ornary 4,2014 - AGLS on Stacks-Introduction

r. Motivation.

Problem: When a moduli problem's objects have automorphisms, typically won't be timely represented by a scheme.

· Obvious guess for a module space of elliptic curves: H! - the j-line.

(2)
$$E_0 \times 14^1 + 50^5$$
, $E_6 = Y^2 = X^3 - 2^3$

 $j(\mathcal{E}_t) = j(\mathcal{E}_0) = 0 \quad \forall t.$ induced morphisms by \oplus and \otimes $A_{t}^{1} : \{0\} \longrightarrow A_{t}^{1} \text{ are constant} = 0.$ Claim: 9+ # Ex Al, 80's as families, in fact generic fikers Pt, C(+) = Fo, C(+). Pf. Fix (lt) e K alg. closure, t'6 EK. Then $y^2 = x^3 - t$. $f: \mathcal{L}_{1, \mathbb{C}(1/6)} \cong F_{0, \mathbb{C}(1/6)}$ $f: (x,y) \longmapsto (f^{-1/3}x, f^{-1/2}y)$ $f: (x,y) \longmapsto (f^{-1/3}x, f^{-1/2}y)$ $f: (x,y) \longmapsto (f^{-1/3}x, f^{-1/2}y)$ Set $T = \{ i \text{ somorphisms } \mathcal{E}_{t,K} \cong E_{c,AK} \}$ free transitive. $\mathcal{E}_{t,K} \cong E_{c,AK} \}$ $\mathcal{E}_{t,K} \cong \mathcal{E}_{t,K} \cong \mathcal{E}_{t,K} \cong \mathcal{E}_{t,K}$ $\mathcal{E}_{t,K} \cong \mathcal{E}_{t,K} \cong \mathcal{E}_{t,K} \cong \mathcal{E}_{t,K}$ $\mathcal{E}_{t,K} \cong \mathcal{E}_{t,K} \cong \mathcal{E}_$ 6=6al(K/(1+1). These actions commute.

=) Suffice to show f not fixed by 6 so that there are no the elements in I defined over (It).

Do this by explicit calculation of action of any (I)

ge6 on f: 6-th not unity,

If g.t' = X(g)+1/6 g.f: (x,y) +> (X(g)² t^{-1/3} x, X(g)³ E^{1/2} y).

Rem. Since H! is a "coarse moduli space" for elliptic corver, I this proves there is no Analyses representing scheme for M
Rem. Many other examples of interesting moduli problems a/auts, Mg, 972, Vect bundles/variety, etc
Ultimate solution: Consider $M_{1,1}: Seh/p \longrightarrow Gpds.$ Show a family actives of auts! To do this in a useful way, need to: Understand in what sense $M_{1,1}$ is a functor (algorite of $M_{1,1}$). 2 Understand in what sense $M_{1,1}$ is algebraic (algebra stacks).

II. Schemes as functors

•	Scheme X/s determineda by hx = Home	(-,X): Sehol-Sets.
	Yoneda lemma #says for cat. 6, 6 -> Fun (6°P, Sets) XI-> hx Hom (hx, F) ~ F(x)	fully faithful

· As a warm up for alg. spaces + Stacks, let's build category of schemes up in terms of affine schemes.

-Silly but instructive.

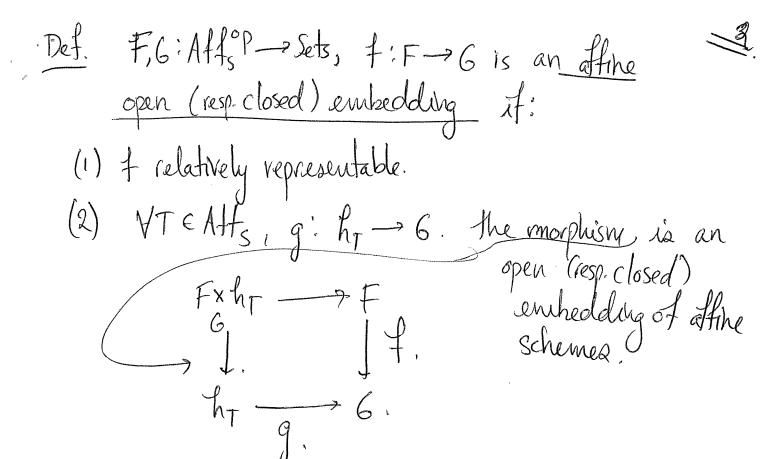
· Set-up: Sæffine scheme. Affs = affine schemes/s.

Q: Given F: Affs Sets, when is it representable by a scheme?

Preliminary dets.

Def. f:F>6 morphism of functors 6°P> Sets, called relatively representable it Yg;h_>6, Te6

FX.h_>F there product FX.h_T is h_D lf. representable



Def. F: Affor -> Sets is a big Zaviski sheaf if

YUEAHTS, U= UU; open cover by affines.

F(u) -> TF(ui) => TF(uinuj)

(ij) EIXI
is exact.

Proposition: F: Att -> Set representable by a separated
Proposition: F: Affs -> Set representable by a separated. S-scheme iff:
(1) F is a big Zeriski sheat.
(2) AND F Xi EAFfs, Ti: hxi -> F affine open enchaddings,
(2) And F Xi EAffs, Ti: hxi — F affine open enchodolings, st. Lihxi — F surjective morphism of Zaucki sheaves.
(3) 1: F→ FXF is an affine closed embedding.
The functor
The functor h_: (separated)
is an equivalence.
cample: Fun to cheek Pro is a scheme by using this

criteria.

Can get a similar criteria for arbetrary schemes.