Chapter 1

The Family of Vehicle Routing Problems

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1.1 ▪ Introduction

A generic verbal definition of the *family of vehicle routing problems* can be the following:

> **Given:** A set of *transportation requests* and a *fleet of vehicles*.

The problem is then to find a plan for the following:

> **Task:** Determine a set of *vehicle routes* to *perform* all (or some) transportation requests with the given vehicle fleet *at minimum cost*; in particular, decide which *vehicle handles which requests in which sequence* so that all *vehicle routes* can be *feasibly* executed.

In this type of problem, subsumed under the term *Vehicle Routing Problem* (VRP), the transportation requests to be served are generally concentrated in specific points of a road network as opposed to the *Arc Routing Problems* (ARP; see the companion book by Corberán and Laporte [35]), where the requests are dispersed along the arcs, i.e., street segments of the underlying road network. The following sections will shed light on the basic VRP components, which are the *transportation requests* and how they can be performed, the *fleet* of vehicles, the related *costs* and *profits* (if relevant), and the *feasibility of routes*.

Before going into the details, however, we discuss the economic relevance of computer-supported vehicle routing. Indeed, the large number of real-world applications have widely shown that the use of computerized solution methods for the solution of VRP, both at the planning and the operational levels, yields substantial savings in the global transportation costs. The success of the utilization of optimization techniques is due not only to the power of the current computer systems and to the full integration of the information systems into the operations and commercial processes, but it can also be attributed to the development of rigorous mathematical models, which are able to take into account almost all the characteristics of the VRP arising in real-world applications. Furthermore, the corresponding algorithms and their computer implementations (*software tools*) play an essential role in finding high-quality feasible solutions for real-world instances within acceptable computing times. Compared to procedures not based on optimization techniques, significant cost savings and a better utilization of the vehicle fleet can be achieved. In addition, by means of such planning software it is possible to improve the automation, standardization, and integration into the organizations' overall planning processes, leading to less time-consuming and more cost-efficient planning processes with respect to manual planning. Moreover, computerized planning allows planners to compare several different planning scenarios, and herewith to choose a best one through a careful and fast evaluation of cost and service-related performance indicators.

In the last years, software tools integrating telematics services (electronic data transmission between vehicles and planners) have been developed with the aim of enabling a faster reaction of planners to the dynamics of the transportation system and to possible disruptions caused by the failure of vehicles or by heavy traffic conditions on the roads. Of course, online planning tools are indispensable for real-time applications such as the control of *automatic guided vehicles*.

More than 50 years have elapsed since the 1959 paper by Dantzig and Ramser [40] which introduced the VRP (then called the *truck dispatching problem*) as a real-world application concerning the delivery of gasoline to gas stations. In this seminal paper the authors proposed the first mathematical programming formulation and algorithmic approach for the VRP. Again, approximately 50 years separate us from the famous paper by Clarke and Wright [33], who in 1964 proposed an effective greedy heuristic for the approximate solution of the VRP. Following such brilliant forerunners, a huge number of papers have been published during the past five decades in the international Operations Research and Transportation Science journals, presenting mathematical models and proposing exact and (meta)heuristic algorithms for the optimal and approximate solution of the different versions of the VRP. Among the journals that regularly publish papers on the VRP, we can mention *Operations Research*, *Transportation Science*, *Computers & Operations Research*, *European Journal of Operational Research*, *EURO Journal on Transportation and Logistics*, *Journal of the Operational Research Society*, *Transportation Research*, *Networks*, and *Journal of Heuristics*. Finally, in all the main international Operations Research conferences there is at least one stream (often containing several sessions) with presentations of new research and application results on the VRP.

The high interest of the international research community in the different variants of VRP is not only motivated by their notorious difficulty as combinatorial optimization problems but also, as previously mentioned, by their practical relevance. As a consequence, researchers from both the academic and the industrial world work on the subject. In North America, the majority of the academics and practitioners working on VRP are members of the *Transportation Science and Logistics society* (TSL) within INFORMS (the Institute for Operations Research and Management Science), while in Europe a new working group of EURO (the association of the European operational research societies), called VeRoLog (Vehicle Routing and Logistics optimization), has recently been founded. Its purpose consists in "favouring the development and application of Operations Research models, methods and tools in the field of vehicle routing and logistics" and in "encouraging the exchange of information among practitioners, end-users and researchers in the area of vehicle routing and logistics, stimulating the work on new and important problems with sound scientific methods"; see http://www.verolog.eu/.

It is finally worth noting that the mathematical models and the exact and metaheuristic algorithms proposed for the VRP and presented in the chapters of this book constitute a reference point not only for the research on this specific area but also for the general domain of combinatorial optimization. Indeed, the VRP, and its variants are often used as benchmarks for the development of new models and algorithmic techniques later successfully applied for the effective solution of other difficult combinatorial optimization problems.

1.2 ▪ The Capacitated Vehicle Routing Problem

The *Capacitated Vehicle Routing Problem* (CVRP) is the most studied version of the VRP, although it has primarily an academic relevance. For the sake of clarity, we start our illustration of the *VRP family* with this basic variant in order to introduce a concrete example. Notation and models discussed here will be helpful to introduce other variants and to clarify the differences between the diverse types of VRPs.

1.2.1 ▪ Problem Statement

In the CVRP, the transportation requests consist of the distribution of goods from a single *depot*, denoted as point 0, to a given set of n other points, typically referred to as *customers*, $N = \{1,2,\dots,n\}$. The amount that has to be delivered to customer $i \in N$ is the customer's *demand*, which is given by a scalar $q_i \geq 0$, e.g., the weight of the goods to deliver. The *fleet* $K = \{1,2,\dots,|K|\}$ is assumed to be *homogeneous*, meaning that $|K|$ vehicles are available at the depot, all have the same capacity $Q > 0$, and are operating at identical costs. A vehicle that services a customer subset $S \subseteq N$ starts at the depot, moves once to each of the customers in S, and finally returns to the depot. A vehicle moving from i to j incurs the *travel cost* c_{ij}.

The given information can be structured using an undirected or directed graph. Let $V = \{0\} \cup N = \{0,1,\dots,n\}$ be the set of *vertices* (or nodes). It is convenient to define $q_0 := 0$ for the depot. In the symmetric case, i.e., when the cost for moving between i and j does not depend on the direction, i.e., either from i to j or from j to i, the underlying graph $G = (V,E)$ is complete and undirected with edge set $E = \{e = \{i,j\} = \{j,i\} : i,j \in V, i \neq j\}$ and edge costs c_{ij} for $\{i,j\} \in E$. Otherwise, if at least one pair of vertices $i,j \in V$ has asymmetric costs $c_{ij} \neq c_{ji}$ then the underlying graph is a complete digraph $G = (V,A)$ with arc set $A = \{(i,j) \in V \times V : i \neq j\}$ and arc costs c_{ij} for $(i,j) \in A$. Note that $|E| = n(n+1)/2$ and $|A| = n(n+1)$ so that both graphs contain $\mathcal{O}(n^2)$ links. Overall, a CVRP instance is uniquely defined by a complete weighted graph $G = (V,E,c_{ij},q_i)$ or digraph $G = (V,A,c_{ij},q_i)$ together with the size $|K|$ of the vehicle fleet K and the vehicle capacity Q.

A *route* (or *tour*) is a sequence $r = (i_0,i_1,i_2,\dots,i_s,i_{s+1})$ with $i_0 = i_{s+1} = 0$, in which the set $S = \{i_1,\dots,i_s\} \subseteq N$ of customers is visited. The route r has cost $c(r) = \sum_{p=0}^{s} c_{i_p,i_{p+1}}$. It is *feasible* if the capacity constraint $q(S) := \sum_{i \in S} q_i \leq Q$ holds and no customer is visited more than once, i.e., $i_j \neq i_k$ for all $1 \leq j < k \leq s$. In this case, one says that $S \subseteq N$ is a *feasible cluster*. A solution to a CVRP consists of $|K|$ feasible routes, one for each vehicle $k \in K$. The routes $r_1,r_2,\dots,r_{|K|}$ and corresponding clusters $S_1,S_2,\dots,S_{|K|}$ provide a *feasible solution* to the CVRP if all routes are feasible and the clusters form a partition of N. Concluding, the CVRP consists of two interdependent tasks:

(i) the partitioning of the customer set N into feasible clusters $S_1,\dots,S_{|K|}$;

(ii) the routing of each vehicle $k \in K$ through $\{0\} \cup S_k$.

The latter task requires the solution of a *Traveling Salesman Problem* (TSP) over $\{0\} \cup S_k$ (see, e.g., Lawler et al. [93] and Gutin and Punnen [77]). Both tasks are intertwined because the cost of a cluster depends on the routing, and the routing needs clusters as an input.

1.2.2 ▪ Models

In this section, we present four important mathematical programming formulations for the CVRP. We have selected these four models in order to provide different views on the CVRP and to modify and extend some parts of the models to better explain VRP variants in later sections. Our choice of models is not intended to provide a comprehensive survey of VRP models, but additional formulations such as commodity-flow formulations can be found in Laporte and Nobert [90] and Toth and Vigo [136].

