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Chapter 10

Vehicle Routing Problems with Profits

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10.1 ▪ Introduction

The key characteristic of the class of *Vehicle Routing Problems with Profits* (VRPPs) is that, contrary to what happens for the most classical vehicle routing problems, the set of customers to serve is not given. Therefore, two different decisions have to be taken: (i) which customers to serve, and (ii) how to cluster the customers to be served in different routes (if more than one) and order the visits in each route. In general, a *profit* is associated with each customer that makes such a customer more or less attractive. Thus, any route or set of routes, starting and ending at a given depot, can be measured both in terms of cost and in terms of profit. The difference between route profit and cost may be maximized, or the profit or the cost optimized with the other measure bounded in a constraint.

There are several types of applications that can be modeled by means of a problem of this class:

- scheduling of the daily operations of a steel rolling mill (see, e.g., Balas [13, 16]);
- design of tourist trips to maximize the value of the visited attractions in a limited period (see, e.g., Vansteenwegen and Van Oudheusden [109]);
- identification of suppliers to visit to maximize the recovered claims with a limited number of auditors (see Ilhan, Iravani, and Daskin [61]);
- recruiting of athletes from high schools for a college team (see Butt and Cavalier [24]);
- planning of the visits of a salesperson (see, e.g., Ramesh and Brown [89]);
- routing of oil tankers to serve ships at different locations (see Golden, Levy, and Vohra [57]);

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- delivery of home heating fuel, where the urgency of a customer request for fuel is treated as a score (see Golden, Assad, and Dahl [56]);
- reverse logistics problem of a firm that aims to collect used products from its dealers (see Aras, Aksen, and Tekin [2]);
- customers selection in less-than-truckload transportation (see Archetti et al. [8]);
- service outsourcing of unprofitable customers (see Chu [31]).

Despite the practical interest in considering profits in vehicle routing, most of the existing literature concentrates on the single-vehicle case of the problem. Clearly, these findings form a valuable starting point also for more general VRPPs, and we therefore examine them in detail in the first part of this chapter.

The most basic problems of this class with only one route are often presented as variants of the *Traveling Salesman Problem* (TSP) (see Fischetti, Salazar González, and Toth [47]). The *Orienteering Problem* (OP) was first studied by Tsiligirides [105] and Golden, Levy, and Vohra [57]. The name is derived from the orienteering sport where each participant has to maximize the total collected prize associated with visited points while returning to the starting point within a given time limit. The OP is also known with the names *Selective Traveling Salesperson Problem* (see, e.g., Laporte and Martello [74], Gendreau, Laporte, and Semet [53], and Thomadsen and Stidsen [102]), *Maximum Collection Problem* (see, e.g., Kataoka and Morito [65] and Butt and Cavalier [24]), and *Bank Robber Problem* (see Awerbuch et al. [12]). The other two basic routing problems with profits and one vehicle only are the *Profitable Tour Problem* (PTP) and the *Prize-Collecting Traveling Salesman Problem* (PCTSP). In the PTP the objective is to maximize the difference between total collected profit and traveling cost with no constraint on the route. In the PCTSP the objective is to minimize the traveling cost with the constraint of collecting a total profit which is at least equal to a given threshold. In the literature, there is no consistent definition of these two problems. In this chapter we will follow the definitions given by Feillet, Dejax, and Gendreau [45] and explain, case by case, the different contributions.

Among the routing problems with profits and multiple vehicles the only one that has been studied in depth is the so-called *Team Orienteering Problem* (TOP). The TOP was introduced by Butt and Cavalier [24] with the name *Multiple Tour Maximum Collection Problem*. The first paper where the name TOP appears is due to Chao, Golden, and Wasil [28].

Previous surveys appeared covering parts of the literature on routing problems with profits. The survey by Feillet, Dejax, and Gendreau [45] is focused on the routing problems with profits with one vehicle only, whereas the more recent survey by Vansteenwegen, Souffriau, and Van Oudheusden [106] covers the OP and the TOP. Finally, the review by Gavalas et al. [51] is focused on the problems arising in the context of tourist trip design.

All the above-mentioned papers consider problems where customers are represented as vertices of a graph. We call these problems *VRP with Profits* (VRPPs). The problems where customers are represented as edges or arcs of a graph are called *arc routing problems with profits* (see Archetti and Speranza [10] for a recent survey).

In this chapter we cover all the VRPPs, from the problems considering a single vehicle to their extensions with multiple vehicles. We cover also the variants, e.g., those considering time windows and those inspired by applications.. We start with problems with one vehicle (the OP in Section 10.2.1, PTP in Section 10.2.2, PCTSP in Section 10.2.3, and variants in Section 10.2.4). We then present the TOP in Section 10.3.1 and its variants in

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10.3.2. Finally, Section 10.3.3 is devoted to the Vehicle Routing Problems with Private Fleet and Common Carrier (VRPPC), a specific class of VRPPs that arise in distribution where subcontracting options are considered.

Following Vansteenwegen, Souffriau, and Van Oudheusden [106] we formulate the problems on a directed graph. We will show how the models change in the case where the graph is undirected. Although different papers often use different notations, for the ease of presentation we have chosen a standard notation that we will use throughout the chapter to present the various problems. A complete graph $G=(V,A)$ is given, where $V=\{0,\dots,n\}$ is the set of vertices and A is the set of arcs. Vertices in $N=V\setminus\{0\}=\{1,\dots,n\}$ correspond to the customers, and vertex 0 corresponds to the depot where the routes start and end. For any subset of vertices $S\subset V$, we define $\delta^+(S)=\{(i,j)\in A: i\in S, j\notin S\}$ and $\delta^-(S)=\{(i,j)\in A: i\notin S, j\in S\}$. For the ease of presentation, in the following we will use the notation $\delta^+(i)$ and $\delta^-(i)$ when $S=\{i\}$. Two nonnegative values may be associated with each arc $(i,j)\in A$: a traveling cost c_{ij} and a traveling time t_{ij}. In most of the problems only one of these two values is relevant. A nonnegative profit p_i is associated with each customer i, while a profit $p_0=0$ is associated with the depot. The profit of each customer can be collected at most once. One or more vehicles are available to collect the profit of a subset of customers. Each vehicle route starts from and ends at the depot. While in some problems a time limit $T_{\max}$ is set on the time duration of a route, in some others a minimum value $p_{\min}$ is imposed on the total profit to be collected.

