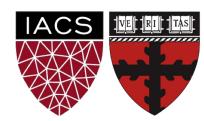
# Lab 11: Reinforcement Learning

With a focus on Homework 8

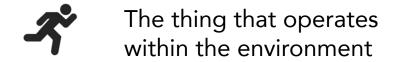
## Harvard IACS

CS109B

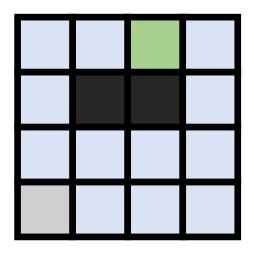
Chris Tanner, Pavlos Protopapas, Mark Glickman



Agent



## **RL** Environment



#### **RL** Environment

Agent

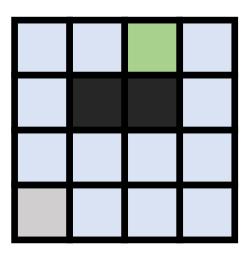


The thing that operates within the environment

States



Static representations that define the current environment



#### **RL** Environment

Agent



The thing that operates within the environment

States

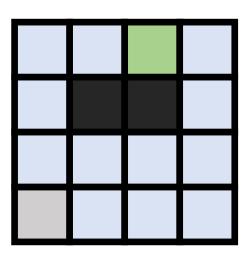


Static representations that define the current environment

Actions



An agent's operation that takes him/her from state **s** to state **s'** 



#### **RL** Environment

Agent



The thing that operates within the environment

States



Static representations that define the current environment

Actions

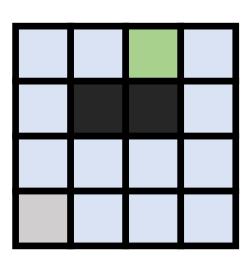


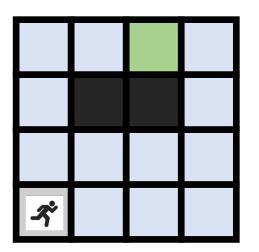
An agent's operation that takes him/her from state **s** to state **s'** 

Reward



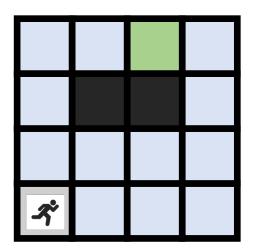
A real-valued # that represents the goodness for the agent's being in a given state **s** 



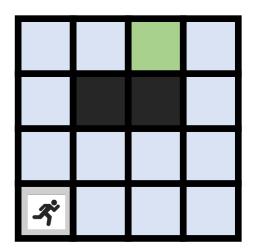






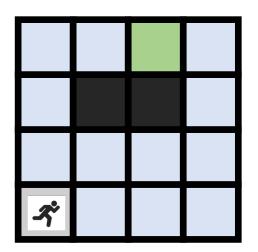


She performs an action  $a \longrightarrow and becomes in state s'$ 



She performs an action  $a \longrightarrow$  and becomes in state s'

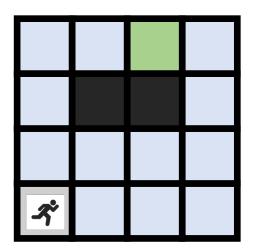
Being in each state s' yields a reward r (e.g, 3.6)

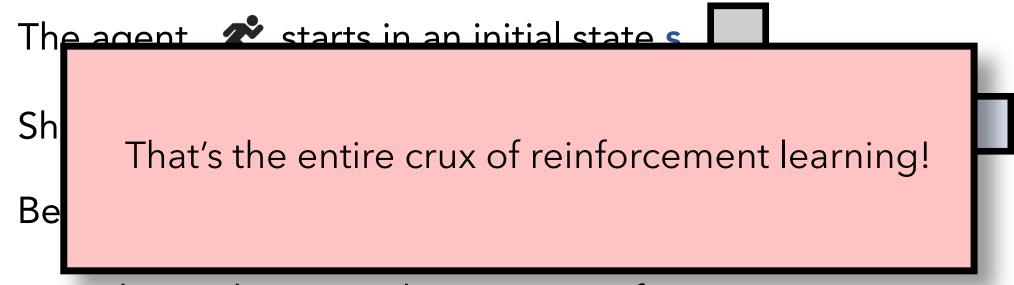


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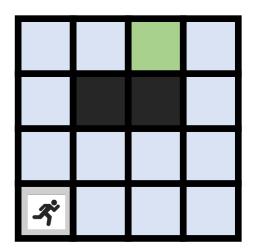
Being in each state s' yields a reward r (e.g, 3.6)

How do we determine how to move from state-to-state so as to receive maximum reward r?





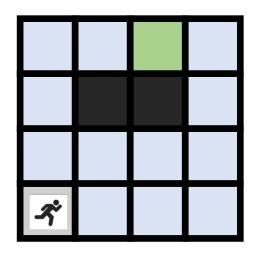
How do we determine how to move from state-to-state so as to receive maximum reward r?



