Chapter 11

Dynamic Vehicle Routing Problems

Tolga Bektaş
Panagiotis P. Repoussis
Christos D. Tarantilis

11.1 ▪ Introduction

When a vehicle routing model is cast and solved, it is normally assumed that the values of all input parameters are known with certainty. However, this is hardly the case in real-life applications where parameters such as customer demands, travel and service times, or even the information of whether a particular customer will require service or not are often incomplete, uncertain, or unknown during the route design phase (see Gounaris et al. [60]). As pointed out by Psaraftis [123], there are two important dimensions of input data, namely *evolution* and *quality of information*. The former implies that the available information is subject to change even after the routing plan is realized, while the latter reflects the possible uncertainties in the available data. What is common in both cases is that the partially known, uncertain, or unknown input parameters are revealed or updated concurrently with the execution of the routing process.

Depending on the availability and quality of a priori information and other characteristics of the problem under consideration, two alternatives emerge for solving the routing problem. Assuming that sufficient information is available (e.g., all input data are known in advance with a predefined degree of uncertainty), the first option is to treat the problem as static and solve it once during the design phase. The goal in this case is to obtain a robust routing plan that will possibly be subject to relatively small changes during the actual execution. This option is named *a priori optimization*, for which anticipating uncertainty is crucial in order to find realizable routing plans, and to avoid hefty penalties, both economic and reputational, when one fails to provide the required level of service (see Gounaris et al. [60]).

The second option is to address the problem in an ongoing and dynamic fashion as new input data arrive or are revealed in real time. While information evolves and decisions must be continuously made in a changing environment, the goal is to react to the new events as well as to anticipate future events, particularly if exploitable stochastic information is available or can be derived from past data. This approach, namely *dynamic optimization*, recognizes the additional decisions that become available during the execution of the routing plan and attempts to handle uncertainty in real time. However, it also requires advanced technological support and real-time communication between the vehicle and the dispatcher.

Based on the above described optimization frameworks, and the quality of advanced information, a possible taxonomy for VRPs can be stated as follows:

- *Static and Deterministic.* All input parameters are known in advance and with certainty, and are assumed not to change during planning and execution. This class of problems can be solved once and before the beginning of the planning horizon, and includes the traditional VRPs discussed in other chapters.

- *Static and Stochastic.* Part of the input parameters are known as random or stochastic variables, for which the actual values are revealed during the execution of the routing process. Unlike static and deterministic problems that have been studied thoroughly, VRPs under uncertainty have received less attention. Most papers address the stochastic *Capacitated VRP* (CVRP) through recourse and chance-constrained models, considering stochastic customers (Cheung, Xu, and Guan [28]), demands (Christiansen and Lysgaard [29]), and/or travel times (Kenyon and Morton [82]). An alternative to stochastic modeling is the use of robust optimization techniques (see Gounaris, Wiesemann, and Floudas [59], Gounaris et al. [60], and Sungur, Ordonez, and Dessouky [138]). For reviews of the literature on stochastic VRPs, we refer the reader to Cordeau et al. [34], Gendreau, Laporte, and Séguin [49], Toth and Vigo [145], and Chapter 8.

- *Dynamic and Deterministic.* As opposed to the above static groups of problems, dynamic problems assume that either part or all input data are not known prior to the execution of the plan, but only become available incrementally over time. Therefore, this group of problems is characterized by total uncertainty, since only probabilistic information is available for future events, and optimization can only be performed as new information arrives. The terms *real-time* and *online* typically refer to this group of problems.

- *Dynamic and Stochastic.* This group can be seen as dynamic problems that cannot be solved once and before the realization of the routing process; however, part of the unknown input data is in the form of stochastic information (e.g., forecasts, range values, and prescribed distributions). In contrast to pure dynamic and deterministic problems, there is a strong incentive to exploit and integrate all available information on foreseen future events in the solution process. This group of problems is usually referred to as *partially dynamic* and *mix dynamic and stochastic*.

This chapter focuses on the latter two groups of the above taxonomy, namely dynamic VRPs either with deterministic and/or stochastic data, in which *dynamic data* or *interaction of activities* over time are considered explicitly. The term dynamic data refers to one or more problem parameters (e.g., customer locations, demands, and travel times) that can be expressed as a function of time, including time-dependent travel times that are known in advance (see Powell, Jaillet, and Odoni [118]). An interacting activity refers to a dynamic event that affects the execution of the routing plan (e.g., service cancellations and vehicle availabilities). In the chapter, emphasis is given to dynamic VRPs where consolidation of requests is allowed (i.e., many customer requests can be served by the same vehicle); however, dynamic vehicle dispatching problems without consolidation (i.e., a vehicle is dispatched to serve a single customer) are also discussed.

Dynamic vehicle routing and dispatching problems have received significant attention in the literature. The early works of Psaraftis [123, 124] provide formal definitions and discuss the differences between dynamic and ordinary static VRPs. Bianchi [18] reviews early solution approaches. Ghiani et al. [53] differentiate between sequential and parallel solution approaches for both single- and multi-vehicle problems, and discuss key implementation features. An excellent review of the more recent approaches, frameworks, and routing strategies is provided by Gendreau and Potvin [52] and Ichoua, Gendreau, and Potvin [77]. The survey papers by Larsen, Madsen, and Solomon [89, 90] discuss technological advances, analyze the degrees of dynamism, different objectives, and performance measures, and suggest a three-echelon classification scheme for real-life applications. The works by Berbeglia, Cordeau, and Laporte [13] and Cordeau et al. [33] focus on dynamic pickup-and-delivery problems. Pillac et al. [111] review recent approaches for both dynamic and partially dynamic problems where some form of advance information is provided for near and long-term future events. Finally, Ritzinger, Puchinger, and Hartl [128] discuss solutions approaches proposed for stochastic VRPs as well as for VRPs that combine dynamic and stochastic information.

The aim of this chapter is to present the latest advances and research trends in the field of dynamic vehicle routing and dispatching. The goal is not only to provide an overview of the relevant literature but also to present the state of the art in frameworks and strategies, to provide useful insights, and to identify directions for further research in the area.

