

Final Exam

Write up your solutions on paper or tablet.

- You must have a cover page with your name printed legibly in the upper left corner and your Penn ID in the upper right corner.
- Each problem must be done on a separate page with the problem number written prominently at the top of the page.

You have 100 minutes to complete the 8 questions. Each question is worth 10 points and will be graded for partial credit.

You must be on Zoom with your camera showing your face the entire time. You will be split up into separate Zoom rooms so that the proctor will have a reasonable number of students to proctor.

We will start at 9:10 am Eastern and end at 10:50 am Eastern. You will have 10 minutes to digitize your work into a single pdf file. This is the only acceptable file format. We will not accept any uploads after 11 am Eastern.

You will upload the document to the Final Exam Upload assignment found on Canvas.

All the best,
N. Rimmer

1. Use the limit definition of the derivative to calculate the derivative of

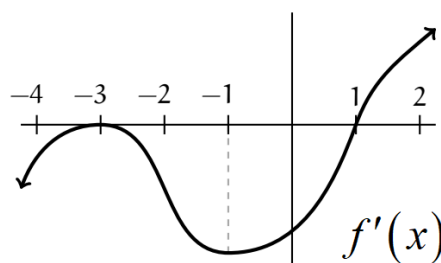
$$f(x) = \frac{3}{4x-3}.$$

Find the equation of the tangent line to $f(x)$ at $x = 1$.

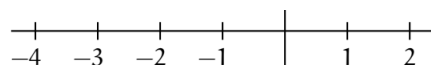
2. The graph of $f'(x)$ (the derivative of $f(x)$) is given below.

Answer the following questions about the function $f(x)$.

Include a **brief** explanation justifying your answer in each part.



a) Fill out the sign chart for $f'(x)$ by putting $+$ and $-$ signs on the number line indicating where $f'(x)$ is positive and where $f'(x)$ is negative.



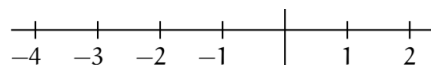
b) Identify the interval(s) where $f(x)$ is

i) increasing

ii) decreasing

c) Identify and classify the x -coordinate of any critical point(s) of $f(x)$.

d) Fill out the sign chart for $f''(x)$ by putting $+$ and $-$ signs on the number line indicating where $f''(x)$ is positive and where $f''(x)$ is negative.



e) Identify the interval(s) where $f(x)$ is

i) concave up

ii) concave down

f) Identify the x -coordinate of any inflection point(s) of $f(x)$.

3. Let $f(x) = \sqrt{1 + 4\sqrt{x}}$.

a. Find $f(36)$

b. Find $f^{-1}(7)$

c. Find $f'(4)$

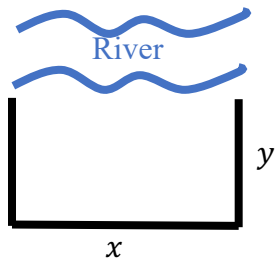
4. Evaluate the following limits.

a. $\lim_{x \rightarrow 0} \frac{x^2 - x - 6}{\sqrt[3]{x + 27}}$

b. $\lim_{x \rightarrow \infty} \frac{6x + 2}{\sqrt{4x^2 - x - 3}}$

c. $\lim_{x \rightarrow 1} \frac{\ln(5 - 4x) + e^{2x - 2} - 1}{\frac{4}{\pi} \arctan(3x - 2) - x}$

5. A farmer wants to fence in a rectangular grazing area next to a river. The grazing area must be 180,000 square meters. What dimensions would require the least amount of fencing if no fencing were required along the river?

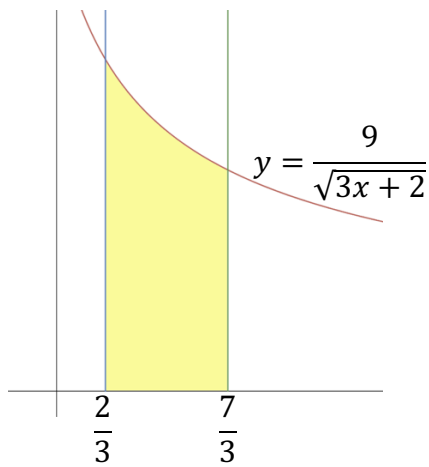


6.

a) Evaluate the integral below

$$\int \frac{\ln(x^2 + 1)}{x^2} dx$$

b) Find the area of the shaded region below



7. A continuous random variable X has the following probability density function

$$f(x) = \begin{cases} \cos x & \text{if } 0 \leq x \leq \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

Find the median.

8.

a. Use the Taylor series centered at 0 (Maclaurin series) for $\cos x$ to approximate $32x \cos \sqrt{x}$ with a cubic polynomial.

b. Use the cubic polynomial found in part a to approximate

$$\int_0^1 32x \cos \sqrt{x} \, dx$$