Ray Tracing

Rendering

Issues

- Visibility

    What objects or parts in the scene are visible?

    Clipping (with respect to the view frustum)

        Done

    Occlusion (with respect to the objects in the scene)

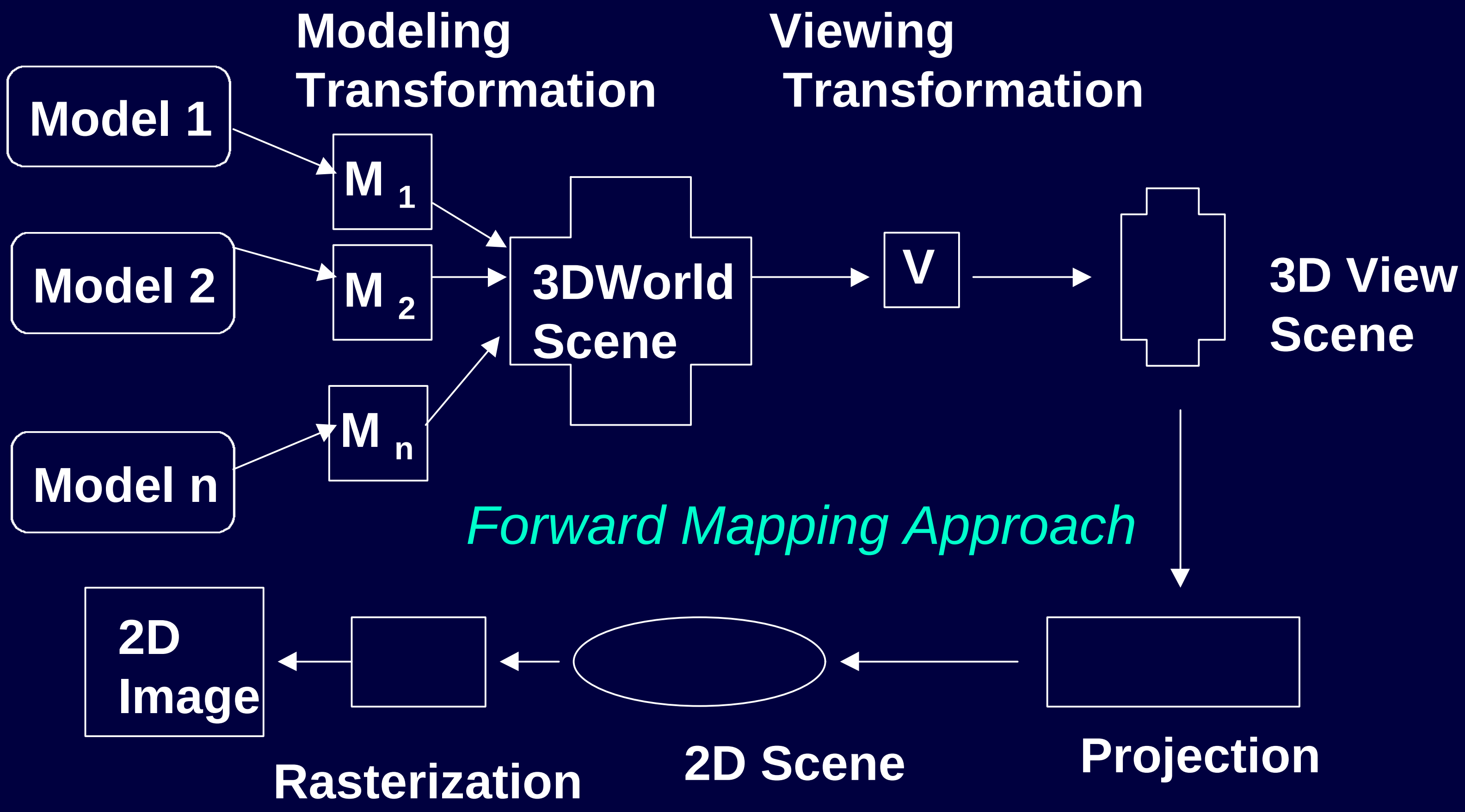
        Hidden surface elimination

- Illumination

    Reflection, Refraction, Transparency, Shadows, etc.