While a chapter on the dynamic VRPs can be organized in a number of ways, we have decided to structure it according to the following outline, as illustrated in Figure 11.1. According to this structure, we start by introducing the concept of degrees of dynamism, discuss the different sources of dynamism, namely *requests*, *travel times*, and *vehicle availability*, present the objectives that are often encountered in dynamic problems, and provide an overview of important problem variants and applications in Section 11.2.

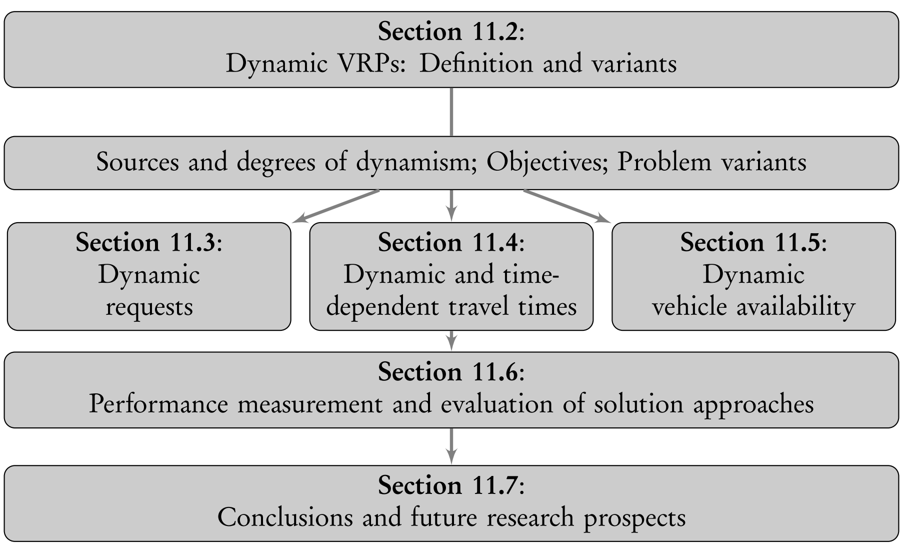

Figure 11.1. *The structure of the chapter.*

The three subsequent sections correspond to the three sources of dynamism. In particular, dynamic requests are discussed in Section 11.3, where we differentiate between pure dynamic problems (a.k.a. dynamic and deterministic) and dynamic problems that incorporate stochastic information (a.k.a. dynamic and stochastic). For the former, we present a review of routing policies, heuristics, reoptimization, and multiple plan approaches, as well as exact techniques based on dynamic and linear programming. As for the latter, we discuss anticipatory and predictive approaches, and review algorithms based on sampling, scenario analysis, and stochastic programming. Problems with dynamic and time-dependent travel times are presented in Section 11.4, and problems with dynamic vehicle availabilities are the subject of Section 11.5. Finally, Section 11.6 discusses performance measurement issues, and Section 11.7 presents the conclusions and offers directions for future research.

11.2 ▪ Definitions, Objectives, and Overview of Problem Variants

11.2.1 ▪ Sources of Dynamism

Based on the definition provided by Psaraftis [123], dynamic VRPs deal with the evolution and manipulation of routes under various operational constraints (e.g., time windows) performed by a fleet of vehicles on the move to serve future or immediate (customer) requests as a function of those inputs (e.g., customer demands, travel times, and on-site service times) that evolve in real time. The term *advance requests* refers to static service requests received before the realization of the routing process, whereas *immediate requests* refers to dynamic service requests revealed over time during the execution of the routing process.

As opposed to a static problem setting, the solution of a dynamic VRP seeks to handle and respond to all dynamic elements of the problem at hand as well as to exploit and integrate the available information on future events (e.g., customer demand forecasts) in an on-going fashion. Hence, the decision making process is executed in a changing and continuously evolving environment. Such an approach allows planners to react to external events and to anticipate the future events and handle uncertainty in real time. It also provides opportunities to reduce costs, improve customer service, and reduce the environmental impacts (Pillac et al. [111]).

Figure 11.2 illustrates the evolution and execution process of a single vehicle with the occurrence of an immediate request. Time t_0 refers to the beginning of the planning horizon, i.e., the time at which the vehicle leaves the depot. At this point, the initial planned route shown by the dotted arrows only contains the requests known a priori, shown by A, B, C, and D. During the execution of the route by the vehicle, one new request denoted by X appears at time t_r and the planned route is reconfigured based on the new input data. In particular, the route is divided into three parts: (i) the part of the route that is already executed and that cannot be modified, (ii) the current movement (position) of the vehicle to reach the next customer, and (iii) the remaining part of the route that will possibly be executed in the future and which can be modified. In this example, the route executed by the end of the planning horizon (t_T) is shown by the sequence A, B, C, X, and D. Note that the planned route can be used not only to decide on the next destination but also to accept or reject an immediate request (Ichoua, Gendreau, and Potvin [77]).

Based on the above dynamic routing scenario, Figure 11.3 describes the timeline of events, the interaction and communication points between the vehicle and the dispatcher, and the type of information and decisions that are exchanged between them.

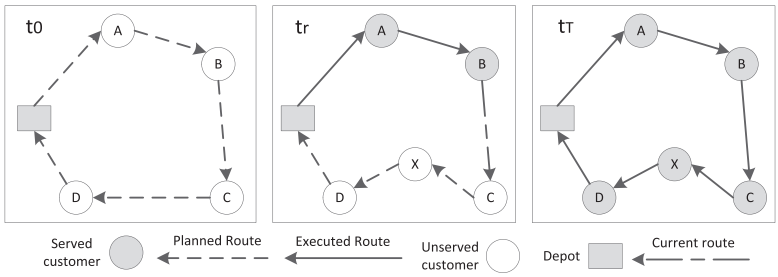

Figure 11.2. *Vehicle routing scenario with advance and immediate requests.*

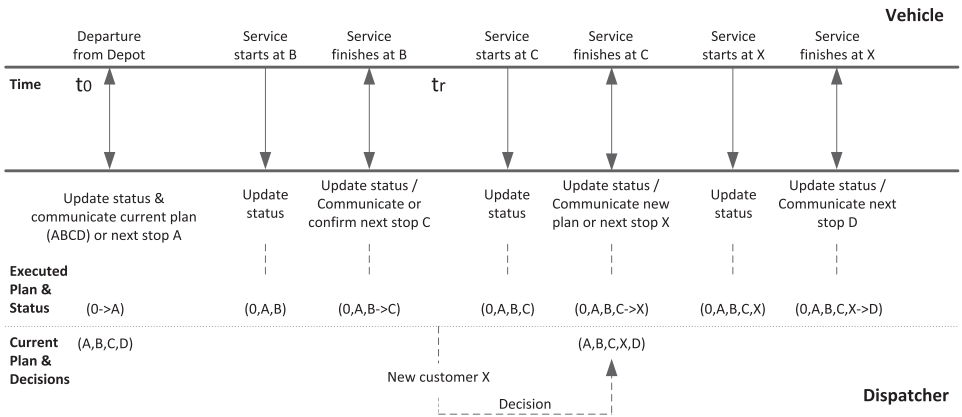

Figure 11.3. *Timeline of events and real-time communication between the vehicle and the dispatcher.*

