Algebraic Spaces II

An alg. space has a dense open subscheme.

Why? 1. Every algebraic space is the quotient of a scheme by an ~~alg.~~ étale equiv. relation.

$W \longrightarrow X$ scheme atlas
$W \times_X W \longrightarrow W \times_S W$ gives the equivalence relation.

2. There's an open set over which this equiv. relation is finite.

3. Quotients of schemes by finite flat groupoids exist when orbits are contained in affines.

Idea: reduce to affine case, $\operatorname{Spec} A_1 \rightrightarrows \operatorname{Spec} A_0$.
Look at equalizer of $A_0 \rightrightarrows A_1$.
Ex. G finite group, we're just taking the ring of invariants.

You can topologize k-pts. of alg. spaces: A subset is closed if it's the set of k-pts. of a closed subspace.

Quasi-coherent sheaves
Have to work w/ étale topology.
Small/big étale site have similar def'n but allow alg. spaces
$Et'(X) \subset Et(X)$ schemes/X. This gives an equiv. topos.

$\mathcal{O}_X \in X_{et}$ structure sheaf. On scheme $Y \in Et'(X)$, gives
$\Gamma(Y_{|zar}, \mathcal{O}_Y)$

Another description:

$R \underset{t}{\overset{s}{\rightrightarrows}} U$ gives X. Let $R' = U \times_X U \times_X U$

we can look at "equivariant" sheaves, i.e. sheaves $\mathcal{F}$ on U w/
an iso. $s^*\mathcal{F} \xrightarrow{\sim} t^*\mathcal{F}$ satisfying a cocycle condition on R'.

$\mathcal{O}_X$-modules can be defined in either setting.

Def. An $\mathcal{O}_X$-module $\mathcal{F}$ is quasi-coherent if $\exists Y \in Et'(X)$ s.t. $\mathcal{F}_Y$ is
quasicoherent. Same w/ coherent.
By descent theory, this gives the usual def'n on schemes.

Pushforwards, pullbacks defined the usual way.
You can define rel. Spec, hence affine morphisms

Stein factorization, Chow's lemma, finiteness of cohomology for
proper things hold.