Chinese Takeaway Restaurant: Delivery Operation and Tipping Patterns Analysis

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Executive Summary

Problem Background

The Chinese Takeaway Restaurant has 1 delivery driver to deliver food to its customers. Our project is to analyse the restaurant's delivery operation based on the data provided by the delivery man.

Problem Statement

The 2 Problem Statements we have are:

1) What is the minimum number of drivers needed to drive the average customers' waiting time in the system to below 30 minutes?
2) What possible strategy can the restaurant take to increase the tips received by the driver?

Data Analysis

The arrival times for each order were modelled using an Exponential Distribution. The same applies for the service times of the delivery driver.

Queue Modelling using M/M/K and CTMC

M/M/K queue models based on Markov Chains were used to obtain estimates of the queuing statistics. Another CTMC model was further developed in an attempt to simulate the real case. Upon comparison, we believe the CTMC model is a better fit. The impact of one more driver on the service time was studied based on the two models, respectively.

Statistical Analysis on Customers' Tipping Patterns

The relationships between the tips received by the driver and customers' payment type, as well as their ethnicity, were analysed. Various statistical methods were adopted including single-factor ANOVA, F-test, T-test, Bonferroni method, Levene's test and Welch Heteroscedastic test. The results indicate that the mean tips paid by customers are positively associated with cash payment, as well as Asians, Whites and Latinos

1. Introduction

Currently, the Chinese takeaway restaurant is often overwhelmed by the number of orders received during lunch (1100 to 1400) and dinner hours (1700 to 2000). Hence, we seek to measure the impact on the waiting time with an additional driver. We expect that by doing so, the average waiting time in the system for customers will drop and the customers' satisfaction will be increased.

Due to the nature of the restaurant, we assume that the food is always ready for collection by the driver. Hence, the total customer waiting time can be split into two parts: the waiting time to be delivered (queuing time), and the delivery time (service time).

2. Data Analysis

2.1 Raw data

The table below briefly describes the critical information provided from our dataset.

Date	Tips	Gender	Payment type	Price	Race	Time-in	Time-out
15-06-2016	3.5	M	Cash	16.7	W	11:04	12:05
17-07-2016	5.6	F	Credit	15.8	A	15:09	15:30
…	…	…	…	…	…	…	…

2.2 Arrival Rate Analysis

First, the interarrival times were obtained by computing the arrival time difference between 2 adjacent orders. To see if it fits exponential distribution, the Chi-Square *Goodness-of-fit test* was conducted for both lunch and dinner time order intervals. Testing at the 5% level of significance, the Chi-Square statistic was found to be smaller than the critical value, indicating that H_0 was not rejected, and the arrival rate follows an exponential distribution. The lunch time order arrival rate is 4.44/h and dinner time is 3.37/h.

		Number of observations	Mean value	Pi_0	Expected	Chi^2
0	10	114	570	0.522577	113.3992351	0.003183
10	20	57	855	0.24949	54.13938938	0.151149
20	30	24	600	0.119112	25.84738318	0.132038
30	40	8	280	0.056867	12.34013211	1.526462
40	50	14	630	0.051953	11.27386022	0.65921
	Total observations	217			Chi^2 statistic	2.472041
	Sample mean	13.52534562			Critical value	7.814728
	Lambda hat	0.073935264				

3. Queuing Models

The M/M/K model is a queuing model with the key assumption that a server can only serve one customer at a time. Since the delivery man delivers multiple orders at once, the orders were grouped by their arrival times when they were near each other.

To fulfil the assumption of the M/M/K model where orders are fulfilled individually, the grouped orders were split into individual orders, where the service time per order is simply the service time of the group of orders, divided by the number of orders in the group.

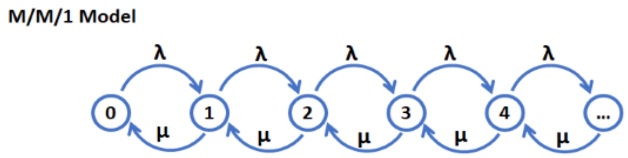
Another CTMC model was explored where the driver could deliver multiple orders at once and did not have to be split into individual orders.

<u>3.1 The M/M/K Model</u>

Arrival Rate(λ): Refer to Section 2.2 for computational details.

