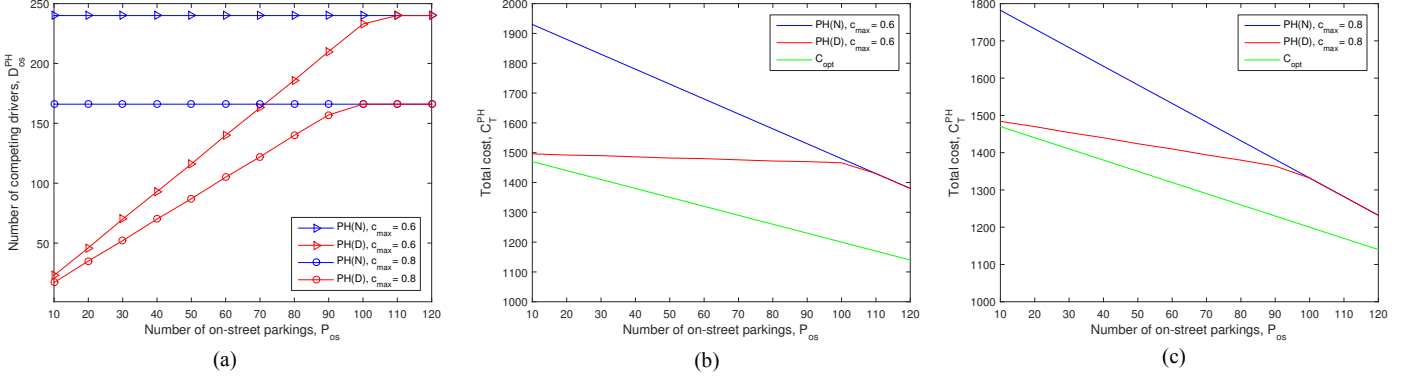


Fig. 3. Effect of the number of on-street parking spaces on the number of competing drivers (a), Total cost as a function of number of on-street parkings (b,c), under fixed $C_{os} = 2$, $C_{priv} = 5$, $C_{exc} = 2$, $D = 300$.

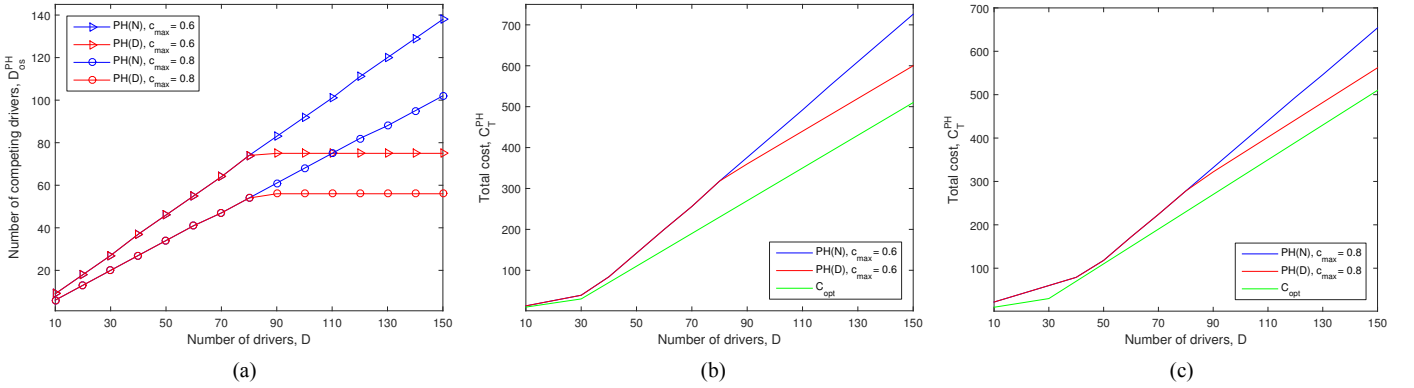


Fig. 4. Effect of the number of total drivers on the number of competing drivers (a), Total cost as a function of number of total driver (b,c), under fixed $C_{os} = 1$, $C_{priv} = 4$, $C_{exc} = 2$, $P_{os} = 30$.

Although this is also true in $\Gamma(D)$ method, the increase in PoA's slope in our suggested model is very insignificant compared to $\Gamma(N)$. From Fig. 1b it can be concluded that that even in this condition $\Gamma(D)$ performs better than $\Gamma(N)$ and is much closer to optimum condition.

Fig. 2, shows the influence of cost difference between on-street and private car parks on the value of PoA in the two methods of $\Gamma(N)$ and $\Gamma(D)$. As can be seen, with the increase in cost difference between on-street and private car parks in both $\Gamma(N)$ and $\Gamma(D)$ methods the value of PoA increases. This happens because the higher prices the private parking costs, the more people guided to get inexpensive on-street parking. But as these inexpensive parking are limited, increase in the number of competitors causes increase in the number of people that fail in the competition. This leads to an increase in drivers' costs. As a result with the increase in costs, the efficiency of

the system decreases and PoA increases. Comparing $\Gamma(N)$ and $\Gamma(D)$ methods shows even in such a condition our suggested model performs better than $\Gamma(N)$.

