Re Quiz 2-102-Solutions

Calculus, Part I (University of Pennsylvania)

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Math 1400 ReQuiz #2 11/11/22 **Quiz Session 0040-102: 10:15am**

1. Define a function F as
$$F(s) = \int_0^\infty e^{-st} f(t)\ dt.$$
where s is any positive constant. Compute F for $f(t) = 1 - 2t$ if it exists.

$$F(s) = \int_0^\infty e^{-st}(1-2t)\ dt$$
$$u = 1-2t,\ dv = e^{-st}\ dt \implies du = -2\ dt,\ v = \frac{-1}{s}e^{-st}$$
$$\begin{aligned} F(s) &= -\frac{1}{s}(1-2t)e^{-st}\ \Big|_0^\infty - \frac{2}{s}\int_0^\infty e^{-st}\ dt \\ &= -\frac{1}{s}(1-2t)e^{-st} + \frac{2}{s^2}e^{-st}\ \Big|_0^\infty \\ &= (0-0) - \left(-\frac{1}{s}(1-0)e^0 + \frac{2}{s^2}e^0\right) = \frac{1}{s} - \frac{2}{s^2} = \frac{s-2}{s^2} \end{aligned}$$

Where the limit as $t \to \infty$ gives 0 since any polynomial is vanishingly small compared to an exponential.

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2. Consider the solid formed by taking the region bounded by the curves $y = \sin x,\ y = \frac{1}{2}, x = \frac{\pi}{6},\ x = \frac{\pi}{2}$ and revolving it about $x = -2$.

   Set up but **do not evaluate** an integral which gives the volume of the solid

   (i) Using horizontal cross sections.

   (ii) Using vertical cross sections.

(i) $dV = 2\pi r h\ dx$ *where* $r = -2 + x$ *and* $h = \sin(x) - \frac{1}{2}$.

$$V = \int_{\pi/6}^{\pi/2} 2\pi(-2+x)\left(\sin(x) - \frac{1}{2}\right)\ dx$$

(ii) $dV = \pi(R^2 - r^2)\ dy$ *where* $r = 2 + \arcsin y$ *and* $R = 2 + \frac{\pi}{2}$. $\sin x$ *is* $\frac{1}{2}$ *at* $x = \pi/6$ *and 1 at* $x = \frac{\pi}{2}$ *so*

$$V = \int_{1/2}^{1} \pi\left(\left(2 + \frac{\pi}{2}\right)^2 - (2 + \arcsin y)^2\right)\ dy$$

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3. Estimate the value of the integral $\int_2^{11} x\cos(x^3)\,dx$ using 3 rectangles of equal width and left endpoints for height. Your final answer may be left as a sum.

The width of the rectangles should be $\Delta x = \frac{b-a}{n} = \frac{11-2}{3} = 3$. *Using left endpoints our representative* x *values should be* $x_1 = 2, x_2 = 2+3 = 5, x_3 = 2+3+3 = 8$. *The approximation is* $f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$. *This gives*

$$\int_2^{11} x\cos(x^3)\,dx \approx 2\cos(2^3)\cdot 3 + 5\cos(5^3)\cdot 3 + 8\cos(8^3)\cdot 3$$

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