Answer pratice 1

Calculus 1 (McGill University)

McGill Math 140 Fall 2023: Multiple choice practice 1.
mailto:dylan.cant@mcgill.ca

Introduction

A selection of multiple choice questions.

1. *Limit practice 1*

1.1. $\lim_{x\to 0} \dfrac{1-1/x}{1+1/x^2}$ equals:

(a) 1

(b) 0

(c) $+\infty$

(d) $-\infty$

(e) DNE

1.2. $\lim_{x\to 0+} x^2\cos(1/x)$ equals:

(a) $1/2$

(b) 1

(c) 0

(d) -1

(e) DNE

1.3. $\lim_{x\to\infty} \dfrac{9^x+3^x}{3^x+1}$ equals:

(a) 3

(b) $(\ln(9)+\ln(3))/(\ln(3)+\ln(1))$

(c) 1

(d) ∞

(e) DNE

1.4. $\lim_{x\to\infty} \dfrac{(4+3x)^{1/2}-4}{x-4}$ equals:

(a) DNE

(b) $3^{1/2}$

(c) 0

(d) ∞

(e) $\frac{1}{2\sqrt{3}}$

2. *Derivative practice*

2.1. If $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ then $f'(1) - f(1)^2$ equals:

(a) $\dfrac{e^1 - e^{-1}}{e^1 + e^{-1}}$

(b) $\dfrac{(e^1 - e^{-1})^2}{(e^1 + e^{-1})^2}$

(c) $\dfrac{e^1 + e^{-1}}{e^1 + e^{-1}} + \dfrac{(e^1 - e^{-1})^2}{(e^1 + e^{-1})^2}$

(d) 1

(e) None of the above.

2.2. If $f(x) = \dfrac{e^x \sin(x)}{1 + e^x \sin(x)}$ then $f'(0)$ equals:

(a) 1/2

(b) 1

(c) 2

(d) 0

(e) None of the above.

2.3. If $f(x) = \dfrac{\sqrt{1 + x + x^2 + x^3}}{1 + \sin(x)}$ then $f'(0)$ equals:

(a) 1

(b) 1/2

(c) 0

(d) $-1/2$

(e) None of the above.

2.4. If $f(x) = \ln(\ln(\ln(2^x)))$ then $\ln(\ln(2))\ln(2)f'(\ln(2))$ equals:

(i) 1

(ii) 1/2

(iii) 0

(iv) $-1/2$

(v) None of the above.

3. *Limit from the derivative*

3.1. Let $f(x) = \dfrac{1}{x+3}$. Then $g(h) = \dfrac{f(1+h) - f(1)}{h}$ equals:

(a) $1/4$
(b) $-1/(h+3)^2$
(c) $-1/4$
(d) $1/(h+4)^2$
(e) None of the above.

$$\frac{\frac{1}{h+4} - \frac{1}{4}}{h} = \frac{-1}{4(h+4)}$$

3.2. Let $f(x) = \dfrac{e^{1+x}}{x}$. Then $\lim_{h\to 0} \dfrac{1}{f(x)} \dfrac{f(x+h) - f(x)}{h}$ equals:

(a) $1 - 1/x$
(b) $-1/x$
(c) $e(e^x x - e^x)/x^2$
(d) $e^x - e^x/x$
(e) None of the above.

$$= \frac{e^h x - x - h}{(x+h)h} = \frac{x}{x+h} \frac{e^h - 1}{h} - \frac{1}{x+h}$$

$$\to 1 - 1/x$$

3.3. Let $f(x) = \dfrac{1}{(x+1)^2}$. Then $\lim_{h\to 0} \dfrac{f(h) - f(0)}{h}$ equals:

(a) -2
(b) -1
(c) 0
(d) 1
(e) None of the above.

3.4. Let $f(x) = e^x + e^{2x} + e^{3x}$. What is $\lim\limits_{h\to 0} \dfrac{f'(h) - 6}{h}$?

(i) 6
(ii) 9
(iii) 11
(iv) 14
(v) None of the above.

Note the "prime" appearing in the limit

4. *Limit practice 2*

4.1. $\lim_{x\to 0-} \dfrac{\sin(2|x|)\sin(3x)}{\cos(x)-1}$ equals:

too hard problem...