There is a significant body of literature on problems dealing with dynamic requests. The requests can be a demand for goods (see Azi, Gendreau, and Potvin [7], Hvattum, Løkketangen, and Laporte [73], Khouadjia et al. [84], Mes, Van der Heijden, and Van Harten [101], and van Hemert and La Poutré [146]) or services (see Bertsimas and Van Ryzin [16], Gendreau et al. [48], and Mitrović-Minić and Laporte [104]), including the cases of on-demand transportation (see Attanasio et al. [6] and Beaudry et al. [9]). A special case is to dynamically consider revealed demands for a given set of customers (see Novoa and Storer [108], Secomandi [133], Secomandi and Margot [134], and Wang and Cao [149]). Besides (and in addition to) dynamic requests, variability in travel times has also been taken into account in various studies (see Fleischmann, Gnutzmann, and Sandvoss [44], Haghani and Jung [61], Jung and Haghani [81], Lorini, Potvin, and Zufferey [96], and Potvin, Xu, and Benyahia [115]). The literature is also rich on time-dependent travel times (see Chen, Hsueh, and Chang [25]). In contrast, only a few papers consider variable service times. However, in most cases, this source of dynamism can be treated as part of the dynamic travel times. The literature is also scarce concerning less predictable events, such as service cancellations, vehicle breakdowns, unexpected congestion and accidents, cargo damages, and unexpected changes in customer locations and demands (see Wang et al. [151]). Such events may disrupt the routing plan significantly, and therefore require a special recovery treatment and recourse strategies to minimize the potential negative effects (see Ichoua, Gendreau, and Potvin [77], and Wang et al. [151]).

Given that routing plans are readjusted over time in dynamic VRPs, this requires the exploitation of advanced telematic technology to support the real-time communication between the vehicle and the dispatcher. Recent advances and the rapid growth in information and communication technologies, such as *Radio Frequency Identification* (RFID), *Global Positioning Systems* (GPS), *Geographical Information Systems* (GIS), *General Packet Radio Services* (GPRS), and 3G/4G cellular networks, have overcome the barriers to accessing real-time data and information sharing, and have allowed for a real-time monitoring of vehicle fleets. Larsen, Madsen, and Solomon [89] highlight that nowadays even medium-sized companies have adopted and implemented advanced GPS/GIS systems coupled with wireless telecommunication facilities and mobile equipment, using them to be able to track the status and current position of their vehicles in real time. For descriptions of and discussions on the technological environment required for dynamic vehicle routing and dispatching problems, we refer the reader to Larsen, Madsen, and Solomon [90] and Giaglis et al. [55] as well as to the books edited by Goel [56] and Zeimpekis et al. [160].

Figure 11.4 sketches the basic flow and exchange of information between a dispatcher and a vehicle, and shows various sources of historical and real-time data. Obviously, the type of technology adopted for transmitting and storing data and the nature of the equipment used (e.g., the communication devices to transfer information between the dispatcher and the vehicles) will determine the quality of not only the data obtained, the frequency of interactions, and the system updates but also of the structure of the optimization system itself. For example, if up-to-date information is provided at any time to the dispatcher about the current positions of the vehicles, then vehicle diversion options become available.

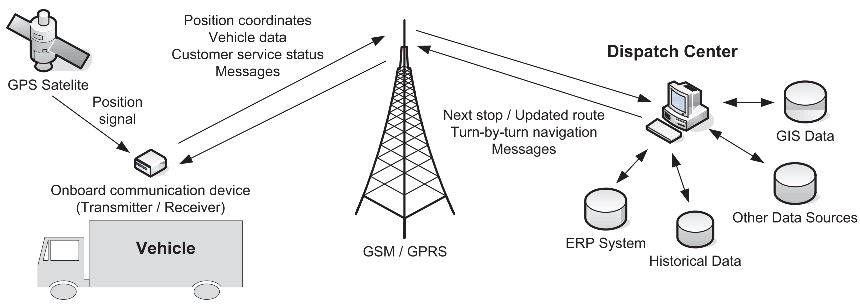

Figure 11.4. *Information flow of a typical GPS-based vehicle routing and monitoring system.*

Finally, access to historical data pertaining to certain problem attributes and/or real-time information about traffic, weather conditions, etc., will allow the use of stochastic methodologies. This is a key element for successfully solving dynamic and stochastic VRPs. As mentioned earlier, the integration of stochastic information about future events (e.g., location of requests, customer demands, travel times, cancellation of requests, and vehicle breakdowns) can significantly increase the look-ahead capability, reliability, and robustness of an optimization system. However, data analysis, data provision, and development of predictive models for estimating future levels and trends for specific problem parameters are often very challenging.

11.2.2 ▪ Degrees of Dynamism

The number of dynamic events as well as the time they actually take place during the planning horizon determine, to a large extent, the *degrees of dynamism*. For problem instances with dynamic requests, the *frequency of changes* and the *urgency of requests* can be used to measure the dynamism of the environment (see Ichoua, Gendreau, and Potvin [77]). The former refers to the rate at which new service requests or their attributes (e.g., demand and time windows) become available or are updated over time. The latter refers to the available *response time*, which can be seen as the time gap between the arrival time of a new request and the latest allowable time at which service begins.

