



103finalfall 2014 withans

Introduction to Calculus (University of Pennsylvania)

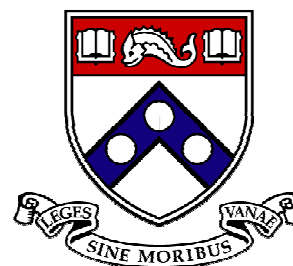


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University of Pennsylvania

Math 103-Final Exam

Fall 2014



Name _____ (PRINT) Penn ID# _____

Professor : (circle one) Rimmer Wong Towsner

Recitation Number _____ Rec. Day _____ Rec. Time _____

This exam has 13 multiple choice questions and 2 open-ended questions. Each question is worth 10 points. Partial credit will be given for the entire exam so be sure to show all work. On the multiple choice questions, circle the correct answer and give supporting work, a correct answer with little or no supporting work will receive little or no credit. Use the space provided to show all work. A sheet of scrap paper is provided at the end of the exam. If you write on the back of any page please indicate this in some way.

You have 120 minutes to complete the exam. You are not allowed the use of a calculator or any other electronic device. You are allowed to use the front and back of a standard 8.5"X11" sheet of paper for handwritten notes. Please silence and put away all cell phones and other electronic devices. When you finish, please stay seated until the entire 120 minutes has elapsed. When time is up continue to stay seated until someone comes by to collect your exam.

Once you have completed the exam, sign the academic integrity statement below.

Do NOT write in the grid below. It is for grading purposes only.

Problem	Points		Problem	Points
1			9	
2			10	
3			11	
4			12	
5			13	
6			14	
7			15	
8				
Total			Total	

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination paper.

Name (printed)

Signature

Date

Final Exam
Score

1. The point $(1,1)$ is on the graph of

$$xy^2 + yx^2 = 2$$

What is $\frac{dy}{dx}$ at this point?

- | | | | |
|---------|---------|---------|--------------|
| a) -3 | c) -2 | e) -1 | g) 0 |
| b) 1 | d) 2 | f) 3 | h) Undefined |

2. Sometimes an antiderivative can't be described using functions you already know. An example of this is $\sin(x^2)$.

The physicist Augustin-Jean Fresnel invented a new function $\text{Fresnel}(x)$ with the property that

$$(\text{Fresnel}(x))' = \sin(x^2).$$

What is $\frac{d}{dx}(\text{Fresnel}(x^2))$?

- | | | | |
|-------------------|-----------------------------------|-------------------|---------------------|
| a) $\sin(x^4)$ | c) $2x \cdot \text{Fresnel}(x^4)$ | e) $2x \cos(x^2)$ | g) $4x^3 \cos(x^4)$ |
| b) $2x \sin(x^4)$ | d) $\cos(x^2)$ | f) $2x \cos(x^4)$ | h) $4x^3 \sin(x^4)$ |

3. An ant is moving along the curve $y = \sqrt{x}$ so that the ant's x -coordinate is increasing at a rate of 2 units per second.

When the x -coordinate of the ant is 1, how fast is the distance between the ant and the origin increasing?

4. For the function

$$f(x) = \ln(x + \sqrt{x^2 + 4})$$

find $f'(2\sqrt{3})$

- | | | | |
|------------------|------------------|------------------|------------------|
| a) $\frac{1}{4}$ | c) $\frac{1}{2}$ | e) $\frac{3}{2}$ | g) $\frac{9}{2}$ |
| b) 0 | d) 1 | f) 2 | h) 12 |

5. For the function

$$f(x) = xe^x$$

find $f''(2)$.

- | | | | |
|----------|-----------|-----------|-----------|
| a) 0 | c) 1 | e) e | g) $2e$ |
| b) e^2 | d) $2e^2$ | f) $3e^2$ | h) $4e^2$ |

6. For the function

$$f(x) = \frac{x^2 - 2x + 6}{x - 3}$$

Let A = the local maximum value of $f(x)$ and B = the local minimum value of $f(x)$

Calculate the value of $A + B$.