B. Comparing Priority Heuristic Methods PH(D) vs. PH(N)

Fig. 3 shows the effect of the number of on-street parking spaces on the number of drivers participating in the competition (a) and related total costs (b,c). As mentioned before, our suggested model (PH(D)) performs more efficiently than PH(N). In fact, in PH(N) method, besides the number of on-street parking spaces, a proportion of the total number of drivers ($CDF_{thr_c}(mid)$) participate in the competition for on-street parking spaces. But in PH(D) method thr_{cmp}^{PH} number of drivers participate in the competition. So until $thr_{cmp}^{PH} < D_{os}^{PH}$ our model performs more efficiently than the other model. As soon as $thr_{cmp}^{PH} = D_{os}^{PH}$ the efficiency of the two models

become the same. The same is true about total costs, because the more drivers participate in the competition, the more of them have to pay extra costs. So the total cost of PH(N) model are definitely higher than our model as far as $D_{os}^{PH} > thr_{cmp}^{PH}$. But after $thr_{cmp}^{PH} = D_{os}^{PH}$ the total costs become the same. It should also be noted that in real life the number of on-street parking spaces is always very limited and therefore by paying attention to total costs' plots we realize that PH(D) has performed much better than PH(N) when on-street parking spaces are limited (Fig 3b,c).

Fig. 4 illustrates the effect of total number of drivers on the number of drivers participating in the competition (a) and total costs caused by it (b,c). As can be seen, with the increase in the total number of drivers, the number of drivers who participate in the competition also experiences an increase. When the total number of drivers is low (less than 80) our model is no different from PH(N). But when the total number of drivers exceeds than 80, our suggested model only allows to thr_{cmp}^{PH} people to participate in the competition. While PH(N) with the increasing

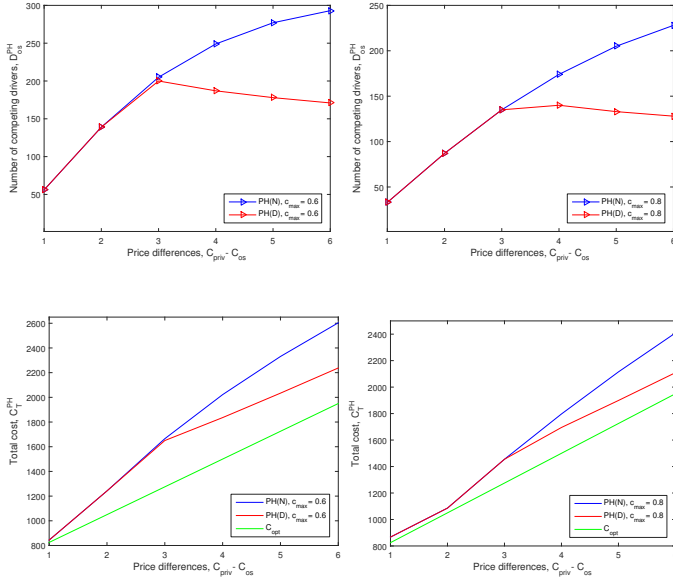


Fig. 5. Effect of the price differences between private and on-street parkigs on the number of competing drivers and related total costs under fixed $D = 300$, $P_{os} = 75$, $C_{os} = 2$, $C_{exc} = 2$ and variable private parking fee.

number of drivers sends more and more drivers to the competition. As a result of this, from $\frac{(C_{exc} + C_{priv})}{C_{priv}} \cdot \frac{P_{os}}{c_{max}}$ point on total costs in PH(N) model becomes greater than that of PH(D) (Fig.4 b,c). This shows higher efficiency of our method when the total number of drivers is great.

Fig. 5 illustrates the influence of cost differences of on-street and private parking ($C_{priv} - C_{os}$) on the number of drivers participating in the competition and the total costs of it. As it is clear, if the drivers' sensitivity on the price differences of the two types of car park is high ($c_{max} = 0.6$), more of them will participate in the competition (for example: when $C_{priv} - C_{os} = 2$ and $c_{max} = 0.6$, 139 drivers participate in the competition. While if $c_{max} = 0.8$ with the same conditions, the

number of competitors is decreased to 87). When the difference between private and on-street parking is small (happening rarely in real life) the two methods act the same way. But the bigger the difference between prices the more we experience higher efficiency of PH(D) in comparison to PH(N). Another point which seems necessary to mention is that when the difference in costs is small, total costs of both methods are close to the optimum value. But with the increase of the value of $C_{priv} - C_{os}$ total costs diverge from the optimum value, in PH(N) with higher magnitude and in PH(D) with lower magnitude.

VI. CONCLUSION

In this paper, two different models of drivers' decision making to choose a parking space were analyzed. The first model was based on the games theory and the second on priority heuristic. Since in the static and strategic game of parking space selection game personal characteristics of drivers were ignored, we decided to investigate the impact of such factors on drivers' decision making method. The designed models were compared to previously designed works on the three aspects of total number of drivers, the number of on-street parking spaces, and the difference in costs between on-street and private parking, and the influence of each factor on the efficiency of the system was compared. In the parking space selection game, since the number of drivers that participate in the competition has been chosen much more optimally in comparison to previous works the efficiency of the model was also much higher than them. In priority heuristic gamble, since in previous works the number of competitors was selected without paying attention to the number of on-street parking spaces, the number of drivers who failed was very high. In our model this weakness is covered and the number of competitors is selected in a smarter way. This can reduce traffic to great extent and fuel consumption considerably in large cities.

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