As described in Larsen, Madsen, and Solomon [89], four factors contribute to the dynamism of a VRP with dynamic requests, namely the number of advance requests, the number of immediate requests, the arrival time, and the response time. Looking myopically at the ratio between the number of immediate requests n_{dr} and the total number of requests n_{tr} (see Lund, Madsen, and Rygaard [97]), the degree of dynamism can be expressed as

$$\alpha = \frac{n_{dr}}{n_{tr}}. \tag{11.1}$$

Metric (11.1) is indicative of the extent of the information received in real time in relation to the overall system information and provides an indication of how dynamic the system actually is (see Larsen, Madsen, and Solomon [89]. Depending on the length T of the planning horizon and the distribution of immediate requests over time, it is also important to consider the arrival times of requests. Clearly, the later the immediate requests are received (e.g., close to T), the more difficult it is to react accordingly. To capture this aspect, Larsen, Madsen, and Solomon [89] proposed the so-called *effective degree of dynamism* α^e. If the arrival time of the ith immediate request is denoted by τ_i, i.e., $0 < \tau_i \leq T$, α^e can then be calculated as the normalized average of the disclosure time (temporal distribution) as follows:

$$\alpha^e = \frac{\sum_{i=1}^{n_{dr}} \frac{\tau_i}{T}}{n_{tr}}. \tag{11.2}$$

For problem instances with service time windows, it is also important to take into account the level of urgency, i.e., the temporal distance between the request arrival time and the latest requested service time. One may expect that the degree of dynamism is not only related to the length of the time windows but also to the available reaction times and the remaining planning time. For this reason, the latest possible service starting time should be also considered. To that end, the effective degree of dynamism, as described in [89], can be extended as follows:

$$\alpha^e_{TW} = \frac{1}{n_{tr}} \sum_{i=1}^{n_{dr}} \left(1 - \frac{r_i}{T}\right), \tag{11.3}$$

where r_i denotes the reaction time that corresponds to the difference between the arrival time τ_i and the latest allowable service time l_i of the ith immediate request, with $l_i - \tau_i \leq T$ for all $i = 1, 2, \ldots, n_{dr}$. Both α^e and α^e_{TW} take values in the interval [0,1], where 0 corresponds to the static case (all requests are received at time 0).

The effective degree of dynamism captures the volume and the temporal composition of immediate requests, and can be used to classify the problem as being either weakly, moderately, or strongly dynamic. As reported in [89] and [90], a VRP is *weakly dynamic* if α^e is below 30%, *moderately dynamic* if between 30% and 80%, and *strongly dynamic* if higher than 80%. This information is useful not only to adopt the right methods but also to select which and how different decision strategies can be applied for obtaining high-quality routing plans. In weakly dynamic environments, more time-consuming and accurate methods can be used to obtain optimal or near-optimal solutions, since interruptions are scarce and such methods can be repeatedly applied for a long period of time. On the other hand, methods able to deliver high quality solutions in short computational times are better suited for strongly dynamic environments where the quality of advance information is low.

Bent and Van Hentenryck [10] and Larsen, Madsen, and Solomon [87, 88] have examined the performance of different solution approaches for varying degrees of dynamism. In particular, they focused on problem instances with different mixes of immediate and future requests, without considering other sources of dynamism. However, they did not consider the spatial distribution of requests, demands and the traveling times, frequency of updates, or the availability of a priori information for future events. An extension to a^e_{TW} that takes into account travel times and the required capacity for the demand is proposed by Wohlgemuth, Oloruntoba, and Clausen [156].

Recently, Ferrucci, Bock, and Gendreau [41] have introduced the *degree of structural diversity* as a measure of the existing variability in the requests. Having divided a given service area into subareas and segments with a spatial and a temporal dimension, the degree of structural diversity is calculated as the sum of the distances between the barycenters of consecutive time periods weighted with the number of arrived requests in the respective time period, divided by the total number of requests times the maximum distance between subareas. Their results show that as structural diversity increases (i.e., larger distances between barycenters and more requests occur) the more valuable is the utilization of (available or derived) stochastic knowledge about future request arrivals.

11.2.2.1 ▪ Objectives

In static VRPs the objective is to minimize the transportation cost, expressed in terms of one-off costs (e.g., fixed fleet acquisition and depreciation costs) and/or recurring costs (e.g., variable costs related to the distance traveled). In cases where the fleet size is not prescribed, a hierarchical objective is generally used, which is to minimize the total number of vehicles, followed by minimizing the total distance traveled for the fixed number of vehicles. In contrast, objective functions used for dynamic VRPs include various fitness criteria and performance measures, relevant to the degrees of dynamism. Furthermore, the objective(s) of dynamic VRPs often differ from one application to another, since various factors are considered to determine the quality of a solution including, among others, routing costs, customer requirements, and company policies.

Below, important attributes when defining the objective(s) are discussed:

- *Transportation Costs.* A successful routing plan must be characterized by short distances between the customers, since the distances traveled often reflect the expected transportation costs (see Montemanni et al. [105]). For this reason, transportation costs are usually included in the objective function, especially for weakly dynamic problems.

- *Service Level.* The service level can be seen as a measure of the response time to changes. From a customer service perspective, this can be translated to minimizing the waiting time and lateness to maximize customer satisfaction, i.e., the delay between the arrival of a request and the time at which it is serviced. However, a fast response to an immediate request is often conflicting with the objective of minimizing transportation costs, as it might result in long detours. In moderately and strongly dynamic environments, the objective is often a weighted sum (or a hierarchical ordering) of routing costs and lateness (see Branchini, Armentato, and Lokketangen [20], Chang, Chen, and Hsueh [24], Ichoua, Gendreau, and Potvin [74], and Chen and Xu [26]). Ferrucci, Bock, and Gendreau [41] considered the objective of minimizing customer inconvenience, calculated as a function of request response time, and studied the effects of using a linear and a quadratic dependency between response time and customer inconvenience. Ghiani et al. [54] seek to minimize the expected inconvenience cost, which is expressed via a non-decreasing convex function.

– *Throughput.* Throughput is typically related to revenue, utilization levels (e.g., minimum idle times for the vehicles), and the ability of serving as many customers as possible. In highly dynamic environments and time-constraint problem settings, such as emergency and on-demand transportation services, maximizing the total number of served requests (or similarly the minimization of the number of rejected requests) is the primary objective. This happens because new customer requests can be either accepted or rejected, and to serve them is either feasible or very costly (see Ichoua, Gendreau, and Potvin [75]). Van Hentenryck and Bent [147] refer to this decision process as *service guarantee*.