Ray Tracing

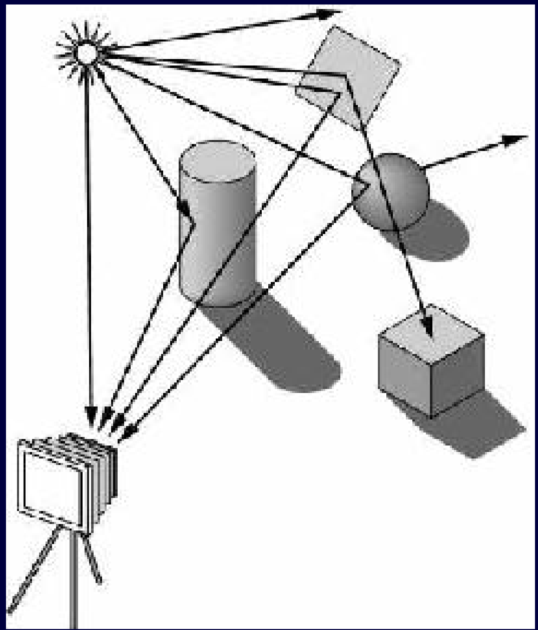
Rendering Pipeline (Revisit)



Ray Tracing

Forward Ray Tracing

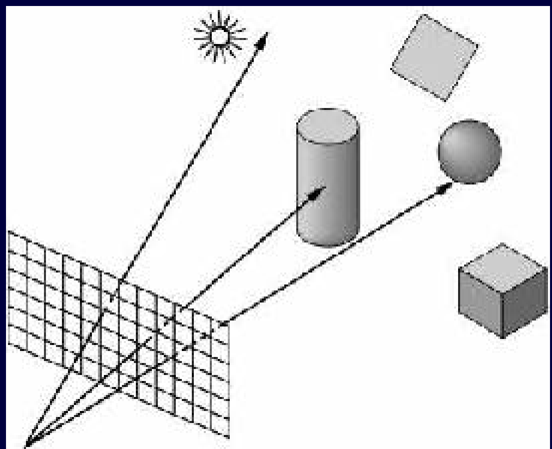
Modeling interaction of light with the objects/surfaces



Problem:
Many rays will not contribute to the image!