Service Rate(μ): The total service time was divided by the number of grouped orders to get the average service time per order. Visualizing the service time per order using Q-Q plots, it was shown to approximate an Exponential Distribution. μ=1/service time **per order**.

State Change Diagram(M/M/1)



$$\pi_0\lambda=\pi_1\mu$$
$$\pi_1\lambda=\pi_2\mu$$
$$\pi_2\lambda=\pi_3\mu$$
$$\vdots$$
$$\pi_0+\pi_1+\pi_2+\ldots+\pi_\infty=1$$

$$\Rightarrow \quad \pi_0 = 1-\frac{\lambda}{\mu}$$
$$\pi_i = \left(\frac{\lambda}{\mu}\right)^i\left(1-\frac{\lambda}{\mu}\right) \text{ where } i>=1$$

The long-run average number of customers in the system: $L=0*\pi_0+1*\pi_1+2*\pi_2+\ldots$

The average time a customer spends in the system: $W=\frac{L}{\lambda}$

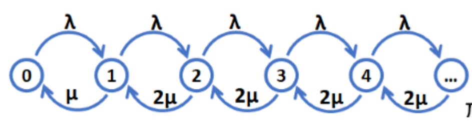
	Arrival Rate(λ)	Service rate(μ)	Long-run average no . of customers(L)	Average time spent in system (W)	Server utilization rate =1 - π0
Lunch Time	4.44/h	5.49/h	4.22	57.1 mins	0.809
Dinner Time	3.73/h	4.89/h	3.22	51.9 mins	0.763

*Table 1: Performance Metrics for **M/M/1 Model***

As seen above, the current steady state waiting time in the system for both mealtimes exceed 30 minutes. Hence, the restaurant may lose potential customers who demand shorter delivery times.

State Change Diagram(M/M/2)

M/M/2 Model



$$\pi_0\lambda=\pi_1\mu$$
$$\pi_1\lambda=\pi_2(2\mu)$$
$$\pi_2\lambda=\pi_3(2\mu)$$
$$\vdots$$
$$\pi_0+\pi_1+\pi_2+\ldots+\pi_\infty=1$$

$$\Rightarrow \pi_0=\frac{2\mu}{\lambda}-1$$
$$\pi_1=2-\frac{\lambda}{\mu}$$
$$\pi_i=\left(\frac{\lambda}{2\mu}\right)^{i-1}\left(2-\frac{\lambda}{\mu}\right) \quad \text{where } i>=2$$

The long-run average number of customers in the system: $L=0*\pi_0+1*\pi_1+2*\pi_2+\ldots$

The average time a customer spends in the system: $W=\frac{L}{\lambda}$

	Arrival Rate(λ)	Service rate(μ)	Long-run average no . of customers(L)	Average time spent in system (W)	Server utilization rate =1 - π0
Lunch Time	4.44/h	5.49/h	0.967	13.1 mins	0.526
Dinner Time	3.73/h	4.89/h	0.893	14.4 mins	0.382

Table 2: Performance Metrics for ***M/M/2 Model***

Result

We note that by hiring another driver, mean customer waiting times in the system will decrease to 13.1 minutes during lunch times and 14.4 minutes during dinner times, achieving our objective of reducing mean waiting times to less than 30 minutes.

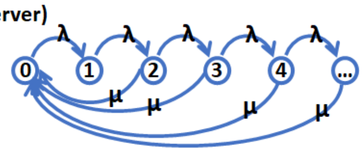
3.2 Modelling the delivery as a Continuous Time Markov Chain (CTMC)

Arrival Rate(λ): Same as M/M/K Model.

Service Rate(μ): Under this CTMC Model, the delivery man can carry as many orders as are available in the queue. The total service times of the grouped orders were computed and visualized using Q-Q plots and the service times were shown to approximate an Exponential Distribution. Thus, μ=1/service time **per grouped order**.