- | | | | |
|-------|-------|------|-------|
| a) -4 | c) -2 | e) 0 | g) 2 |
| b) 4 | d) 6 | f) 8 | h) 10 |

7. Find the absolute minimum value of

$$f(x) = \frac{4}{3}\sqrt[3]{x} + \frac{16}{3x^{2/3}}$$

on the interval $[1, 27]$

- | | | | |
|-------------------|---------------------|-------|------------------|
| a) $\frac{20}{3}$ | c) $\frac{127}{24}$ | e) 8 | g) $\frac{9}{2}$ |
| b) 4 | d) 3 | f) 27 | h) 1 |

8. Evaluate

$$\lim_{x \rightarrow 0} \frac{6 \ln(\cos x)}{3x^2}$$

- | | | | |
|-------|-------|------|------|
| a) -2 | c) -1 | e) 0 | g) 1 |
| b) 2 | d) 3 | f) 4 | h) 5 |

9. A builder wishes to fence in $30,000 \text{ m}^2$ of land in a rectangular shape. For security reasons, the fence along the front part of the land will cost \$50 per meter, while the fence for the other three sides will cost \$10 per meter. How much of each type of fence should the builder buy to minimize the cost of the fence?

Answer :

_____ m of the \$50 per meter fence.

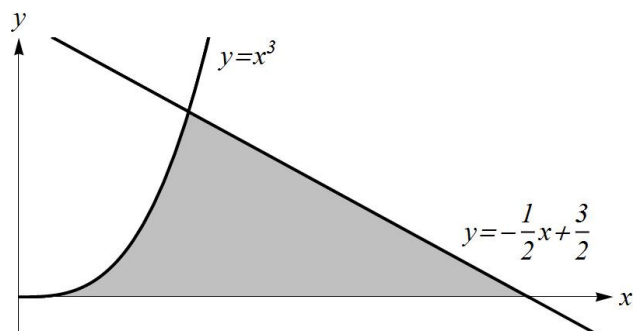
_____ m of the \$10 per meter fence.

10. Find the value of the sum

$$\sum_{k=1}^{12} \left(\frac{k^3}{13^2} - \frac{k^2}{26} \right)$$

- | | | | |
|--------|------|------|-------|
| a) -16 | c) 1 | e) 6 | g) 11 |
| b) 0 | d) 3 | f) 9 | h) 15 |

11. Find the area of the shaded region below



- a) $\frac{1}{4}$ c) $\frac{1}{2}$ e) 1 g) $\frac{5}{4}$
 b) $\frac{3}{2}$ d) 2 f) $\frac{9}{4}$ h) None of these

12. Evaluate the integral

$$\int_0^{\frac{1}{2}} \frac{\arcsin(x)}{\sqrt{1-x^2}} dx.$$

- a) $-\frac{\pi^2}{8}$ c) $-\frac{\pi^2}{18}$ e) $-\frac{\pi^2}{72}$ g) 0
 b) $\frac{\pi^2}{72}$ d) $\frac{\pi^2}{18}$ f) $\frac{\pi^2}{8}$ h) None of these

13. Define

$$G(x) = \int_x^{x^2} \frac{1}{t} \sin\left(\frac{\pi t}{2}\right) dt$$

Find $G'(-1)$.

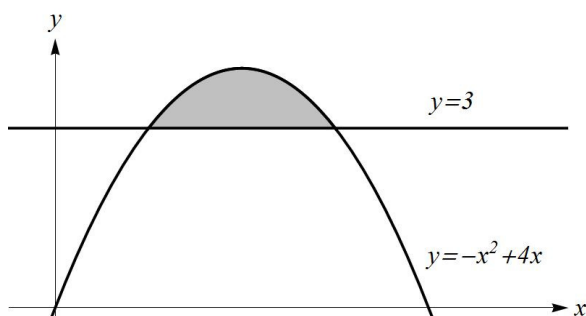
- a) -3 c) -2 e) -1 g) 0
 b) 1 d) 2 f) 3 h) None of these

14. Find

$$\int_{-1}^1 x^2 \sin^5(\pi x) dx.$$

- a) $-\infty$ c) -2 e) -1 g) 0
b) 1 d) 2 f) ∞ h) None of these

15. Find the area of the shaded region below.



- a) $\frac{1}{3}$ c) $\frac{2}{3}$ e) $\frac{4}{3}$ g) $\frac{5}{3}$
b) $\frac{1}{2}$ d) 1 f) $\frac{3}{2}$ h) None of these

ANSWERS:

1. e
2. b
3. $\frac{3\sqrt{2}}{2}$ units/sec.
4. a
5. h
6. f
7. b
8. c
9. 100 m of the \$50/m, 700 m of the \$10/m
10. g
11. g
12. b
13. a
14. g
15. e