



103Final 2012 Awans - Finals Practice Questions

Introduction to Calculus (University of Pennsylvania)



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1. Suppose that f and g are integrable and that

$$\int_1^2 f(x) dx = -4, \quad \int_1^5 f(x) dx = 6, \quad \int_1^5 g(x) dx = 8.$$

Find

I)

$$\int_5^1 g(x) dx$$

II)

$$\int_2^5 f(x) dx$$

III)

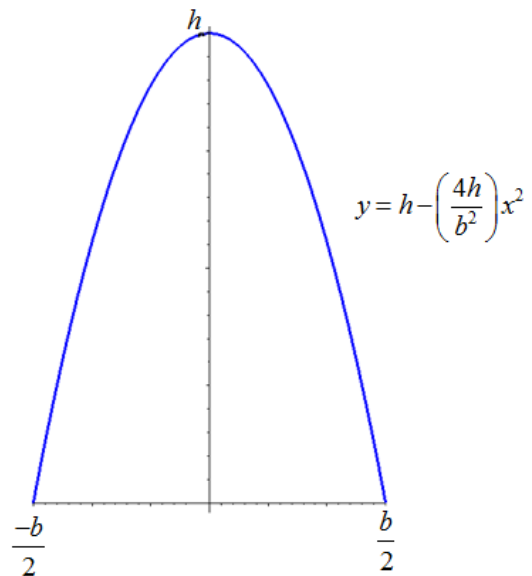
$$\int_1^5 [4f(x) - g(x)] dx$$

2. Archimedes (287-212 B.C.), inventor, military engineer, physicist, and the greatest mathematician of classical times in the Western world, discovered that area under a parabolic arch is two-thirds the base times the height. Let h = height and b = base

Use calculus to find the area and verify Archimedes' discovery for the parabola

$$y = h - \left(\frac{4h}{b^2} \right) x^2$$

whose graph is given to the right.
given that $h = 8$ and $b = 3$.



3. Let

$$y = \int_0^{e^{x^2}} \frac{1}{\sqrt{t}} dt$$

- | | |
|----------|-----------|
| A) e | E) $2e^2$ |
| B) e^2 | F) $4e^2$ |
| C) e^4 | G) 2 |
| D) $2e$ | H) 4 |

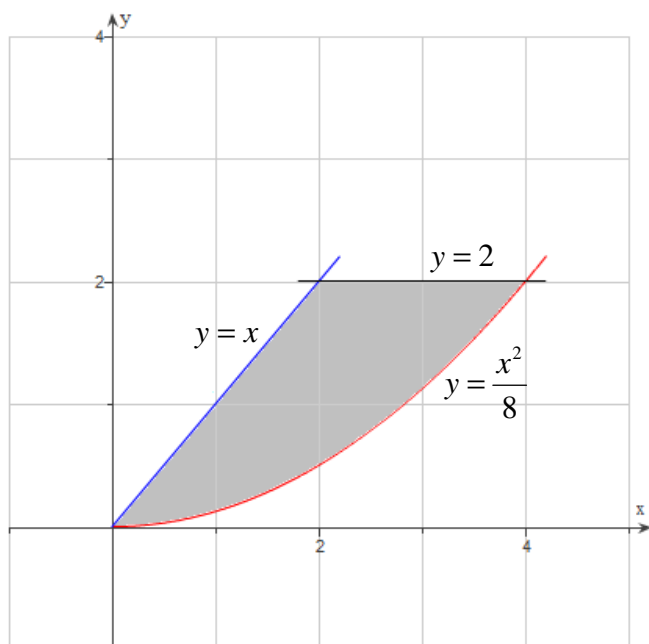
Find $y'(2)$.