In Table 10.1 we summarize the basic VRPPs and their characteristics. In the first column we indicate whether it is a single- or multiple-vehicle problem; then we give the name we use for the problem and, in parentheses, the other names used in the literature, if any. In the following columns, we show the objective and the kind of constraints. For short we write “max profit” to mean “maximize profit collected”. When we simply write “min cost” we mean “minimize the routing cost”. While the first three problems

Table 10.1. *Summary of VRPPs.*

Vehicles	Problem name	Objective	Constraints
single	Orienteering Problem (OP) (Selective TSP, Maximum Collection Problem, Bank Robber Problem)	max profit	route duration
single	Profitable Tour Problem (PTP)	max (profit - cost)	–
single	Prize Collecting TSP (PCTSP)	min cost	route profit
multiple	Team Orienteering Problem (TOP) (Multiple Tour Maximum Collection Problem)	max profit	route duration
multiple	Capacitated PTP (CPTP)	max (profit - cost)	capacity
multiple	Capacitated Prize Collecting VRP (CPCVRP)	min cost	route profit capacity
multiple	VRP with Private Fleet and Common Carrier (VRPPC)	min cost	limited fleet capacity

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with multiple vehicles are uniquely defined problems, extensions of their single vehicle counterparts listed in the upper part of the table, the last problem, the VRPPC, refers to a class of distribution problems where customers are considered for outsourcing.

10.2 ▪ Single-Vehicle Case

In this section we survey the literature dedicated to the three basic VRPPs with a single vehicle: the *Orienteering Problem* (OP, Section 10.2.1), the *Profitable Tour Problem* (PTP, Section 10.2.2), and the *Prize-Collecting Traveling Salesman Problem* (PCTSP, Section 10.2.3). We first present a general formulation for the VRPP with a single vehicle and then specify, in each of the following sections, how each basic problem can be obtained from it.

We introduce the following variables:

- y_i = binary variable equal to 1 if vertex $i \in V$ is visited by the vehicle route, and 0 otherwise;
- x_{ij} = binary variable equal to 1 if arc $(i,j) \in A$ is traversed by the vehicle, and 0 otherwise.

The mathematical programming formulation of the directed version of the VRPP with a single vehicle is the following, where α is a nonnegative value:

$$\text{maximize } \alpha \sum_{i \in V} p_i y_i - \sum_{(i,j)\in A} c_{ij} x_{ij} \tag{10.1}$$

$$\text{s.t.} \quad \sum_{(i,j)\in\delta^+(i)} x_{ij} = y_i \qquad \forall\, i \in V, \tag{10.2}$$

$$\sum_{(j,i)\in\delta^-(i)} x_{ji} = y_i \qquad \forall\, i \in V, \tag{10.3}$$

$$\sum_{(i,j)\in\delta^+(S)} x_{ij} \ge y_h \qquad \forall\, S \subseteq V \setminus \{0\},\ h \in S, \tag{10.4}$$

$$\sum_{(i,j)\in A} t_{ij} x_{ij} \le T_{\max}, \tag{10.5}$$

$$\sum_{i\in V} p_i y_i \ge p_{\min}, \tag{10.6}$$

$$y_i \in \{0,1\} \qquad \forall\, i \in V, \tag{10.7}$$

$$x_{ij} \in \{0,1\} \qquad \forall\, (i,j) \in A. \tag{10.8}$$

The objective function (10.1) maximizes the difference between collected profit, multiplied by α, and traveling cost. Constraints (10.2) and (10.3) ensure that one arc enters and one arc leaves each visited vertex. Subtours are eliminated through (10.4). Constraint (10.5) is the maximum duration constraint on the route, while (10.6) imposes collecting a profit not smaller than $p_{\min}$. Finally, (10.7) and (10.8) are variable definitions.

In case of an undirected graph $G = (V,E)$, a variable x_e is defined for each edge $e = (i,j) \in E$. In this case we define c_e and t_e as the traveling cost and the traveling time associated with each edge $e \in E$, respectively. Moreover, let us define $\delta(S) = \{e = (i,j) \in$

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$E : (i \in S, j \notin S)$ or $(i \notin S, j \in S)\}$ and $\delta(i)$ when $S = \{i\}$. The formulation changes as follows:

$$\text{maximize } \alpha \sum_{i \in V} p_i y_i - \sum_{e \in E} c_e x_e \tag{10.9}$$

$$\text{s.t.} \quad \sum_{e \in \delta(i)} x_e = 2y_i \qquad \forall\, i \in V, \tag{10.10}$$

$$\sum_{e \in \delta(S)} x_e \geq 2y_h \qquad \forall\, S \subseteq V \setminus \{0\},\ h \in S, \tag{10.11}$$

$$\sum_{e \in E} t_e x_e \leq T_{\max}, \tag{10.12}$$

$$\sum_{i \in V} p_i y_i \geq p_{\min}, \tag{10.13}$$

$$y_i \in \{0,1\} \qquad \forall\, i \in V, \tag{10.14}$$

$$x_e \in \{0,1\} \qquad \forall\, e \in E \setminus \delta(0), \tag{10.15}$$

$$x_e \in \{0,1,2\} \qquad \forall\, e \in \delta(0). \tag{10.16}$$

The meaning of the objective function and of the constraints is analogous to the case of a directed graph.

10.2.1 ▪ The Orienteering Problem

In this section we introduce the OP and we overview the exact, heuristic, and approximation algorithms proposed in the literature for its solution.

In the OP, one vehicle is available at the depot with maximum route duration $T_{\max}$. The OP is the problem of finding a vehicle route that maximizes the total collected profit while satisfying the maximum duration constraint on the route. The formulation for the OP is obtained from (10.1)–(10.8) by setting $\alpha = 1$, $c_{ij} = 0$ for each $(i,j) \in A$, and $p_{\min} = 0$.

We now present the solution methods described in the literature to solve the OP. We first focus on exact algorithms, then on heuristics, and finally on approximation algorithms.

Exact Algorithms. Hayes and Norman [59] proposed the first exact algorithm to solve the OP which is based on dynamic programming. The algorithm was tested on a real problem related to the sport of orienteering in a mountain area where traveling times were approximated through an estimation which takes into account uphill and downhill steps.