(a) a finite positive number

(b) a finite negative number

(c) zero

(d) $+\infty$

(e) $-\infty$

4.2. $\lim_{x\to -\infty} \dfrac{\sqrt{1-x^3}}{x^2-x-1}$ equals:

(a) a finite positive number

(b) a finite negative number

(c) zero

(d) $+\infty$

(e) $-\infty$

4.3. $\lim_{x\to 0+} \dfrac{(1^x-2^x)(3^x-4^x)}{5^x-6^x}$ equals:

(a) a finite positive number

(b) a finite negative number

(c) zero

(d) $+\infty$

(e) $-\infty$

4.4. $\lim_{x\to 0+} \dfrac{1+|x-1|-|2-3x|}{x}$ equals:

(a) a finite positive number

(b) a finite negative number

(c) zero

(d) $+\infty$

(e) $-\infty$

4.5. $\lim_{x\to 0+} \dfrac{x\sin(1/x)}{\sqrt{1+x^2}}$ equals:

(a) a finite positive number

(b) a finite negative number

(c) zero

(d) $+\infty$

(e) $-\infty$

4.6. $\lim_{x\to -\infty} \dfrac{1+\frac{1}{1+x}}{1-\frac{x}{x+1}}$ equals:

(a) a finite positive number

(b) a finite negative number

(c) zero

(d) $+\infty$

(e) $-\infty$

4.7. $\lim_{x\to \infty} \dfrac{(1+\cos(x))^3}{\cos(x)+\ln(1+x^2)}$ equals:

(a) a finite positive number

(b) a finite negative number

(c) zero

(d) $+\infty$

(e) $-\infty$

4.8. $\lim_{x\to 4+} \dfrac{\sqrt{x^2-3x-4}}{\sqrt{x^2-8x+16}}$ equals:

(a) a finite positive number

(b) a finite negative number

(c) zero

(d) $-\infty$

(e) $+\infty$

4.9. $\lim_{x\to 0} \dfrac{\sin(x)(e^x-1)}{\cos(x)(e^{2x}-1)}$ equals:

(a) a finite positive number

(b) a finite negative number

(c) zero

(d) $-\infty$

(e) $+\infty$

5. *Word problems*

5.1. A particle travels along the y-axis according to the formula $y(t) = t^3 - 2t$. Find the average speed of the particle over the interval $[0, 2^{1/2}]$.

(i) $1/2^{1/2}$

(ii) 0

(iii) $2^{1/2}$

(iv) 1

(v) None of the above.

5.2. A particle travels along the x-axis according to $x(t) = \ln(2 + \cos(t))$. Find the average speed of the particle over the interval $[0, 2\pi]$.

(i) $\ln(3)/\pi$

(ii) $\ln(3^{1/2})/\pi$

(iii) $[\ln(3) - \ln(2)]/2\pi$

(iv) The function is discontinuous on the interval and the average speed cannot be computed.

(v) None of the above.

5.3. Let $a(t) = (1+0.05)^t$ be the amount of money (in millions) in an offshore investment account, where t is measured in months. Let $A = a'(0)$, $B = a'(12)$ and let C be the average rate of change of $a(t)$ over $[0, 12]$. Which is correct:

(i) $A < B < C$

(ii) $C < A < B$

(iii) $A < C < B$

(iv) $B < C < A$

(v) None of the above.

5.4. Let $B(t) = \gamma^t/(1 + \gamma^t)$ be the amount of bacteria in a petri dish, where t is measured in hours and $B(t)$ is measured in milligrams, and $\gamma = 2$ is the bacterial growth coefficient. Out of the options, what is the *smallest* time the scientist must wait until $B(t) \geq 2\,\text{mg}$?

(i) 1 hour is enough ($t = 1$)

(ii) 2 hours is enough ($t = 2$)

(iii) 3 hours is enough ($t = 3$)

(iv) 10 hours is enough ($t = 10$) *you may use* $2^{10} = 1024$

(v) None of the above.

$B(t) < 2$ for all time.

6. *Implicit differentiation*

6.1. If $f(x)^2 + x^4 = 5$ where f is a differentiable function satisfying $f(1) = 2$, what is $f'(1)$?

(i) 2

(ii) -1

(iii) 1

(iv) -2

(v) None of the above.

6.2. What is the slope of the tangent line to the curve $xy + y^2 = 2$ at $(1, 1)$?

(i) $-1/6$

(ii) $-1/3$

(iii) $-1/2$

(iv) Cannot be determined with the information given.

(v) None of the above.

6.3. Let $g : (0, 1) \to \mathbb{R}$ be a differentiable function satisfying $\sin(g(x)) = x$ and $g(0.5) = \pi/6$. What is $g'(0.5)$?

(i) $2^{-1/2}$

(ii) $3^{1/2}/2$

(iii) $-2^{1/2}$

(iv) $-2/3^{1/2}$

(v) None of the above.

$+ \frac{2}{3^{1/2}}$

6.4. Let $g : (0, 1) \to \mathbb{R}$ be a differentiable function satisfying $\cos(g(x)) = x$ and $g(0.5) = 2\pi/3$. What is $g'(0.5)$?

(i) $-2^{1/2}/3$

(ii) $3/2^{1/2}$

(iii) $-3/2^{1/2}$

(iv) $2^{1/2}/3$

(v) None of the above.

