CSE 21 Spring 2022
Homework 2
Due date: Wed April 13, 2022 at 11:59pm

INSTRUCTIONS

This homework should be done in **groups** of one to four students, without assistance from anyone besides the instructional staff and your group members. Homework must be submitted through Gradescope by a **single representative** of your group and received by **11:59pm** on the due date. There are no exceptions to this rule.

You will be able to look at your scanned work before submitting it. You must **type** your solutions. (hand-drawn diagrams are okay.) Your group representative can resubmit your assignment as many times as you like before the deadline. Only the most recent submission will be graded.

Students should consult their textbook, class notes, lecture slides, podcasts, group members, instructors, TAs, and tutors when they need help with homework. You may ask questions about the homework in office hours, but questions on Piazza should be private, visible only to instructors.

This assignment will be graded for not only the *correctness* of your answers, but on your ability to present your ideas clearly and logically. You should explain or justify, present clearly how you arrived at your conclusions and justify the correctness of your answers with mathematically sound reasoning (unless explicitly told not to). Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to **convince the reader** that your results and methods are sound.

KEY CONCEPTS Binomial coefficient identities, stars and bars, basic probability principles, independence, conditional probability, Bayes' Theorem.

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1. (10 points)

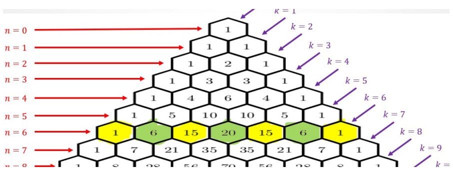
   Consider the function: $flip : \{0,1\}^n \to \{0,1\}^n$ $(n \geqslant 1)$ that flips the first bit from a 0 to a 1 or a 1 to a 0.

   For example: $flip(00110101) = 10110101$

   Take a look at the nth row of Pascal's triangle $(n \geqslant 1)$ and notice that if you sum the even binomial coefficients then it equals the sum of the odd binomial coefficients:

   For example, the 6th row:

$$\binom{6}{0} + \binom{6}{2} + \binom{6}{4} + \binom{6}{6} = \binom{6}{1} + \binom{6}{3} + \binom{6}{5}$$



   Exercise: Use the function $flip$ to prove that the sum of the even binomial coefficients is equal to the sum of the odd binomial coefficients.

   i.e.

$$\sum_{i=0}^{n} \binom{2n}{2i} = \sum_{i=1}^{n} \binom{2n}{2i-1}$$

   and

$$\sum_{i=0}^{n} \binom{2n+1}{2i+1} = \sum_{i=0}^{n} \binom{2n+1}{2i}.$$

2. (9 points) Compute the number of *integer solutions* for each equation:

   (a)

$$a_1 + a_2 + a_3 + a_4 + a_5 = 30$$

   $1 \leqslant a_1$
   $2 \leqslant a_2$
   $3 \leqslant a_3$
   $4 \leqslant a_4$
   $5 \leqslant a_5$

   (b)

$$a_1 + a_2 + a_3 + a_4 + a_5 = 30$$

   $0 \leqslant a_1$
   $0 \leqslant a_2$
   $0 \leqslant a_3$
   $0 \leqslant a_4$
   $0 \leqslant a_5 \leqslant 10$

   (c)

$$a_1 + a_2 + a_3 + a_4 + a_5 = 30$$

   $0 \leqslant a_1$
   $0 \leqslant a_2$
   $0 \leqslant a_3 \leqslant 10$
   $0 \leqslant a_4 \leqslant 10$
   $0 \leqslant a_5 \leqslant 10$

3. (10 points) Suppose your friend is at a chess tournament that has 6 different opponents. One of the opponents has an equal rating to your friend (which means that if your friend plays this opponent, then your friend has a $1/2$ chance of winning.) The other 5 opponents have higher rating than your friend, specifically, your friend has a $1/4$ chance of winning against the each of the other 5 opponents.

   (a) Your friend plays an opponent selected uniformly at random from the 6 opponents.

   Your friend tells you that after playing 10 games, your friend won 5 games and lost 5 games. What is the probability that your friend played the opponent with equal rating.

   (b) Your friend plays an opponent selected uniformly at random from the 6 opponents.

   Your friend tells you that after playing 10 games, your friend won 10 games and lost 0 games. What is the probability that your friend played the opponent with equal rating.

   (Assume that there is always a winner and a loser and there is no ties. Also assume that each match is independent meaning that the result of one match does not affect the next match.)

4. (12 points) Suppose you are playing a game involving 4 fair 10-sided dice (each die has the values $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and you can assume each die is a different color $\{black, green, red, yellow\}$.)

   (a) (3 points) Throw all 4 dice. What is the probability that all dice are different values?

   (b) (3 points) Throw all 4 dice. What is the probability that you roll a "pair"? (meaning that 2 of the dice share a value and the other two dice are different than the pair and different than each other.)

   (c) (3 points) Choose 2 of the dice. Throw them. Then throw the remaining 2 dice. What is the probability that the first 2 dice share the same value AND the remaining 2 dice are different than the pair and different than each other?)

   (d) (3 points) Throw all 4 dice, what is the probability that they sum up to exactly 10?