Stanford CME 241 (Winter 2021) - Assignment 4

Frog on Lilypad Revisited (code part)

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In [1]:

```
import sys
sys.path.append('/Users/chih-hsuankao/Desktop/CME241/RL-book/')

import matplotlib.pyplot as plt
from rl.distribution import Categorical, Constant
from rl.dynamic_programming import (
    evaluate_mrp_result,
    policy_iteration,
    policy_iteration_result,
    value_iteration,
    value_iteration_result
)
from rl.markov_decision_process import (
    FiniteMarkovDecisionProcess,
    FinitePolicy,
    StateActionMapping
)
from rl.markov_process import (
    Transition,
    RewardTransition,
    FiniteMarkovProcess,
    Optional,
    FiniteMarkovRewardProcess
)
import time
```

```
/Users/chih-hsuankao/.pyenv/versions/anaconda3-2019.03/lib/py
thon3.7/site-packages/scipy/__init__.py:137: UserWarning: Num
Py 1.16.5 or above is required for this version of SciPy (det
ected version 1.16.2)
  UserWarning)
```

In [2]:

```
sys.path.append('..')
import Assignment3.Q3_Frog_croak_lilypad as frog
```

In [3]:

```
gamma = 0.8
pad = 10

si_mdp: frog.FiniteMarkovDecisionProcess[frog.FrogState, int] = frog.FrogM
```

```
DP(num_pad = pad)

print("MDP Transition Map")
print("------------------")
print(si_mdp)
```

```
MDP Transition Map
------------------
From State FrogState(position=1):
  With Action A:
    To [State FrogState(position=0) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=2) and Reward 0.000] with Pr
obability 0.900
  With Action B:
    To [State FrogState(position=0) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=2) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=3) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=4) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=5) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=6) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=7) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=8) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=9) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=10) and Reward 1.000] with P
robability 0.100
From State FrogState(position=2):
  With Action A:
    To [State FrogState(position=1) and Reward 0.000] with Pr
obability 0.200
    To [State FrogState(position=3) and Reward 0.000] with Pr
obability 0.800
  With Action B:
    To [State FrogState(position=0) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=1) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=3) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=4) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=5) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=6) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=7) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=8) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=9) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=10) and Reward 1.000] with P
```

```
    To [State FrogState(position=10) and Reward 1.000] with P
robability 0.100
From State FrogState(position=3):
  With Action A:
    To [State FrogState(position=2) and Reward 0.000] with Pr
obability 0.300
    To [State FrogState(position=4) and Reward 0.000] with Pr
obability 0.700
  With Action B:
    To [State FrogState(position=0) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=1) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=2) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=4) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=5) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=6) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=7) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=8) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=9) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=10) and Reward 1.000] with P
robability 0.100
From State FrogState(position=4):
  With Action A:
    To [State FrogState(position=3) and Reward 0.000] with Pr
obability 0.400
    To [State FrogState(position=5) and Reward 0.000] with Pr
obability 0.600
  With Action B:
    To [State FrogState(position=0) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=1) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=2) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=3) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=5) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=6) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=7) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=8) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=9) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=10) and Reward 1.000] with P
robability 0.100
From State FrogState(position=5):
  With Action A:
    To [State FrogState(position=4) and Reward 0.000] with Pr
obability 0.500
```

```
    To [State FrogState(position=6) and Reward 0.000] with Pr
obability 0.500
  With Action B:
    To [State FrogState(position=0) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=1) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=2) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=3) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=4) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=6) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=7) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=8) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=9) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=10) and Reward 1.000] with P
robability 0.100
From State FrogState(position=6):
  With Action A:
    To [State FrogState(position=5) and Reward 0.000] with Pr
obability 0.600
    To [State FrogState(position=7) and Reward 0.000] with Pr
obability 0.400
  With Action B:
    To [State FrogState(position=0) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=1) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=2) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=3) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=4) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=5) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=7) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=8) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=9) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=10) and Reward 1.000] with P
robability 0.100
From State FrogState(position=7):
  With Action A:
    To [State FrogState(position=6) and Reward 0.000] with Pr
obability 0.700
    To [State FrogState(position=8) and Reward 0.000] with Pr
obability 0.300
  With Action B:
    To [State FrogState(position=0) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=1) and Reward 0.000] with Pr
obability 0.100
```

```
obability 0.100
    To [State FrogState(position=2) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=3) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=4) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=5) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=6) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=8) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=9) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=10) and Reward 1.000] with P
robability 0.100
From State FrogState(position=8):
  With Action A:
    To [State FrogState(position=7) and Reward 0.000] with Pr
obability 0.800
    To [State FrogState(position=9) and Reward 0.000] with Pr
obability 0.200
  With Action B:
    To [State FrogState(position=0) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=1) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=2) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=3) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=4) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=5) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=6) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=7) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=9) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=10) and Reward 1.000] with P
robability 0.100
From State FrogState(position=9):
  With Action A:
    To [State FrogState(position=8) and Reward 0.000] with Pr
obability 0.900
    To [State FrogState(position=10) and Reward 1.000] with P
robability 0.100
  With Action B:
    To [State FrogState(position=0) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=1) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=2) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=3) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=4) and Reward 0.000] with Pr
```

```
obability 0.100
    To [State FrogState(position=5) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=6) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=7) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=8) and Reward 0.000] with Pr
obability 0.100
    To [State FrogState(position=10) and Reward 1.000] with P
robability 0.100
FrogState(position=10) is a Terminal State
FrogState(position=0) is a Terminal State
```