Basic Notation. Let $S \subseteq V$ be an arbitrary subset of vertices. For undirected graphs, the *cut set* $\delta(S) = \{\{i,j\} \in E : i \in S, j \notin S\}$ (set $E(S) = \{\{i,j\} \in E : i,j \in S\}$) is the set of edges with exactly one (both) endpoint(s) in S. For directed graphs $G = (V,A)$, the *in-arcs* and *out-arcs* of S are defined as $\delta^-(S) = \{(i,j) \in A : i \notin S, j \in S\}$ and $\delta^+(S) = \{(i,j) \in A : i \in S, j \notin S\}$, respectively. It has become a standard to define $\delta(i) := \delta(\{i\})$ for singleton sets $S = \{i\}$ (similarly, $\delta^+(i)$ and $\delta^-(i)$). Moreover, $A(S) = \{(i,j) \in A : i,j \in S\}$ is the set of all arcs connecting vertices in S.

We will use a condensed notation in models, where for any vector of variables or coefficients x indexed by $i \in J$ and any subset $I \subseteq J$, the term $x(I)$ means $x(I) = \sum_{i \in I} x_i$. However, for the sake of completeness, for the first model we will also describe it with the more traditional notation, which makes explicit use of summations. Both notations will be used throughout the book.

For a customer subset $S \subseteq N$, let $r(S)$ be the minimum number of vehicle routes needed to serve S. In the CVRP, the number $r(S)$ can be computed by solving a *bin packing problem* (see Martello and Toth [98]) with items N of weight $q_i, i \in N$, and bins of size Q. A lower bound, often used instead of $r(S)$, is given by $\lceil q(S)/Q \rceil$.

Compact Formulations. We present here the classical *compact formulations* for the CVRP, i.e., (Mixed) Integer Programming (MIP and IP) models which have a polynomial number of variables with respect to $n = |N|$ and $|K|$. We start with two *vehicle-flow formulations* that have an exponential number of constraints, and we briefly discuss modeling techniques to reduce the cardinality of the constraint sets to a polynomial number. Compact models are particularly well suited to solve simple VRP variants with mathematical programming based techniques, i.e., the direct use of a MIP solver or Branch-and-Cut algorithms; see Chapter 2. By "simple variant" we refer to those variants of the VRP in which the objective function and the constraints are expressed as summations over the visited vertices and traversed links (as opposed to, e.g., load-dependent costs).

A first important class of models has integer decision variables x_{ij} for $\{i,j\} \in E$ (or $(i,j) \in A$) indicating how often a vehicle directly moves between i and j (from i to j). Since the variables have two indices, the formulations are known as *two-index (vehicle-flow) formulations*.

The model for directed CVRP, denoted here as VRP1, was introduced by Laporte, Mercure, and Nobert [89]. As previously discussed, we first give the model for directed graphs in the traditional notation, which reads as follows:

$$
\begin{array}{llll}
(1.1) & \text{(VRP1)} & \text{minimize} \sum_{(i,j)\in A} c_{ij}x_{ij} & \\
(1.2) & \text{s.t.} & \sum_{j\in\delta^+(i)} x_{ij} = 1 & \forall i \in N, \\
 & & \sum_{i\in\delta^-(j)} x_{ij} = 1 & \forall j \in N, \\
(1.3) & & \sum_{j\in\delta^+(0)} x_{0j} = |K|, & \\
(1.4) & & \sum_{(i,j)\in\delta^+(S)} x_{ij} \geq r(S) & \forall S \subseteq N, S \neq \emptyset, \\
(1.5) & & x_{ij} \in \{0,1\} & \forall (i,j) \in A.
\end{array}
$$

Next, we present the same model in more condensed form using vectors and the above condensed notation for summations. Moreover, we set formulation VRP1 side by side with the two-index formulation for the undirected CVRP (denoted here as VRP2) introduced by Laporte, Nobert, and Desrochers [92] in order to highlight their similarities and differences:

$$
\begin{array}{lllll}
 & & \text{(VRP1)} & \text{(VRP2)} & \\
(1.1) & & \text{minimize } c^\top x & \text{minimize } c^\top x & \\
(1.2) & \text{s.t.} & x(\delta^+(i)) = 1 & x(\delta(i)) = 2 & \forall i \in N, \\
 & & x(\delta^-(j)) = 1 & & \forall j \in N, \\
(1.3) & & x(\delta^+(0)) = |K| & x(\delta(0)) = 2|K|, & \\
(1.4) & & x(\delta^+(S)) \geq r(S) & x(\delta(S)) \geq 2r(S) & \forall S \subseteq N, S \neq \emptyset, \\
(1.5) & & x_a \in \{0,1\}\ \forall a \in A & x_e \in \{0,1,2\}\ \forall e \in \delta(0), & \\
 & & & x_e \in \{0,1\}\ \forall e \in E \setminus \delta(0). &
\end{array}
$$

In both models, the objective (1.1) is the minimization of the overall routing costs. Constraints (1.2) state that in a route, each customer vertex is connected to two other vertices, which are its *predecessor* and *successor*. Similarly, constraints (1.3) ensure that exactly $|K|$ routes are constructed. Therefore, the depot has $|K|$ successor vertices and is connected to $2|K|$ customer vertices. If more vehicles than needed are available (i.e., $|K| > r(N)$), one can replace the equalities (1.3) with inequalities of type "$\leq$". Note that a solution with $|K| = r(N)$ may have larger routing costs than one where more routes are allowed. In fact, fleet size minimization and routing cost minimization are conflicting objectives. However, by adding fixed costs for routes (altering the cost coefficients c_{0i}), both objectives can be integrated.

Constraints (1.4) serve at the same time as *capacity constraints* and *Subtour Elimination Constraints* (SECs), which can be seen as follows: First, consider an infeasible route over the cluster $S \subseteq N$ with a demand $q(S) > Q$. Due to $r(S) > 1$, at least two routes must connect S with its complement $V \setminus S$, so that any capacity-infeasible route is excluded. Second, any subtour over a non-empty subset $S \subseteq N$ (S does not contain the depot) fulfills $x(\delta^{(+)}(S)) = 0$. Due to $r(S) \geq 1$ this subtour is also eliminated. In the directed case, the SEC $x(\delta^+(S)) \geq r(S)$ are equivalent to $x(\delta^-(S)) \geq r(S)$ and $x(A(S)) \leq |S| - r(S)$. In the undirected case, the SEC $x(\delta(S)) \geq 2r(S)$ are equivalent to $x(E(S)) \leq |S| - r(S)$. This can be proven easily by summing up the degree constraints (1.2) for $i \in S$ and $j \in S$. In all cases, the number of constraints grows exponentially with the number of vertices.

There are two basic techniques to handle this exponential number of constraints. On the one hand, leaving out all or some SEC gives a relaxation of the CVRP. The resulting LP-relaxation can be solved with cutting-plane algorithms in which at each iteration the violated SEC are identified (using a so-called *separation procedure*) and added to the relaxation until no more violated SEC are found. Branch-and-Cut algorithms include procedures for the separation of other classes of valid inequalities for the associated integer polyhedron (see Naddef and Rinaldi [104] and Chapter 2). On the other hand, when considering the directed model VRP1 the above SEC can be replaced by another set of constraints using additional variables. The model is known as the *MTZ-formulation* as introduced by Miller, Tucker, and Zemlin [99] for the TSP. More precisely, for the directed model VRP1, the additional variables $u = (u_1, \ldots, u_n)^\top$ indicate the accumulated demand u_i already distributed by the vehicle when arriving at customer $i \in N$. Constraints (1.4) can be replaced by MTZ-specific SEC

$$u_i - u_j + Qx_{ij} \leq Q - q_j \qquad \forall (i,j) \in A(N) \tag{1.6}$$

and capacity constraints

$$q_i \leq u_i \leq Q \qquad \forall i \in N. \tag{1.7}$$

Note that $x_{ij} = 1$ implies $u_j \geq u_i + q_j > u_i$. Hence, the presence of a subtour $(i, j, \ldots, i)$ not containing the depot leads to the contradiction $u_i > u_j > \cdots > u_i$. The advantage of the MTZ-formulation is that it has $\mathcal{O}(n^2)$ variables and constraints. On the downside, however, its linear relaxation generally produces a significantly weaker lower bound compared to those produced by model VRP1 (see, e.g., Desrochers and Laporte [45] and Padberg and Sung [108]). Thus, there is a clear trade-off between the strength of the linear relaxation and the size of the underlying MIP model.

The two-index formulations VRP1 and VRP2 model the underlying fleet only implicitly. Even if we know that a specific edge $\{i, j\}$ or arc (i, j) is used in a solution, it is not clear which vehicle k will travel between i and j. Hence, two-index formulations cannot model vehicle-specific characteristics such as different capacities, associated depots, and costs (see Section 1.3.4), which is a clear disadvantage. However, models VRP1 and VRP2 have the advantage of providing non-redundant representations, i.e., no symmetric solutions resulting from renumbering the vehicles, such that there is a one-to-one correspondence between feasible VRP solutions and feasible vectors x.