State Change Diagram (CTMC (1 server))

Modelling the delivery as a Continuous Time Markov Chain (CTMC) (1 server)



$$\pi_0\lambda=(\pi_2+\pi_3+\ldots)\mu$$
$$\pi_1\lambda=\pi_0\lambda$$
$$\pi_2(\lambda+\mu)=\pi_1\lambda$$
$$\vdots$$
$$\pi_0+\pi_1+\pi_2+\ldots+\pi_\infty=1$$

$$\Rightarrow \pi_0=\frac{\mu}{\lambda+2\mu}$$
$$\pi_i=\left(\frac{\lambda}{\lambda+\mu}\right)^{i-1}\frac{\mu}{\lambda+2\mu} \quad \text{where } i>=1$$

The long-run average number of customers in the system:

$$L=0*\pi_0+1*\pi_1+2*\pi_2+\ldots$$

The average time a customer spends in the system:

$$W=\frac{L}{\lambda}$$

The driver always leaves with more than one order in hand during lunch and dinner times, hence the CTMC model developed reflects this, with there being no state change from 1 to 0.

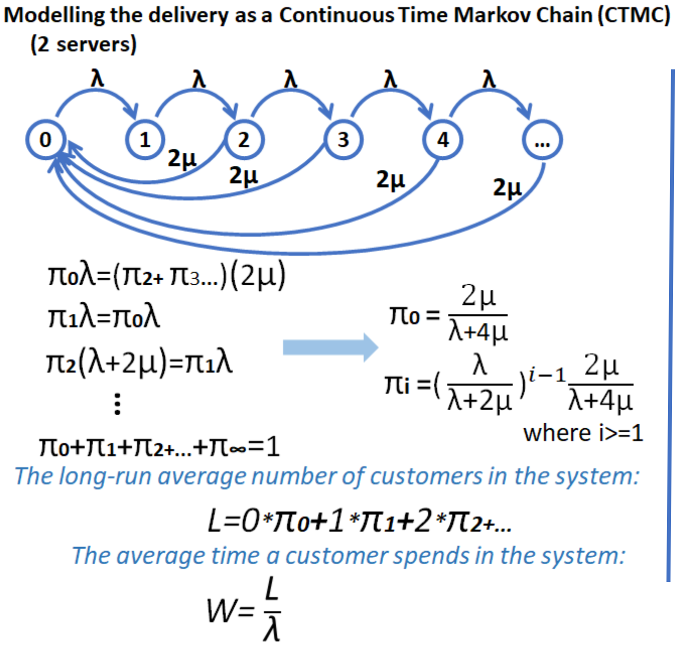
This model is used to model the reality that orders are delivered in groups. The corresponding service times used to derive μ are the times taken to deliver each group of orders.

	Arrival Rate(λ)	Service rate(μ)	Long-run average no . of customers(L)	Average time spent in system (W)	Server utilization rate =1 - π0
Lunch Time	4.44/h	1.37/h	3.15	42.6 mins	0.809
Dinner Time	3.73/h	1.41/h	2.74	44.1 mins	0.785

*Table 3: Performance Metrics for the CTMC Model (**1 server**)*

The result obtained was similar to the M/M/1 model, where the average time spent in the system for both mealtimes exceed 30 minutes.

State Change Diagram (CTMC (2 servers))



Result

This caused the mean customer waiting times in the system to decrease to 25.2 minutes during lunch times and 26.1 minutes during dinner times, achieving our objective of reducing mean waiting times to less than 30 minutes.

	Arrival Rate(λ)	Service rate(μ)	Long-run average no . of customers(L)	Average time spent in system (W)	Server utilization rate =1 - π0
Lunch Time	4.44/h	2.74/h	1.88	25.2 mins	0.724
Dinner Time	3.73/h	2.82/h	1.62	26.1 mins	0.699

*Table 4: Performance Metrics for CTMC Model (**2 servers**)*

3.3 Comparing the M/M/K model with our CTMC

The calculated expected waiting time from the CTMC model is closer to our real dataset. This is because it models the real situation more accurately, where multiple orders can be delivered together. On the other hand, the M/M/K model required that customers were served one at a time. This potentially led to higher deviations in the performance metrics as delivering orders individually would take a longer time, and service times cannot be accurately determined by dividing service times by the number of orders in each group. Therefore, we conclude that the CTMC model is a better fit for our situation.

Result

Either way, hiring one more driver is shown to be crucial in decreasing the average customers' waiting time in the system to less than 30 minutes.

