

As-Rigid-As-Possible Stereo under Second Order Smoothness Priors

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Problem Definition

INPUT: A pair of stereo images

OUTPUT: Dense correspondences, or disparity map

Motivation: Combine Two Priors

Segment-wise *As-Rigid-As-Possible* Prior

- Pros: Long-Range Constraints
- Cons: Bad alignment on boundaries

Pixel-wise *Second Order Smoothness* Prior

- Pros: Flexible, has not alignment problem
- Cons: Too Local, sometimes over flexible

Algorithm

Algorithm 1 As-Rigid-As-Possible Stereo

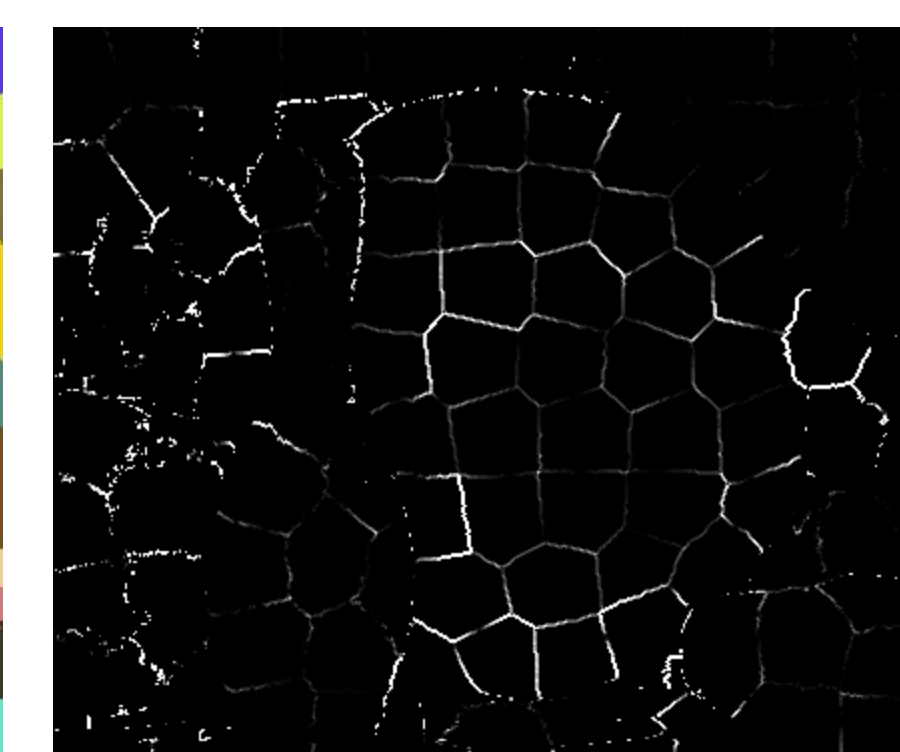
```

compute Laplacian matrix  $L_i$ 
set  $\theta$  to zero
repeat
  repeat
    minimize  $E_{\text{DATA}}$  w.r.t.  $\Lambda$ 
      by a derivative-free algorithm
    minimize  $E_{\text{SMOOTH}}$  w.r.t.  $\mathbf{u}$ 
      by sparse Cholesky decomposition
  until converged
  increase  $\theta$ 
until converged
  