Ray Tracing

Backward Ray Tracing



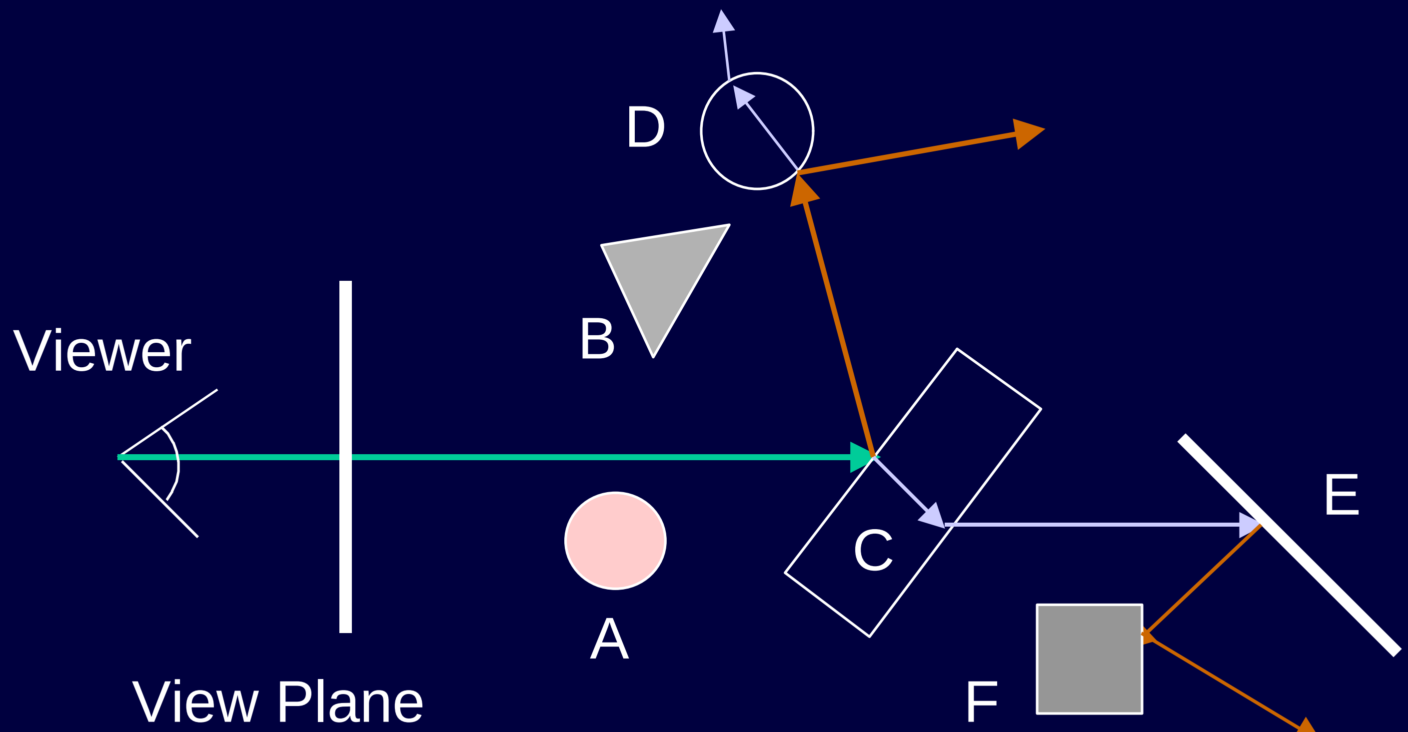
Rays from camera (viewer) through each pixel to the scene