A combination of the objectives described above can also be considered for dynamic VRPs. However, multiple objectives are often conflicting, for example in terms of speed and flexibility or reliability and robustness. Therefore, a compromise or different hierarchy levels among them might be considered. Additional objectives or a different mix are usually adopted for solution approaches specifically designed for dealing with less predictable events, such as vehicle breakdowns and accidents. The goal of handling such disruption events is to alleviate the negative impact. To this end, the cost of delaying a service request, the deviation from the original plan, and the inconvenience to the customers are often adopted as primary or secondary objectives.

As mentioned by Pillac et al. [111] the requirement for real-time decision making within an evolving environment often compromises solution quality to better react to changes in the input data. Therefore, short-term decisions should maintain a reasonable tradeoff between accuracy and speed, while long-term decisions need to ensure an adequate level of solution quality. To that end, important elements for defining the objectives properly are the size of the dispatching area and the length of the planning horizon. For example, local area dispatching systems (e.g., repair and courier services) often exhibit highly dynamic environments that evolve over narrow planning horizons, requiring prioritization of high service levels and quick response times over, for example, transportation costs (see Gendreau and Potvin [52]).

11.2.3 ▪ Overview of Problem Variants and Important Applications

Dynamic vehicle routing and dispatching problems have been studied extensively and linked with numerous important applications, encountered in a large variety of practical contexts including, among others, maintenance operations, courier services, on-demand dial-a-ride systems, emergency services, and pickup and/or delivery of goods. In the literature, several problem variants and models have been defined with various combinations of operational requirements and constraints. Below, we briefly discuss the various problem classes that have emerged. The main goal here is to provide a link between problem settings and practical applications, and also to identify key structural characteristics.

Initially, one may distinguish between problems with and without consolidation, as in, for example, less-than-truckload trucking and truckload trucking, respectively. In the former, where many customer requests are served by a single vehicle, the main challenge is to determine the "right" sequence of visits. In the latter, one vehicle is dispatched to visit a single customer. These problems are of an assignment nature, where the main challenge is the repositioning of idle vehicles in anticipation of future requests, which are particularly relevant in the planning of emergency services.

Dynamic VRPs with consolidation are often the most difficult to solve, and they have been studied assuming various operational settings and constraints pertaining to, among others, time windows, capacity, and route durations. The typical setting involves a set of capacitated or uncapacitated vehicles which all start from and end at a depot, and have to visit a set of geographically scattered locations. These problems can be classified into four groups as follows:

- *One-to-many* and *many-to-one* problems. A single location is associated with each request. A request can be either for collecting or delivering a product or providing a service.

- *One-to-one* problems. A pickup and a delivery location are associated with each request; i.e., products or people are moved from a given origin to a given destination. In this case, a pickup must precede a delivery for each request.

- *One-to-many-to-one* problems. All delivery demands (shipments to linehaul customers) are initially located at the depot, and all pickup demands (collections from backhaul customers) are returned to the depot.

- *Many-to-many* problems. Any node can serve as a destination or as a source for any product.

Collection or delivery systems of various products, including variants of the dynamic *Traveling Salesman Problem* (TSP), are typical examples of one-to-many and many-to-one problems (see Hvattum, Løkketangen, and Laporte [73], Khouadjia et al. [84], and Montemanni et al. [105]). Maintenance operations and repair services that respond to immediate requests for providing service (e.g., equipment and facility maintenance or providing utility services) at the customer's premises (see Madsen, Tosti, and Voelds [99]) also fall into this category. A typical example is the dynamic *Traveling Repairman Problem* (TRP) (see Bertsimas and Van Ryzin [15] and Larsen, Madsen, and Solomon [87]). Note that in some of these problems the only requirement is to visit the customer location, possibly within a time window, without any constraints on vehicle capacity. Time windows can be either hard or soft in the sense that they can be violated to allow either early or late arrivals. Newspaper delivery (see Ferrucci, Bock, and Gendreau [41]) and courier services (see Gendreau et al. [48]), where parcels and mail are consolidated in a central location for further processing, can be also classified as one-to-many or many-to-one problems. As described by Kilby, Prosser, and Shaw [85], it is important to highlight the difference between one-to-many and many-to-one problems in a dynamic environment. In particular, it is physically impossible to add a delivery service to a vehicle that has already left the depot, whereas a pickup service can be added to an existing route (see Azi, Gendreau, and Potvin [7]). Therefore, one-to-many problems can be much harder to solve compared to many-to-one problems, where service guarantee issues often prevail. On the other hand, dynamic multi-period one-to-many and many-to-one problem settings appear in the works of Wen et al. [155] and Angelelli et al. [1], respectively.

One-to-one problems include virtually all variants of the dynamic *Dial-a-Ride Problems* (DARP) for on-demand transportation and door-to-door services (see Attanasio et al. [6] and Psaraftis [122]), such as transportation of elderly, handicapped, or disabled people (see Madsen, Ravn, and Rygaard [98]). All dynamic *Pickup-and-Delivery Problems* (PDPs) arising in local less-than-truckload applications, such as local express mail delivery services in urban areas (see Gendreau et al. [47]), pickup-and-delivery systems for different kinds of products and goods (see Wohlgemuth, Oloruntoba, and Clausen [156]), and multi-cab metropolitan transportation services (see Caramia et al. [23]), share the same properties. Where the problem concerns transportation of people, additional constraints might need to be taken into account relevant to passenger waiting and travel times.

In contrast to the two groups of problems discussed above, only few papers have looked at dynamic one-to-many-to-one problems. Wang and Cao [149] studied dynamic requests and service cancellations in the context of a VRP with Time Windows and Backhauls (VRPBTW). Chang, Chen, and Hsueh [24] investigated the dynamic *VRP with Simultaneous Pickups and Deliveries* (VRPSPD). Finally, we are not aware of any work addressing dynamic many-to-many problems.

The dynamic VRPs described above seek to determine the assignment of customer requests to the vehicles as well as the sequencing of visits over the planning horizon. The possibility of serving several customers with a single vehicle implies consolidation of goods or services. In contrast, problems without consolidation where a vehicle serves only one request at a time are also referred to as *resource allocation*. Dynamic fleet management problems as well as dynamic vehicle scheduling and rescheduling problems also fall into this category. Throughout this chapter, we will refer to this class of problems as dynamic vehicle dispatching problems.