Given a state  $\mathbf{s}$ , and an action  $\mathbf{a}$ , estimate the reward  $\mathbf{r}$ .

A policy  $\pi(s)$  takes a state s and executes an action a

$$\pi(\mathbf{s}) \rightarrow \mathbf{a}$$



So, there can be many policies  $\pi_1, \pi_2, ..., \pi_n$  (some better than others).

# which policy to execute?

How do we determine how to move from state-to-state so as to receive maximum reward r?

The one that gives us the highest reward! Let's estimate each policy  $\pi_i$  via a "value-function"  $v_\pi(s)$ , where s is our starting state

$$v_{\pi}(s) = \mathbb{E}_{\pi} \left[ G_t | S_t = s \right] = \mathbb{E}_{\pi} \left[ \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} | s \right]$$

$$=\sum_{a}\pi(a|s)\sum_{s'}\sum_{r}p(s',r|s,a)[r+\gamma v_{\pi}(s')]$$

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Our starting state s

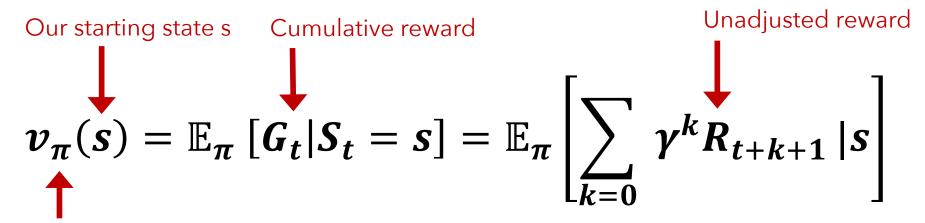
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Our starting state s Cumulative reward

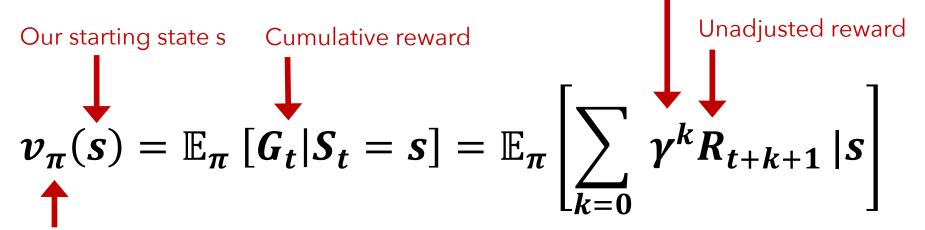
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$$= \sum_{a} \pi(a|s) \sum_{s'} \sum_{r} p(s',r|s,a) [r + \gamma v_{\pi}(s')]$$

Weakened based on how far into the future it is



$$=\sum_{a}\pi(a|s)\sum_{s'}\sum_{r}p(s',r|s,a)[r+\gamma v_{\pi}(s')]$$

$$v_{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} \sum_{r} p(s', r|s, a) [r + \gamma v_{\pi}(s')]$$

state-to-state transitions T

reward R

T=

R=

$$v_{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} \sum_{r} p(s',r|s,a) [r + \gamma v_{\pi}(s')]$$

- Estimate  $v_{\pi}$
- The above equation is general and works for stochastic situations.
- We have fixed state-transitions and rewards though (we can make our life easier).

$$v_{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} \sum_{r} p(s',r|s,a) [r + \gamma v_{\pi}(s')]$$
$$= \sum_{a} \pi(a|s) \sum_{s'} p(s',r|s,a) (R(s,a) + \gamma v_{\pi}(s'))$$

- Estimate  $v_{\pi}$
- The above equation is general and works for stochastic situations.
- We have fixed state-transitions and rewards though (we can make our life easier).
- What can we define as a constant?

$$v_{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} \sum_{r} p(s',r|s,a) [r + \gamma v_{\pi}(s')]$$
$$= \sum_{a} \pi(a|s) \sum_{s'} p(s',r|s,a) (R(s,a) + \gamma v_{\pi}(s'))$$

· As a sanity check, a geometric series is defined to have a sum:

$$a + ar + ar^2 + ar^3 + ar^4 + \dots = \sum_{k=0}^{\infty} ar^k = rac{a}{1-r}, \; ext{for} \; |r| < 1.$$

$$v_{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} \sum_{r} p(s',r|s,a) [r + \gamma v_{\pi}(s')]$$

$$= \sum_{a} \pi(a|s) \sum_{s'} p(s',r|s,a) (R(s,a) + \gamma v_{\pi}(s'))$$

• What is a Q-function?

$$v_{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} \sum_{r} p(s',r|s,a) [r + \gamma v_{\pi}(s')]$$

$$= \sum_{a} \pi(a|s) \sum_{s'} p(s',r|s,a) (R(s,a) + \gamma v_{\pi}(s'))$$

• What is a Q-function?