

## HW7

1. The objective is to maximize the following,

$$E \left[ \int_t^T e^{-p(s-t)} \cdot \log(c_s) + e^{-p(T-t)} \cdot \log(B(T) \cdot W_T) ds \mid W_t \right]$$

The reward at time  $t$  is  $\int_t^T e^{-p(s-t)} \cdot \log(c_s) ds$ . Action is  $[\pi_t, c_t]$ .

Hence, we further write down objective as,

$$V^*(t, W_t) = \max_{\pi, c} E_t \left[ \int_t^{t_1} e^{-p(s-t)} (c_s) ds + e^{-p(t_1-t)} V^*(t_1, W_{t_1}) \right]$$

HJB:

$$\max_{\pi_t, c_t} E_t [dV^*(t, W_t) + \log(c_t)] = p * V^*(t, W_t) dt$$

3.

States are roughly be consist of a boolean indicating if employed; and a level scale that indicates the skill.

Action is a binary boolean

Transition Probabilities:

$$\begin{aligned} \mathcal{P}(x, a, r, x') &= h(s), \text{ if } x = (0, s), x' = (1, s), r = 0 \\ \mathcal{P}(x, a, r, x') &= 1 - h(s), \text{ if } x = (0, s), x' = (0, s \cdot e^{-\lambda}), r = 0 \end{aligned}$$