Laporte and Martello [74] defined a scheme to obtain an upper bound on the optimal value of the OP based on the solution of the following knapsack problem:

$$\text{maximize } \sum_{i \in V} p_i y_i \tag{10.17}$$

$$\text{s.t.} \quad \sum_{i \in V \setminus \{0\}} w_i y_i \leq T_{\max} - w_0, \tag{10.18}$$

$$y_i \in \{0,1\} \qquad \forall\, i \in V \setminus \{0\}, \tag{10.19}$$

where $w_i = \alpha \min_{j \neq i}\{t_{ij}\} + (1-\alpha) \min_{k \neq i}\{t_{ki}\}$. This upper bound was used in an enumerative algorithm based on the idea of extending a simple path emanating from the depot

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through a *Branch-and-Bound* (BB) scheme. The authors presented also two simple heuristic algorithms to obtain lower bounds based on the application of the classical nearest neighbor and cheapest insertion algorithms for the TSP. These algorithms were adapted to the OP by inserting a stopping criterion which checks the violation of the maximum route duration constraint. Instances with a number of vertices between 10 and 90 were solved to optimality. Millar and Kiragu [80] presented a mathematical formulation with a polynomial number of constraints. The formulation is enhanced by introducing an upper bound on the number of vertices visited in an optimal solution. This bound is obtained through a modification of the bounding scheme developed by Laporte and Martello [74]. Instances with 10 vertices were solved through the use of CPLEX. Leifer and Rosenwein [75] proposed new valid inequalities that were added to the formulation presented in [74]. They developed a procedure to obtain upper bounds by solving three successive linear programs. Tests were made on instances introduced in Tsiligirides [105] and showed that the deviation between the value of the upper bound and the best known solution was always lower than 5%.

Ramesh, Yoon, and Karwan [90] developed an exact algorithm using a Lagrangian relaxation where flow conservation and maximum route duration constraints were relaxed. The solution algorithm is a BB where the problem is iteratively solved improving the value of Lagrangian multipliers in the first phase while branching is performed in the second phase. They solved instances with up to 150 vertices. Kataoka, Yamada, and Morito [66] proposed a new relaxation scheme to the OP based on the *Minimum Directed* 1*-Subtree Problem* (MD1SP). Two methods were developed to improve the lower bound given by the solution of the MD1SP: the first one is based on the iterative introduction of violated valid inequalities, while the second one is based on Lagrangian relaxation. The two methods were combined together in a solution procedure which was tested on instances with up to 1000 customers. Results showed that the proposed relaxation was superior to the ordinary assignment relaxation. Fischetti, Salazar González, and Toth [46] presented a *Branch-and-Cut* (BC) algorithm using several families of valid inequalities. They solved instances with up to 500 vertices.

Heuristic Algorithms. The literature on heuristic algorithms for the OP is broader than the one on exact approaches. Tsiligirides [105] presented the first heuristic algorithms for the OP, which were a deterministic and a stochastic one. In the stochastic algorithm, different routes are generated using a Monte Carlo process and the best one is chosen. Each route is constructed by inserting customers sequentially on the basis of a desirability measure, given by the ratio between the profit and the closeness to the last inserted customer. The deterministic algorithm builds the route by dividing the Euclidean space in different areas. All customers in a given area are inserted, if feasible, before moving to a different area. Tests were made on instances with up to 33 vertices and showed that the stochastic algorithm is superior. Golden, Levy, and Vohra [57] used a center of gravity heuristic composed by three steps: construction, improvement, and calculation of the center of gravity. In the construction phase, a solution is built by inserting customers sequentially following a given order. In the improvement phase, the 2-opt procedure, proposed by Lin [77] for the TSP, is applied followed by cheapest insertion. Finally, the center of gravity of the route is calculated. Customers are then ordered on the basis of their distance with respect to the center of gravity and the procedure is repeated. Computational tests were made on the instances introduced by Tsiligirides [105] and showed that the center of gravity heuristic was better than both algorithms given in [105]. Keller [69] proposed a new construction heuristic which inserts a customer at each iteration. The customer is chosen on the basis of the ratio between the profit and the distance with

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respect to the last customer inserted. The algorithm was compared with those by Tsiligirides [105] and by Golden, Levy, and Vohra [57]. Computational results showed that the new algorithm is the best among the competitors. Golden, Wang, and Liu [58] introduced a heuristic that combines previous algorithmic concepts, along with learning capabilities. The heuristic was compared with the algorithms proposed in [105] and [57] on the instances from [105]. The results showed that it beat both previous approaches and it was much faster than the center of gravity algorithm in [57]. Ramesh and Brown [89] developed a four-phase algorithm where the phases are vertex insertion, improvement, vertex deletion, and maximal insertions. The insertion of a vertex is made with a rule which is similar to the one by Tsiligirides [105]. The improvement phase uses the k-opt algorithm for the TSP. The algorithm was shown to beat the heuristics given in [105] and [57]. Chao, Golden, and Wasil [29] introduced a new heuristic composed by two phases: initialization and improvement. The initialization phase constructs different routes with the cheapest insertion method where the first customer inserted is changed from one solution to another. Then, in the improvement phase customer exchanges between different routes or intra-route moves are performed. Tests were made on the instances given in [105] and on new instances. Despite its simplicity, the heuristic compared favorably with respect to all previous heuristic algorithms.

All the heuristic methods cited above use classical local ascent schemes and thus tend to become trapped in local optima. In the following years researchers concentrated on metaheuristic schemes for the OP which could overcome this problem. A neural network based heuristic was proposed by Wang et al. [111], where a state matrix was used to generate solutions and perturbed at each iteration in order to obtain different solutions. Tests were made on the instances from Tsiligirides [105]. Results showed that the neural network heuristic performed much better than the stochastic heuristic in [105] and slightly worse than that of Chao, Golden, and Wasil [29]. The first tabu search heuristic for the OP was presented by Gendreau, Laporte, and Semet [52]. The algorithm iteratively inserts clusters of customers or removes chains of vertices. The algorithm was tested on randomly generated instances with up to 300 vertices. Results were compared with optimal solutions provided by the BC algorithm described in Gendreau, Laporte, and Semet [53] and proved that the tabu search was able to find optimal or near optimal solutions in a very short computing time. The gap with respect to the optimum was typically less than 1%. Liang, Kulturel-Konak, and Smith [76] proposed an ant colony and a tabu search heuristic. Both algorithms are based on standard schemes for ant colony optimization and tabu search. Tests were made on the instances of Tsiligirides [105]. Results showed that the solution quality was comparable to the one produced by the heuristic of Chao, Golden, and Wasil [29]. A heuristic, based on the *Greedy Randomized Adapted Search Procedure* (GRASP) and the path relinking methods, was proposed by Campos et al. [27]. Computational results show good quality solutions obtained in short computing times.