4. Evaluate

$$\int_0^4 \frac{6x}{\sqrt{x^2 + 9}} dx$$

- | | |
|------------------|-------|
| A) $\frac{1}{2}$ | E) 6 |
| B) 1 | F) 8 |
| C) 2 | G) 12 |
| D) 4 | H) 16 |

5. Find the area of the shaded region



- | | |
|-------------------|-------------------|
| A) $\frac{13}{4}$ | E) $\frac{11}{4}$ |
| B) $\frac{10}{3}$ | F) $\frac{14}{5}$ |
| C) $\frac{16}{5}$ | G) 3 |
| D) $\frac{8}{3}$ | H) $\frac{19}{6}$ |

6. For what values of a, m and b does the function

$$f(x) = \begin{cases} 3, & x = 0 \\ -x^2 + 3x + a, & 0 < x < 1 \\ mx + b, & 1 \leq x \leq 2 \end{cases}$$

Satisfy the hypotheses of the Mean Value Theorem on the interval $[0, 2]$?

7. Suppose that $f'(x) = 2x$ for all x and that $f(-2) = 3$.

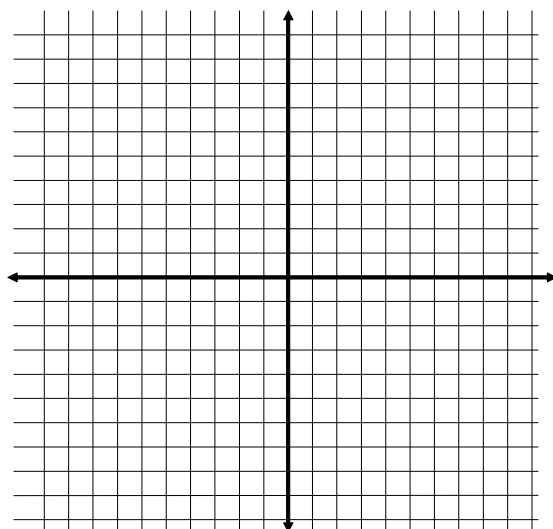
Find $f(2)$.

- | | |
|-------|------|
| A) -2 | E) 2 |
| B) -1 | F) 3 |
| C) 0 | G) 4 |
| D) 1 | H) 5 |

8. Let

$$f(x) = \frac{x^2 - 3}{x - 2}.$$

- I) Find the interval(s) where the functions is increasing and where the function is decreasing.
- II) Find the critical points of $f(x)$, if any, identify whether these lead to local maximum values, local minimum values, or neither.
- III) Find the interval(s) where the function is concave up and where the function is concave down.
- IV) Find the inflection point(s) of $f(x)$, if any.
- V) Sketch the graph of $f(x)$. Take into account the domain, symmetry, intercepts, asymptotes.

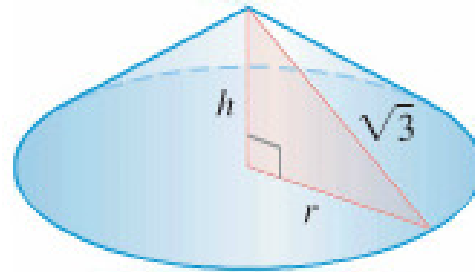


9. Evaluate the limit

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin(x)}$$

- | | |
|------------------|------|
| A) $\frac{1}{2}$ | E) 0 |
| B) $\frac{3}{4}$ | F) 1 |
| C) $\frac{1}{4}$ | G) 2 |
| D) $\frac{2}{3}$ | H) 3 |

10. A right triangle whose hypotenuse is $\sqrt{3}$ m long is revolved about one of its legs to generate a right circular cone. Find the radius, height, and volume of the cone of greatest volume that can be made this way.



11. Let

$$y = (1 + \cos 2t)^{-4}$$

Find $y'\left(\frac{\pi}{4}\right)$.

- | | |
|------------------|------------------|
| A) 4 | E) 8 |
| B) $\frac{1}{3}$ | F) 2 |
| C) $\frac{2}{3}$ | G) $\frac{1}{4}$ |
| D) $\frac{1}{2}$ | H) $\frac{1}{8}$ |

12. Find the equation of the tangent line to the curve

$$6x^2 + 3xy + 2y^2 + 17y - 6 = 0$$

at $(-1, 0)$.

A) $y = \frac{1}{7}x + \frac{1}{7}$

E) $y = \frac{2}{7}x + \frac{2}{7}$

B) $y = \frac{3}{7}x + \frac{3}{7}$

F) $y = \frac{4}{7}x + \frac{4}{7}$

C) $y = \frac{5}{7}x + \frac{5}{7}$

G) $y = \frac{6}{7}x + \frac{6}{7}$

D) $y = \frac{-1}{7}x - \frac{1}{7}$

H) $y = \frac{-3}{7}x - \frac{3}{7}$

13. Let

$$y = \frac{\ln x}{x}$$

Find $y'(\sqrt{e})$.

