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Math 1400 Quiz #3 09/30/22 **Quiz Session 0040-102: 10:15am**

1. (a) Use a local linear approximation to approximate the value of $\sqrt[4]{15}$.
   (b) At which x value did you center the approximation? Explain your choice.
   (c) Do you expect your result to be an overestimate or an underestimate? Justify your response.

(a) Let $f(x) = \sqrt[4]{x}$. Find the local linear approximation at $x = 16$.

$$f(16) = 2, \quad f'(x) = \frac{1}{4\sqrt[4]{x^3}}, \quad f'(16) = \frac{1}{32}$$

$$\sqrt[4]{x} \approx f(16) + f'(16)(x-16) = 2 + \frac{1}{32}(x-16)$$

$$\sqrt[4]{15} = 2 - \frac{1}{32}$$

(b) We must know the value of $f(a)$ and $f'(a)$. We must also choose a to be close to the value which we are approximating.

(c) $f''(x) = -\frac{3}{16\sqrt[4]{x^7}}$ so $f''(16) < 0$ so we expect that we have an overestimate.

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2. Find $g'(x)$ where $g(x) = \frac{\sqrt[3]{1+x+x^2}\sqrt[6]{1+5x+10x^2}}{(3x+1)^7(x+1)^3}$.

$$\ln g(x) = \frac{1}{3}\ln(1+x+x^2) + \frac{1}{6}\ln(1+5x+10x^2) - 7\ln(3x+1) - 3\ln(x+1)$$

$$\frac{g'(x)}{g(x)} = \frac{1}{3}\cdot\frac{1+2x}{1+x+x^2} + \frac{1}{6}\cdot\frac{5+20x}{1+5x+10x^2} - 7\cdot\frac{3}{3x+1} - 3\cdot\frac{1}{x+1}$$

$$g'(x) = \frac{\sqrt[3]{1+x+x^2}\sqrt[6]{1+5x+10x^2}}{(3x+1)^7(x+1)^3}\left(\frac{1}{3}\cdot\frac{1+2x}{1+x+x^2} + \frac{1}{6}\cdot\frac{5+20x}{1+5x+10x^2} - 7\cdot\frac{3}{3x+1} - 3\cdot\frac{1}{x+1}\right)$$

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3. Let $f(x) = e^{-(x-1)^2/2}$. Let $F(x)$ be the anti-derivative of $f(x)$ such that $F(1) = -2$. Find the Taylor series for $F(x)$ at $x = 1$. Express your answer in summation notation. [*Hint:* You do not know the anti-derivative of f.]

$$e^u = \sum_{n=0}^{\infty} \frac{u^n}{n!}$$
$$e^{-(x-1)^2/2} = \sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^{2n}}{2^n n!}$$
$$F(x) = \int e^{-(x-1)^2/2}\, dx = C + \sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^{2n+1}}{(2n+1)2^n n!}$$

When $x = 1$ all terms in the sum are 0 so we can see that $C = -2$.

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