Next we present a *three-index (vehicle-flow) formulation*, which is based on a directed graph $G = (V, A)$ in which the depot 0 is replaced by two vertices o and d representing the startpoint and endpoint of a route. The new definition of the vertex and arc sets is

$$V := N \cup \{o, d\} \quad \text{and} \quad A := (V \setminus \{d\}) \times (V \setminus \{o\}).$$

As their name suggests, three-index formulations have binary variables of the form x_{ijk} modeling the movement of the vehicles over the arcs; in other words, $x_{ijk} = 1$ if and only if vehicle $k \in K$ moves over the arc $(i, j) \in A$. Moreover, binary variables y_{ik} indicate whether or not vehicle k visits the vertex $i \in V$. In this case, u_{ik} is (a lower bound on) the load in vehicle k directly before reaching i. For completeness, we define $q_0 = q_d = 0$. The three-index formulation, originally proposed by Golden, Magnanti, and Nguyen [72] in a slightly different form, is

$$
\begin{aligned}
&(1.8) \quad \text{(VRP3)} \quad && \text{minimize} \sum_{k \in K} c^\top x_k && \\
&(1.9) && \text{s.t.} \sum_{k \in K} y_{ik} = 1 && \forall i \in N, \\
&(1.10) && x_k(\delta^+(i)) - x_k(\delta^-(i)) = \begin{cases} 1, & i = o, \\ 0, & i \in N, \end{cases} && \forall i \in V \setminus \{d\}, k \in K, \\
&(1.11) && y_{ik} = x_k(\delta^+(i)) && \forall i \in V \setminus \{d\}, k \in K, \\
&(1.12) && y_{dk} = x_k(\delta^-(d)) && \forall k \in K, \\
&(1.13) && u_{ik} - u_{jk} + Q x_{ijk} \le Q - q_j && \forall (i,j) \in A, k \in K, \\
&(1.14) && q_i \le u_{ik} \le Q && \forall i \in V, k \in K, \\
&(1.15) && x = (x_k) \in \{0,1\}^{K \times A}, && \\
&(1.16) && y = (y_k) \in \{0,1\}^{K \times V}. &&
\end{aligned}
$$

The objective (1.8) minimizes the overall routing costs, and constraints (1.9) ensure that each customer is served exactly once. The path-flow constraints (1.10) imply that $x_k(\delta^+(d)) - x_k(\delta^-(d)) = -1$ holds. Therefore, for a fixed $k \in K$, the arc set $\{(i,j) \in A : x_{ijk} = 1\}$ induces an o-d-path, i.e., the route performed by vehicle k. Constraints (1.11)–(1.12) couple routing variables x_{ijk} with indicator variables y_{ik}. Vehicle-specific MTZ and capacity constraints are given by (1.13) and (1.14).

We will see in Section 1.3.4 that formulation VRP3 enables simple modifications to take vehicle-specific characteristics into account. However, in particular for large fleets K, the model suffers from its inherent symmetry with respect to the numbering of the vehicles, since any solution to VRP3 has $|K|!$ equivalent solutions permuting the vehicle indices. Even with the addition of symmetry breaking constraints, such as those proposed by Fischetti, Salazar-González, and Toth [58], the solution of VRP3 with enumerative or MIP-based approaches remains often intractable.

Extensive Formulation. An *extensive formulation* for the CVRP was first proposed by Balinski and Quandt [8] and is based on an extended set partitioning or set covering model. The idea is that feasible routes are the basic objects to work with, and a solution to the model directly decides about the routes to include in the solution. Let Ω be the set of feasible CVRP routes. Each route $r \in \Omega$ is of the form $r = (i_0, i_1, i_2, \ldots, i_s, i_{s+1})$ with $i_0 = o$ and $i_{s+1} = d$ so that we can assign the cost $c_r = \sum_{j=0}^{s} c_{i_j, i_{j+1}}$ to it. Furthermore, the coefficient $a_{ir} \in \{0,1\}$ is equal to 1 if and only if the route r visits customer $i \in N$, i.e., if $i \in \{i_1, i_2, \ldots, i_s\}$. Finally, binary variables λ_r indicate whether or not route $r \in \Omega$ is selected. The extensive formulation is then

$$
\begin{aligned}
&(1.17) \quad \text{(VRP4)} \quad && \text{minimize } c^\top \lambda && \\
&(1.18) && \text{s.t.} \sum_{r \in \Omega} a_{ir} \lambda_r = 1 && \forall i \in N, \\
&(1.19) && \mathbb{1}^\top \lambda = |K|, && \\
&(1.20) && \lambda \in \{0,1\}^{\Omega}. &&
\end{aligned}
$$

The cost of the selected routes is minimized by (1.17). The set partitioning constraints (1.18) stipulate that each customer is visited once by one route, and the constraint (1.19) requires that the complete fleet be utilized.

If the routing costs fulfill the triangle inequality, i.e., $c_{hi}+c_{ij} \geq c_{hj}$ for all $(h,i),(i,j) \in A$, then the set partitioning constraints (1.18) can be replaced by set covering constraints (i.e., $= 1$ becomes ≥ 1). The reason is that removing one or several customers from a feasible route results in a subroute that is feasible and less (or equally) costly. Moreover, if fewer than $|K|$ vehicles suffice to serve all customers, and leaving a vehicle unused is feasible, then one can replace $=$ by $\leq$ in (1.19).

The extensive formulation has two major advantages over compact formulations. First, it typically provides excellent lower bounds by solving its linear relaxation; see Bramel and Simchi-Levi [17]. Second, the costs and constraints that describe the feasibility of a route are implicitly hidden in the definition of the set Ω. Therefore, highly complex non-linear cost functions and any type of intra-route constraints (even beyond those discussed in Sections 1.3.3 and 1.3.6) can also be used in the definition of c_r and Ω. Moreover, model VRP4 does not suffer from symmetry problems. On the downside, the number $|\Omega|$ of feasible routes can easily grow to dimensions that prohibit the explicit instantiation and (even more) the direct solution of (1.17)–(1.20). Therefore, one has to rely on algorithms that implicitly work on the huge set Ω such as Lagrangian relaxation and column generation techniques (see Bramel and Simchi-Levi [18]). In fact, model VRP4 results from a Dantzig–Wolfe decomposition (Dantzig and Wolfe [41]) and a commodity aggregation of the three-index formulation VRP3 (see, e.g., Desaulniers et al. [44] and Lübbecke and Desrosiers [97]).

1.3 ▪ The Family of VRP

In this section, we give a broad overview of the most important variants of the VRP presented in the literature during the more than 50 years of its history. Far from being exhaustive, our presentation mainly aims at introducing a classification perspective which may help in identifying the specific characteristics of a VRP that one wants to model and solve. In particular, we classify problems according to

- the (road) network structure (Section 1.3.1),
- the type of transportation requests (Section 1.3.2),
- the constraints that affect each route individually (Section 1.3.3),
- the fleet composition and location (Section 1.3.4),
- the inter-route constraints (Section 1.3.5), and
- the optimization objectives (Section 1.3.6).

Finally, Section 1.3.7 reviews some further aspects arising when integrating additional logistics issues such as inventory management and synchronization.

1.3.1 ▪ Network Characteristics

In the CVRP, the transportation tasks are related to *points in space*, i.e., locations to which goods have to be delivered. Since we typically model these points as vertices of a graph, the corresponding VRP is a so-called *node routing problem*. In contrast, the tasks in *Arc Routing Problems* (ARP; see Dror [53], Wøhlk [141], and Corberán and Laporte [35]) are services to be performed on *street segments*, also called *connections* or *links*. Examples are the routing of vehicles for street sweeping and inspections as well as winter services with salt gritting and snow plowing or removal. In many urban areas, services such as mail delivery, garbage collection, and meter reading are often also considered as tasks on edges or arcs because households are typically located densely along a street segment. Even a mixture of tasks on vertices and arcs or edges is possible, leading to *General Routing Problems* (GRP; see Orloff [107]).

If the underlying data for travel costs are symmetric (more complex variants of VRP also consider travel times or more general resource consumptions), the problem is symmetric and it is possible to model it by means of an undirected graph. If the movement between two vertices is restricted to one direction or any relevant cost or resource consumption is asymmetric, the underlying graph is either a directed, or mixed, or *windy* (see Guan [75]) graph. Asymmetric models are suitable in these cases. But even in symmetric problems sometimes modeling or solution techniques themselves require the underlying graph being directed. An example is the MTZ-formulation for the CVRP discussed in Section 1.2.2.

A point that distinguishes VRP and ARP/GRP is the *granularity of the data and network*. Typically, edges and arcs in an ARP or GRP represent individual street segments. In VRP, by contrast, edges and arcs result from paths between two points, each consisting of a possibly larger number of street segments. Thus, the distance and travel time information for the paths are to be computed by the routing component of a *Geographical Information System* (GIS; see Chang [26]). Usually, these paths are computed as shortest paths with respect to a weighted combination of distance and travel time so that the resulting cost and travel time matrices do not need to satisfy the triangle inequality. Even worse, relying on a single shortest path means that many other paths with Pareto-optimal resource consumption may simply be neglected. For this reason, Garaix et al. [60] model and solve VRP on multi-graphs in which parallel arcs represent the different Pareto-optimal paths. Summarizing, the ARP and the GRP are closer to GIS raw data, and their underlying graphs are sparse. In contrast, VRPs often have dense or complete underlying graphs (as the CVRP and several other variants discussed in the following), where edges and arcs represent shortest paths. Furthermore, in a real-world setting, a shortest path between two points (with respect to travel time or cost) can heavily depend on the time of day when the vehicle drives between the points. In *dynamic VRP*, some data is not known in advance but becomes available during operation. Also, if some data (cost, travel time, demand, etc.) are not known in advance, but are described by a random variable with a given *probability distribution*, the VRP is *stochastic* (we come back to both aspects in the next section). Obviously, the CVRP is neither dynamic nor stochastic but *static* and *deterministic*.