```

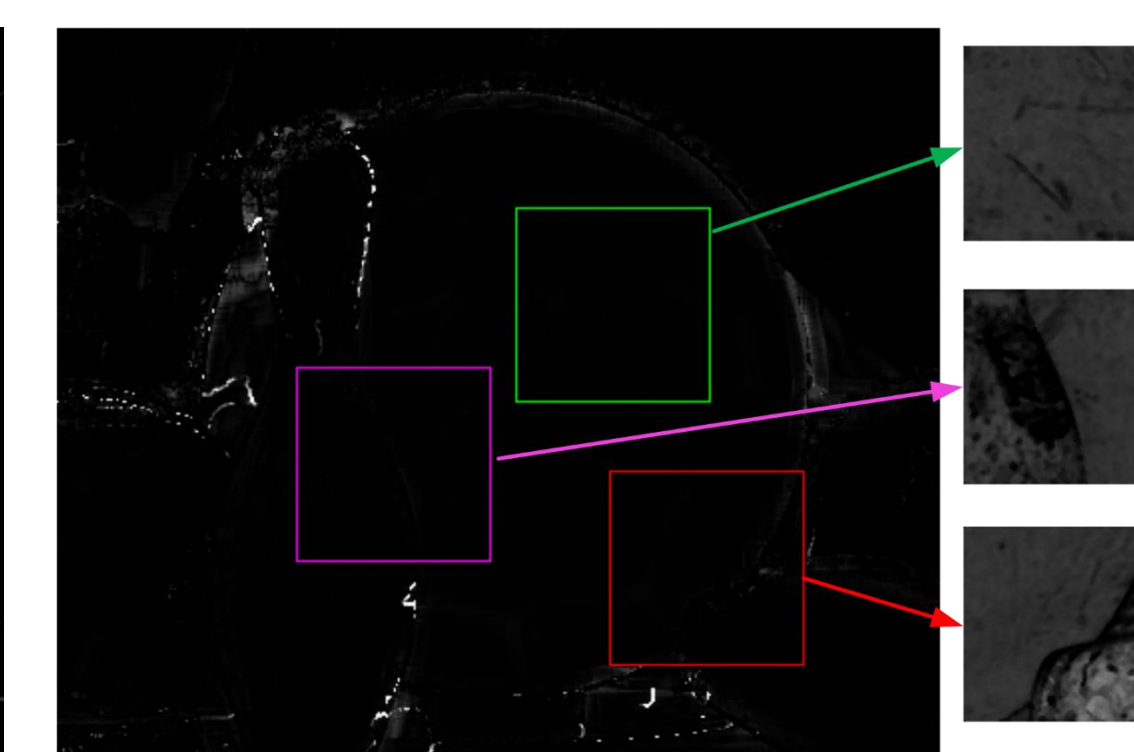
Optimization Process



SLIC Segmentation

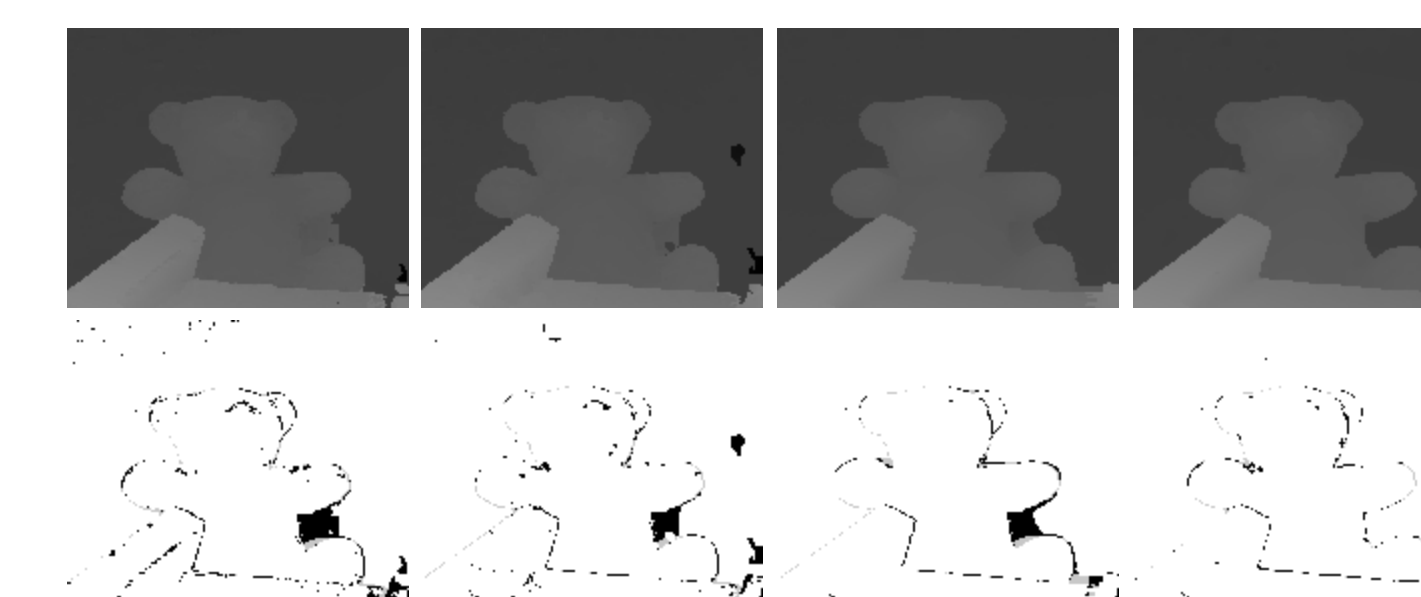


2nd order smooth cost
before optimization

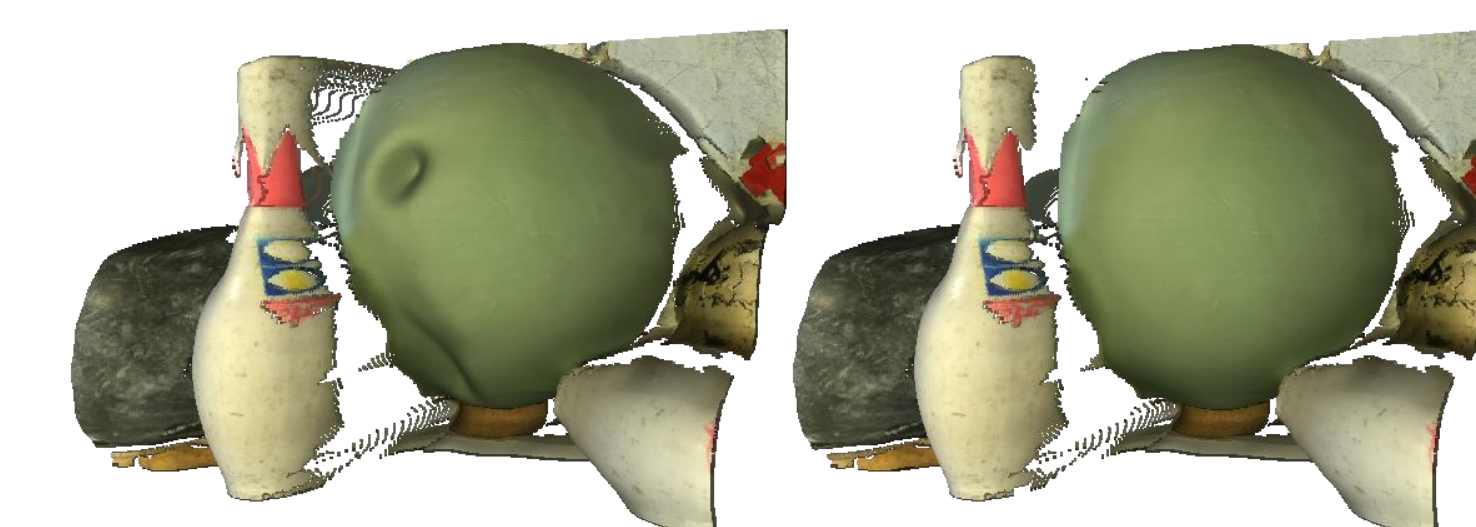


2nd order smooth cost
after optimization

Comparisons

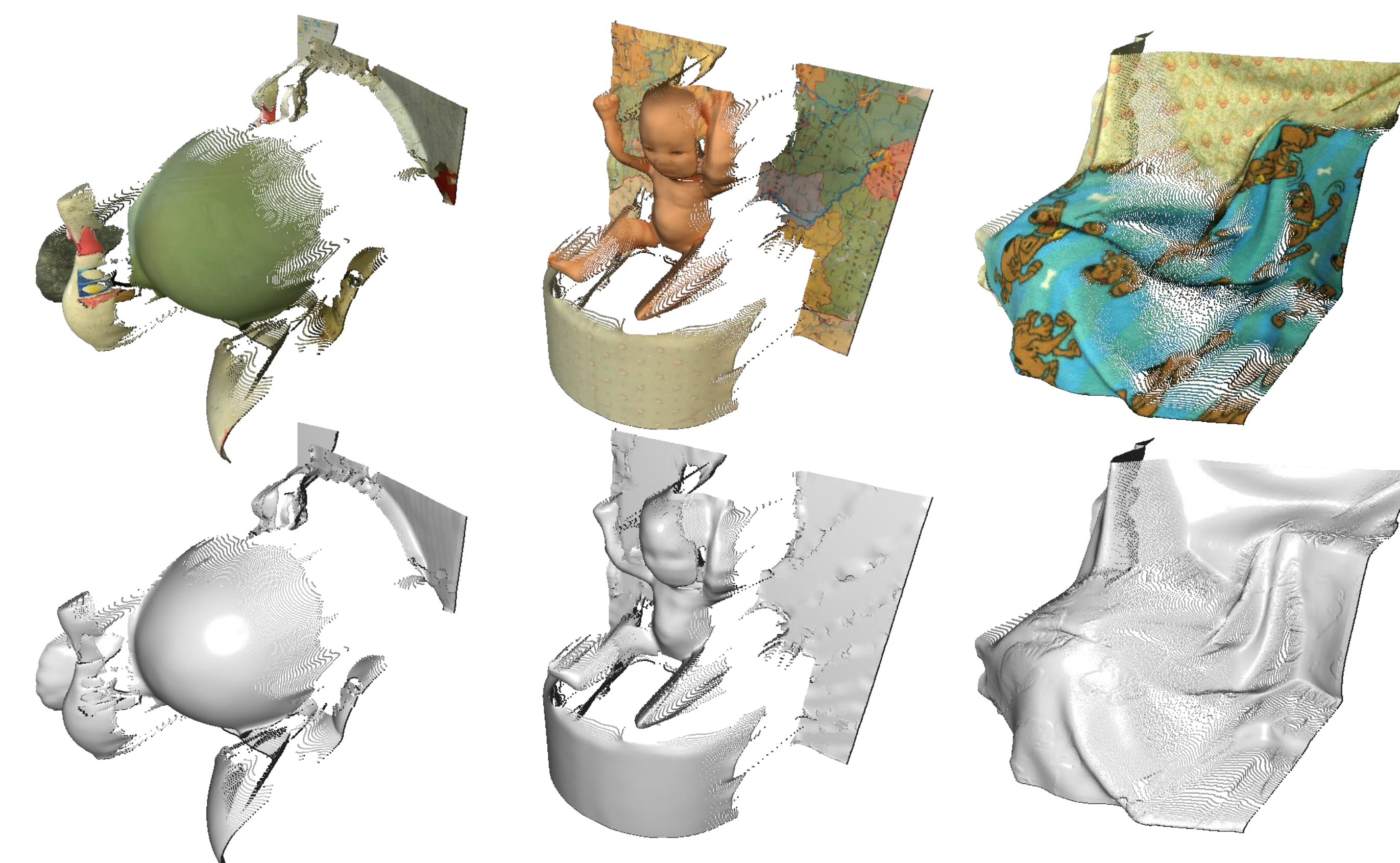


PatchMat. PMBP PmHuber Ours



PatchMatch Ours

Point Clouds of Ours



Conclusions

- Two priors complement each other
- Generate “seamless” accurate disparity maps
- Fast global method, 10s per pair

Formulation: Jointly Optimize Quadratic Surface Coefficients and Disparity Map

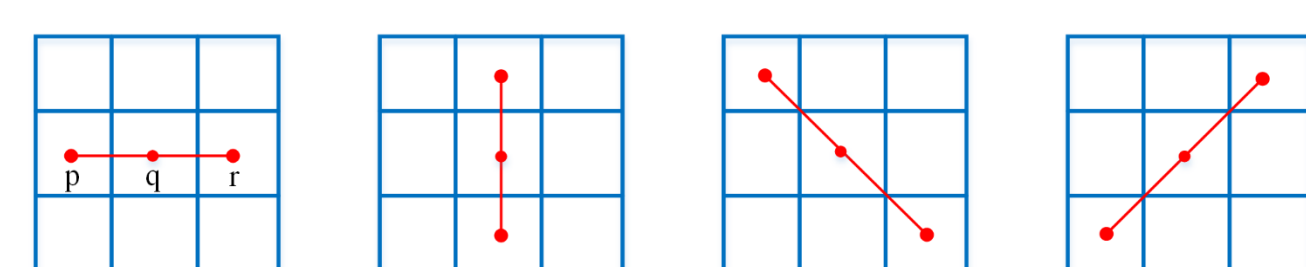