Approximation Algorithms. The first constant-factor approximation algorithm for the OP can be found in Blum et al. [20]. The approximation ratio depends on the approximation factor for the min-excess path problem, for which a constant factor approximation algorithm was proposed in the same paper. Awerbuch et al. [12] introduced an approximation algorithm for the *unrooted* OP, i.e., the problem where no starting and ending vertex is defined. The authors proposed an algorithm which is able to find a route of length $O(T_{\max} \log^2(\min(n,R)))$, where R is the desired value of the total profit to be collected. An approximation result for the directed OP is due to Nagarajan and Ravi [82], who presented an $O(\frac{\log^2(n)}{\log\log n})$ algorithm. Approximation results both for the directed and

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the undirected case of the OP can be found in Chekuri, Korula, and Pál [30]. A $(2+\epsilon)$-approximation algorithm was presented for undirected graphs while a $O(\log^2(OPT))$ algorithm was described for the directed case, where OPT is the number of nodes in the optimal solution. A more detailed discussion of these approximation results for the OP can be found in Gavalas et al. [51].

Angelelli et al. [1] studied the undirected OP together with the variant where a service time is associated with the visit of a customer. They studied the complexity of classes of instances corresponding to special structures of the underlying graph, like paths, cycles, stars, and trees. They proved that the problem with service time is always NP-hard, while, when no service time is considered, the problem is polynomially solvable on cycles and on paths and it is NP-hard on stars and trees. Polynomial algorithms for the polynomially solvable cases and fully polynomial-time approximation schemes for some NP-hard cases were presented.

10.2.2 ▪ The Profitable Tour Problem

In this section we present the PTP and overview the solution algorithms proposed in the literature.

The PTP is the problem of finding a vehicle route that maximizes the difference between the total collected profit and the total traveling cost. No constraint is imposed on the vehicle route. The formulation for the PTP is obtained from (10.1)–(10.8) by setting $\alpha = 1$, $p_{\min} = 0$, and $T_{\max} = +\infty$.

Note that

$$\text{maximize} \sum_{i \in V} p_i y_i - \sum_{(i,j) \in A} c_{ij} x_{ij}$$

is equivalent to

$$-\text{minimize}\left(\sum_{(i,j)\in A} c_{ij} x_{ij} - \sum_{i \in V} p_i y_i + \sum_{i \in V} p_i \right) \tag{10.20}$$

$$= -\text{minimize}\left(\sum_{(i,j)\in A} c_{ij} x_{ij} + \sum_{i \in V} p_i (1 - y_i) \right).$$

Thus, maximizing the difference between the total collected profit and the total traveling cost is equivalent to minimizing the sum of the total traveling cost and the total uncollected profit. Moreover, note that, by subtracting the profit of a customer to each arc outgoing (or ingoing) from each customer, the objective function of the PTP becomes

$$-\text{minimize} \sum_{i \in V} \tilde{c}_{ij} x_{ij},$$

where $\tilde{c}_{ij} = c_{ij} - p_i$, $i \in V$. With the objective function (10.21), the formulation of the PTP corresponds to the one of the *Elementary Shortest Path Problem* (ESPP), as discussed in Drexl and Irnich [40].

To the best of our knowledge, neither exact approaches nor computational analysis of heuristic algorithms were specifically proposed for the PTP. The only exception is the work by Dell'Amico, Maffioli, and Väbrand [37], where a bounding procedure based on Lagrangian relaxation was introduced. Tests were made on instances with up to 500 vertices.

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The PTP received, instead, attention in terms of approximation results, probably due to its simple structure. Several approximation results appeared in the literature under the assumption that the c_{ij} satisfy the triangle inequality. Bienstock et al. [19] provided a 2.5-approximation algorithm which was then improved by Goemans and Williamson [55], where a 2-approximation algorithm was provided. Archer et al. [3] were the first to break the barrier of 2 and provide an improvement of the primal-dual algorithm of Goemans and Williamson [55]; their performance guarantee was $2-\epsilon$. Goemans [54] showed that, by combining the rounding algorithm given in [19] and the primal-dual algorithm in [55], it was possible to obtain a guarantee $\frac{1}{1-\frac{2}{3}e^{-1/3}}$. A $\lceil \log(n) \rceil$-approximation algorithm for the directed version of the PTP was recently presented by Nguyen [83] which improved a previous result proposed by Nguyen and Nguyen [84]. Angelelli et al. [1] studied the undirected PTP together with the variant where a service time is associated with the visit of a customer. Polynomially solvable classes of instances, corresponding to special structures of the underlying graph, were also identified and studied.

10.2.3 ▪ The Prize-Collecting Traveling Salesman Problem

In this section we introduce the PCTSP and overview the algorithms developed for this problem. We adopt the definition of PCTSP used by Feillet, Dejax, and Gendreau [45] and in Vansteenwegen, Souffriau, and Van Oudheusden [106].

The PCTSP is the problem of finding a vehicle route that minimizes the total traveling cost with the constraint that the total collected profit is at least $p_{\min}$. The formulation for the PCTSP is obtained from (10.1)–(10.8) by setting $\alpha = 0$ and $T_{\max} = +\infty$.

In the paper which first introduced the name PCTSP, due to Balas [13], the objective function is the sum of the total traveling cost and a total penalty for the non-visited customers. Denoting by γ_i the penalty associated with customer i, $i \in V \setminus \{0\}$, to distinguish this more general problem we call it PCTSP *with penalties*. The objective function becomes

$$\text{minimize} \sum_{(i,j)\in A} c_{ij}x_{ij} + \sum_{i\in V\setminus\{0\}} \gamma_i(1-y_i), \tag{10.21}$$

while the constraints remain the same.

The objective function (10.21) is equivalent to the maximization of the difference between the penalties (that can be interpreted now as profits) of the visited customers and the total distance traveled (see the transformation (10.20)). Then, the problem turns out to be a generalization of the PTP, obtained when $p_{\min} = 0$, and of the PCTSP, obtained when $\gamma_i = 0$ for all i.

As in Balas [13], also in Dell'Amico, Maffioli, and Väbrand [37] two different values were associated with each customer i: a profit used in constraint (10.6), and a penalty used in the objective function.

Balas [13] studied the structural properties of the PCTSP with penalties, while in Balas [14] the same author provided a polyhedral study of the problem. He derived different classes of facet-defining inequalities from facet-defining inequalities for the asymmetric TSP. Balas [15] studied the complexity of the PCTSP with penalties and additional constraints on the structure of the route. In particular, he studied different precedence constraints among customers and showed that in all cases the problem can be solved in polynomial time.