To our knowledge, Powell [116] is among the first to address dynamic vehicle dispatching problems in the context of long-haul truckload trucking. In particular, this work deals with a dynamic fleet management problem for truckload motor carriers and seeks to dynamically assign incoming requests to a fleet of vehicles. Each request is characterized by its start and ending time window, origin and destination, duration (including travel time if origin and destination are different), and requirements on the driver for dealing with a particular request. Drivers are, in turn, characterized by their time of availability, location, and factors pertaining to hours of service, desired time off, etc.

Besides truckload trucking applications, various dynamic vehicle dispatching problems have been defined in other application areas with similar features and structure. One example concerns the emergency vehicle dispatching problems, including ambulance location and relocation models for which a review is provided by Brotcorne, Laporte, and Semet [22]. Here, the existing models are differentiated with respect to their deterministic, probabilistic, and dynamic natures. For other applications arising in the context of emergency vehicles, the reader is referred to Gendreau, Laporte, and Semet [50] and Haghani and Yang [63]. One further example arises in the context of real-time scheduling of automated guided vehicles in airports, for which an agent-based approach is described in Mes, Van der Heijden, and Van Harten [101].

11.3 ▪ Dynamic Requests

This section discusses methods and algorithms proposed for solving dynamic vehicle routing and dispatching with dynamic requests. We distinguish between *dynamic and deterministic* and *dynamic and stochastic* problems, and have divided the rest of this section into two parts, one for each class of problems. The former class deals with pure dynamic problem settings, while the latter is characterized by dynamic requests with stochastic information. From a methodological perspective, the treatment of these two classes of problems differs significantly, since the advance knowledge for future events leads to distinct solution frameworks and routing strategies.

11.3.1 ▪ Dynamic and Deterministic Problems

Solution approaches for problems with dynamic requests must follow the online routing process where, at the beginning of the planning horizon, an initial plan is constructed based on the advanced requests. This base plan can be followed without any modifications, until a new customer request is received. In this case, there is always a chance that the new customer can be inserted into the existing planned routes without affecting the order of subsequent customers and with minimal delay. However, it is more likely that the insertion of new requests into the existing route will require either partial or full rescheduling of the vehicle route.

The common practice for generating a base routing plan is to use exact or metaheuristic algorithms already developed for the corresponding static problem. These algorithms can be applied in a *rolling horizon* basis to reoptimize the existing solution when there is a new (immediate) request or an update to the input data. In this case, optimization is performed at discrete time intervals called *decision epochs*. Reoptimization approaches have the drawback of repeatedly solving difficult optimization problems, which may require excessive computational times. Note also that exact approaches can provide optimal solutions for the current state only (unless information is available over the entire planning horizon in advance). In this case, any solution at hand may be suboptimal once new data arrives. We refer the reader to Psaraftis [123] for an example where insertion of a new customer into an existing optimal tour renders the tour suboptimal. Despite this drawback, empirical evidence shows that the use of exact approaches often results in better overall solutions compared to using heuristics for the same purpose (see Yang, Jaillet, and Mahmassani [158] and Chen and Xu [26]).

Reoptimization approaches solve and resolve either part or the entire problem at each decision epoch. Depending on the way the time intervals for reoptimization are defined, one may distinguish between *instant* and *periodic* approaches. Instant reoptimization is performed whenever there is an update to the input data. This scheme is suitable for weakly dynamic systems, given that the time elapsed between subsequent updates determines the time available for reoptimization. On the other hand, periodic reoptimization schemes incorporate the latest updates only after a predefined time period, and any new requests (or other changes in the input data) are kept until the beginning of the next decision epoch. An advantage of this scheme is that the time available for optimization can be predefined. However, it is less suitable for settings where immediate actions are required (e.g., emergency services; see Hvattum, Løkketangen, and Laporte [73]).

As mentioned above, heavy computational requirements might hinder the use of reoptimization procedures, especially in highly dynamic environments where the problem quickly becomes more complex with arrivals of new information (see Ichoua, Gendreau, and Potvin [77]). In this case, an alternative approach is to locally *update* the existing solution. For this purpose, a wide variety of local update and instant reaction heuristic methods (e.g., insertion heuristics) have been proposed. Although these heuristic methods are likely to run in short computational times, the solution quality is often poor and necessitates the additional use of more enhanced reoptimization methods. For this reason, the majority of reoptimization approaches found in the literature are hybrid in the sense that they utilize local update techniques to react to the incoming information, while reoptimization is periodically applied to obtain further improvement.

Instead of reacting to the changes in the environment myopically as done by the above schemes, one may resort to routing strategies that aim to anticipate future events. One way to follow is to use relocation strategies with the aim of (re)positioning idle vehicles to strategic locations. Another effective approach is to make use of waiting strategies, which look at delaying commitments in an attempt to take advantage of future opportunities. For example, it might be beneficial for a vehicle to wait at its current location, as opposed to returning to the depot or to commit early, if there is an expected wait at its next visit dictated by, e.g., time-window restrictions (see Branchini, Armentato, and Lokketangen [20], Chang, Chen, and Hsueh [24], Gendreau et al. [48], and Kilby, Prosser, and Shaw [85]).

Another issue with local update heuristics and myopic reoptimization approaches is guaranteeing robustness, especially for dynamic problems. A methodological framework that seeks to provide reliability and stability as well as a proactive way of avoiding infeasibility is the so-called multiple plan approach. The main idea is to generate and maintain throughout the planning horizon a pool of solutions that correspond to alternative plans. These solutions are coherent with the current state, and one of them can be selected as the current plan at each epoch through a consensus function.

The flexibility of selecting and altering the current master routing plan is a key methodological differentiator between dynamic and a priori routing. In the latter, the base routing plan (a.k.a. skeleton routes, backbones routes, semi-fixed routes, standard routes, master plan, etc.) is typically used to determine the customer assignment and sequencing decisions at the first stage so as to avoid the complexity of full reoptimization in later stages. Therefore, a priori routing approaches can be effective in multi-period problem settings only if rich stochastic information is available and the customer demand patterns are relatively stable.

