MATHEMATICS 104, Fall 2021
Final Exam
December 20, 2021

Directions:

- You will have 100 minutes to complete this exam.

- There are eight problems. Each problem is worth four points. You will earn an additional two points if you fill out your cover sheet properly and submit all pages in the correct order in a single pdf file.

- Please write in pencil (preferred), or else a dark color of ink such as blue or black.

- Show all work to justify your solutions. Wrong solutions may receive partial credit for progress towards a correct solution. Correct answers without justification may be denied credit.

- You do not need to copy the questions onto your test.

- You may use one double-sided sheet of handwritten notes. No calculators or other resources are permitted.

- For each problem, you must use a separate sheet of paper. We will only grade one page of work per problem, so write all your work on the page for that problem. We are unable to consider work outside this space.

- Submit your work as a single pdf file, with your cover sheet first, followed by exactly one page per problem, in the correct order. If you do not answer a question, submit a blank or crossed-out page in its place. If you use a note sheet, scan it in as the last two pages of your test.

0. Create a cover sheet for your exam. In the top half of the page, write your full name. In the bottom half of the page, but not in the bottom two inches, write your 8-digit student ID number. Sign your name in the bottom inch of the page to indicate that you will comply with Penn's code of academic integrity. Other than your name, ID number, and signature, do not write anything on this page.

   Do not write your name nor ID number on any other page of your exam.

   You must submit your exam as a single pdf file, with the cover sheet first and the remaining pages in the correct order. You may include your note sheet as the last two pages.

   You will receive two points for correctly following these instructions.

1. Evaluate

$$\int \frac{-2\cos(x)}{(\sin x)^2 + 4\sin(x) + 3} \, dx.$$

2. Find the general solution of the differential equation

$$\frac{y'}{x} = y \sinh(x).$$

3. The function $F(x)$ is given by the following formula

$$F(x) = 3 + \int_2^{2x} \cos(t^2 - 4) \, dt.$$

Use a linear approximation to estimate $F(1.1)$.

4. Suppose $f(x) = 12 + 5(x - 1) + O((x - 1)^2)$ near $x = 1$. Evaluate

$$\lim_{h \to 0} \frac{f(1 + h) - 12}{h}.$$

5. The random variable X takes values on the interval $[1, \infty)$ and has the following probability density:

$$f(x) = \frac{3}{2} x^{-5/2} \, dx.$$

Calculate the mean μ and variance σ^2 for X.

6. Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 1$ and $y = x + 1$ around the line $y = 4$. You must complete all integration, but you do not need to simplify your final numerical answer.

7. Find the full interval of convergence for the following power series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n 4^n (x-2)^n}{n}.$$

8. Use the integral test to determine how many terms of the series

$$\sum_{n=1}^{\infty} \frac{6}{n^4}$$

need to be added together to guarantee an error less than 0.002.