

Functional Dependencies

1. Introduction:

- In a relational database, functional dependencies (FDs) are crucial for understanding the relationships between attributes within a table.
- A functional dependency between two attribute sets, X and Y, denoted as $X \rightarrow Y$, indicates that the values of X uniquely determine the values of Y.

2. Armstrong's Axioms:

- Armstrong's axioms are fundamental rules used for reasoning about functional dependencies.
- These axioms guide us in deriving and understanding the relationships between attributes.

a. Reflexivity Axiom:

- If Y is a subset of X, then $X \rightarrow Y$.
- This axiom reflects the idea that any subset of attributes is functionally dependent on the whole set of attributes.

Example:

Consider a relation R with attributes A, B, and C. If $A \rightarrow B$ holds, then it's also true that $AC \rightarrow B$.

b. Augmentation Axiom:

- If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any attribute set Z.
- This axiom shows that adding attributes to both sides of a functional dependency maintains its validity.

Example:

If $\text{Name} \rightarrow \text{Age}$, then $\text{Name Address} \rightarrow \text{Age Address}$ holds true.

c. Transitivity Axiom:

- If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$.
- This axiom implies that if a functional dependency can be derived indirectly, it can be inferred directly.

Example:

If $\text{Course} \rightarrow \text{Department}$ and $\text{Department} \rightarrow \text{Faculty}$, then $\text{Course} \rightarrow \text{Faculty}$ can be inferred.

3. Example Scenarios:

a. Student Table:

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Roll No	Name	Age	Course
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101	Alice	20	CS
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102	Bob	22	ECE
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103	Carol	21	CS
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- In the above table, $\text{Roll No} \rightarrow \text{Name}$ because each Roll No corresponds to a unique student's name.
- $\text{Roll No} \rightarrow \text{Age}$ because each student's Roll No uniquely determines their age.
- $\text{Course} \rightarrow \text{Roll No}$ because each course maps to multiple Roll Nos.

b. Course Enrollment Table:

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Roll No	Course
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101	DBMS
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102	OS
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103	DBMS
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- In this table, $\text{Roll No} \rightarrow \text{Course}$ because each Roll No maps to a specific course.

4. Inference Rules:

a. Union Rule:

- If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$.

Example:

If $\text{Roll No} \rightarrow \text{Name}$ and $\text{Roll No} \rightarrow \text{Age}$, then $\text{Roll No} \rightarrow \text{Name Age}$.

b. **Decomposition Rule:**

- If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$.

Example:

If $\text{Roll No} \rightarrow \text{Name Age}$, then $\text{Roll No} \rightarrow \text{Name}$ and $\text{Roll No} \rightarrow \text{Age}$.

c. **Pseudo-Transitivity Rule:**

- If $X \rightarrow Y$ and $WY \rightarrow Z$, then $WX \rightarrow Z$.

Example:

If $\text{Course} \rightarrow \text{Department}$ and $\text{CourseFaculty} \rightarrow \text{Office}$, then $\text{CourseFaculty} \rightarrow \text{Office}$ can be inferred as $\text{Course} \rightarrow \text{DepartmentOffice}$.

5. Conclusion:

- Functional dependencies play a pivotal role in maintaining the accuracy and integrity of relational databases.
- Armstrong's axioms provide a systematic approach to understanding and deriving functional dependencies.
- These concepts are essential for database normalization and the efficient design of relational databases.