In the following sections we discuss various methods and algorithms that are proposed for pure dynamic problems. Most sections include a tabulated summary of the papers reviewed. These tables are not intended to serve as an exhaustive coverage of all publications on each topic, but are rather provided to describe the landscape of research pertaining both to problem variants and to solution algorithms.

11.3.1.1 ▪ Routing Policies and Local Update Heuristics

Local update heuristics refer to a set of heuristic rules, which specify what actions should be taken with respect to the current state and the new information at hand. In the literature, various *routing policies* (a.k.a. dispatching policies, online algorithms) have been proposed, ranging from local assignment rules, such as First-Come-First-Served, Nearest Neighbor, Stochastic Queue Median, and Partitioning Policies, to more elaborate algorithms. Local rules can be used to assign customers to a vehicle by considering a queue of pending requests and to build routes sequentially. Therefore, no explicitly planned routes are constructed a priori, although they can be traced back a posteriori (see Ichoua, Gendreau, and Potvin [77]). Routing policies have often been inspired by queuing theory, and they can perform very well even if the rate of immediate requests is very high and the system is congested. So far, studies that analytically examine routing policies have only been investigated for a limited number of dynamic problems, which are discussed in Section 11.3.2.3.

Jaillet and Wagner [78] examined various routing policies for dynamic TSPs with disclosure dates for the requests. Larsen, Madsen, and Solomon [87] studied the partial dynamic TRP with a mix of advance and immediate requests, and provided an evaluation of several routing policies proposed earlier by Bertsimas and Van Ryzin [15] for different degrees of dynamism. Later, Yang, Jaillet, and Mahmassani [158] addressed the multi-vehicle truckload PDP, and compared five rolling horizon strategies based on reoptimization and simple heuristic rules under varying traffic intensities, degrees of advance information, and degrees of flexibility for rejection decisions. Competitive analysis studies for various routing policies can be found in Ascheuer, Krumke, and Rambau [4], Feuerstein and Stougie [42], Hauptmeier, Krumke, and Rambau [68], and Lipmann et al. [95] for the dynamic DARPs, as well as in Angelelli, Savelsbergh, and Speranza [3] for a dynamic multi-period uncapacitated VRP. We refer the reader to Jaillet and Wagner [79] for a survey on complexity results and competitive and analytical studies for dynamic VRPs.

Instead of using local assignment rules that perform well under specific assumptions, ordinary *insertion procedures* and well-known construction heuristics have often been employed in the literature for various problem settings under several operational constraints. In this case, planned routes are constructed for all known requests. Besides flexibility, one other advantage of using insertion procedures to react to incoming immediate requests is that the planned routes can be also used for later decisions. Furthermore, insertion procedures are sufficiently fast and can also be used in real time to accept or reject a request or to specify a time window for a customer visit (see Ichoua, Gendreau, and Potvin [77]).

From an implementation point of view, whenever an immediate request is received (or at regular time intervals) the effort is initially to find feasible insertion positions, or to dispatch a new vehicle, for the new requests in the existing plan. At this point, *accept* or *reject* decisions also occur (see Gendreau et al. [48] and Ichoua, Gendreau, and Potvin [74]). Subsequently, depending on the objective(s), the best feasible insertion position(s) is selected and the new requests are incorporated into the routing plan. Although various fitness criteria and metrics have been proposed (e.g., related to the geographical proximity, the temporal closeness, the latency, the response times, and the smallest disruption), the most used methods adopt an insertion position that results in the shortest detour over a subset of vehicle routes. Insertion procedures for one-to-many and many-to-one problems are relatively simple, since they look at the insertion of single locations. However, for one-to-one problems, one needs to handle the insertion of pairs of pickup and delivery locations. Readers are referred to Berbeglia, Cordeau, and Laporte [13], Cordeau and Laporte [32], Cordeau et al. [34], and Madsen, Ravn, and Rygaard [98] for insertion procedures as applied in PDPs.

One drawback of the myopic savings-based insertion procedures is that they fail to introduce sufficient slack to accommodate future requests. To that end, Mitrović-Minić, Krishnamurti, and Laporte [103] extended the ordinary rolling horizon approach (where all known requests between decision epochs are treated equally or only those requests that are sufficiently close to the current time are considered) to that of *double horizon*. In particular, they considered both short- and long-term planning horizons. The main effort was to alleviate the adverse long-term effects of good short-term decisions. For this problem, it is preferable to put emphasis on minimizing the traveling distance in the short-term since the later portions of the routes are likely to change in the future, and on maximizing the slack time in the long-term, thus favoring the accommodation of new future requests.

In a dynamic VRP, as mentioned earlier, a vehicle at a given point in time is either serving (or waiting to start service) a customer, idle (waiting) at some location, or moving to serve a customer. Whenever a vehicle finishes serving a customer, there are two decisions to be made: (i) *wait* at the current location, or (ii) *move* towards a known (or dummy) but unserved customer. A wait decision after service or a planned wait at a "strategic" location is very important and can be used as a way of anticipating future requests. Similarly, request buffering (holding) techniques (see Pureza and Laporte [125]) can be applied to prioritize more urgent requests. Section 11.3.1.4 describes various routing strategies for the anticipation of future requests and discusses vehicle diversion strategies as well.

11.3.1.2 ▪ Instant, Periodic, and Continuous Reoptimization Approaches

Various reoptimization approaches have appeared in the literature for solving VRPs with dynamic requests. This section presents an overview of such approaches.

Kilby, Prosser, and Shaw [85] presented a periodic reoptimization approach wherein the time horizon is divided into fixed time slots, the size of which are determined based on the degree of dynamism. A rolling horizon with fixed decision intervals for solving the dynamic VRP with and without time windows is described by Montemanni et al. [105], Gambardella et al. [46], and Rizzoli et al. [129]. These papers assume an event manager who receives the immediate requests and keeps track of the served customers, as well as the position and the remaining capacity of each vehicle. Based on this information, an *Ant Colony Optimization* (ACO) algorithm is used to solve the static VRP instances. A pheromone conservation mechanism is used to determine the good solution parts and to transfer this information from one time slot to another. Later, Euchi, Yassine, and Chabchoub [39] extended the work of Montemanni et al [105] and enhanced the ACO system by using a local search algorithm.

