From ☆ we also get:

$$\mathrm{Sel}^{bal}(\mathbb{Q}, \mathbb{V}^*) \cong S^{-}_{\bar{\mathfrak{p}},\lambda} \oplus S^{-}_{\mathfrak{p},\lambda\psi} \oplus \text{finite}$$

$S^{-}_{\bar{\mathfrak{p}},\lambda}$ same as $S^{-}_{\mathfrak{p},\lambda}$ but with conditions @ $\mathfrak{p}$ & $\bar{\mathfrak{p}}$ reversed

and $$\mathrm{Sel}^{bal}(\mathbb{Q}, \mathbb{V}) \cong \check{S}^{-}_{\bar{\mathfrak{p}},\lambda} \oplus \check{S}^{-}_{\mathfrak{p},\lambda\psi} \oplus \text{finite}$$

Define κ_λ^{diag} by $\kappa_\infty^{diag} \longmapsto (\kappa_\lambda^{diag}, \text{—})$, where κ_∞^{diag} is written under ψ.

$$\kappa_{\alpha\alpha} \neq 0 \Longrightarrow \dim_{\mathbb{Q}_p} \mathrm{Sel}(\mathbb{Q}, V_pE) = 2.$$

$$\kappa_{\alpha\alpha} \neq 0 \Longrightarrow \mathrm{cork}_{\mathbb{Z}_p} \mathrm{Sel}_{0,\phi}(K,\lambda) = 1 \Longrightarrow \dim_{\mathbb{Q}_p} \mathrm{Sel}(\mathbb{Q}, V_pE) = 2 \quad (\#Ш(E/\mathbb{Q})[p^\infty] < \infty)$$

anticyclotomic ES $\left\{ AJ_p(\Delta^{fg_k h_k}) \right\}_{k \geq 2}$

$$\Longrightarrow \gamma - 1 \nmid \mathrm{char}_\Lambda\left(\left[(S^-_{\bar{\mathfrak{p}},\lambda})^\wedge \right]_{\Lambda\text{-tors}} \right)$$

$$\Longrightarrow \kappa_\lambda^{\mathrm{diag}} \text{ has nonzero image}$$

$$\check{S}^-_{\bar{\mathfrak{p}},\lambda}/(\gamma-1) \hookrightarrow \check{\mathrm{Sel}}_{0,\phi}(K,\lambda) \subset \mathrm{Sel}(\mathbb{Q}, V_pE)$$

$$(\kappa_\lambda^{\mathrm{diag}} \bmod \gamma - 1) \longmapsto \kappa_{\alpha\alpha}$$

Moreover, $\mathrm{res}_{\mathfrak{p}}(\kappa_{\alpha\alpha}) = 0$, so $\log_{\mathfrak{p}}(\kappa_{\alpha\alpha}) = 0$.