Final exam 1 19 April Winter 2018, questions

Calculus 1 (McGill University)

McGill

April 2018
Final Examination

VERSION # :

Calculus I

MATH 140

19 April, 9:00-12:00

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STUDENT NAME:		**McGILL ID:**									

INSTRUCTIONS:

EXAM:	CLOSED BOOK ☒ OPEN BOOK ☐
	SINGLE-SIDED ☒ PRINTED ON BOTH SIDES OF THE PAGE ☐
	MULTIPLE CHOICE ANSWER SHEETS ☐ NOTE: The Examination Security Monitor Program detects pairs of students with unusually similar answer patterns on multiple-choice exams. Data generated by this program can be used as admissible evidence, either to initiate or corroborate an investigation or a charge of cheating under Section 16 of the Code of Student Conduct and Disciplinary Procedures. ANSWER IN BOOKLET ☒ EXTRA BOOKLETS PERMITTED: YES ☒ NO ☐ ANSWER ON EXAM ☐
	SHOULD THE EXAM BE: RETURNED ☐ KEPT BY STUDENT ☒
CRIB SHEETS:	NOT PERMITTED ☒ PERMITTED ☐ e.g. one 8 1/2X11 handwritten double-sided sheet Specifications:
DICTIONARIES:	TRANSLATION ONLY ☒ REGULAR ☐ NONE ☐
CALCULATORS:	NOT PERMITTED ☒ PERMITTED (Non-Programmable) ☐
ANY SPECIAL INSTRUCTIONS: e.g. molecular models	Special Instructions

Question 1 (9 marks, 3 per part)

Compute the following limits and derivatives.
a) Find

$$\lim_{x\to 5}\left(\frac{x^2-8x+15}{x^2-25}\right).$$

b) Find

$$\frac{d}{dx}\left(\log_3(x)+4^x-x^2+17\right).$$

c) Find

$$\frac{d}{dx}\left(\frac{1+e^x}{x^2+9}\right).$$

Question 2 (6 marks)

Find the derivative of

$$\sin\left(e^{3x}+17\ln(x^2+1)\right).$$

Question 3 (5 marks)

Find

$$\lim_{x\to 2}\left(\tan\left(\frac{(x^3-x^2-x-2)\pi}{x^2+17x-38}\right)\right).$$

Question 4 (6 marks)

Compute the derivative of

$$x^3+17x+9$$

from first principles, i.e. from the definition of the derivative as a limit.

Question 5 (8 marks)

Find the domain of the function

$$\ln\left(\cot^{-1}\left(x^2\right)-\frac{\pi}{6}\right).$$

Question 6 (6 marks)

Compute the following limit

$$\lim_{x\to 1}\left(\frac{4\tan^{-1}(x)-\pi}{x^2+8x-9}\right).$$

Question 7 (14 marks)

Let $f(x)=(x^2-6x+9)e^x$.
a) Find the maximum of $f(x)$ in the interval $[0,2]$.
b) Find where $f(x)$ is concave down.
c) Find the horizontal asymptote(s) of $f(x)$.

Question 8 (5 marks)

Let

$$f(x)=5x^{10}+3x^3-1.$$

a) Show that $f(x)$ has at least one zero in $[0,1]$.
b) Show that this root is unique, i.e. that $f(x)$ cannot have more than one zero in the same interval.

Question 9 (9 marks)

Find the derivative of

$$\cosh\left((\sqrt[3]{x})^{\sec(x)}\right).$$

Question 10 (6 marks)

An object moves on the curve defined by the implicit relation

$$4x^3+y^2+5x^2y=0.$$

When reaching $x=1$ and $y=-4$, the rate of change in the x-direction is given by $\frac{dx}{dt}=3$. What is $\frac{dy}{dt}$ at this point?

Question 11 (6 marks)

Find the derivative of

$$\ln\left(\frac{x^9(x^4+6)^3}{x^8+20x^2+7}\right)$$

when $x=1$.

Question 12 (10 marks)

The graph of the curve defined by the implicit equation

$$3y^3 - 2xy = x^2$$

is given below.
a) Find $\frac{dy}{dx}$.
b) Find the equation in the form $y = mx + c$ of the tangent line at $(-3, 1)$.
c) There are two points where the graph turns in the horizontal direction. Find the x and y coordinates of these two points.