Backward Ray Tracing = Ray Tracing

Ray Tracing

Backward Ray Tracing

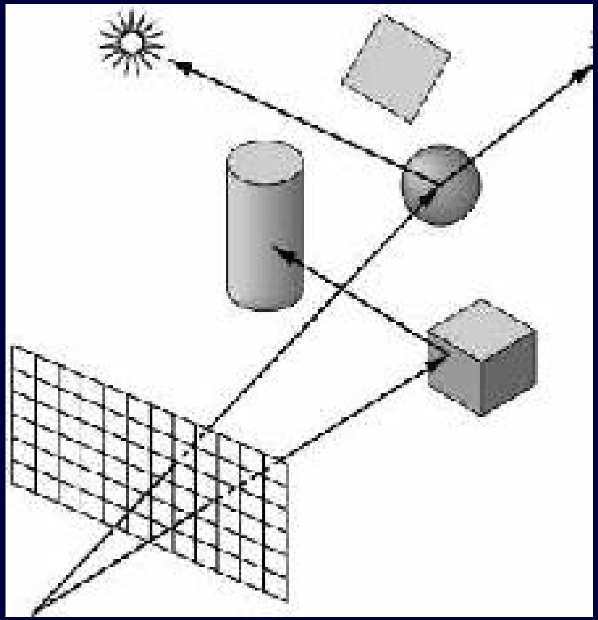
Primary and Secondary Rays



Ray Tracing

Backward Ray Tracing

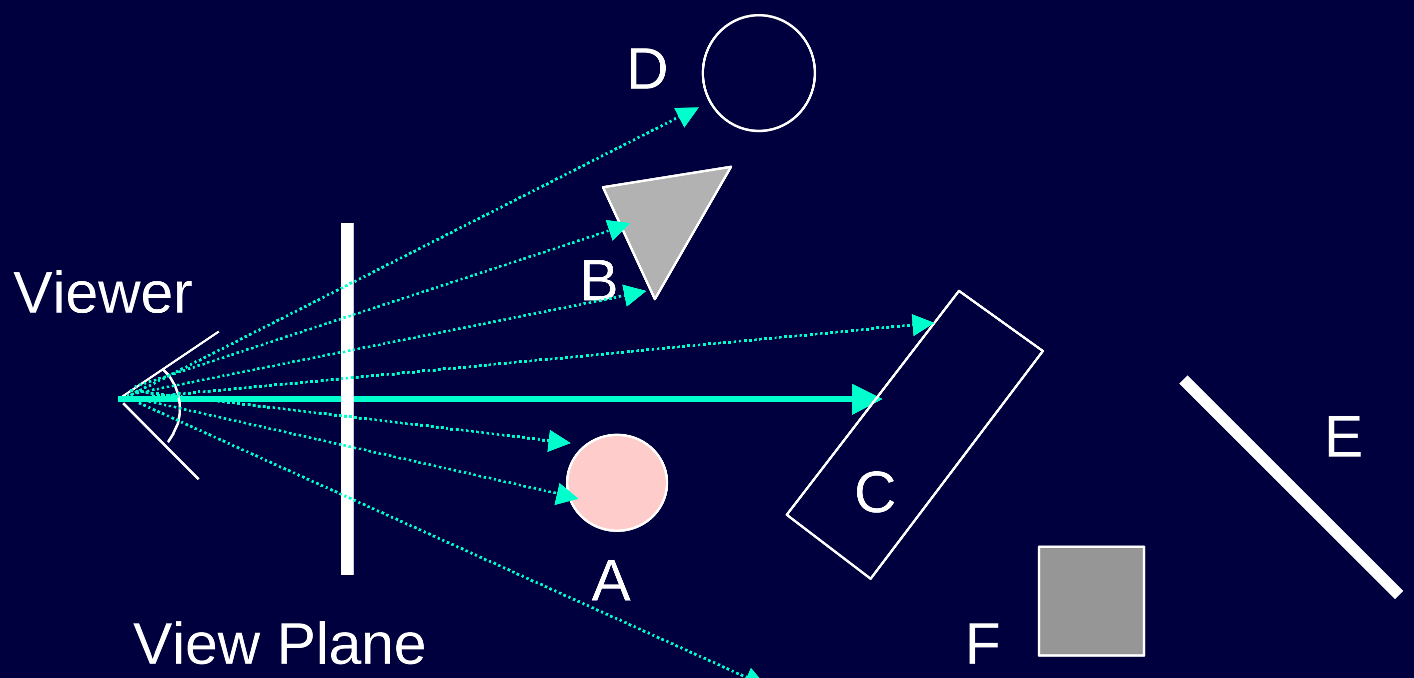
Shadow Rays



Visibility check with respect to the light source

Ray Tracing

Ray Casting



Ray Tracing

Two Issues

Ray-object intersection

Visibility test: Closest to the viewer

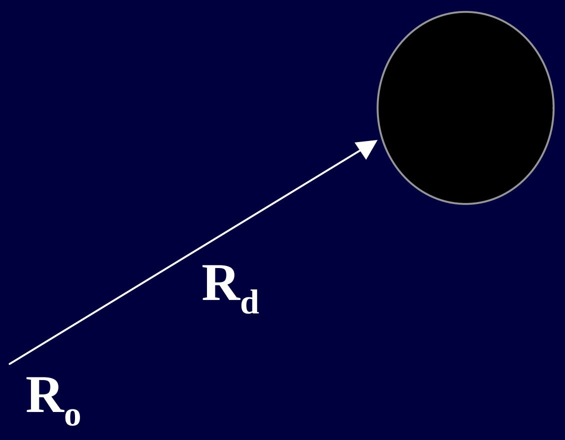
Pixel color determination (shading)