We now review the solution approaches proposed in the literature.

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Exact Algorithms. Fischetti and Toth [48] introduced different mathematical formulations for the problem and an additive approach to obtain lower bounds from various problem relaxations. These bounds were then embedded in a BB algorithm which was used to solve instances with up to 200 customers. Dell'Amico, Maffioli, and Väbrand [37] introduced the same relaxation illustrated above for the PTP to obtain a bound also for the PCTSP and tested it on the same set of instances. Bérubé, Gendreau, and Potvin [17] proposed a BC algorithm for the PCTSP where different classes of valid inequalities were used to strengthen the formulation. Tests were made on instances generated from standard VRP and TSP benchmark instances with up to 532 vertices. Computational results showed that the solution time was strictly related to the value of the minimum value of the profit required in constraint (10.6).

Heuristic Algorithms. To the best of our knowledge, the only heuristic algorithm available in the literature for the PCTSP with penalties is a Lagrangian heuristic proposed by Dell'Amico, Maffioli, and Sciomachen [36]. It starts from a lower bound to the problem and makes the solution feasible. The heuristic was evaluated on randomly generated instances and real-world ones with up to 500 vertices. The solution was compared with the optimal one, when available, or with an upper bound. The performance of the heuristic improved when the minimum prize required in constraint (10.6) increased.

Approximation Algorithms. An approximation algorithm for the undirected PCTSP with penalties was presented by Awerbuch et al. [12]. The result was derived from previous results for the minimum-weight k-tree problem presented in Ravi et al. [91]. The authors, as done for the OP, proposed an approximation algorithm for the unrooted OP, i.e., the problem where no starting and ending vertex is defined. The algorithm is able to find a route of length $O(T_{\max} \log^2(\min(n, R)))$, where R is the desired value of the total profit to be collected. Angelelli et al. [1] studied the undirected PCTSP together with the variant where a service time is associated with the visit of a customer. They studied the complexity of classes of instances corresponding to special structures of the underlying graph, like paths, cycles, stars, and trees. They proved that the problem with service time is always NP-hard, while, when no service time is considered, the problem is polynomially solvable on cycles and on paths and it is NP-hard on stars and trees. Polynomial algorithms for the polynomially solvable cases and fully polynomial-time approximation schemes for some NP-hard cases were presented.

Ausiello, Bonifaci, and Laura [11] studied the online version of the PCTSP with penalties where customers are disclosed over time. For this problem they derived a 7/3 approximation algorithm which is not far from the best possible one since they proved that the competitive ratio of any approximation algorithm for the online PCTSP is at least two.

10.2.4 ▪ Variants

Wang, Golden, and Wasil [112] and Silberholz and Golden [95] considered a variant of the OP that differs from the basic problem mainly in the objective function that is a nonlinear function of the profits collected from the visited vertices. In this problem each city is assigned a number of scores for different attributes and the overall function to optimize is a function of these attribute scores. A different variant of the OP, where a limited resource is consumed on the traversed arcs and on the visited nodes, was studied by Pietz and Royset [86], who presented a specialized BB algorithm. Two variants of the OP with variable profits were introduced by Erdoǧan and Laporte [42]. In the first variant part of the profit of a vertex is collected at each visit if a predefined amount of time is spent at

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the vertex, and in the second one the collected profit is a function of the time spent at the vertex. They proposed a BC algorithm that was tested on randomly generated instances obtained from benchmark test problems.

Kantor and Rosenwein [64] studied the *OP with Time Windows* (OPTW) and proposed a heuristic. Righini and Salani [92] introduced an exact algorithm for the OPTW which is based on a bi-directional and bounded dynamic programming algorithm with state space relaxation. Fomin and Lingas [49] studied a different generalization of the OP, called *Time-Dependent OP*, which is the problem where the traveling time between two locations depends on the starting time. They proposed an approximation algorithm. The same problem was considered by Verbeeck et al. [110], who proposed a heuristic that combines the principle of an ant colony system with a time-dependent local search procedure. Erkut and Zhang [43] developed a penalty-based greedy heuristic and a BB algorithm for a variant of the OP where the profit is a time-dependent decreasing linear function. Deitch and Ladany [35] studied a more general problem than the OP, called the *One-Period Bus Touring Problem*, where an attractiveness, that we can also see as a profit, is associated also with edges. They presented a transformation of the problem into the OP together with a heuristic that was compared with one of the heuristics proposed by Tsiligirides [105]. The OP with a given set of compulsory vertices is studied by Gendreau, Laporte, and Semet [53]. A BC algorithm based on several families of valid inequalities was proposed that can solve to optimality instances with up to 300 vertices.

The *OP with stochastic profits* is the problem of finding a route that visits a subset of vertices within a pre-specified time limit and maximizes the probability of collecting more than a pre-specified target profit level. This variant of the OP was studied by Ilhan, Iravani, and Daskin [61]. Campbell, Gendreau, and Thomas [26] introduced a variant of the OP where travel and service times are stochastic. Special cases of the problem were solved exactly and heuristics presented for the general problem. Different ways to approximate the objective function are compared by Papapanagiotou, Montemanni, and Gambardella [85]. Stochastic travel times were also considered by Evers et al. [44], who formulated a two-stage stochastic model with recourse.

The only studied variant of the PTP is the *Capacitated PTP* (CPTP), where each customer has a demand and the vehicle has a prefixed capacity which must not be exceeded by the route. Jepsen [62] proposed a BC algorithm for the solution of the undirected version of the CPTP where instances with up to 800 vertices were solved to optimality. Tang and Wang [100] recently introduced an iterated local search heuristic for the Capacitated PCTSP with penalties to model a real-world application related to the optimal scheduling of rolling mills in the steel industry.

Finally, some authors considered variants with multiple objectives. The bi-objective version of the OP was studied by Jozefowiez, Glover, and Laguna [63], who developed a heuristic to find the efficient frontier. The same problem was considered in Bérubé, Gendreau, and Potvin [18]. Schilde et al. [94] studied a multi-objective version of the OP where each vertex may provide different benefits belonging to different categories (e.g., culture, history, leisure).