Compute the Optimal Value Function and Optimal Policy using Policy Iteration algorithm

In [4]:

```
print("Policy Iteration: Optimal Value Function and Optimal Policy")
print("------------------------------------------------------------")

opt_vf_pi, opt_policy_pi = frog.policy_iteration_result(si_mdp, gamma=gamm
a)
print(opt_vf_pi)
print(opt_policy_pi)
```

```
Policy Iteration: Optimal Value Function and Optimal Policy
------------------------------------------------------------
{FrogState(position=1): 0.2824061058293976, FrogState(positio
n=2): 0.2824061058293976, FrogState(position=3): 0.2824061058
293976, FrogState(position=4): 0.2824061058293976, FrogState
(position=5): 0.2824061058293976, FrogState(position=6): 0.28
24061058293976, FrogState(position=7): 0.2824061058293976, Fr
ogState(position=8): 0.2824061058293976, FrogState(position=
9): 0.30332433866457054}
For State FrogState(position=1):
  Do Action B with Probability 1.000
For State FrogState(position=2):
  Do Action B with Probability 1.000
For State FrogState(position=3):
  Do Action B with Probability 1.000
For State FrogState(position=4):
  Do Action B with Probability 1.000
For State FrogState(position=5):
  Do Action B with Probability 1.000
For State FrogState(position=6):
  Do Action B with Probability 1.000
For State FrogState(position=7):
  Do Action B with Probability 1.000
For State FrogState(position=8):
  Do Action B with Probability 1.000
For State FrogState(position=9):
  Do Action A with Probability 1.000
```

Compute the Optimal Value Function and Optimal Policy using Value Iteration algorithm

In [5]:

```
print("Value Iteration: Optimal Value Function and Optimal Policy")
print("-----------------------------------------------------------")

opt_vf_vi, opt_policy_vi = frog.value_iteration_result(si_mdp, gamma=gamma
)
print(opt_vf_vi)
print(opt_policy_vi)
```

```
Value Iteration: Optimal Value Function and Optimal Policy
-----------------------------------------------------------
{FrogState(position=1): 0.2824061058293976, FrogState(positio
n=2): 0.2824061058293976, FrogState(position=3): 0.2824061058
293976, FrogState(position=4): 0.2824061058293976, FrogState
(position=5): 0.2824061058293976, FrogState(position=6): 0.28
24061058293976, FrogState(position=7): 0.2824061058293976, Fr
ogState(position=8): 0.2824061058293976, FrogState(position=
9): 0.30332433866457054}
For State FrogState(position=1):
  Do Action B with Probability 1.000
For State FrogState(position=2):
  Do Action B with Probability 1.000
For State FrogState(position=3):
  Do Action B with Probability 1.000
For State FrogState(position=4):
  Do Action B with Probability 1.000
For State FrogState(position=5):
  Do Action B with Probability 1.000
For State FrogState(position=6):
  Do Action B with Probability 1.000
For State FrogState(position=7):
  Do Action B with Probability 1.000
For State FrogState(position=8):
  Do Action B with Probability 1.000
For State FrogState(position=9):
  Do Action A with Probability 1.000
```

Analyze the computational efficiency of Policy Iteration versus Value Iteration in terms of speed of convergence to the Optimal Value Function. Plot some graphs of convergence speed for different values of n (the number of lilypads).

n = 5

In [6]:

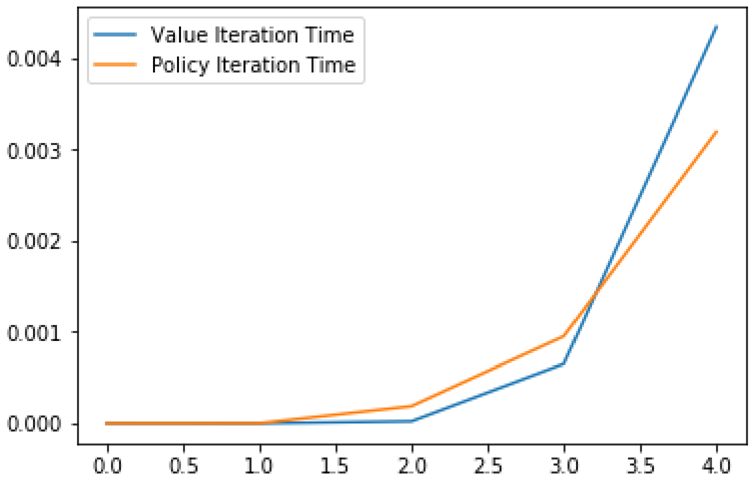
```
val_iter_time = []
pol_iter_time = []

for n in range(0, 5):

    # Value Iteration
```

```
    frog_MDP = frog.FrogMDP(n)
    last = {}
    v_i = value_iteration(mdp=frog_MDP, gamma=1)
    t_0 = time.time()

    for vi in v_i:
        if vi == last: break
        last = vi
    val_iter_time.append(time.time() - t_0)

    # Policy Iteration
    p_i = policy_iteration(mdp=frog_MDP, gamma=1)
    t_0 = time.time()
    last = {}

    for pi, pp in p_i:
        if pi == last: break
        last = pi
    pol_iter_time.append(time.time() - t_0)

print('Value Iteration Time: ', val_iter_time)
print('Policy Iteration Time: ', pol_iter_time)
```

```
Value Iteration Time:  [9.5367431640625e-07, 0.0, 2.217292785
6445312e-05, 0.0006499290466308594, 0.004345893859863281]
Policy Iteration Time:  [9.5367431640625e-07, 1.1920928955078
125e-06, 0.00018787384033203125, 0.00095367431640625, 0.00319
7908401489258]
```

In [7]:

```
x_axis = [i for i in range(0, 5)]
plt.plot(x_axis, val_iter_time, label="Value Iteration Time")
plt.plot(x_axis, pol_iter_time, label="Policy Iteration Time")
plt.legend()
plt.show()
```



n = 10

In [8]:

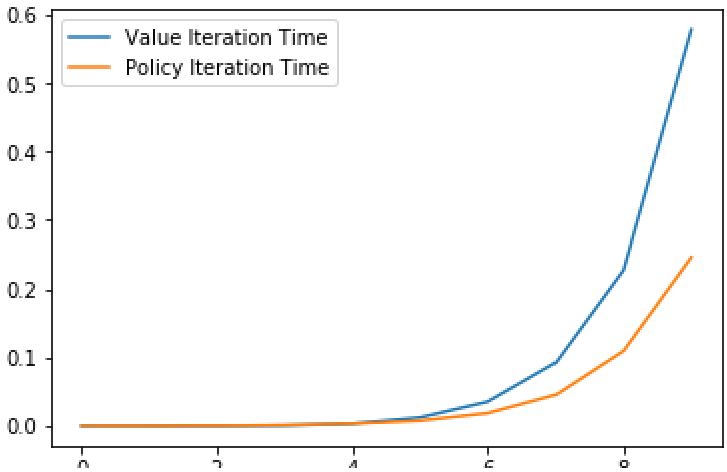
```
val_iter_time = []
```

```
pol_iter_time = []

for n in range(0, 10):

    # Value Iteration
    frog_MDP = frog.FrogMDP(n)
    last = {}
    v_i = value_iteration(mdp=frog_MDP, gamma=1)
    t_0 = time.time()

    for vi in v_i:
        if vi == last: break
        last = vi
    val_iter_time.append(time.time() - t_0)

    # Policy Iteration
    p_i = policy_iteration(mdp=frog_MDP, gamma=1)
    t_0 = time.time()
    last = {}

    for pi, pp in p_i:
        if pi == last: break
        last = pi
    pol_iter_time.append(time.time() - t_0)

print('Value Iteration Time: ', val_iter_time)
print('Policy Iteration Time: ', pol_iter_time)
```

```
Value Iteration Time:  [2.1457672119140625e-06, 0.0, 2.503395
0805664062e-05, 0.0006420612335205078, 0.003234148025512695 3,
0.012189865112304688, 0.03522777557373047, 0.092207908630371

1, 0.22800922393798828, 0.5789268016815186]
Policy Iteration Time:  [2.1457672119140625e-06, 1.1920928955
078125e-06, 0.00019979476928710938, 0.0011451244354248047, 0.
0032908916473388 67, 0.007607936859130859, 0.01857185363769531
2, 0.0452570915222168, 0.10977029800415039, 0.246401071548461
91]
```

In [9]:

```
x_axis = [i for i in range(0, 10)]
plt.plot(x_axis, val_iter_time, label="Value Iteration Time")
plt.plot(x_axis, pol_iter_time, label="Policy Iteration Time")
plt.legend()
plt.show()
```



As illustrated above, we could see that the policy iteration algorithm is more efficient in terms of speed of convergence.

In [ ]:

```

```