In this book, we will restrict ourselves to node routing problems because the ARP and the GRP are covered by the companion volume edited by Corberán and Laporte [35].

1.3.2 ▪ Types of Transportation Requests

We have already described the CVRP as a variant of VRP in which all transportation requests consist of the *distribution of goods from a depot to customers*. We start by classifying other types of transportation requests in the node routing context.

Delivery and Collection. The counterpart of delivery to customers is *collection from customers*, where all tasks involve the movement of goods or waste from a point to the depot. Collections are often called *pickups*. Associated routing problems often occur either at the beginning of a supply chain, e.g., for raw-milk collection (see, e.g., Sankaran and Ubgade [120]), or at the very end of it, e.g., in reverse logistics where returned empties have to be collected, or waste has to be disposed (see Golden, Assad, and Wasil [71] and Chapter 15 in [35]). The equivalence of pure distribution and pure collection VRP turns out to be obvious when one reverses the routes so that collection becomes distribution and vice versa.

Different variants of VRP result when both *collection and distribution* (or *pickup and delivery*) of material occur together in a route. We still assume that distribution starts and collection ends at the depot. Therefore, problems with collection are also known as *many-to-one* VRP and problems with distribution as *one-to-many VRP*. The first and probably simplest variant is the *VRP with Backhauls* (VRPB; see Chapter 9). When, for example, transporting bulky material, all deliveries to the so-called *linehaul customers* must be performed first so that the vehicle is empty when it arrives at the first collection point, called a *backhaul customer*. Because any movement from a backhaul customer to a linehaul customer is forbidden, model VRP1 remains applicable if the corresponding arcs are removed from the arc set A (alternatively, one can set the costs of these arcs to a sufficiently large number M). Backhauling constraints result from the difficulty of rearranging the loaded items inside the vehicle. If the loading space allows rearrangements, e.g., because the vehicle can be loaded from rear and front or all sides, the resulting problem is a *VRP with mixed deliveries and collections*, or simply *Mixed VRPB* (MVRPB; see Wade and Salhi [139]). Here, the vehicle capacity must be checked for each edge (or arc) traversed; i.e., the load already collected from backhaul customers plus the load to be delivered to linehaul customers can never exceed the given capacity.

While each customer in the VRPB and MVRPB requires either a delivery or a collection, but not both, the *VRP with Simultaneous Pickup and Delivery* (VRPSPD; see Min [100] and Chapter 6) comprises two transportation requests for each customer, namely a delivery from depot to customer and a pickup from customer to depot. Both transportation requests must be performed by one vehicle in a single visit. This situation is common in several real-world applications, such as the delivery of beverages and simultaneous collection of empty bottles, and the bus transportation of newly arriving and leaving hotel guests between a local airport and several hotels (this is common practice of tour operators in many holiday regions). Again, the capacity constraint ensures that no vehicle is overloaded at any point. Interestingly, there exists a simple criterion to check whether or not a given customer set $S \subseteq N$ can be feasibly served by a single vehicle: A feasible route exists if neither the overall amount to be delivered nor that to be collected exceeds the vehicle capacity. One simply has to build the route so that customers with a higher amount to be delivered than to be collected are visited first in the route. A relaxation of the VRPSDP is the *VRP with Divisible Deliveries and Pickups* (VRPDDP). Here, delivery and pickup at a customer may be performed in a single visit or in two separate visits by the same vehicle. Since less capacity is needed when the delivery properly precedes the pickup, cost savings compared to the VRPSDP are possible in VRPDDP. The form of routes is more general and includes so-called *lasso routes* and others; see Gribkovskaia et al. [73]. The VRPSDP and VRPDDP should not be mixed up with the problems presented in a following paragraph on point-to-point transportation, which are also known as pickup-and-delivery problems.

Simple Visits and Vehicle Scheduling. Sometimes the service is neither collection nor delivery, but merely consists of *visiting a customer* or location. For example, service technicians repair or install something, and nurses take care of elderly people at their homes. Another specific case is that route segments have to be followed in a priori given sequences (and schedules). This is the class of *Vehicle Scheduling Problems* (VSPs), which arise in public transport when scheduling buses, trams, trains, etc., in urban, local, and national transport systems. We exclude these problems from our consideration because the major routing decisions are determined beforehand. Surveys on VSP are compiled by Desrosiers et al. [46] and Bunte and Kliewer [22], while school bus routing and other VSP applications for passenger transportation are discussed in Chapter 7.

Alternative and Indirect Services. There exist situations in which a service can be performed in alternative ways and modes. For example, the delivery of a parcel can either be made to a person's working place (during office hours), to his or her private home, or to an automated parcel terminal (see Drexl [49]). If the service provider can choose between different alternatives, the simultaneous service choice and routing of vehicles can help in consolidating tours (see Cardeneo [24]).

In the *Multi-Vehicle Covering Tour Problem* (MVCTP; see Hachicha et al. [78]), the service to customers consists of visiting a location that is close enough to the customer. Applications of the MVCTP are the simultaneous location of postboxes at given potential sites and the construction of optimal collection routes. Moreover, in some developing countries, mobile health care delivery teams operate at a selected number of sites, to which the supported people must travel. The arc routing pendant of the problem occurs in automatic meter reading (see Shuttleworth et al. [125] and Chapter 13 in [35]).

Point-to-Point Transportation. Pickup-and-delivery problems are VRPs in which the transportation requests consist of *point-to-point transports*. More precisely, each transportation request consists of the movement of goods or people between two particular locations, one where someone or something is picked up, and a corresponding location for the delivery. Generally, neither of these locations is a depot so that these problems are also referred to as *many-to-many VRP*. The problem is called a *Pickup-and-Delivery Problem* (PDP; see Desaulniers et al. [43], Parragh, Doerner, and Hartl [109], and Chapter 6) in the context of goods transportation. Applications include freight transportation, like those described by Savelsbergh and Sol [122]. In the context of passenger transportation, the problem is known as the *Dial-a-Ride Problem* (DARP; see Chapter 7). There exist applications in bus routing for pupils, patients, handicapped persons, and elderly (between their individual homes and schools, care facilities, or hospitals). Almost all DARP variants include time-window constraints. Often service levels and user (in)convenience is taken into account either with constraints or in the objective function (see Sections 1.3.3 and 1.3.6).

Repeated Supply. In the context of goods delivery, customers may require that they repeatedly receive deliveries of goods. Considering a longer planning horizon, a customer might be unconcerned about the specific day when receiving a shipment as long as he or she does not run out of stock. In the *Periodic VRP* (PVRP; see Cordeau, Gendreau, and Laporte [37], Mourgaya and Vanderbeck [103], and Chapter 9), there are two planning levels: At the first level, a feasible *visiting pattern* for each customer has to be selected from a given set of admissible ones. For example, a customer may have agreed to receive two or three deliveries per week, either on days Mon-Thu, Tue-Fri, Mon-Wed-Fri. Note that the customer demand may then depend on the particular visiting pattern and day. At the second level, a VRP must be solved for each day, where the subset of customers to be visited and their demands result from the first-level decisions.

Another form of repeated supply occurs in *Inventory Routing Problems* (IRPs; see Campbell, Clarke, and Savelsbergh [23], Bertazzi and Speranza [12], and Coelho, Cordeau, and Laporte [34]). The fundamental difference with respect to the VRP variants discussed so far is that there are no customer orders in an IRP. Instead, the delivery company decides when to visit a customer and how much to deliver so that no stock-out occurs. Based on these decisions and assuming that the planning horizon spans several days, an optimal routing of the vehicles has to be determined for every day. Thus, daily routing decisions depend on the selected subset of customers to visit and quantities to deliver on that particular day, and vice versa. Moreover, the amount to deliver to a customer may be restricted by a maximal storage level. The objective in IRP variants can differ, but it normally includes routing costs and inventory holding costs.

There is a growing interest in IRP in *Supply Chain Management* (SCM). A mean to reduce the bull-whip effect (see Lee, Padmanabhan, and Whang [94]) is *Vendor Managed Inventory* (VMI; see Disney and Towill [47]), where the supplier monitors the customer inventory levels and is responsible for the timely resupply. Advantages often cited in the SCM literature are a faster and more reliable information exchange between customer and supplier, shorter lead times, lower inventory levels, and higher service levels. Examples of real-world applications of IRP include deliveries to supermarkets or grocery stores, parts distribution in the automotive industries, refill of vending machines, e.g., with beverages, and fuel delivery to gas stations.

The *PVRP with service choice*, where the delivery frequency has an impact on the demand and the service level, as introduced by Francis, Smilowitz, and Tzur [59], can be seen as an intermediate between PVRP and IRP. Extensions of IRP cover aspects such as stochastic daily demand (for an overview on stochastic IRP see Bertazzi and Speranza [13]) and combined production planning and IRP as described by Adulyasak, Cordeau, and Jans [1].

