Math 140 Tutorial2

Calculus 1 (McGill University)

Main problems:

1. For this question, reference the graph of $f(x)$ below and compute the following quantities. If they do not exist, write DNE.

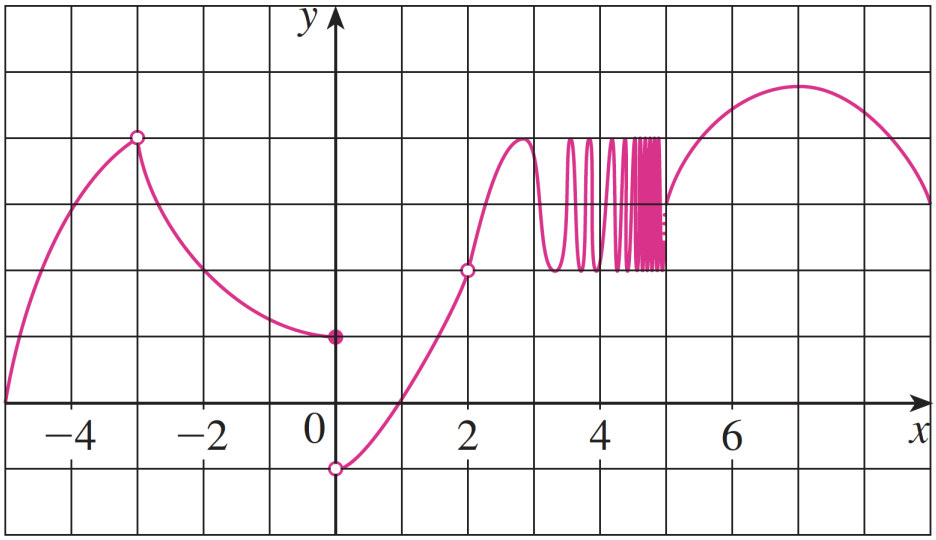

Figure 1: A graph of the function $f(x)$

(a) $\lim_{x \to -3^+} f(x) =$

(b) $\lim_{x \to -3^-} f(x) =$

(c) $\lim_{x \to -3} f(x) =$

(d) $\lim_{x \to 0^-} f(x) =$

(e) $\lim_{x \to 0} f(x) =$

(f) $\lim_{x \to 5^-} f(x) =$

(g) $\lim_{x \to 5^+} f(x) =$

(h) $f(2) =$

2. Using limit laws and squeeze theorem compute the following: Clearly state if the limit exists, diverges to infinity, diverges to negative infinity or if the limit does not exist.

   (a) $\lim_{t\to 1} \dfrac{t}{t^3 - t^2}$ (b) $\lim_{x\to 0^-} \left(\dfrac{1}{x} - \dfrac{1}{|x|}\right)$ (c) $\lim_{t\to 0} t^2 \cos\left(\dfrac{1}{t^2}\right)$

3. Using the ϵ, δ definition of a limit, show that $x^2 + 1$ approaches 26 as x approaches 5.

Extra practice problems

These problems are for you to get supplementary practice on the topics covered by the Main problems. Work on them at home and use any of the available resources to check your work (Instructor/TAs office hours, Frezca and the 9th floor help desk). You will be responsible for them in your future examinations.

1. Using limit laws and squeeze compute the following: Clearly state if the limit exists, diverges to infinity, diverges to negative infinity or if the limit does not exist.

   (a) $\lim_{x\to -3} \dfrac{x^2 + x - 6}{x + 3}$ (b) $\lim_{x\to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{|x|}\right)$ (c) $\lim_{s\to 0^+} \sqrt{s}\, e^{\sin\left(\frac{1}{s}\right)}$

2. Given any integer $m > 0$, use the fact that $\lim_{x\to 0} \dfrac{\sin(x)}{x} = 1$ (shown later in the course) to show that

$$\lim_{t\to 0} \frac{\sin(t)}{\sin(mt)} = \frac{1}{m}$$

3. Using the ϵ, δ definition of a limit, show that $\sqrt{x+1}$ approaches 2 as x approaches 3

Challenge problems

Is there a number a such that

$$\lim_{x\to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$$

exists and is finite (i.e exists but is not $\pm\infty$)? If that's the case find such an a and the value of the limit.