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HW10

1. The market maker's Value Function at time t is given by the Expected Utility at time T:
$V\left(t, S_t, W, I\right)=\mathbb{E}\left[-e^{-\gamma \cdot\left(W+I \cdot S_T\right)} \mid\left(t, S_t\right)\right]$
This could be further expressed as
$-e^{-\gamma \cdot W} \mathbb{E}\left[e^{-\gamma I \cdot S_T} \mid\left(t, S_t\right)\right]$
$=-e^{-\gamma \cdot W-\gamma I S_t+\frac{\sigma^2 \gamma^2 I^2(T-t)}{2}}$
Also by definition, the indifference bid Price obtained as solved by $V\left(t, S_t, W-Q^{(b)}\left(t, S_t, I\right), I+1\right)=V\left(t, S_t, W, I\right)$ where the latter term is the market maker's value function as indicated above. Specifically be like $-e^{-\gamma \cdot\left(W-Q^{(b)}\left(t, S_t, I\right)\right)-\gamma(I+1) S_t+\frac{\sigma^2 \gamma^2(I+1)^2(T-t)}{2}}=-e^{-\gamma \cdot W-\gamma I S_t+\frac{\sigma^2 \gamma^2 I^2(T-t)}{2}}$ by evaluating both sides.
Similarly, the indifferent ask as also obtained upon solving the following,
$V\left(t, S_t, W+Q^{(a)}\left(t, S_t, I\right), I-1\right)=V\left(t, S_t, W, I\right)$