Illumination model

Ray Tracing

Ray Object Intersection

Sphere



$$R_o = [X_o \; Y_o \; Z_o] \qquad \textit{(Ray Origin)}$$

$$R_d = [X_d \; Y_d \; Z_d] \qquad \textit{(Ray Direction)}$$

$$X_d^2 + Y_d^2 + Z_d^2 = 1$$

Parametric Form

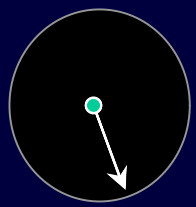
$$R(t) = R_o + R_d t \qquad t > 0$$

Ray Tracing

Ray Object Intersection

Sphere

Implicit Form



Center $S_c = [\ X_c\ Y_c\ Z_c\ ]$
Radius S_r
Surface Point $[\ X_s\ Y_s\ Z_s\ ]$

$$(X_s - X_c)^2 + (Y_s - Y_c)^2 + (Z_s - Z_c)^2 = S_r^2$$

Ray Tracing

Ray Object Intersection

Sphere

To solve the intersection problem the ray equation is substituted into the sphere equation and the result is solved for *t*

That is

$$(X_o + X_d t - X_c)^2 + (Y_o + Y_d t - Y_c)^2 + (Z_o + Z_d t - Z_c)^2 = S_r^2$$

Ray Tracing

Ray Object Intersection

Sphere

$$At^2 + Bt + C = 0$$

where

$$A = X_d^2 + Y_d^2 + Z_d^2 = 1$$

$$B = 2(X_d(X_o - X_c) + Y_d(Y_o - Y_c) + Z_d(Z_o - Z_c))$$

$$C = (X_o - X_c)^2 + (Y_o - Y_c)^2 + (Z_o - Z_c)^2 - S_r^2$$

Ray Tracing

Ray Object Intersection

Sphere

$$At^2 + Bt + C = 0$$

$$t_0 = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$

$$t_1 = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

Smaller positive among t_0 and t_1 gives the closest intersection point

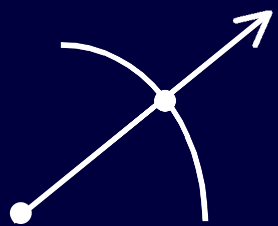
$$[\ X_i\ Y_i\ Z_i\ ] = [\ X_o + X_d t\ ,\ Y_o + Y_d t\ ,\ Z_o + Z_d t\ ]$$

Ray Tracing

Ray Object Intersection

Sphere

Normal



$$n = \left[\frac{(X_i - X_c)}{S_r}, \frac{(Y_i - Y_c)}{S_r}, \frac{(Z_i - Z_c)}{S_r} \right]$$

Ray Tracing

Ray Sphere Intersection

Sum up

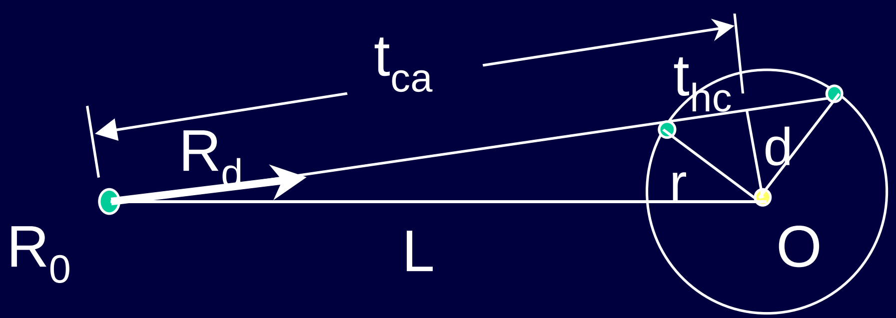
- Calculate A B C
- Compute the discriminant
- Calculate min (t_0 , t_1)
- Compute the intersection point
- Compute the normal

