CSE 21 Spring 2022
Homework 1
Due date: Wed, April 6 2022 at 11:59pm

INSTRUCTIONS

This homework should be done in **groups** of one to four students, without assistance from anyone besides the instructional staff and your group members. Homework must be submitted through Gradescope by a **single representative** of your group and received by **11:59pm** on the due date. There are no exceptions to this rule.

You will be able to look at your scanned work before submitting it. You must **type** your solutions. (hand-drawn diagrams are okay.) Your group representative can resubmit your assignment as many times as you like before the deadline. Only the most recent submission will be graded.

Students should consult their textbook, class notes, lecture slides, podcasts, group members, instructors, TAs, and tutors when they need help with homework. You may ask questions about the homework in office hours, but questions on Piazza should be private, visible only to instructors.
This assignment will be graded for not only the *correctness* of your answers, but on your ability to present your ideas clearly and logically. You should explain or justify, present clearly how you arrived at your conclusions and justify the correctness of your answers with mathematically sound reasoning (unless explicitly told not to). Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to **convince the reader** that your results and methods are sound.
KEY CONCEPTS Product Rule, Power Rule, Permutations, Combinations, Inclusion/Exclusion, symmetries.
(Note: For this homework, you can leave your answers in terms of exponentials, factorials, binomial coefficients, etc.)

(Note: for justifying counting arguments, a good rule of thumb is to explain how you came up with every term and factor of your answer. You can leave your answer in terms of exponentials, factorials and binomial coefficients rather than compute the exact numerical value.)

1. A password is a string over the alphabet of the 26 uppercase letters, the 26 lowercase letters, the 10 digits, and the 10 special characters: $\{!, @, \#, \$, \%, \&, ?, +, =, -\}$

   (a) (4 points) How many 8-character passwords start and end with a special character (not necessarily the same one.)?

   (b) (4 points) How many 8-character passwords avoid having both at least one lowercase letter and at least one uppercase letter?

   (c) (4 points) How many 8-character passwords avoids having the word COUNT (with any combination of upper and lowercase letters, i.e., avoiding count, Count, CoUnT, coUNT,...)?

   (d) (4 points) How many 8-character passwords have at least one uppercase letter, at least one digit and at least one special character?

   (e) (4 points) How many 8-character passwords consist of three different characters with 3 copies of two characters and 2 copies of the other?

   (f) (4 points) How many 8-character passwords consist of 8 different letters that are in alphabetical order such that each letter can be uppercase or lowercase?

2. Suppose you have 15 different jellybeans and 5 friends.

   (a) (4 points) How many ways can you distribute the 15 jellybeans to your 5 friends if you don't necessarily give away all 15 jellybeans?

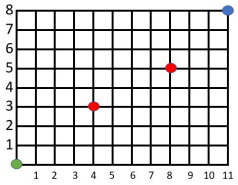
   (b) (4 points) How many ways can you distribute the 15 jellybeans to your 5 friends if you give away all 15 jellybeans to 3 of your friends each getting exactly 5 and the other two get 0?

   (c) (4 points) How many ways can you distribute all of the 15 jellybeans to your 5 friends if at least one of your friends gets exactly 8 jellybeans?

   (d) (4 points) How many ways can you distribute (not necessarily all) the 15 jellybeans to your 5 friends such that each friend gets the same number of jellybeans?

   (e) (4 points) How many ways can you distribute all the 15 jellybeans to your 5 friends if exactly two friends get 0 jellybeans (and the other three get at least one each)?

3. Suppose that you are on the bottom left corner (green) of a 8×11 grid of city blocks.



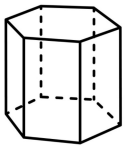
   (a) (4 points) How many different ways can you get from the bottom left corner (green position (0,0)) to the top right corner (blue position (8,11)) by using only North and East moves such that you go through the red dot (located at position (3,4)) and you go through the red dot (located at position (5,8))?

   (b) (4 points) How many different ways can you get from the bottom left corner (green position (0,0)) to the top right corner (blue position (8,11)) by using only North and East moves such that you go through at least one of the red dots?

   (c) (4 points) How many different ways can you get from the bottom left corner (green position (0,0)) to the top right corner (blue position (8,11)) by using only North and East moves such that you go through at most one of the red dots?

4. (a) (4 points)
   How many different ways are there to arrange the letters in HIPPOPOTAMUS?

   (b) (4 points)
   How many different ways are there to color the 8 *faces* of a hexagonal prism with 8 different colors?



   (c) (4 points)
   How many different ways are there to color the 6 *edges* of a tetrahedron with 6 colors? (A tetrahedron is a solid geometrical shape made out of 4 equilateral triangles.