Non-split and Split Services. Until now, we have assumed that all service tasks are performed by a single vehicle in one service operation, i.e., services are *non-split*. However, there are two reasons for splitting some services: On the one hand, if demand exceeds the vehicle capacity, more than one visit is unavoidable. On the other hand, splitting services into several smaller service requests can produce significant cost savings. The *Split Delivery VRP* (SDVRP; see Dror and Trudeau [54], Dror and Trudeau [55], and Chapter 9) allows, in principle, that each demand be split into arbitrarily many smaller demands served by different vehicles.

Combined Shipment and Multi-modal Service. Whereas in the SDVRP individual deliveries are split into smaller parts, *combined shipments* leave the individual shipments intact, but several vehicles transport the shipment from its supplier to the customer using intermediate transfer points or consolidation centers. This is a common practice in multi-modal transportation, where different types of vehicles and transportation modes are used at various steps: for example, large trucks for the long-distance full truckload transfer from factories to distribution centers and small trucks for the less-than-truckload transportation to final customers. Several variants arise depending on the specific distribution network structure, such as *hub-and-spoke* or *crossdocking*, and the presence of distribution routes or direct shipping for the *last mile* distribution. A recent survey on these problems can be found in Guastaroba, Speranza, and Vigo [76], whereas a thorough illustration of the case in which in-transit consolidation is carried out at intermediate consolidation centers is given by Song, Hsu, and Cheung [128]. Another example that has recently attracted some interest due to its applications in *city logistics* is the so-called *2-Echelon VRP* (2E-VRP; see Laporte and Nobert [91] and Perboli, Tadei, and Vigo [110]), where the delivery from a single depot to the customers is managed using *intermediate depots*, also called *satellites*.

Routing with Profits and Service Selection. With a limited fleet (see Section 1.3.4) it may be impossible to service all transportation requests. Then, just a subset of requests has to be fulfilled. More generally, by optimizing routing and request selection simultaneously, a company may gain additional revenues compared to traditional two-staged decisions, where the request acceptance precedes the route planning step. In order to control this request selection process, one can either set constraints on service levels and costs, respectively, or penalize unmet requests or reward the ones met. Note that if penalties are chosen as negative profits, then both approaches are equivalent.

Most of the problems of this type were first introduced by considering the single-vehicle case, i.e., as variants of the TSP, and later extended to the VRP. In addition, these problems are known under a variety of names such as the *selective TSP/VRP* and the *Maximum Collection TSP/VRP*. Following the taxonomy introduced by Feillet, Dejax, and Gendreau [57], there are three categories of problems:

(i) If routing costs and profits are combined into one objective, the single-vehicle routing problem is a *Profitable Tour Problem* (PTP; see Chapter 10). The VRP variant has no consistent naming, but can be found as *Capacitated PTP* (CPTP); see Archetti et al. [3].

(ii) The variant where the route length is bounded from above and the objective is profit maximization is called the *Team Orienteering Problem* (TOP; see Archetti, Hertz, and Speranza [4] and Chapter 10). The single-vehicle case is known as the *Orienteering Problem* (OP) and is discussed in Chapter 10.

(iii) Finally, if there exists a lower bound on the profit to be collected and the goal is to find a least cost routing, the problem is a *Prize-Collecting VRP* (PCVRP; see Tang and Wang [132]). The variant with only one vehicle, known as *Prize-Collecting TSP* (PCTSP), is analyzed in Chapter 10.

A related variant that attracted some interest in the recent literature is the so-called *VRP with Private fleet and Common carrier* (VRPPC; see Chu [32] and Potvin and Naud [112]), where customers may either be served by using owned vehicles with traditional routes or be assigned to a common carrier, which serves them directly at a prefixed cost. This problem can be easily seen as a PCVRP and takes into account the option of subcontracting unprofitable customers, as common in the parcel delivery service. The multiple depot case is studied by Stenger et al. [129].

The *Multiple Vehicle Traveling Purchaser Problem* (MV-TPP) (see Choi and Lee [30]) is yet another variant with service selection. We are given a set of marketplaces, where a set of goods is available at specified prices, and the demands are also known. The MV-TPP requires the determination of routes for capacitated vehicles that have to select and visit a subset of the marketplaces so that the required demand is collected and the overall sum of routing and purchase costs is minimized. Recently, the MV-TPP was used by Riera-Ledesma and Salazar-González [118] to model a school bus route design problem, whereas a distance-constrained variant of the problem is studied by Bianchessi, Mansini, and Speranza [15].

Dynamic and Stochastic Routing Important VRP variants arise with the consideration of uncertainty and variability of system conditions. In general, a problem is

- *dynamic* if parts or all relevant information about the system conditions become available during operation;
- *stochastic* if system conditions are uncertain, but uncertainty is described by a given probability distribution.

In *dynamic VRP*, the information generally revealed over time consists of customers' locations and demands (see Tillman [135], Psaraftis [114], and Chapter 11): some of them are possibly known in advance, but for the remaining customers only probabilistic information is given. This type of problem is also called *online* when the emphasis is given to the development of heuristic methods with a performance guarantee (see Jaillet and Wagner [84] for an overview). Two other types of dynamic problems have been studied: The first one considers the time dependency of travel duration (see Haghani and Jung [79] and Chapter 11). In fact, vehicle speed along roads is not only affected by the variability of traffic volumes, which may change considerably during the day, but is also subject to irregular congestion phenomena caused, for example, by car accidents, street maintenance, and illegal parking of vehicles reducing the streets' capacity. In the second type, the availability of vehicles is a dynamic component of the problem, which requires rescheduling of the routes when vehicle breakdowns and delays occur (see, e.g., Li, Mirchandani, and Borenstein [96] and Chapter 11).

In *stochastic VRP*, some problem components, such as customer demand and travel times, are uncertain (see Bertsimas [14], Gendreau, Laporte, and Séguin [62], and Chapter 8) and described as random variables. As a result, planned routes may incur delayed service at customers and may be terminated prematurely when the vehicle capacity has been reached. The focus of stochastic VRP is, therefore, on analyzing the impact of uncertainty on the resulting costs and service levels.

1.3.3 ▪ Intra-route Constraints

A key aspect to consider when defining different VRP variants is the type of constraints that determine whether or not a route is *feasible*. In this section, we discuss issues related to loading, route length, reuse of vehicles, time schedules, and the various combinations of these types of constraints occurring in practice. Common to these so-called *intra-route constraints* (also called *local constraints*) is that they can be checked once an individual route (the vertex sequence) is known, independently from what the other routes are.

Loading. The *capacity constraints*, as presented for the CVRP, belong to the simplest type of constraints, since they can be written as an overall bound on a resource that is consumed at every vertex the vehicle reaches:

$$\sum_{i \in V} q_i y_{ik} \leq Q \qquad \forall k \in K. \tag{1.21}$$

Note that we will refer to the three-index model VRP3 in the following, since many modifications and extensions can be described conveniently in that case. There may exist *several capacity constraints* relating to weight, space (e.g., pallets), and volume (e.g., m^3 for liquid goods) that can individually restrict the loading. Instead of (1.21), several constraints with their corresponding coefficients q_i and Q must then be added to the model.

More complex loading constraints occur when both the shipments and the cargo compartments are described either by 2-dimensional or 3-dimensional quantities. In these VRP variants, multi-dimensional packing problems and VRPs are combined. In the *CVRP with 2-dimensional Loading constraints* (2L-CVRP; see Iori, Salazar-Gonzalez, and Vigo [83]), shipments are rectangular items that have to be feasibly assigned to a rectangular compartment. Moreover, the items sent to the same customer must arrive with the same vehicle (*item clustering constraint*), and the items may or may not be rotated (*item orientation constraint*). When delivering an item, no items of customers served later along the route may lay, not even partially, in the rectangular area between that item and the door of the vehicle. This is a *sequential loading constraint*. As in the CVRP, a capacity constraint with respect to the weight (kg) has to be taken into account as well. The problem with *3-dimensional Loading constraints* is the 3L-CVRP as described by Gendreau et al. [61], where additional operational constraints are introduced to ensure the stability of stacked boxes, the secure transportation of fragile boxes, and the easy unloading of boxes at the customer locations. In the *Pallet-Packing VRP* (PPVRP; see Zachariadis, Tarantilis, and Kiranoudis [144]), the shipments are 3-dimensional boxes that have to be feasibly stacked onto pallets before they are loaded into the vehicles.

Interesting additional complications arise if vehicles have more than one compartment. In the *VRP with Compartments* (VRPC; see Derigs et al. [42]), the vehicle capacity, modeled as a one-dimensional quantity Q, is partitioned into a given number of smaller compartments. The compartments are either fixed units, such as tanks in a road tanker, or result from the use of dividers that split the available space. In the latter case, the divider's position may or may not be completely or partially flexible. The *compatibility constraints* refer to two aspects: Only compatible items can share a compartment (*item-item compatibility*), e.g., the smell of washing powder spoils some food, and items can only be assigned to compatible compartments (*item-compartment compatibility*), e.g., refrigerated cargo must go into cooled compartments. Compatibility between groups of items is also considered in some industrial variants (see, e.g., Xu et al. [143]).