Ray Tracing

Ray Object Intersection

Sphere

Geometric Approach

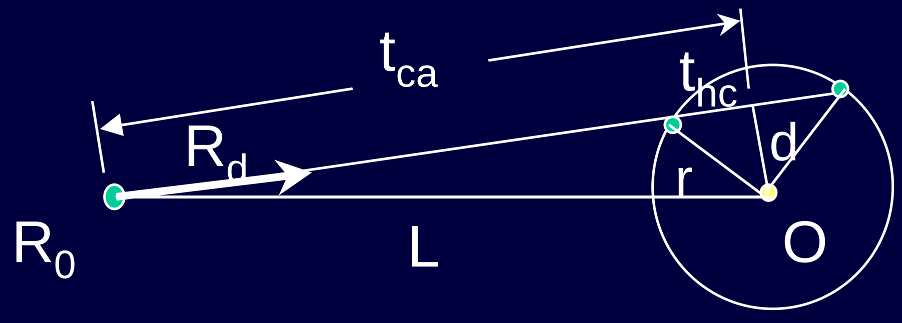


Ray Tracing

Ray Object Intersection

Sphere

Geometric Approach



$$L = O - R_0$$

$$t_{ca} = L^T R_d$$

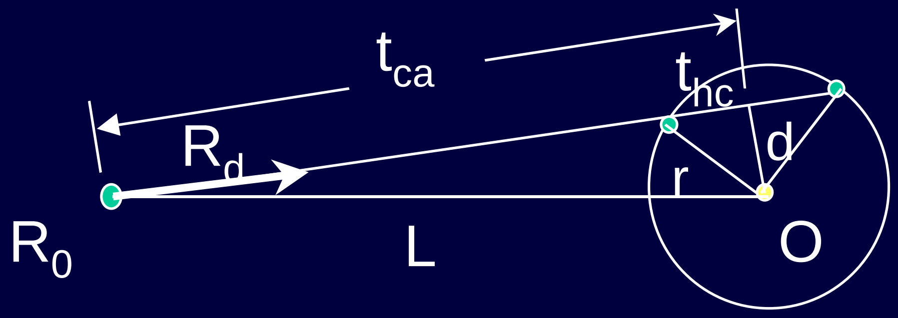
$$t_{ca} < 0 \text{ } \textit{no intersection}$$

Ray Tracing

Ray Object Intersection

Sphere

Geometric Approach



$$L = O - R_0$$

$$t_{ca} = L^T R_d$$

$$t_{ca} < 0 \text{ no intersection}$$

$$d = L^T L - t_{ca}^2$$

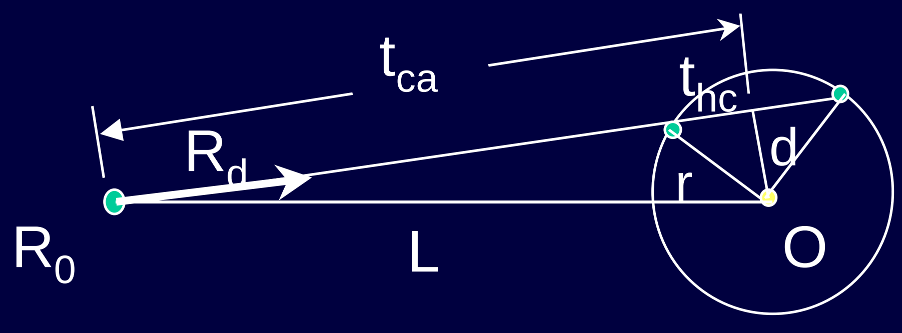
$$\text{if } d > r \text{ no intersection}$$

Ray Tracing

Ray Object Intersection

Sphere

Geometric Approach



$$t_{hc} = \sqrt{r^2 - d^2}$$

$$t = t_{ca} - t_{hc} \text{ and } t_{ca} + t_{hc}$$

smaller t

Ray Tracing

Ray Plane Intersection

Ray

$Ro = [\ X_o\ Y_o\ Z_o\ ]$ (ray origin)
$Rd = [\ X_d\ Y_d\ Z_d\ ]$ (ray direction)
$X_d^2 + Y_d^2 + Z_d^2 = 1$ (normalized)
$R(t) = R_o + R_d t$ $t > 0$

Plane

$P : Ax + By + Cz + D = 0$
$A^2 + B^2 + C^2 = 1$
$P_{normal} = P_n = [\ A\ B\ C\ ]$
D : Distance from origin