$$E_1(\mathbf{u}, \Lambda) = \mathbf{u}^\top \left(\sum_i L_i^\top L_i \right) \mathbf{u} + \lambda \sum_{\mathbf{s}} \sum_{p \in \mathbf{s}} \rho(p, p - D_{\mathbf{s}}(p)) \quad s.t. \quad \mathbf{u} = \mathbf{D}(\Lambda)$$

disparity map ← quadratic surface coefficients

2nd order prior

data cost with built-in ARAP prior

L_i are four directional
Laplacian operators:



Surface equation:
 $D_{\mathbf{s}}([x, y]^\top) = dx^2 + ey^2 + ax + by + c$

Relax

$$E_2(\mathbf{u}, \Lambda) = \mathbf{u}^\top \left(\sum_i L_i^\top L_i \right) \mathbf{u} + \theta (\mathbf{u} - \mathbf{v})^\top G (\mathbf{u} - \mathbf{v}) + \lambda \sum_{\mathbf{s}} \sum_{p \in \mathbf{s}} \rho(p, p - D_{\mathbf{s}}(p))$$

Alternative Optimization

$$E_{\text{DATA}}(\Lambda) = \lambda \sum_{\mathbf{s}} \sum_{p \in \mathbf{s}} \rho(p, p - D_{\mathbf{s}}(p)) + \theta (\mathbf{u} - \mathbf{D}(\Lambda))^\top G (\mathbf{u} - \mathbf{D}(\Lambda))$$

Find new surfaces by
PatchMatch-like
Nelder-Mead search

$$E_{\text{SMOOTH}}(\mathbf{u}) = \mathbf{u}^\top \left(\sum_i L_i^\top L_i \right) \mathbf{u} + \theta (\mathbf{u} - \mathbf{v})^\top G (\mathbf{u} - \mathbf{v})$$

$$\left(\sum_i L_i^\top L_i + \theta G \right) \mathbf{u} = \theta G \mathbf{v}$$

E_{SMOOTH} has closed form
solution. But we use
gradient decent for
speed