$- \frac{2}{3^{1/2}}$

6.5. Find the ~~equation~~ of the tangent line to $x = y^3 + y^2$ at the point $(2, 1)$.

(i) 1

(ii) 5

(iii) 1/5

(iv) $-1/5$

(v) None of the above.

slope

7. *Continuity*

A *discontinuity* of an expression $f(x)$ is a point a which is not in the maximal domain for which $f(x)$ is well-defined, or for which $\lim_{x\to a} f(x) \neq f(a)$.

If $f(a)$ can be redefined so that $\lim_{x\to a} f(x) = f(a)$, then we say a is a removable discontinuity.

If $\lim_{x\to a+} f(x)$ and $\lim_{x\to a-} f(x)$ both exist and are different, then we say a is a jump discontinuity.

7.1. Let:

$$f(x) = \frac{x^2 + 3x - 3\sqrt{2} - 2}{x^2 + 6x + 7}.$$

How many discontinuities does this expression have, how many are removable, and how many are jump discontinuities?

(i) 2 and 1 and 0

(ii) 1 and 0 and 0

(iii) 2 and 2 and 0

(iv) 2 and 1 and 1

(v) None of the above

7.2. Let:

$$f(x) = \frac{|1 - x|}{1 - x} + \frac{|2 - x|}{2 - x}.$$

How many discontinuities does this expression have, how many are removable, and how many are jump discontinuities?

(i) 2 and 2 and 0

(ii) 2 and 0 and 2

(iii) 2 and 1 and 1

(iv) 2 and 0 and 0

(v) None of the above

7.3. Let:

$$f(x) = \frac{e^{\ln(2)+x} - e^{\ln(2)}}{x} + \frac{\sin(x-1)}{x-1}.$$

How many discontinuities does this expression have, how many are removable, and how many are jump discontinuities?

(i) 2 and 2 and 0

(ii) 2 and 0 and 2

(iii) 2 and 1 and 1

(iv) 2 and 0 and 0

(v) None of the above

8. *Asymptotes*

8.1. Find all the asymptotes of the expression $f(x) = (e^x + 1)/(e^x - 1)$, and add them up (i.e., if $y = a_1, a_2, \ldots$ and $x = b_1, b_2, \ldots$ are the horizontal and vertical asymptotes, you should return $(a_1 + a_2 + \ldots) + (b_1 + b_2 + \ldots)$). If there are no asymptotes, return NA.

(a) -1
(b) 0
(c) 1
(d) NA
(e) None of the above.

8.2. Find all the asymptotes of the expression $f(x) = (x^3+2x+1)/(x^3+x^2)$, and add them up. If there are no asymptotes, return NA.

(a) -1
(b) 0
(c) 1
(d) NA
(e) None of the above.

8.3. Find all the asymptotes of the expression $f(x) = (2x\sin(x)+1)/(x^2+1)$, and add them up. If there are no asymptotes, return NA.

(a) -1
(b) 0
(c) 1
(d) NA
(e) None of the above.

8.4. Find all the asymptotes of the expression $f(x) = x(x-1)^{-1} - (x-2)^{-1}$, and add them up. If there are no asymptotes, return NA.

(a) -1
(b) 0
(c) 1
(d) NA
(e) None of the above.

9. *Piecewise functions*

9.1. Find values of $m, b \in \mathbb{R}$ so that the following function is continuous.

$$f(x) = \begin{cases} (\cos(3x) - 1)/x & \text{for } x < 0 \\ mx + b & \text{for } x \in [0, 1] \\ \sin(3x - 3)/(x - 1) & \text{for } x > 1 \end{cases}$$

What is $m + b$?

(a) 1
(b) 2
(c) 3
(d) 4
(e) None of the above.

9.2. Find values of $k, m, \ell \in \mathbb{R}$ so that the following function is differentiable.

$$f(x) = \begin{cases} \dfrac{1 - e^{-2x}}{1 - e^{-x}} + \ell x & \text{for } x < 0 \\ m & \text{for } x = 0 \\ \dfrac{1 - e^{2x}}{1 - e^{x}} + k & \text{for } x > 0 \end{cases}$$

What is $k + m + \ell$?

(a) 1
(b) 2
(c) 3
(d) 4
(e) None of the above.

9.3. Let $f(x) = |1 - |1 - |x|||$. It can be shown that:

$$f(x) = \begin{cases} ax + b & \text{for } x \leq -2 \\ cx + d & \text{for } -2 < x \leq -1 \\ ex + f & \text{for } -1 < x \leq 0 \\ gx + h & \text{for } 0 < x \leq 1 \\ ix + j & \text{for } 1 < x \leq 2 \\ kx + \ell & \text{for } 2 < x \end{cases}$$

What is $a + c + e$?

(a) -2
(b) -1
(c) 0
(d) 1
(e) None of the above.