Ray Tracing

Ray Plane Intersection

Substituting ray equation in plane's equation

$$A (X_o + X_d t) + B (Y_o + Y_d t) + C (Z_o + Z_d t) + D = 0$$

Solving for *t*

$$t = -\frac{AX_0 + BY_0 + CZ_0 + D}{AX_d + BY_d + CZ_d}$$

$$t = -\frac{P_n \cdot R_0 + D}{P_n \cdot R_d}$$

Ray Tracing

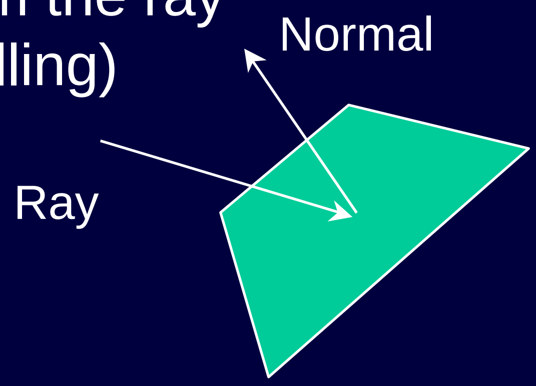
Ray Plane Intersection

Let

$$V_d = P_n \cdot R_d = AX_d + BY_d + CZ_d$$

If V_d = 0 then the ray is parallel to the plane
(no intersection)

V_d > 0 normal is pointing away from the ray
(may be used for back-face culling)



Ray Tracing

Ray Plane Intersection

Let

$$V_0 = -(P_n \cdot R_0 + D) = (AX_0 + BY_0 + CZ_0 + D)$$

$$t = \frac{V_0}{V_d}$$

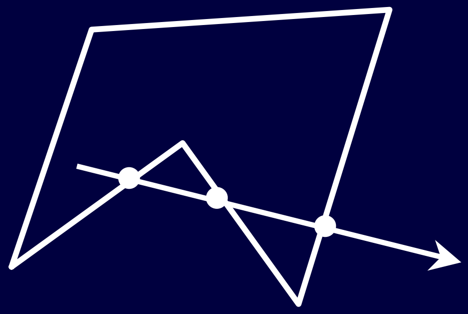
If $t < 0$ then plane is behind ray's origin
else compute intersection

$r_i = [\ X_i\ Y_i\ Z_i\ ] = [X_o + X_d t\ ,\ Y_o + Y_d t\ ,\ Z_o + Z_d t\ ]$
$r_{normal} = P_n$

Ray Tracing

Polygon Intersection

Containment Test



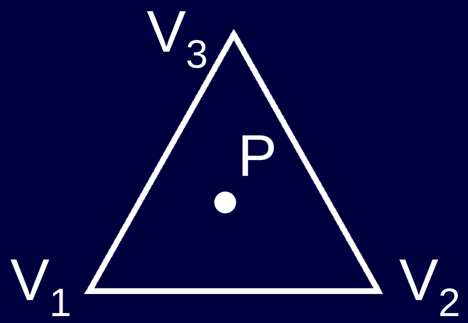
Parity Test: If the number of intersection is odd then point is inside (special case for vertices)