10.3 ▪ Multiple-Vehicle Case

As mentioned in the introduction, the study of multiple VRPs with profits has started relatively recently and the area is still open for research. We overview in this section the main results presented in the literature which mainly focus on the extension to the case with multiple vehicles of the OP and some of its variants and on the *VRP with Private Fleet and Common Carrier* (VRPPC). The only contributions that do not fall in these

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categories concern the capacitated and multiple-vehicle version of the PTP (CPTP) that was considered by Archetti et al. [8] and, more recently, by Archetti, Bianchessi, and Speranza [6]. In both papers exact BP algorithms were presented and tested on benchmark instances derived from instances of the VRP with up to 200 vertices. To the best of our knowledge, no work has been done on the *Capacitated Prize Collecting VRP* (CPCVRP), which is the extension to the multiple-vehicle case of the PCTSP. In the CPCVRP a demand is associated with each customer, vehicles are capacitated, and a minimum profit must be collected, either by each route or in total.

10.3.1 ▪ The Team Orienteering Problem

The multiple-vehicle extension of the OP was first introduced by Butt and Cavalier [24], who called it the *Multiple Tour Maximum Collection Problem*. The name *Team Orienteering Problem* (TOP) was coined by Chao, Golden, and Wasil [28] to highlight the connection with the more widely studied single-vehicle case. More precisely, given a set K of vehicles, the TOP calls for the determination of at most $|K|$ vehicle routes that maximize the total collected profit, while satisfying a maximum duration constraint for each route. Several real-world applications of the TOP are mentioned in the literature, such as athlete recruiting (Chao, Golden, and Wasil [28]), technician routing (Tang and Miller-Hooks [99]), and tourist trip planning (Vansteenwegen and Van Oudheusden [109]).

Vehicle flow models, extending those presented in Section 10.2.1, are proposed in the literature (see, e.g., Tang and Miller-Hooks [99]). We present here a formulation for the directed case of the TOP that is an extension of the one presented for the directed OP in Section 10.2.1 and uses the following decision variables:

- y_{ik} = binary variable equal to 1 if vertex $i \in V$ is visited by vehicle route $k \in K$, and 0 otherwise;
- x_{ijk} = binary variable equal to 1 if arc $(i,j) \in A$ is traversed by vehicle route $k \in K$, and 0 otherwise.

The mathematical programming formulation for the directed TOP is as follows:

$$\text{(TOP1) maximize } \sum_{i \in V} p_i \sum_{k \in K} y_{ik} \tag{10.22}$$

$$\text{s.t. } \sum_{j \in V} x_{ijk} = y_{ik} \qquad \forall\, i \in V,\ k \in K, \tag{10.23}$$

$$\sum_{j \in V} x_{jik} = y_{ik} \qquad \forall\, i \in V,\ k \in K, \tag{10.24}$$

$$\sum_{k \in K} y_{0k} \le |K|, \tag{10.25}$$

$$\sum_{k \in K} y_{ik} \le 1 \qquad i \in V \setminus \{0\}, \tag{10.26}$$

$$\sum_{(i,j) \in \delta^+(S)} x_{ijk} \ge y_{hk} \qquad \forall\, S \subseteq V \setminus \{0\},\ h \in S,\ k \in K, \tag{10.27}$$

$$\sum_{(i,j) \in A} t_{ij} x_{ijk} \le T_{\max} \qquad \forall\, k \in K, \tag{10.28}$$

$$y_{ik} \in \{0,1\} \qquad \forall\, i \in V,\ k \in K, \tag{10.29}$$

$$x_{ijk} \in \{0,1\} \qquad \forall\, (i,j) \in A,\ k \in K. \tag{10.30}$$

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The objective function and most of the constraints are extensions to the multiple-vehicle case of those presented for the OP. Constraint (10.25) limits the number of routes to be at most $|K|$, while constraints (10.26) impose that each customer be visited at most once. Finally, the constraints (10.27) impose that each route be connected and (10.28) limit the maximum distance for each route.

Several authors based their exact methods on route-based formulations, as an alternative to vehicle flow formulations. Let Ω be the set of all feasible routes, i.e., elementary routes with length at most T_{max}, and let A be a matrix of binary coefficients a_{ir} describing such routes. More precisely, for each route $r \in \Omega$ the coefficient a_{ir} takes value 1 if vertex $i \in N$ is included in route r, and 0 otherwise. Moreover, let P_r be the total profit of route r, defined as the sum of the profits of its customers. The set-packing model for the TOP (denoted as TOP2), proposed by Butt and Ryan [25], uses a binary variable λ_r for each route $r \in \Omega$ which takes value 1 if the route is selected in the optimal solution, and 0 otherwise:

$$\text{(TOP2) maximize} \sum_{r\in\Omega} P_r \lambda_r \tag{10.31}$$

$$\text{s.t.} \quad \sum_{r\in\Omega} a_{ir}\lambda_r \le 1 \qquad \forall i \in N, \tag{10.32}$$

$$\sum_{r\in\Omega} \lambda_r \le |K|, \tag{10.33}$$

$$\lambda_r \in \{0,1\} \qquad \forall r \in \Omega. \tag{10.34}$$

The objective function (10.31) maximizes the total profit. Constraints (10.32) ensure that each vertex is visited by at most one route, and constraint (10.33) limits the number of used routes.

Testing of algorithms is generally performed by using a large benchmark set proposed by Chao, Golden, and Wasil [28] (called CGW hereafter) and obtained by adapting instances originally proposed for the OP. The original set contained 353 instances with 21 to 102 customers and up to four vehicles, but was later enlarged to 387 instances in total (see, e.g., Boussier, Feillet, and Gendreau [23]) by considering additional values for $T_{\max}$.

Exact Algorithms. The first exact solution procedure for the TOP was proposed by Butt and Ryan [25] for a heterogeneous fleet variant of the problem. They started from the set-packing formulation TOP2, and their algorithm makes use of both column generation and constraint branching. Pricing in the column generation step is performed through complete enumeration of the routes. Therefore, the algorithm is able to solve instances with up to 100 potential customers only when routes include a few customers each. A more effective BP algorithm was introduced by Boussier, Feillet, and Gendreau [23]. They again used as a base model TOP2 but defined the pricing subproblem as a *Resource Constrained Elementary Shortest Path Problem* (RCESPP) and then solved it by dynamic programming. Using various acceleration procedures in the column generation step, the algorithm is able to solve 270 instances with up to 100 potential customers from the set of 387 CGW benchmark instances. An effective bi-directional dynamic programming procedure and relaxation-based dominance procedures are the base of an enhanced BP approach by Keshtkaran et al. [70] which solves 10% more instances than the previous approaches (i.e., 301 out of the 387 instances of the benchmark).