A) $\frac{1}{e^2}$

E) $\frac{e}{8}$

B) $\frac{1}{2e}$

F) $\frac{\sqrt{e}}{4}$

C) $\frac{e}{2}$

G) $\frac{2}{\sqrt{e}}$

D) $\frac{1}{2}$

H) 0

14. Let

$$y = \ln(\arctan x)$$

Find $y'(1)$.

A) $\frac{1}{2}$

E) 1

B) $\frac{\sqrt{3}}{2}$

F) $\frac{4}{\pi}$

C) π

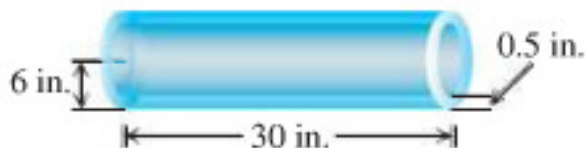
G) $\frac{3}{\pi}$

D) $\frac{\pi}{2}$

H) $\frac{2}{\pi}$

15. Charlotte flies a kite at a height of 300 ft, the wind carries the kite horizontally away from her at a rate of 25 ft./sec. How fast must she let out the string when the length of the string is 500 ft.?

16. Estimate the volume of material in a cylindrical shell with length 30 in., radius 6 in., and shell thickness 0.5 in. using differentials.



17. Find the slope of the tangent line to the function $y = 5 - x^2$ at the point $(1, 4)$ **using the definition of the derivative.**

18. Let $f(x) = \sqrt{19 - x}$, $x_0 = 10$, and $\varepsilon = 1$. Find $L = \lim_{x \rightarrow x_0} f(x)$. Then find a number $\delta > 0$ such that for all x

$$0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$$

19. For what value of a is

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$$

continuous at every x ?

20. Evaluate

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 3x} - \sqrt{x^2 - 2x})$$

Answers:

1. I) -8 II) 10 III) 16

2. 16

3. F

4. G

5. B

6. $a = 3, m = 1, b = 4$

7. F

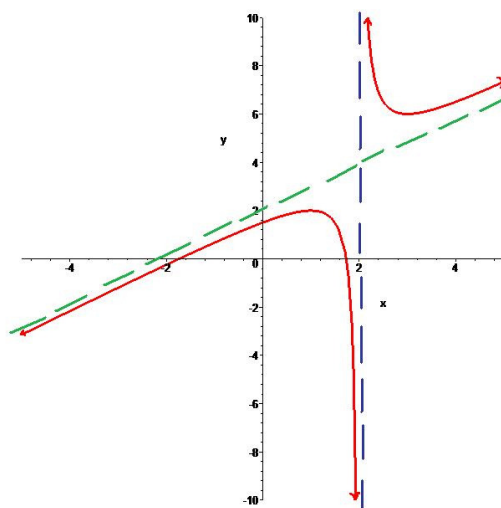
8. I) Increasing: $(-\infty, 1) \cup (3, \infty)$ Decreasing: $(1, 3)$

II) Critical points: $x = 1$ leads to a local maximum value,
 $x = 3$ leads to a local minimum value

III) Concave Up: $(2, \infty)$ Concave Down: $(-\infty, 2)$

IV) No inflection points

V)



Domain: $(-\infty, 2) \cup (2, \infty)$, No Symmetry,

Vertical asymptote $x = 2$, Slant asymptote $y = x + 2$

x -int. $(\sqrt{3}, 0), (-\sqrt{3}, 0)$, y -int. $(0, \frac{3}{2})$

9. G

10. $h = 1, r = \sqrt{2}, V = \frac{2\pi}{3}$

11. E

12. G

13. B

14. H

15. 20 ft/s

16. $180\pi \text{ in}^3$

17. $m = -2$

18. $L = 3, \delta = 5$

19. $a = \frac{4}{3}$

20. $\frac{5}{2}$