(all morphisms are iso)

$\mathcal{M}_{1,1}$ should be a "pseudo" functor $Sch/_{\mathbb{Z}} \to$ Groupoids

with (1) Gluing property (families glue together)

$S \mapsto$ families of elliptic curves w/ isomorphisms

(2) Algebraic (in a precise sense of admitting a surjection from a certain algebraic space

Model: <u>Schemes as functors.</u>

Recall: $X \in \mathcal{C}$

$$\mathcal{C} \to \mathrm{Fun}(\mathcal{C}^{op}, \mathrm{Set})$$
$$X \mapsto h_X \text{ is ff.}$$

Now let S affine scheme, $\mathcal{C} = \mathrm{Aff}_S$ affine S-schemes

Now regard $h_X : \mathcal{C} = \mathrm{Aff}_S^{op} \to \mathrm{Set}$.

Say $f : F \to G$, $F, G : \mathcal{C}^{op} \to \mathrm{Set}$, f is <u>relatively representable</u> if $\forall g : h_T \to G$ ~~the fiber product~~

$$\begin{array}{ccc} F \times_G h_T & \to & F \\ \downarrow & & \downarrow f \\ h_T & \xrightarrow{g} & G \end{array}$$

if $F \times_G h_T$ representable.

<u>Def</u> $f : F \to G$, $F, G \in \mathcal{C}^{op} \to \mathrm{Set}$

f is affine open /resp closed embedding if f is relatively representable, and

$$\begin{array}{ccc} F \times_G h_T & \to & F \\ \downarrow & & \downarrow \\ h_T & \longrightarrow & G \end{array}$$

$\to$ comes from an open/resp closed morphism of affines.

<u>Def</u> $F : \mathrm{Aff}^{op}/_S \to \mathrm{Set}$ is a <u>big Zariski sheaf</u> if $\forall U \in \mathrm{Aff}_S$ and every $U = \bigcup U_i$ affine cover,

$$\mathcal{F}(U) \to \prod_i \mathcal{F}(U_i) \rightrightarrows \prod_{i,j} F(U_i \cap U_j)$$

is an equalizer.

<u>Prop</u> $F : (\mathrm{Aff}/S)^{op} \to \mathrm{Set}$ is represented by a separated S-scheme iff

(1) F Zariski sheaf,

(2) $F \xrightarrow{\Delta} F \times F$ is an affine closed embedding,

(3) $\exists X_i \in \mathrm{Aff}_S$, $\pi_i : h_{X_i} \to F$ open embeddings st $\coprod h_{X_i} \to F$ surjective morphism of (Zariski) sheaves.

Thus get (Sep S-schemes) $\to$ (functors with 1-3) and is an equiv of categories.