3.4 Conclusion on queuing models

After modelling the dataset with the M/M/K and CTMC models, the results showed that both models yield an average customers' waiting time in the system that is shorter than 30 minutes after one more driver is hired. The comparison between the average waiting time in the system derived from the two models and the actual mean value from the data demonstrated that the CTMC model simulates the real situation more accurately, and hence can be used by the restaurant to make better decisions on the number of drivers it should hire in the future.

4. Statistical Analysis on Tipping Patterns

4.1 Choice of Statistical Methods

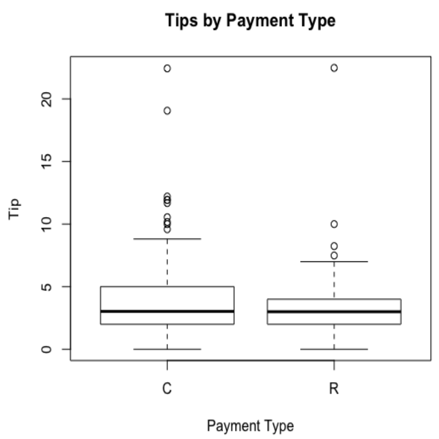
We would like to investigate whether two factors, **payment methods** with two levels and **ethnicity** with 4 levels affect the tips given to the driver. Ideally, a 2-way ANOVA should be conducted to determine if any interactions exist. However, the unequal sample sizes with a range of 7 to 176 demonstrated an unbalanced design and so we decided to conduct 2 separate tests, using an independent samples design as well as a one-way ANOVA instead.

Observations with missing values for tips, payment methods, and ethnicities were **excluded** as they only accounted for a small percentage of observations

4.2 Tips and Payment Methods

Statistical methods summary:
Independent samples design, F test and T test

The restaurant accepts 2 types of payment methods, cash and credit. We suspect people may tip less when using credit payment as no physical transaction takes place, reducing the pressure to tip. Thus, the hypothesis we would like to test is if the mean tips given via cash payments is more than that of credit payments.



The boxplot for each payment method is shown on the left, in which C represents cash payment and R represents credit payment.

Independent Samples Design method was chosen as there were two types of samples with different sizes which were not matched together. Since the population variances were unknown, we started by investigating the equality of their variances using a two tailed **Snedecor's F distribution**

$H_0: \frac{\sigma_1^2}{\sigma_2^2} = 1, \qquad H_A: \frac{\sigma_1^2}{\sigma_2^2} \neq 1$

Groups	*Counts*	*df*	*Sum*	*Average*	*Variance*
Cash Payment	258	257	938.06	3.636	8.777
Credit Payment	269	268	787.29	2.927	4.495

Table 5: Statistical summary on both payment types

Testing at **α = 1%** significance level.
The rejection region is given by:

$$f \le f_{1-\frac{\alpha}{2}, n_1-1, n_2-1} = 0.726531 \; or \; f \ge f_{\frac{\alpha}{2}, n_1-1, n_2-1} = 1.375133$$

The F test statistic value calculated from table 5 is 1.95, which falls in the rejection region. Hence, there is sufficient evidence at the 1% level of significance that the **variances** of cash and credit payments are significantly **different**.

With this result, the Independent Samples Design for the two-sample t-test was conducted with the assumption of **unequal** variances. Testing at **α = 5%** significance level.

$$H_0: \mu_{cash} = \mu_{credit}, \qquad H_A: \mu_{cash} > \mu_{credit}$$

The t test statistic is calculated to be 3.15 and the corresponding P-value is 0.00087

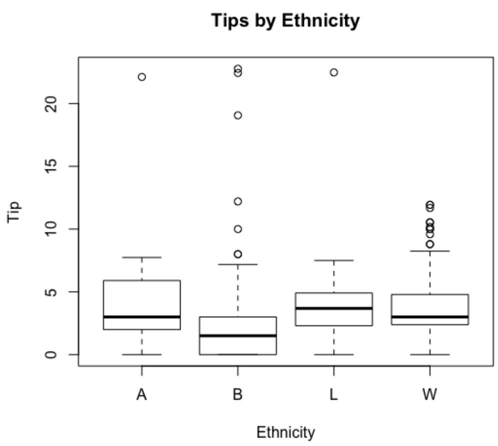
Result

Since the P-value is lower than 0.05 thus, there is sufficient evidence at the 5% level of significance to reject H_0, concluding that the mean tips paid with cash is higher than credit.