Ray Tracing

Triangle Intersection

Containment Test

Triangle: Barycentric Coordinates



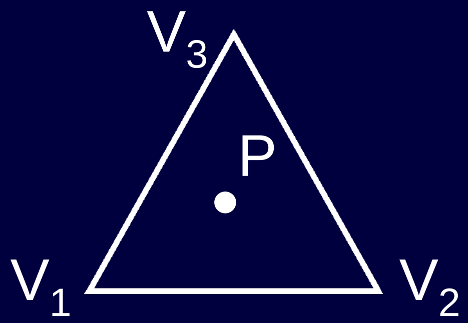
$$P = uV_1 + vV_2 + wV_3$$

Ray Tracing

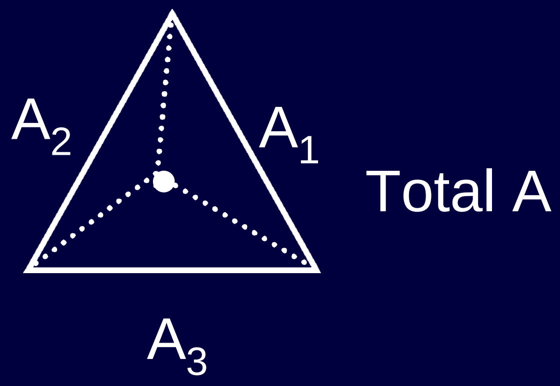
Triangle Intersection

Containment Test

Triangle: Barycentric Coordinates



$$P = uV_1 + vV_2 + wV_3$$



Area Coordinates

Ray Tracing

Triangle Intersection

Containment Test

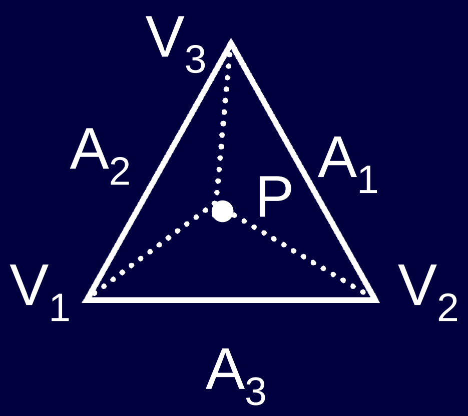
Triangle: Barycentric Coordinates

$$u = \frac{A_1}{A}, v = \frac{A_2}{A}, w = \frac{A_3}{A}$$

$$u + v + w = 1$$

$$u \geq 0, v \geq 0, w \geq 0$$

$$P = uV_1 + vV_2 + wV_3$$



Ray Tracing

Ray Quadric Intersection

Quadrics:

Cylinders, Cone, Sphere, Ellipsoids,
Paraboloids, Hyperboloids, etc.

Implict form $f(X,Y,Z) = 0$

$Ax^2 + 2Bxy + 2Cxz + 2Dx + Ey^2 + 2Fyz + 26y + Hz^2 + 2Iz + J = 0$

Ray: *Parametric form*

$Ro = [\ X_o\ Y_o\ Z_o\ ]$ (ray origin)
$Rd = [\ X_d\ Y_d\ Z_d\ ]$ (ray direction)
$X_d^2 + Y_d^2 + Z_d^2 = 1$ (normalized)
$R(t) = R_o + R_d t \quad t > 0$

Ray Tracing

Ray Quadric Intersection

Matrix Form

f(X,Y,Z) = 0

$$\begin{bmatrix} X & Y & Z & 1 \end{bmatrix} \begin{bmatrix} A & B & C & D \\ B & E & F & G \\ C & F & H & I \\ D & G & I & J \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = 0$$

Ray Tracing

Ray Quadric Intersection

Substituting

$$A_q t^2 + B_q t + C_q = 0$$

$If\ A_q \neq 0$

$$t_0 = \frac{-B_q - \sqrt{B_q{}^2 - 4A_qC_q}}{2A_q}$$

$$t_1 = \frac{-B_q + \sqrt{B_q{}^2 - 4A_qC_q}}{2A_q}$$

$If\ A_q = 0$

$$t = -\frac{C_q}{B_q}$$

Ray Tracing

Ray Quadric Intersection

Normal

$$n = \left[\frac{\partial F}{\partial X_i}, \frac{\partial F}{\partial Y_i}, \frac{\partial F}{\partial Z_i} \right]$$

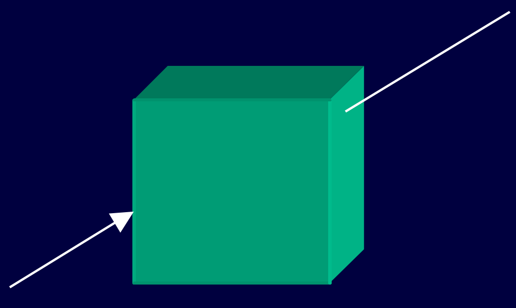
$$n_x = 2(AX_i + BY_i + CZ_i + D)$$

$$n_y = 2(BX_i + EY_i + FZ_i + G)$$

$$n_z = 2(CX_i + FY_i + HZ_i + I)$$

Ray Tracing

Ray Box Intersection



3D Clipping: Cyrus Beck/Liang Barsky