McGill

April–May 2020
Final Examination

VERSION #: 1

CALCULUS 1

MATH 140 SECTION: 001

DATE: MONDAY, APRIL 20, 2020 TIME: 9:00 AM

EXAMINER: Dr. Jérôme Fortier ASSOC. EXAMINER: Dr. Sidney Trudeau

STUDENT NAME:		**McGILL ID:**	

EXAM:	CLOSED BOOK ☐ OPEN BOOK ☒
	SINGLE SIDED ☐ PRINTED ON BOTH SIDES ☐
	MULTIPLE CHOICE ANSWER SHEETS YES ☐ NO ☒ NOTE: The Examination Security Monitor Program detects pairs of students with unusually similar answer patterns on multiple-choice exams. Data generated by this program can be used as admissible evidence, either to initiate or corroborate an investigation or a charge of cheating under Section 16 of the Code of Student Conduct and Disciplinary Procedures. ANSWER BOOKLET REQUIRED: YES ☐ NO ☒ EXTRA BOOKLETS PERMITTED: YES ☐ NO ☒ ANSWER ON EXAM: YES ☒ NO ☐
	SHOULD THE EXAM BE RETURNED ☐ KEPT BY STUDENT ☒
CRIB SHEETS:	PERMITTED ☒ Specifications: The exam is open-book, so... NOT PERMITTED ☐
DICTIONARIES:	TRANSLATION ONLY ☐ REGULAR ☒ NONE ☐
CALCULATORS:	NOT PERMITTED ☐ PERMITTED (Non-Programmable) ☒ PERMITTED (Programmable) ☒
ANY SPECIAL INSTRUCTIONS:	See instructions on Page 2.

INSTRUCTIONS

- You have until April 23 at 9:00 AM to complete the exam and submit it on myCourses. No late submissions will be accepted.
- All solutions should be your own: you are not allowed to ask for anyone's help in order to solve the problems.
- Show and justify each step in the solution and simplify the answers, unless instructed otherwise.
- You may answer the questions directly in the exam: either on-screen with annotation tools, or by printing and scanning it. If not possible, then you can simply write your answers on any piece of paper and take pictures, but please collate those pictures into a single PDF file in order to submit them.
- Good luck! Stay safe and enjoy the summer!

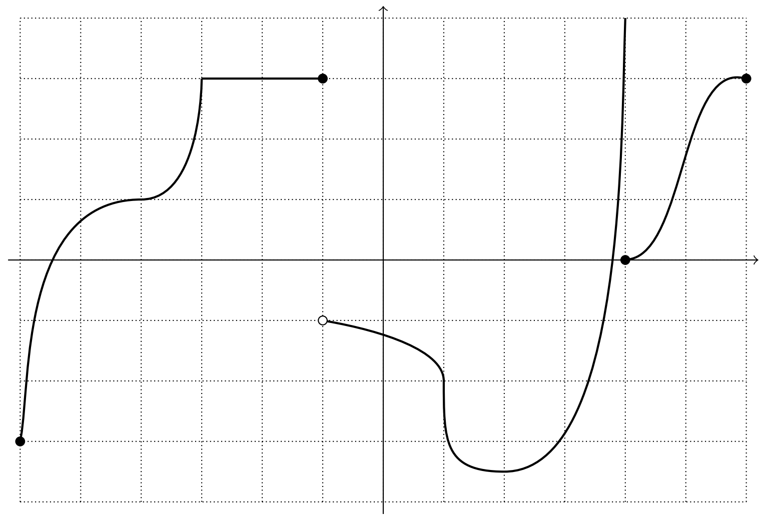
Question 1. (6 marks)

Find a and b such that the following function is differentiable everywhere.

$$f(x) = \begin{cases} x^2 + 6x + 12, & \text{if } x < -2 \\ ax + b, & \text{if } x \geq -2 \end{cases}$$

Question 2. (6 marks)

Consider the function $f(x)$ whose graph is given below. Find the set of all numbers x satisfying the following properties. *No justifications required.*



(a) f is not differentiable at x.

(b) $f'(x) < 0$.

(c) $f''(x) = 0$.

(d) $f''(x) > 0$.

(e) f has an absolute minimum at x.

(f) f has a local maximum at x.

Question 3. (18 marks)

Find the derivative of the following functions.

(a) $\sqrt{\cosh(2x)}$

(b) $\dfrac{\arcsin(x)}{\arccos(x)}$

(c) $e^{(x^e)} + x^{(e^x)}$

Question 4. (18 marks)

Find the following limits.

(a) $\displaystyle\lim_{x\to 0} \frac{5^{(1/x)}}{2^{(1/x^2)}}$

(b) $\lim_{x \to 0} \dfrac{\sinh(x) - x}{\sin(x) - x}$

(c) $\lim_{x \to \frac{\pi}{2}} \sec(x) - \tan(x)$

Question 5. (8 marks)

(a) Find the equation of the line tangent to $f(x) = \tan(x)$ at $x = \dfrac{\pi}{4}$.

(b) Deduce a linear approximation of $\tan\left(\dfrac{\pi}{5}\right)$.

Question 6. (10 marks)

The volume of a right circular cylinder of height h and radius r is $V = \pi r^2 h$. At the instant when $h = 6$ cm and increasing at the rate of 2 cm per second, we observe that the radius r is 10 cm and is decreasing at the rate of 1 cm per second. How fast is the volume changing at this time?

Question 7. (14 marks)

Given: $f(x) = \left(\dfrac{x+3}{x+1}\right)^2$, find the following.

(a) The domain of f.

(b) Critical numbers.

(c) Possible points of inflection.

(d) Horizontal asymptotes

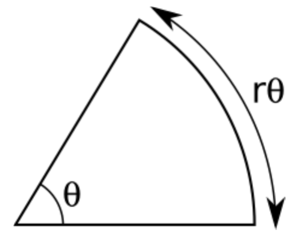
(e) Vertical asymptotes

(f) Fill a table of signs with all the information above in order to find the intervals where f is increasing/decreasing and the intervals of concavity.

(g) Sketch the graph of f, labelling all asymptotes, extrema and points of inflection on your picture.

Question 8. (10 marks)

Your neighbourhood pizza place sells pizza slices "by the perimeter". You can only afford a pizza slice with a perimeter of 60 cm. What radius should your pizza have in order to maximize its area? Don't forget to show that your answer is the maximum.



Hint: *The area of a sector of angle θ radians in a circle of radius r is $A = \frac{1}{2}\theta r^2$, and the arc length of this sector is $r\theta$.*

Question 9. (10 marks)

Consider the following equation: $3 - 2x = e^x - \cos(x)$.

(a) Use the Intermediate Value Theorem to show that the equation has *at least* one solution.

(b) Use the Mean Value Theorem to show that the equation has *at most* one solution.