Last-In-First-Out (LIFO) loading constraints mean that, by considering all shipments that are currently on board a vehicle, one must deliver the shipment that is most recently picked. LIFO loading constraints were mainly considered in the single-vehicle routing context (see Cordeau et al. [38]) but apply to the PDP as well (see Cherkesly, Desaulniers, and Laporte [29]). LIFO loading is intended to reduce loading and unloading times to a minimum because for vehicles that are loaded from one side very few rearrangements need to take place.

Route Length. Another simple type of constraints results from global bounds that limit the consumption of a resource consumed on edges or arcs. The addition of distance constraints to the CVRP yields the *Distance-constrained CVRP* (DCVRP; see Christofides, Mingozzi, and Toth [31] and Laporte, Desrochers, and Nobert [88]). In the following, we assume that $t_{ij} > 0$ is the distance between vertex i and vertex j for all $(i,j) \in A$. If $L > 0$ is a given upper bound on the length of a route, the distance constraints are

$$\sum_{(i,j)\in A} t_{ij} x_{ijk} \le L \qquad \forall k \in K. \tag{1.22}$$

Not only bounds on the *spatial distance* can be modeled, but so can constraints on the *route duration* (with travel times t_{ij}), on the routing costs, or on the number of connections with a certain property (with indicators $t_{ij} \in \{0,1\}$). For additional examples we refer the reader to Avella, Boccia, and Sforza [5].

Multiple Use of Vehicles. The standard assumption for many VRP variants is that each vehicle performs only one route over the planning horizon T. In the *VRP with Multiple use of vehicles* (VRPM; see Taillard, Laporte, and Gendreau [131]), vehicles may perform several routes. Given some routes, with durations $T_1, T_2, \ldots, T_p$, a single vehicle may perform them if $T_1+T_2+\cdots+T_p \le T$ holds. In particular, if the vehicle capacity Q is relatively small or other constraints impose a small number of services per route, feasible solutions with a limited fleet of size $|K|$ can only be achieved when vehicles are reused. Note that routing with unlimited fleet first and subsequently packing the resulting routes into the planning horizon generally yields suboptimal solutions to VRPM. In some cases, overtime for the drivers is permitted with a penalty (see, e.g., Brandão and Mercer [19]).

The VRPM is also known as the *Multi-Trip VRP* (MTVRP) and recently attracted new research efforts in the context of real-world applications (see Battarra, Monaci, and Vigo [10]) and city logistics (see Cattaruzza et al. [25]). An exact algorithm has been proposed by Mingozzi, Roberti, and Toth [102], while metaheuristics have been recently proposed by Olivera and Viera [106], Petch and Salhi [111] and Alonso, Alvarez, and Beasley [2].

The large recent diffusion of alternative-fuel vehicles, e.g., electrical cars, has stimulated interest of the research community in the variants of VRP that incorporate the environmental issues. The main characteristic is the currently limited autonomy of vehicles due to restricted battery capacities, which forces the visit of refuel or recharge stations during the travel. VRP with the possibility or need of refueling during the trip have been studied by Erdogan and Miller-Hooks [56] and by Schneider, Stenger, and Goeke [123] for the case with time windows.

Time Windows and Scheduling Aspects. Some very practically relevant constraints present in most VRP variants are those related to *scheduling*, i.e., requiring the consideration of *travel, service, and waiting times* together with *time-window* constraints. In the *VRP with Time Windows* (VRPTW; see Cordeau et al. [36] and Chapter 5), a traversal time t_{ij} for each arc $(i,j)\in A$ and a time window $[a_i, b_i]$ for each vertex $i \in V$ are given. A schedule, i.e., a combination of start times T_{ik} for the service at a vertex $i \in V$ when visited by vehicle $k \in K$, is considered feasible if

$$a_i \le T_{ik} \le b_i \qquad \forall i \in V, k \in K \tag{1.23}$$

(if vehicle k does not visit vertex i, the time T_{ik} is irrelevant) and

$$x_{ijk} = 1 \quad \Rightarrow \quad T_{ik} + t_{ij} \le T_{jk} \qquad \forall (i,j) \in A, k \in K \tag{1.24}$$

holds. Model VRP3 along with (1.23)–(1.24) is a three-index formulation for the VRPTW. The latter constraints couple routing decisions with the time schedule. They can be linearized by means of MTZ-like constraints of the form

$$T_{ik} - T_{jk} + M x_{ijk} \le M - t_{ij} \qquad \forall (i,j) \in A, k \in K.$$

Note that, with the above definition, time windows are asymmetric in the sense that arriving at vertex i before time a_i is allowed, in which case the vehicle has to wait until time a_i, while arriving later than time b_i is prohibited. Some authors also add *service times* s_i at vertices to their models. This is only a minor extension, since these can be included by properly redefining the travel times and time windows.

Time-window constraints can be altered and generalized in various ways: *Multiple time windows* mean that (1.23) has to be replaced by $T_{ik} \in \bigcup_{\ell=1}^{p} [a_i^\ell, b_i^\ell]$ when $[a_i^1, b_i^1], \ldots, [a_i^p, b_i^p]$ are p alternative possible intervals for the start of service at vertex i. Disjunctive constraints have to be used for linearizing the above conditions.

Travel times may depend on *the time of the day* so that t_{ij} have to be replaced by travel *time functions* $t_{ij}(T_i)$, also called *time-dependent travel times* (see, e.g., Haghani and Jung [79] and Chen, Hsueh, and Chang [28]). Note also that, in the dynamic context, time-dependent travel times may be present even without time windows.

The route performed by vehicle k *ends* at time T_{dk} and has a *duration* $T_{dk} - T_{ok}$ (both plus a possible service time at d). If $x_{ijk} = 1$, i.e., vehicle k visits vertex i directly before arriving at vertex j, then the *waiting time* at vertex j is $w_j = (a_j - T_{ik} - t_{ij})^+$, where $x^+ =$

$\max\{0, x\}$. Some interesting VRPTW variants result when these and other quantities related to the schedule are either bounded or if they contribute to the objective. In the *VRP with Soft Time Windows* (VRPSTW; see Taillard et al. [130]), a convex function $p_i : [a_i, b_i] \to \mathbb{R}$ is used to generate solutions in which service is provided at a time close to the minimum of p_i. The most studied variants correspond to linear penalty functions of the form $p_i(T_{ik}) = p(T_{ik} - b_i')^+$, where p is a penalty factor and $a_i \leq b_i' < b_i$ defines a linear *penalty on late services*, and $p_i(T_{ik}) = p(a_i' - T_{ik})^+ + p(T_{ik} - b_i')^+$ with $a_i < a_i' \leq b_i' < b_i$ for linear *penalties on both early and late services*. In the first case, the favored start time is in the time interval $[a_i, b_i']$, while in the second it is in $[a_i', b_i']$, and deviating from it is penalized linearly. Other VRP variants require waiting times to be bounded at each vertex, per vehicle, or over all vehicles, or waiting is penalized (see also Section 1.3.4). The route completion times T_{dk} can be bounded individually, and in other applications the *makespan* $\max_{k \in K} T_{dk}$ is a component of the objective.

Probably the most complex scheduling constraints are related to driving rules and schedule regulations. For example, the very advanced approach of Kok et al. [87] takes into account the driving rules appointed by European Regulation (EC) No. 561/2006, Directive 2002/15/EC, and several modified rules in these laws. These include rules that describe in detail restrictions on *driving periods* (max. 4.5 hours), *daily driving times* (max. 9 hours, but twice a week max. 10 hours), *weekly driving times* (max. 56 hours and max. 90 hours within two weeks), *breaks to end a driving period* (min. 45 minutes; also possible 30 minutes plus an earlier break of min. 15 minutes), *daily rest periods* (min. 11 hours; might be reduced to 9 hours under various preconditions), and *weekly rest periods* (min. 45 hours; might be reduced to 24 hours in every second week if compensated by an equally longer rest; max. 144 hours between two weekly rests). Obviously, the consideration of all these rules and, in particular, the possibilities to deviate from default requirement creates highly intricate VRPTW variants. Even checking the feasibility of a given route (as a vertex sequence), i.e., checking whether or not there exists a feasible time schedule for driving, breaks, and rests, is highly complex. Similar settings were handled in general and for Australia, Canada, Europe, the U.S., and in an international comparison by Goel [64], Goel, Archetti, and Savelsbergh [66], Goel and Rousseau [69], Goel [65], Prescott-Gagnon et al. [113], Goel and Kok [67], Rancourt, Cordeau, and Laporte [115], and Goel and Vidal [70].

The DARP (see Section 1.3.2 and Chapter 7) is the pickup-and-delivery problem for passenger transportation. A transportation request, given by a pair of pickup-and-delivery vertices $(i, i+n)$, comes with two time windows: one window $[a_i, b_i]$ for the pickup operation and one window $[a_{i+n}, b_{i+n}]$ for the delivery operation. The ride time R_i is therefore bounded by $b_{i+n} - a_i$ and may be further controlled by imposing a so-called *ride-time constraint* of the form $T_{i+n} - T_i \leq R_i$, or by penalizing user inconvenience through a non-decreasing function in $T_{i+n} - T_i$ in the objective.

Many relevant aspects related to timing problems, their modeling, and efficient algorithmic treatment are covered in a recent paper by Vidal et al. [138].