Poggi de Aragão, Viana de Freitas, and Uchoa [87] defined a *Branch-and-Cut-and-Price* (BCP) algorithm based on a pseudo-polynomial formulation in which binary variables are associated with the visit of vertices and the traversal of an arc at a given time instant

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between 0 and T_{max}. A Dantzig–Wolfe decomposition scheme was adopted to handle the potentially huge number of arc variables, and the resulting column generation algorithm was enhanced through the addition of min-cut and clique cuts. The computational tests showed that the root node upper bounds computed with this approach are generally better than those of Boussier, Feillet, and Gendreau [23] and that the additional cuts help in reducing the gap with respect to the BP bound. This is confirmed by the results obtained by more recent BC and BCP approaches. In particular, Dang, El-Hajj, and Moukrim [33] defined a BC approach based on model TOP1 which uses Generalized Subtour Elimination and Clique cuts to strengthen the linear relaxation and is able to close some open instances that were hard for BP. The subset-row inequalities are instead used by the BCP of Keshtkaran et al. [70] which, although on average is inferior with respect to the pure BP, is capable of solving some instances that were not solvable by BP.

The complementarity of the different exact approaches available for TOP is further illustrated by Table 10.2. For each existing algorithm we report in the last line the total number of solved instances within 2 hours of computing time, out of the 387 of the extended CGW benchmark set. The BCP of Poggi de Aragão, Viana de Freitas, and Uchoa [87] is not included in the table since the paper does not report complete results on the benchmark set. In the first four lines of the table we make a direct comparison of the algorithms by providing, for each ordered pair, the number of instances that the first algorithm is able to solve and the other is not. For example, the BC by [33] can solve 13 test instances the BP by [70] cannot solve, whereas this latter solves 36 instances unsolved by the BC. By observing the table we note that even though BCP approaches are not currently the best available ones, it is likely that a more effective integration of cuts within the BP may be the path to follow for future research, as happened for other variants of the VRP.

Table 10.2. *Comparison between exact approaches for TOP on the benchmark of Chao, Golden, and Wasil* [28] *including* 387 *instances.*

	BP [23]	BP [70]	BC [33]	BCP [70]
BP [23]	-	+8	+21	+6
BP [70]	+39	-	+36	+16
BC [33]	+29	+13	-	+19
BCP [70]	+27	+6	+32	-
Total solved instances	270	301	278	291

Heuristic Algorithms. The first heuristic proposed for the TOP is a simple construction algorithm introduced by Butt and Cavalier [24] and tested on small-sized instances with up to 15 vertices by comparing the results with a mathematical programming model similar to TOP1. Chao, Golden, and Wasil [28] developed a more sophisticated construction heuristic in which the initial solution was refined through customer moves and exchanges and various restart strategies. The resulting algorithm was tested on the original CGW set of 353 instances with up to 102 customers and four vehicles.

More recently, several metaheuristics were applied to the TOP, starting from the tabu search algorithm introduced by Tang and Miller-Hooks [99] which is embedded in an adaptive memory procedure that alternates between small and large neighborhoods during the search and outperforms previous heuristics. Archetti, Hertz, and Speranza [9] proposed two variants of a generalized tabu search algorithm and a variable neighborhood search algorithm. Ke, Archetti, and Feng [67] used an ant colony optimization approach which uses four different methods to construct candidate solutions. Other metaheuristic

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paradigms were successfully applied to the TOP, such as guided local search (Vansteenwegen et al. [107]), path relinking (Souffriau et al. [96]), memetic algorithms (Bouly, Dang, and Moukrim [22]), particle swarm optimization-based memetic algorithm (Dang, Guibadj, and Moukrim [34]), and augmented large neighborhood search (Kim, Li, and Johnson [71]), the latter two being the current best in class.

10.3.2 ▪ Variants of the Team Orienteering Problem

Archetti et al. [8] studied the capacitated version of the TOP, called *Capacitated TOP* (CTOP). In this problem a demand is associated with each customer and each vehicle has a maximum capacity. The objective is to maximize the total collected profit while satisfying the capacity and duration constraint for each route. As previously mentioned in this paper also the CPTP was studied. The authors proposed a BP algorithm adapted from the one defined by Boussier, Feillet, and Gendreau [23] for the TOP. The algorithm is able to solve instances derived from classical VRP instances and including up to 200 customers. Heuristic algorithms were also proposed for both problems adapted from the heuristics described by Archetti, Hertz, and Speranza [9] for the TOP. An improved BP for both CTOP and CPTP was developed by Archetti, Bianchessi, and Speranza [6]. The algorithm implements the bi-directional dynamic programming and decremental state space relaxation of Righini and Salani [92] and clearly obtains better results than those of Archetti et al. [8]. Two new heuristic algorithms for the CTOP were simultaneously proposed by Luo et al. [79] and Tarantilis, Stavropoulou, and Repoussis [101] and compared with the heuristics presented in Archetti et al. [8]. In Luo et al. [79] the algorithm uses the ejection pool framework with an adapted strategy and a diversification mechanism and finds 16 new best solutions among 120 benchmark instances. Tarantilis, Stavropoulou, and Repoussis [101] proposed a method that adopts a hierarchical bi-level search framework. The best tested version of the method finds 18 new best solutions on benchmark instances.

Archetti, Bianchessi, and Speranza [7] studied a variant of the CTOP where incomplete service to the customers is allowed and developed a BP algorithm for its exact solution. In Archetti et al. [4] split deliveries are allowed in CTOP, but each customer has to be either entirely served or not served, while in Archetti et al. [5] incomplete service is also allowed. A BP exact algorithm and heuristic algorithms were proposed for the solution of both problems.

The *TOP with Time Windows* (TOPTW) received considerable attention from the heuristic community in the last few years. Vansteenwegen et al. [108] proposed an iterated local search which quickly produces good solutions on large instances derived from VRPTW ones. Montemanni and Gambardella [81] developed an ant colony optimization approach which was later improved by Gambardella, Montemanni, and Weyland [50]. Other metaheuristic approaches for the TOPTW were recently introduced and overall obtained very good average results on benchmark instances. Tricoire et al. [104] applied a *Variable Neighborhood Search* (VNS) developed for the multi-period variant of the problem. Labadie, Melechovský, and Wolfler Calvo [73] used a hybrid approach which combines a greedy randomized adaptive search procedure and an evolutionary search, whereas Labadie et al. [72] presented a VNS and a hybrid approach which combines VNS with the granular search by Toth and Vigo [103]. Lin and Yu [78] used two heuristics based on the simulated annealing paradigm. The most recent heuristic for the solution of the TOPTW is due to Hu and Lim [60], who proposed a three-component heuristic. The first two components are a local search procedure and a simulated annealing procedure, whereas the third component recombines the routes. The computational results show

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that their algorithm outperforms previous approaches. They obtained 35 new best solutions.

