Penn ID:

Math 1400 Quiz #4 10/21/22 **Quiz Session 0040-102: 10:15am**

1. Evaluate the following integral.
$$\int \frac{8e^{2x} - 3e^x}{e^{2x} + 3e^x - 4}\ dx$$

Let $u = e^x$ so $du = e^x\ dx$. Substitute and use partial fractions.

$$\begin{aligned}\int \frac{(8u-3)\ du}{u^2+3u-4}\ du &= \int \frac{8u-3}{(u+4)(u-1)}\ du \\ &= \int \frac{7}{u+4} + \frac{1}{u-1}\ du \\ &= 7\ln|u+4| + \ln|u-1| + C \\ &= 7\ln|e^x+4| + \ln|e^x-1| + C\end{aligned}$$

2. The *Lotka-Volterra model* of predation describes the population of a prey species over time. The rate of change of population of the prey species is determined by two competing factors: natural population growth and predation. Population growth is proportional to both the current prey population and how far below the carrying capacity the prey population is. Population loss to predation is proportional to both the prey population and the predator population. In a particular species, the carrying capacity for the prey species is 300, the predator population is 5, all constants of proportionality are 1, and prey population starts at 150. Write down a differential equation and any necessary initial conditions which would allow you to find the prey population at any time. **You do not need to solve the differential equation.**

   Let $U(t)$ and $V(t)$ represent the size of the prey and predator species respectively at time t. We have initial conditions $U(0) = 150,\ V(0) = 5$. We can model $\frac{dU}{dt}$ as the rate of growth of the prey population minus the rate at which $U(t)$ is decreased due to predators. The former is $U(C - U)$ where C is the carrying capacity. The latter is UV. Since $C = 300$ we have the following initial value problem to solve for $U(t)$.

$$\begin{cases} \dfrac{dU}{dt} = U(300 - U) - UV \\[2ex] U(0) = 150 \end{cases}$$

   Here $V = 5$ is constant giving us

$$\begin{cases} \dfrac{dU}{dt} = U(300 - U) - 5U \\[2ex] U(0) = 150 \end{cases}$$

Penn ID:

Math 1400 Quiz #4 10/21/22 **Quiz Session 0040-102: 10:15am**

3. Find the general solution to the following differential equation. Write it explicitly as a function of x.

$$x\frac{dy}{dx} + \frac{1}{2}y = x\ln x$$

Rewrite in standard form.

$$\frac{dy}{dx} + \frac{1}{2x}y = \ln x$$

Find the integrating factor $\mu = e^{\int \frac{1}{2x}\ dx} = e^{\ln x^{1/2}} = \sqrt{x}$. *Multiply and integrate.*

$$\begin{aligned}\sqrt{x}\frac{dy}{dx} + \frac{\sqrt{x}}{2}y &= \sqrt{x}\ln x\\ \frac{d}{dx}\left[y\sqrt{x}\right] &= \sqrt{x}\ln x\\ y\sqrt{x} &= \int \sqrt{x}\ln x\ dx\end{aligned}$$

Integrate by parts with $u = \sqrt{x},\ dv = \ln x\ dx$.

$$\begin{aligned}y\sqrt{x} &= \frac{2}{9}x^{3/2}\left(3\ln(x) - 2\right)\right) + C\\ y &= \frac{2}{9}x\left(3\ln(x) - 2\right)\right) + \frac{C}{\sqrt{x}}\end{aligned}$$

Name:

Penn ID:

Math 1400 Quiz #4 10/21/22

Quiz Session 0040-102: 10:15am