1.3.4 ▪ Fleet Characteristics

So far we have discussed VRPs with identical vehicles based at the same depot. In this section, we turn to fleets of vehicles that are stationed at different depots and vehicles having different characteristics concerning capacity, costs, speed, and the ability to load material and access locations. Moreover, VRP with trucks and trailers consider autonomous and non-autonomous vehicles, where the latter need to be pulled for moving from one place to another and can be decoupled to reach otherwise inaccessible locations.

Multiple Depot VRP. If the fleet of vehicles is homogeneous, but vehicles start and end their routes at different depots, the resulting problem is known as the *Multi(ple) Depot VRP* (MDVRP; see Renaud, Laporte, and Boctor [116]). Vehicle specific locations for beginning and terminating a route can easily be incorporated in model VRP3: Replace the vertices o and d (or the corresponding entries in the cost matrix) of the single-depot case with vehicle specific vertices o_k and d_k (with new entries). In principle, every vehicle $k \in K$ may have its own starting and ending locations. However, in the MDVRP, groups of vehicles are assigned to a (smaller) number of depots. Depots may have a limited capacity and host a limited or an unlimited subfleet. Vidal et al. [137] present a review of the recent literature on the MDVRP as well as an effective heuristic for the problem and its periodic variants.

The variant of the MDVRP, where depots can act as intermediate replenishment facilities along the route of a vehicle, is considered by Crevier, Cordeau, and Laporte [39] and Tarantilis, Zachariadis, and Kiranoudis [134]. This problem is strongly related with the multiple use of vehicles discussed in Section 1.3.3.

Heterogeneous or mixed Fleet VRP. The class of *Heterogeneous or mixed Fleet VRP* (HFVRP; see Baldacci, Battarra, and Vigo [6] and Chapter 9) considers groups or types of vehicles that can differ in capacity, variable and fixed costs, speeds, and the customers that they can access. The fleet K is partitioned into $|P|$ subsets of homogeneous vehicles $K = K^1 \cup K^2 \cup \cdots \cup K^{|P|}$ (also called *vehicle types*). All vehicles $k \in K^p$ from the pth type ($p = 1, \dots, |P|$) are characterized by a capacity $Q_k = Q^p$, variable routing costs $c_{ijk} = c_{ij}^p$, *fixed costs* $FC_k = FC^p$, and the subset $N_k = N^p \subseteq N$ of *accessible* customers. Individual travel times $t_{ijk} = t_{ij}^p$ can replace standard travel times t_{ij}.

Fixed costs become relevant only if not all vehicles need to be utilized. In this case, fixed costs often reflect the effort required to provide a driver and to prepare and maintain the vehicle. The inclusion of overhead costs is critical because these cannot be assigned fairly to a single route according to the cost-by-cause principle. Generally, the magnitude of the fixed costs strongly depends on whether the fleet is owned by the decision maker or whether the transport service is performed by a third party.

It is easy to modify model VRP3 to take vehicle-specific attributes into account. One simply has to replace the general coefficients with vehicle-specific coefficients, e.g., the capacity Q with Q_k. *Vehicle-dependent routing costs* arise when c_{ij} is replaced with different c_{ijk} for all $(i,j) \in A$. Fixed costs can be incorporated into the routing costs by replacing c_{oj} with $c_{ojk} + FC_k$ for all $(o,j) \in \delta^+(o)$. Also, to model inaccessible customers $j \in N \setminus N_k$, one can set c_{ijk} to a sufficiently large number M for all $(i,j) \in \delta^-(j)$.

Another important aspect is whether the fleet and groups of vehicles are limited or not. In principle, model VRP3 is a limited-fleet model because the fleet consists of exactly $|K| = |K^1| + |K^2| + \cdots + |K^{|P|}|$ vehicles. However, using sufficiently large numbers for $|K^p|$, unlimited fleet models result.

A classification scheme for heterogeneous fleet VRP is offered by Baldacci, Battarra, and Vigo [6]. In *Heterogeneous VRP* (HVRP) and *site-dependent VRP*, the fleet is always limited, while for *Fleet Size and Mix VRP* (FSM) the fleet is unlimited. In FSM, the planning task is more strategic or tactical, since it is typically related to an optimal acquisition of vehicles. In the site-dependent VRP, site dependencies are the only characteristic in which vehicles differ. Therefore, no other vehicle-dependent costs or fixed costs are considered and vehicles have the same capacity. In contrast, HVRP and FSM exist in both variants where fixed costs and vehicle-dependent routing costs are either considered or not. More details on this class of problems, including the above-mentioned classification, can be found in Chapter 9.

Routing of Trucks and Trailers. The *Truck-and-Trailer Routing Problem* (TTRP; see Chao [27]) considers a fleet with at least two groups of vehicles: *Single Trucks* (ST), i.e., normal vehicles without trailer, and *Truck-and-Trailer Combinations* (TTC). While a TTC is attractive due to larger overall capacity, there exist site-dependency conditions stating that some customers are not accessible by a TTC. Insufficient maneuvering space at a customer location is a typical reason for this type of inaccessibility. Customers inaccessible by TTC are called *truck customers*. All others are *regular customers*. Hence, three types of routes are possible:

(i) an ST route is performed by a ST and visits any type of customer;

(ii) a pure TTC route is performed by a TTC and visits only regular customers;

(iii) a mixed TTC route is also performed by a TTC but visits both truck and regular customers. Here, the TTC travels on a main route between regular customers. In order to reach the truck customers, the truck performs one or several subtours, where the trailer is first decoupled at a regular customer, and the truck returns to the same place at the end of the subtour before continuing the main route with the coupled trailer. Note that the demand collected on a truck subtour must fit into the ST. Only at regular customers can the truck transfer all or some of its load into the trailer so that the joint capacity of the TTC can be used.

The TTRP is generalized by Drexl [48] with respect to three aspects: First, variable and fixed costs for trucks and trailers are considered as in the HFVRP. Second, in addition to locations of regular customers, there are optional locations for parking trailers and performing the load transfer. Third, time-window constraints have to be respected at all locations.

A much more intricate routing problem is the *VRP with Trailers and Transshipments* (VRPTT; see Drexl [51]). The main difference with respect to the TTRP is that in the VRPTT there is no fixed assignment of trailers to trucks. Instead, a trailer may be pulled by one or several trucks on parts of its itinerary. This offers the possibility that different trucks, even those that just meet but do not pull the trailer, transfer all or parts of their collected load into the trailer. Drexl [51] extends the VRPTT by integrating several other constraints and options arising, e.g., in applications such as raw-milk collection: There exist several classes of trucks and trailers either compatible or incompatible with respect to load transfer and/or the possibility that the truck moves the trailer. Moreover, *support trucks* cannot access any customers but serve as mobile depots to which other vehicle can transfer their load. The TTRP is a prime example of a VRP with synchronization constraints discussed later in Section 1.3.7.

1.3.5 ▪ Inter-route Constraints

Many VRP variants have the property that a given set of routes provides a feasible solution whenever the routes are feasible and the given transportation requests are partitioned accordingly. The only dependency between the routes consists of the partitioning of the transportation requests. Thus, feasibility exclusively depends on the intra-route constraints discussed in Section 1.3.3. In this section, we present examples of so-called *inter-route constraints* (also *global constraints*), where the feasibility of a solution depends on how the routes (and their schedules) are combined.

A first example are *balancing constraints* that often result from fairness considerations. They state, e.g., that the *difference between maximum and minimum route duration in a solution* should not exceed a given threshold (see, e.g., Bodin, Maniezzo, and Mingozzi [16]). Thus, the workload can be evenly distributed among the drivers. Balancing route duration also makes sense in connection with time windows. The constraints generalize simpler types of balancing, e.g., when the number of stops in a route, the route length, or the delivery amount of a route is taken as a balancing criterion.

A second example are *inter-route resource constraints* which occur in VRPs when different vehicles compete for globally limited resources (see Hempsch and Irnich [82]). The simplest type is a constraint limiting the number of available vehicles that can be allocated to a specific depot (the case of a limited fleet was discussed in Section 1.3.4). One may also wish to restrict the number of routes having a specific characteristic, such as the number of long routes (with respect to distance, number of stops, late arrival, etc.) or the number of routes crossing an area. Other interesting examples are a limited number of docks at a depot or hub (see also Rieck and Zimmermann [117]) and a limited processing capacity for incoming goods at a destination depot. The latter arises in mail collection, where mail is collected from customers and postboxes and is then processed and sorted at the depot before a given cut-off time (as for parcels). A limited processing capacity means that the vehicles need to arrive in a staggered sequence, where feasibility depends on both the arrival time and on the amount that remains to be processed before cut-off (this is the main application in [82]).

The third example is related to synchronization issues. Here also the routes and schedules of different vehicles are interdependent and need to be coordinated. A first systematic study on *VRP with Multiple Synchronization constraints* (VRPMS) was provided by Drexl [50]. He offers the following (not exclusive) classification of synchronization:

(i) *Task synchronization:* One must decide which vehicle or vehicles jointly fulfill each task. Basically, this is a problem of clustering tasks, but recall from Section 1.3.2 that the tasks may also be split by load or volume, as in the SDVRP, or by periods, as in the PVRP or the IRP, or may be transshipped between vehicles, as in the 2E-VRP.

