AGS lec 3 (Adrian).

Descent $\mathcal{M}_{1,1}: Sch^{op} \to Set$ not representable

So consider the category

$\mathcal{M}_{1,1}$: objects $(S, (E,e))$ family of elliptic curves over S

morphisms: (f, g) where $f: S' \to S$, $g: E' \to E$,

s.t.
$$\begin{tikzcd} E' \arrow[r] \arrow[d] \arrow[dr, phantom, "\boxtimes"] & E \arrow[d] \\ S' \arrow[r] \arrow[u, bend left, "e'"] & S \arrow[u, bend left, "e"] \end{tikzcd}$$

and have forgetful functor $\mathcal{M}_{1,1} \to Sch$

$(S, (E,e)) \mapsto S$ forgetting E

So $\mathcal{M}_{1,1}$ fibered category / Sch.

fibers = families, isos amongst families.

So it's a category fibered in groupoids.

Recall
$$\begin{array}{c} F \\ p\downarrow \\ C \end{array}$$
$X \in Ob\, C$,

Consider C/X.

2-Yoneda: $Hom(C/X, F) \to F(X)$ sending $(C/X \xrightarrow{g} F) \mapsto g(id_X)$

is an equivalence of categories.

In our example $C = Sch$, $F = \mathcal{M}_{1,1}$, $X \in Sch$

e.g. $\left(\begin{matrix}\text{schemes}\\ \text{over } X\end{matrix} \xrightarrow{S} \left\{\begin{matrix}\text{families}\\ \text{of ell curves}\end{matrix}\right\}/S\right) \longrightarrow$ {families over X}.

For a quasi inverse, pull back along a morphism $Y \to X$.

Def (fppf site) X, Sch_X Define a Grothendieck topology on X by $\{U_i \to U\}$ is a covering if each $U_i \to U$ flat & locally of finite presentation, and $\bigcup U_i \to U$ is surjective.

$X \in Sch_S$, functor of pts $h_X: Sch_S^{op} \to Set$

Main technical theorem.

h_X is a ~~se~~ sheaf in Sch_S w/ the fppf topology. big étale site

Cor $X \to h_X$ defines a fully f. functor $Sch \to$ (sheaves on $Sch_{ét}$)

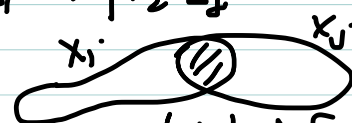
ie extend category of schemes in this way.)

General setup for descent.

C cat with finite fiber products
$p: F \to C$ fibered cat. eg $M_{11} \to Sch$
for each $f: X \to Y \in Mor\, C$, choose pullback functor $f^*: F(Y) \to F(X)$

<u>Def</u> For $\{X_i \to Y\}_{i\in I}$ morphisms in C define $F(\{X_i \to Y\})$ to be the following category

<u>Objects</u>: $(\{E_i\}_{i\in I}, \{\sigma_{ij}\}_{i,j\in I})$
where $E \in F(X_i)$ and $\sigma_{ij}: pr_1^* E_i \to pr_2^* E_j$
is an iso in $F(X_i \times_Y X_j)$



restricted families are the same

and a cocycle condition...
Morphisms are $\{g_i: E_i' \to E_i\}$ in $F(X_i)$ st

$$\begin{tikzcd} pr_1 E_i' \arrow[r] \arrow[d] & pr_1 E_i \arrow[d] \\ pr_2 E_j' \arrow[r] & pr_2 E_j \end{tikzcd}$$
commutes.

There is a (restriction) functor
$\mathcal{E}: F(Y) \to F(\{X_i \to Y\})$
<u>Def</u> $\{X_i \to Y\}$ is of effective descent for Y if $\mathcal{E}$ is an equiv of cats.

<u>Ex.</u> Sheaves on a top sp (the effective descent says you can glue sheaves)
$\mathcal{M}_{1,1}$

<u>Def</u> Let C be a site. A cat fibered in groupoids is a <u>stack</u> if $\forall X \in Ob(C)$ and any covering $\{X_i \to X\}$, the functor
$$F(X) \to F(\{X_i \to X\})$$
is an equiv of categories.