CSE 21 Spring 2022
Homework 3
Due date: Wed April 20, 2022 at 11:59pm

INSTRUCTIONS

This homework should be done in **groups** of one to four students, without assistance from anyone besides the instructional staff and your group members. Homework must be submitted through Gradescope by a **single representative** of your group and received by **11:59pm** on the due date. There are no exceptions to this rule.

You will be able to look at your scanned work before submitting it. You must **type** your solutions. (hand-drawn diagrams are okay.) Your group representative can resubmit your assignment as many times as you like before the deadline. Only the most recent submission will be graded.

Students should consult their textbook, class notes, lecture slides, podcasts, group members, instructors, TAs, and tutors when they need help with homework. You may ask questions about the homework in office hours, but questions on Piazza should be private, visible only to instructors.

This assignment will be graded for not only the *correctness* of your answers, but on your ability to present your ideas clearly and logically. You should explain or justify, present clearly how you arrived at your conclusions and justify the correctness of your answers with mathematically sound reasoning (unless explicitly told not to). Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to **convince the reader** that your results and methods are sound.

KEY CONCEPTS Random Sampling, Random Variables, Expected value, linearity of expectation, sorting.

1. (12 points)

   Suppose you want to use a random process to generate an 4-character password (using only uppercase letters and digits) with at least one uppercase letter.

   You will generate the password by selecting one of the 4 positions $\{1, 2, 3, 4\}$ uniformly at random. Then you will select an uppercase letter uniformly at random. Then you will generate a sequence of 3 characters (uppercase or digits) uniformly at random.

   Then you will put the selected uppercase letter in the selected position and fill in the remaining 3 positions with the sequence of 3 characters.

   For example if you chose position 3, uppercase letter P and 3 character sequence $R99$, then the resulting password would be $R9P9$.

   It should be clear that this random process indeed will generate any possible 4 character string with at least one uppercase letter.

   (a) (3 points) What is the probability that you generate the password $A111$?

   (b) (3 points) What is the probability that you generate the password $ABCD$?

   (c) (3 points) What is the probability that you generate a password that starts with an uppercase letter?

   (d) (3 points) Is this a uniform random sampling? (In other words, is each 4-character password with at least one uppercase letter equally likely?) why or why not?

2. (10 points) Suppose you are at a casino and you have 1 dollar. You find a game that costs 1 dollar to play. The game is simple. You roll a fair nine-sided die. If you roll a $\{1, 2, 3, 4\}$ then you win 2 dollars (one dollar profit and one dollar back that you spent.) If you roll any other value $\{5, 6, 7, 8, 9\}$ then you lose. (You lose the dollar that you spent.)

   You play until you run out of money.

   (a) What is the expected number of times you lose?

   (b) What is the expected number of times you win?

3. (18 points)

   (a) Assume you have 10 different jellybeans labeled $1, 2, \ldots, 10$. You distribute the jellybeans to your 6 friends $\{A, B, C, D, E, F\}$ in the following way. For each jellybean starting with jellybean 1, you roll a fair 6-sided die. If the die is 1, you give the jellybean to A, 2 to B and so on.

      i. What is the expected number of jellybeans given to A?

      ii. What is the expected number of friends to get 0 jellybeans?

      iii. What is the expected number of friends to get exactly 1 jellybean? (HINT: this is the same as the expected number of jellybeans that go to a friend who only gets one jellybean.)

   (b) Assume you have 10 different jellybeans labeled $1, 2, \ldots, 10$. You distribute the jellybeans to your 6 friends $\{A, B, C, D, E, F\}$ uniformly at random (so that each distribution of all 10 jellybeans to your 6 friends is equally likely.)

      How many jellybeans is A expected to get?

      i. What is the expected number of jellybeans given to A?

      ii. What is the expected number of friends to get 0 jellybeans?

      iii. What is the expected number of friends to get exactly 1 jellybean?

4. (8 points) Recall the algorithm for SelectionSort from class:

   $SelectionSort(a[1], \ldots, a[n])$:

   1. **for** $i = 1, \ldots, n - 1$:
   2. $\quad a[m] = minimum(a[i], a[i + 1], \ldots, a[n])$
   3. $\quad$ swap $a[m]$ and $a[i]$

   Assuming that the input is a random permutation of $(a[1], \ldots, a[n])$ (selected uniformly at random), what is the expected number of entries that never get swapped during SelectionSort?