Focused model comparison with bootstrapping and alternative loss functions

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Abstract

This vignette briefly illustrates a bootstrap procedure for focused model comparison which allows very general loss functions, though is based on different asymptotic assumptions. This procedure is experimental and has not been studied in detail.

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Take the low birth weight example, described in detail in the main FIC package vignette. We estimate the probability of low birth weight for smokers using a logistic regression, and compare 64 subsets of the wide model with different combinations of covariates. Focused model comparison statistics are computed, as before.

```
library(fic)
wide.glm <- glm(low ~ lwtkg + age + smoke + ht + ui + smokeage + smokeui,
                data=birthwt, family=binomial)
vals.smoke <-    c(1, 58.24, 22.95, 1, 0, 0, 22.95, 0)
X <- vals.smoke
inds0 <- c(1,1,0,0,0,0,0,0)
combs <- all_inds(wide.glm, inds0)
ficres <- fic(wide=wide.glm, inds=combs, inds0=inds0,
              focus=prob_logistic, X=X)
```

These focused model comparison statistics are computed using formulae derived under an asymptotic framework where we assume that the data are generated from a model with parameters which depart from the parameters of the narrow model by an amount $\boldsymbol{\delta}/\sqrt{n}$ that depends on the sample size n.

An alternative approach might be to assume that the data were generated under a wide model that does not depend on the sample size. Then, to compute the mean square error of the estimate of a focus quantity $\mu(\boldsymbol{\theta}, \boldsymbol{\gamma})$ under a submodel with estimated parameters $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}_S, \boldsymbol{\gamma} = \hat{\boldsymbol{\gamma}}_S$, we generate a large number B of alternative parameter estimates $(\boldsymbol{\theta}^{(r)}, \boldsymbol{\gamma}^{(r)})$ from the multivariate normal sampling distribution defined by the maximum likelihood estimates and covariance matrix from the wide model.

The mean square error of the focus is then estimated as

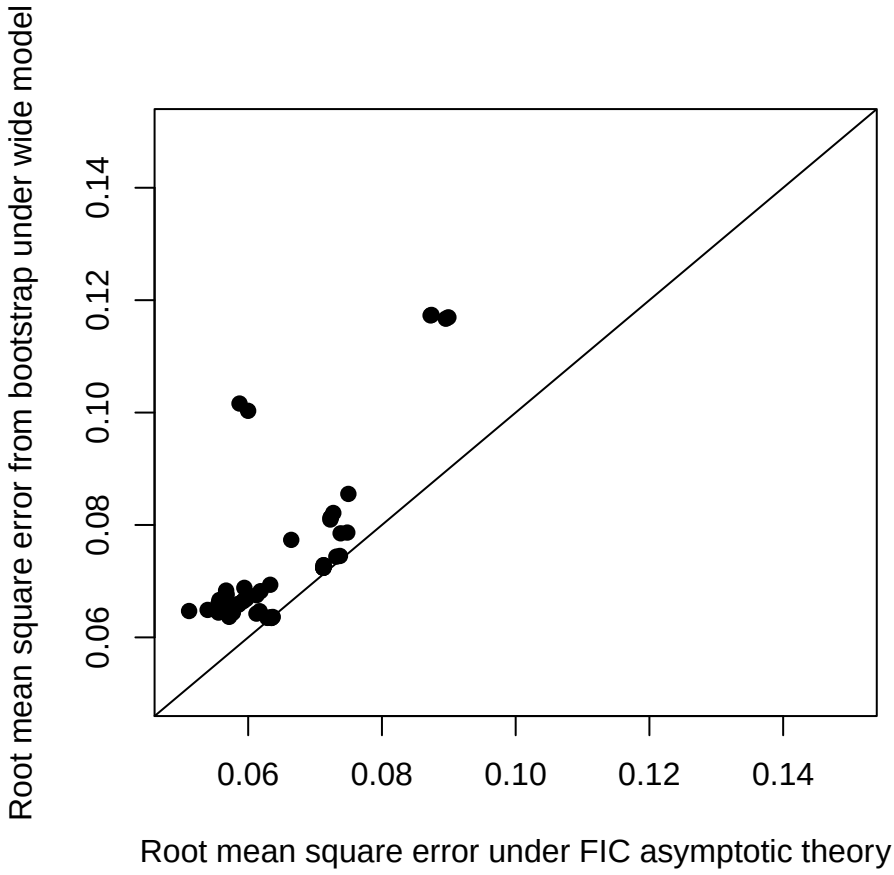
$$\frac{1}{B}\sum_{r=1}^{B}\left(\mu(\boldsymbol{\theta}^{(r)}, \boldsymbol{\gamma}^{(r)}) - \mu(\hat{\boldsymbol{\theta}}_S, \hat{\boldsymbol{\gamma}}_S)\right)^2$$

This is implemented in the `fic` function by supplying a `B` argument giving the number of bootstrap samples.

```
ficboot_mse <- fic(wide=wide.glm, inds=combs, inds0=inds0,
                   focus=prob_logistic, X=X, B=1000)
```

The root mean square errors for each of the 64 models are compared here to those estimated under the standard framework with a sample-size dependent true model. The model preference is similar between the two methods, with a small handful of submodels where the methods give different estimates of error.

```
plot(ficres$rmse, ficboot_mse$loss, xlim=c(0.05,0.15),
     ylim=c(0.05,0.15), pch=19,
     xlab = "Root mean square error under FIC asymptotic theory",
     ylab = "Root mean square error from bootstrap under wide model")
abline(a=0, b=1)
```



This framework allows alternative loss functions, for example, the absolute error loss:

$$\frac{1}{B}\sum_{r=1}^{B}\left|\mu(\boldsymbol{\theta}^{(r)},\boldsymbol{\gamma}^{(r)})-\mu(\hat{\boldsymbol{\theta}}_S,\hat{\boldsymbol{\gamma}}_S)\right|$$

Alternative loss functions can be supplied to `fic` as functions of the (scalar) submodel estimate and the vector of bootstrapped wide model estimates, illustrated here for the absolute error:

```
loss_abserror <- function(sub, wide){
    mean(abs(sub - wide))
}
ficboot_abs <- fic(wide=wide.glm, inds=combs, inds0=inds0,
                   focus=prob_logistic, X=X, B=1000, loss=loss_abserror)
```

While the bootstrap approach is computationally convenient, the relative properties of the two different asymptotic frameworks have not been studied.