A variant of the TOP called *Multi-District TOP* (MDTOP) was studied by Salazar-Aguilar, Langevin, and Laporte [93]. In this problem a set of mandatory and optional tasks located in several districts must be scheduled over a planning horizon to maximize the total collected profit. A sequence of tasks assigned to a group of workers cannot have a duration that exceeds a time limit. When interpreted in terms of routing problem with profits, in the MDTOP a set of customers is mandatory and incompatibility constraints are present among customers.

A variant of the TOP where rewards are linearly decreasing over time was studied by Ekici and Retharekar [41]. A heuristic, called cluster-and-route algorithm, was proposed and tested on randomly generated instances obtained by modifying TOP instances.

Vansteenwegen and Van Oudheusden [109] introduced a class of problems, called *Tourist Trip Design Problems* (TTDPs), that is associated with the application of creating a feasible plan for tourists to visit attractions within the available time span. Such a problem class, also considered in Souffriau et al. [97], generalizes the TOP by considering time windows and other practical constraints. The OP with hotel selection introduced by Divsalar, Vansteenwegen, and Cattrysse [38] belongs to this class and is aimed at determining a fixed number of connected trips starting and ending in one of the hotels. A memetic algorithm was proposed for this problem by Divsalar et al. [39]. An extended survey on the class of TTDPs is presented in Gavalas et al. [51].

Uncertainty in the TOP was modeled through robust optimization by Ke et al. [68]. They allowed all data to vary in an interval and proposed a BP algorithm to solve the resulting robust counterpart of the TOP.

10.3.3 ▪ The VRP with Private Fleet and Common Carrier

An important application area of VRPs with profits arises in the context of the *Small Package Shipping* (SPS) industry and is related to service outsourcing of unprofitable customers. In fact, large SPS companies outsource depot operations and last-mile deliveries of unprofitable areas to small regional suppliers, called subcontractors. Subcontracted services are generally efficient in rural areas characterized by few customers and long distances, where a subcontractor may operate profitably by bundling deliveries. In general, SPS companies pay subcontractors per parcel delivered; hence the cost for the SPS company is independent from routing decisions of subcontractors.

The minimization of the overall cost of an SPS company using subcontractors must consider two elements at a time. The first one is the identification of the unprofitable customers that are outsourced at a fixed cost, and the second is the routing for the private fleet required to serve the remaining customers. Despite the practical interest, the literature on routing with outsourcing decision is still relatively scarce. A single-depot routing problem with outsourcing options was first introduced by Chu [31]. The problem, which was later named *VRP with Private Fleet and Common Carrier* (VRPPC), considers a private fleet of vehicles with limited capacity and fixed cost per use. A set of customers with known demand can be served by the private fleet which then incurs travel costs as in standard VRP. As an alternative, customers may be outsourced to a common carrier, and in such a case only fixed service costs must be paid. The objective is to minimize the total cost involving fixed costs for vehicles, variable travel costs, and fixed costs for orders performed by the common carrier. The VRPPC can be actually considered as a class of problems since some variants were also studied in the literature. Chu [31] proposed a simple heuristic based on a modified savings algorithm that was tested on five instances. Bolduc et al. [21] showed

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that the VRPPC can be modeled as a heterogeneous VRP and presented a metaheuristic based on a perturbation procedure. The algorithm greatly improved the results of [31] and was also tested on two new benchmark sets, one with homogeneous and one with heterogeneous fleet, with up to 483 customers. Two tabu search algorithms for VRPPC were proposed by Côté and Potvin [32] and Potvin and Naud [88]: the latter is based on ejection chain neighborhoods and is able to obtain very good results on both the homogeneous and the heterogeneous versions of the problem within a quite large computing time with respect to that required by Bolduc et al. [21].

Considering a single depot may be not realistic in the case of many SPS companies which manage several depots to serve huge areas including large, medium, and small towns. To optimize the whole delivery network, interdependencies between customer assignment and routing decisions of different self-owned depots have to be considered. Furthermore, instead of simply assuming the existence of a common carrier it is more appropriate to consider the location of potential subcontractors involved which are normally small regional carriers that have a small established depot characterized by a restricted delivery radius and a limited capacity. To this aim Stenger et al. [98] introduced a multiple-depot version of the problem denoted as MDVRPPC. For such a problem they defined a variable neighborhood search algorithm which implemented an effective adaptive mechanism to select routes and customers involved in the shaking step. The resulting algorithm was tested on a benchmark set of instances derived from *Multi-Depot VRP* (MDVRP) ones showing the potential benefits associated with subcontracting. The algorithm is also capable of obtaining state-of-the-art results on both the single depot VRPPC and on the MDVRP.

10.4 ▪ Conclusions and Future Research Directions

In this chapter we reviewed the large family of vehicle routing problems with profits. These problems are widely studied due to the practical relevance of the applications they model and the scientific interest of their structure in which not all customers have to be served. We provided a homogeneous description of the main problems of the family since in the literature some discrepancies in problem definitions and naming are encountered. Then for each problem and variants we reviewed the most relevant results with particular attention to the computational testing of the proposed methods.

In this area most of the research has been devoted to the single-vehicle case and to the so-called orienteering variants, and relatively little attention was given to multiple-vehicle extensions. Therefore, there is still considerable room for valuable and systematic research in this field, particularly for unified approaches capable of successfully tackling several variants. Furthermore, many additional characteristics, such as multiple depots, heterogeneous fleet, and customer clustering may be added to the basic problems to model specific real-world applications such as SPS and tourist trip determination.

As a general comment on the solution approaches proposed in the literature for vehicle routing problems with profits, we can say that, concerning the heuristic algorithms, the most successful ones combine classical procedures for the TSP and the VRP, aimed at optimizing the route length, with the use of neighborhoods which determine the insertion or the removal of customers from the routes. If we instead focus on exact approaches, BC is the leading methodology for the solution of single-vehicle problems, while BP is the most applied procedure for the multiple-vehicle case. In all the BP algorithms described in the literature, the choice of the set of customers to be served is made in the master problem, while the subproblem turns out to have the same structure as the subproblem obtained in column generation for the VRP; i.e., it is an elementary shortest path problem with resource constraints. Recently, these approaches turned out to be competitive when cuts were added.

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