4.3 Tips and Ethnicity

Statistical methods summary:
Single factor ANOVA, Levene's test,
Welch Heteroscedastic test, Bonferroni method

We further investigated the relationship between tips and the customers' ethnicity. From the dataset, four different ethnicities were recorded (A for Asian, B for Black, L for Latino, W for White).



Ethnicity	**Count**	**Sum**	**Average**	**Variance**
Asian	22	93.93	4.267	20.698
Black	175	365.55	2.089	10.710
Latino	29	117.53	4.052	15.797
White	419	1542.45	3.681	3.7273

Table 6: Statistical Summary on Tips VS Ethnicity

To analyse if the mean tips paid by each of these four ethnicities are equal, we applied the method of single factor ANOVA as shown in table 7 below. Testing at the 5% level of significance:

$$H_0: \mu_A = \mu_B = \mu_L = \mu_W, \quad H_A: at\ least\ two\ \mu_i's\ are\ different$$

Source of Variation	SS	df	MS	F	P-value	F critical
Between groups	345.595	3	118.198	17.626	5.27E-11	2.619
Within groups	4298.492	641	6.706			
Total	4653.087	644				

Table 7: Single Factor ANOVA Test Summary

The single factor ANOVA shows that the p-value is less than 0.05. Hence, there is sufficient evidence at 5% level of significance to reject H_0, concluding that mean tips given by four ethnicity groups are **not equal**.

The ANOVA test requires the assumption of **equal** variances and we wanted to verify if our data fulfils this assumption. To do this, we applied the **Levene's test** at the 1% level of significance**.** This test was conducted using the car library in R, measuring the absolute deviations of residuals with the mean.

```
Levene's Test for Homogeneity of Variance (center = mean)
       Df F value    Pr(>F)
group   3  5.6182 0.0008342 ***
      641
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

The Levene's test shows p-value = 0.00083. Thus, there is sufficient evidence at the 1% level of significance that we can reject H_0, concluding that the variances are **unequal** across different ethnicity groups.

The violation of this assumption might result in our single factor ANOVA test being less accurate. Therefore, we deployed the **Welch Heteroscedastic** test which is robust to unequal variances to further enhance our analysis. This test was conducted using the onewaytests library in R, at the 5% level of significance.

```
  Welch's Heteroscedastic F Test (alpha = 0.05)
-------------------------------------------------------------
  data : Sample and Group

  statistic  : 12.19268
  num df     : 3
  denom df   : 58.11345
  p.value    : 2.710905e-06

  Result     : Difference is statistically significant.
-------------------------------------------------------------------
```

The result still shows that the mean tips between the four groups were significantly **different** at the 5% level of significance. This conclusion is the same as the ANOVA test conducted.

Therefore, we proceeded with **Bonferroni method** to find out which groups have significantly different means statistically.

Pairs of Ethnicities	Absolute Difference in Means		Critical Value	Reject?
AB	2.181	>	1.550	Yes
AL	0.217	<	1.938	No
AW	0.588	<	1.499	No
BL	1.964	>	1.374	Yes
BW	1.592	>	0.617	Yes
LW	0.372	<	1.316	No

Table 8: Pairwise comparison among ethnicities

Result

Table 8 shows that the mean tips given by the Whites, Latinos and Asians are statistically higher compared with the remaining ethnicity group.

4.4 Conclusions and limitations

The analysis of tips and payment methods show that cash payments are associated with higher tips than credit payments. With this knowledge, the restaurant can consider encouraging cash payments to increase the benefits given to their driver. The result of analysis of tips against ethnicities shows that the Whites, Latinos and Asians pay more tips in general, which can serve as guidance for the restaurant's marketing strategy.

However, we also acknowledge some potential limitations for our analysis. According to the driver, not all the orders delivered by him were recorded in the dataset due to his busy schedule, and in the dataset, we also noticed that the sample size for different ethnicity groups varies significantly. These are some factors that may affect our result's accuracy.

Finally, to bring our analysis to the next level, some future work for us could be to incorporate more possible relationships between tips and other factors in our dataset such as weekdays and customer gender.