(ii) *Operation synchronization:* Different vehicles at the same or different locations are needed to fulfill a task, where the operations performed by the individual vehicles need to take place either at the same time or with precedence, where so-called *dynamic time windows* may bound the time lag. For example, pairs of service technicians carry out the setup of some services for customers, where a first technician must establish the supply at an associated supply site of the provider before the second can establish the service by performing some operations at the customers site (see Goel and Meisel [68]).

(iii) *Movement synchronization:* Two or more vehicles must move along a part of their itinerary together. For example, a trailer has to be pulled by a truck (Section 1.3.4), and for winter road maintenance, when routing snow plows, the clearing of a two-lane or three-lane street requires that several snow plows follow one another to clear the snow away (see Salazar-Aguilar, Langevin, and Laporte [119]).

(iv) *Load synchronization:* The right amount of load is collected, delivered, and transshipped at all locations and for all vehicles and their combinations when interacting.

(v) *Resource synchronization:* At any point in time, the total utilization or consumption of resources by all vehicles must respect the available capacities. This is similar to the above-mentioned inter-route resource constraints.

Several forms of synchronization can occur together: In the VRPTT (Section 1.3.4), trailers are pulled by trucks (movement sync.), a truck transfers some or all of its load to a trailer (task and load sync.), at which time the trailer is exclusively available for this operation (operation and resource sync.), and the transfer time may depend on the amount that is transferred (also operation and resource sync.).

The most studied VRPMS are, according to Drexl [50], the N-echelon VRP and *location routing problems* (see Perboli, Tadei, and Vigo [110], Baldacci et al. [7], and Section 1.3.7), the PDPTW with transshipments (see Wen et al. [140]), and the simultaneous vehicle and crew routing and scheduling problems (see Drexl et al. [52]).

1.3.6 ▪ Objectives

We have introduced VRPs as pure routing cost minimization problems. However, the objectives may model various goals. In order to structure the discussion about the goals in vehicle routing, we will start with a discussion of single objectives before hierarchical objectives and multi-criteria problems are sketched.

Single Objective Optimization. The simplest modification to the objective (1.8) is setting some of the routing costs c_{ijk} to zero or to a sufficiently large number (such modifications using big-M exclude infeasible or undesirable arcs or edges). Recall that the VRPB and the site-dependent VRP can be transformed into a standard CVRP in this way. Moreover, the *Open VRP* (OVRP; see Li, Golden, and Wasil [95]) models that a vehicle does not return to the depot after servicing the last customer on the route, or likewise the return trip to the depot is not charged.

Whenever it is possible to select among the tasks being serviced, the objective can also include a profit component (to be maximized) for covered tasks, as discussed for the VRP with profits in Section 1.3.2.

For heterogeneous fleet VRP, we mentioned that not only variable routing costs c_{ijk} but also fixed costs FC_k can be relevant when deciding about how many vehicles to acquaint and to use for fulfilling the given transportation requests.

The costs for performing a route may have the following additional components: In the scheduling and time-windows context, i.e., when time schedules (T_{ik}) have to be determined, the costs for employing drivers may be relevant so that costs depend on the *route durations* $T_{dk}-T_{ok}$ (see Savelsbergh [121]) or the *time of finishing routes*, i.e., T_{dk} (see Jozefowiez, Semet, and Talbi [85]). In service industries, *customer satisfaction* is often crucial, and can be incorporated with a cost component depending on a (weighted) latency, i.e., a term of the form $p_i \max(T_i - a_i)^+$ (where the weight p_i is related to the importance of customer i and a_i is the earliest possible service time). Humanitarian applications provide another real-world example of a latency objective, where in case of a disaster, a limited fleet of vehicles has to reach cities or affected areas as early as possible (see Chapter 11). Also waiting times w_i at the customers might be undesirable and incur additional costs (see, e.g., Solomon [127] and Bräysy and Gendreau [20]). Another objective related to customer dissatisfaction is related to the use of soft time windows (for both waiting times and soft time windows, see Section 1.3.3). Finally, in the *delivery man problem with time windows* (see Heilporn, Cordeau, and Laporte [81]) the objective is minimizing the duration that every delivery request is on board a vehicle, i.e., $\sum_{k\in K}\sum_{i\in N}(T_{ik} - T_{ok})$. Applications include passengers or perishable goods distribution, school bus routing and scheduling, and the transportation of disabled persons.

As a substitute for balancing constraints (see Section 1.3.5), one may apply a *min-max objective*, e.g., in order to minimize the length (or duration, or workload) of the longest route (more common in the arc routing context; see Chapter 11 in [35]). Note that a balancing objective alone almost never makes sense because a perfectly balanced solution may contain highly inefficient routes.

From a more general point of view, costs can depend on any type of attribute related to a resource that is consumed or utilized when the vehicle travels. In the soft time-windows example, this resource is time and costs depend on the time of service. Obviously, considering resources such as load on board the vehicle and the distances traveled allow for complex objectives that model real-world *transport tariffs*. In these tariffs, costs often depend in a non-linear way on distances, load (weight, volume, number of pieces, etc.), time, and the particular itinerary. Non-trivial cost functions can also be found in hazardous materials transportation (see, e.g., Tarantilis and Kiranoudis [133]).

Many metaheuristics explicitly consider feasible and infeasible solutions in order to allow neighborhood operators to faster reach high-quality solutions (see Gendreau et al. [63]). *Penalties* are a means of guiding these metaheuristics towards feasible solutions. In the VRP context, these penalties generally depend on combinations of attributes changing along the traveled route. Thus, complex objectives may result from both complicated real-world cost functions and the VRP solution method.

A recently introduced and interesting area of research incorporates energy consumption and pollutant emission control in the routing context (see Bektaş and Laporte [11]). Generally, such aspects are incorporated in the problem as specific and complex objective functions. A comprehensive review of these problems, which are broadly called *green vehicle routing problems*, is given in Chapter 15.

Hierarchical Objectives. Minimizing route length, duration, and completion time are generally conflicting objectives. Moreover, the minimization of the number of vehicles employed also conflicts with these goals. Since the use of vehicles and drivers typically incurs high route-related fixed costs, a common hierarchical way of optimizing is to minimize the number of vehicles first and then, with this number of vehicles fixed, a secondary objective is optimized. For example, the heuristics literature on VRPTW predominantly uses this hierarchical optimization model with route length minimization as the second objective (see Bräysy and Gendreau [20, 21] and Chapter 5).

Multi-criteria Optimization. The survey by Jozefowiez, Semet, and Talbi [85] provides a comprehensive overview of multi-objective optimization in routing problems. The same authors apply in [86] a bi-objective optimization approach to find a compromise between the overall route length and balancing of routes.

1.3.7 ▪ Other Extensions

We have seen that interesting problems result from the simultaneous consideration of vehicle routing and other logistics activities: If the latter are replenishment activities, the class of IRP results (Section 1.3.2). The addition of production decisions leads to the class of production routing problems (see Adulyasak, Cordeau, and Jans [1]). Location Routing Problems (LRPs; see Min, Jayaraman, and Srivastava [101] and Nagy and Salhi [105]) simultaneously seek optimal location and routing decisions. We already discussed VRP with loading constraints in Section 1.3.3, where routing and packing are jointly taken into account.

An important class of VRP variants considers *route consistency* aspects when the service occurs repeatedly or regularly. For example, Gröer, Golden, and Wasil [74] define the *Consistent VRP* (ConVRP), where it is required that the same driver visit the same customers at roughly the same time on each day whenever this customer needs service. Smilowitz, Nowak, and Jiang [126] examine the interaction between routing and workforce management by taking into account drivers' familiarity with customers and delivery regions. Often, such consistency requirements are combined with those of *route compactness* (a.k.a. *visual beauty*), where compact *service territory* are determined and used as a base for daily route construction (see Wong and Beasley [142]). Within these settings, more recent articles examine the impact of demand and customer's location uncertainty on service territory construction (see Zhong, Hall, and Dessouky [145], Haugland, Ho, and Laporte [80], and Schneider et al. [124]). A different way to enforce route compactness is represented by the so-called *Clustered VRP* (CluVRP), where customers are partitioned into clusters: once a vehicle enters a cluster it must serve all its customers before leaving it (see, e.g., Battarra, Erdogan, and Vigo [9]).

It is clear that such a compilation of VRP variants and extensions can never be exhaustive: We had to select the most relevant material to include, and the research field is developing very fast. Even within the time between finishing our chapter and the publication of the book, new models and powerful solution approaches will certainly emerge. This is, to a large extent, driven by the need to better solve real-world VRPs: Important criteria are a *higher solution quality*, not only measured by the objective but also by realistically integrating all relevant aspects, and *faster solution algorithms* which are capable of solving *larger size instances*. For the sake of brevity, we almost completely omitted a discussion of algorithms here. The other chapters of this book will complement this introduction with additional descriptions and references.

We hope that the chapter at hand, with the presented topics and their classification, will prove a helpful introduction and will provide an informative overview of the many relevant aspects arising when modeling VRP. For the future, we expect to see new challenging combinations of problem features and further interesting extensions of the many facets describing the *Family of the Vehicle Routing Problems*.

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