

Constructing \overline{M}_g

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S scheme, k alg closed, $g \geq 2$

Def A prestable curve over S is a proper flat morphism $\pi: C \rightarrow S$ s.t.
 \forall points $\bar{s} \rightarrow S$, $C_{\bar{s}}$ is a connected nodal curve

π is stable if for all components C_i of $C_{\bar{s}}$, $2(g(C_i)) - 2 + \# \text{ nodes} > 0$

Def Let \overline{M}_g be the fibered category:

Objects $(S, f: C \rightarrow S)$ S scheme, $f: C \rightarrow S$ stable curve of genus g .

Morphisms $(S', f': C' \rightarrow S') \rightarrow (S, f)$ are cartesian diagrams

$$\begin{array}{ccc} C' & \longrightarrow & C \\ \downarrow \ulcorner & & \downarrow \\ S' & \longrightarrow & S \end{array}$$

Thm \overline{M}_g is a Deligne-Mumford Stack

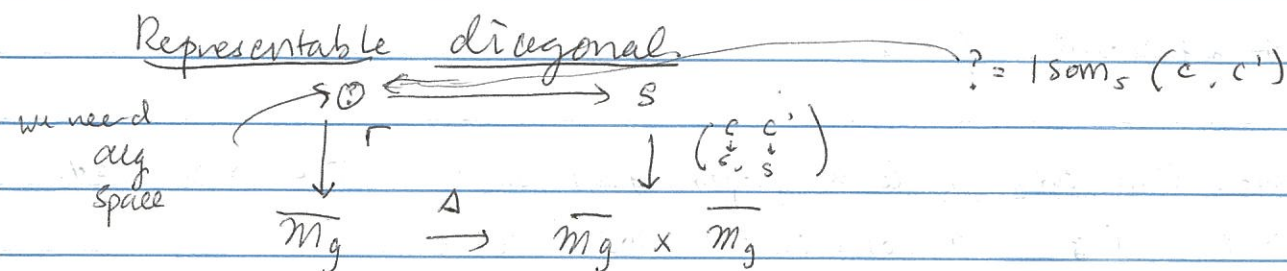
Step 1 \overline{M}_g is a stack.

prop Let $\pi: C \rightarrow S$ a family of stable curves. Then $\omega_{C/S}^{\otimes 3}$ is relatively very ample.

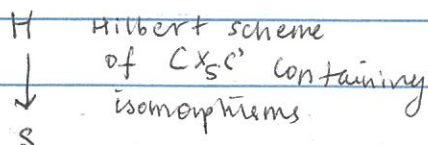
prop 4.4.12 + $\omega_{C/S}^{\otimes 3}$ very ample \Rightarrow satisfies descent.

(embed objects in a Hilbert scheme, which satisfies descent)

Step 2 \overline{M}_g is an algebraic stack.



So we want to show: $\text{Isom}_s(c, c')$ is representable.



Isomorphisms $\sigma: C_s \rightarrow C'_s \iff \Gamma \subseteq (C \times_S C')_s$

s.t. $\Gamma \xrightarrow{\sim} C_s$
 $\Gamma \xrightarrow{\sim} C'_s$ } open condition

scheme
 \downarrow

Therefore $\exists H^0 \xrightarrow[\text{open}]{} H$ s.t. $H^0 \cong \text{Isom}_s(c, c')$

\Rightarrow Diagonal is representable

Step 3. \overline{M}_g admits a smooth cover

If C is smooth, then $\omega_C^{\otimes 3}$ has degree $3(2g-2) \Rightarrow h^0(\omega_C^{\otimes 3}) = 6g-6-g+1 = 5g-5$.

Look at $\mathcal{H}' =$ Hilbert scheme of genus g curves in \mathbb{P}^{5g-6}