A *Large Neighborhood Search* (LNS) is proposed by Hong [69] for the dynamic *VRP with Time Windows* (VRPTW). LNS approaches are based on the so-called ruin-and-recreate principle. More specifically, part of a given solution is destroyed by removing customers from their current positions; then the feasibility is restored by inserting the removed customers into different positions in the solution. Here, all immediate requests and customers already routed but hitherto unserved are treated as "removed" customers. The removal and reinsertion processes are repeated until the next triggering event. The approach has been shown to be an effective one, with the added advantage that only a part of the corresponding static problem is solved at each LNS iteration.

Angelelli et al. [1] proposed short-term solution strategies for solving an uncapacitated multi-period VRP with dynamic pickup requests. In every period, a fixed fleet of vehicles services a set of advance offline and immediate online requests. Each online request has a service deadline of at most two consecutive periods after its arrival, and depending on the length of the deadline, the online request can be also postponable. To that end, an additional decision occurs, which is whether or not to serve the postponable requests, in addition to the immediate requests that may appear later the same day or the day after. The objective is a hierarchical one; i.e., it requires the maximization of the number of served requests (primary) and the minimization of traveling distance (secondary). A periodic reoptimization scheme is adopted, combined with a *Variable Neighborhood Search* (VNS) algorithm. Various computational experiments are reported by varying the criteria to evaluate reoptimized solutions, the length of the look-ahead period (i.e., the period of time which reoptimization will be applied), and the reoptimization intervals.

Wen et al. [155] proposed a three-phase rolling horizon approach for a dynamic multi-period VRP motivated by a large distributor operating in Sweden. Immediate requests are revealed over a multi-period time horizon. The objectives are to minimize cost and customer waiting times as well as to balance the daily workload. Initially, a subset of requests is selected for a given (current) day and for a number of days ahead using a time-space correlation analysis. A routing plan is then generated by solving the corresponding static Periodic VRP (PVRP) with service frequencies equal to one (and visit combinations made up of consecutive days) using VNS. Finally, a *Tabu Search* (TS) post-optimization procedure is applied to minimize the total travel time of each day.

Angelelli, Mansini, and Speranza [2] developed a VNS algorithm for a courier service application. The service area is divided into zones, each with a central hub. In this problem, customer requests must be served within a time window, which might be either pickup or delivery. When the destination zone of a request is different from its origin, the transhipment between the hubs is performed overnight. Customer requests cannot be rejected, but are allowed to be allocated to future shifts. VNS and LNS local search heuristics have been also applied in the dynamic context by Goel and Gruhn [57] for the so-called Generalized VRP (GVRP).

Chang, Chen, and Hsueh [24] proposed a TS algorithm for the dynamic VRPTW with pickup and delivery demands. The objective is to minimize a weighted function of traveling and waiting times. Insertion-based heuristics are employed for the construction of the starting solution as well as for the insertion of new requests. Reoptimization is interrupted at checkpoints, i.e., either if a new request occurs or if the earliest departure time of the last node being served or scheduled to be served has arrived. Beaudry et al. [9] addressed a patient transportation problem between several locations in a hospital campus. This problem can be seen as a dynamic DARP with various side constraints, such as multiple degrees of urgencies, different equipment requirements, soft time windows, and multiple transportation modes. For solving this problem a TS algorithm has been developed, coupled with a local update insertion procedure. Computational experiments on real data are reported. Attanasio et al. [6] proposed different variations of a parallel TS heuristic for the dynamic DARP, which was described earlier by Cordeau and Laporte [31] for the corresponding static problem. In the proposed implementation, immediate requests are randomly inserted into the existing solution and the TS is used to restore feasibility. If a feasible solution is obtained, the request is accepted and TS is applied again as a post-optimization procedure for further improvement.

Du, Wang, and Lu [38] proposed a local search re-optimization approach, coupled with First-Fit and Best-Fit insertion procedures, for solving a two-level VRP with pickups and deliveries that involves transportation of products between suppliers and customers via a distribution center. Pickup-and-delivery orders are placed dynamically, and vehicles can be used for both pickups and deliveries; however, pickup products cannot be sent to customers before being deposited to the distribution center. The problem includes three types of service time windows, namely hard, soft, and mixed, and the objective seeks to minimize both the service penalty (i.e., delay in service) and traveling costs.

A dynamic PDP motivated by a multi-cab metropolitan transportation service company has been studied by Caramia et al. [23]. A cab is allowed to carry up to six customers. Each customer requests a pickup and delivery location, a pickup time window, and a so-called stretch-factor that denotes the maximum deviation from the shortest travel time the customer accepts. Later, Fabri and Recht [40] extended this work, and replaced the stretch-factor with a delivery time window, and allowed cabs to wait at customer locations. For solving the problem, a hybrid TS heuristic combined with an A^*-algorithm is proposed.

A hybrid reoptimization approach is proposed by Berbeglia, Cordeau, and Laporte [14] for the dynamic DARP combining an exact constraint programming algorithm with a TS heuristic. The primary role of the former is to determine whether it is feasible to insert a new request into the existing plan. The role of the latter is to continuously improve the existing solution as well as to insert new incoming requests. For this purpose, the algorithm is equipped with three scheduling procedures. The constraint programming algorithm is executed in parallel to the TS heuristic, either to find a feasible solution or to prove that no feasible compatible solution exists. Experiments indicated that the proposed hybrid scheme outperformed each of the two algorithms when executed independently.

Evolutionary algorithms have also been applied to this class of problems. These algorithms maintain a population of individuals, but when used to solve dynamic problems the population needs to be consistent with the current state of information, and must be updated at least periodically in line with the changes in the input data and the currently executed plan. One advantage of using a population is that it can provide useful information when the search is restarted. To that end, Khouadjia et al. [84] proposed *Particle Swarm Optimization* (PSO) and VNS algorithms for the dynamic VRP. They report that VNS is more accurate than PSO, but PSO is more stable with respect to the changes in the environment. In contrast, the PSO seemed to work better when the objective is to serve as many customers as possible, for increasing degrees of dynamism.

Table 11.1 provides an overview of reoptimization approaches proposed for various dynamic and deterministic problem classes.

Table 11.1. *Overview of reoptimization approaches.*

References	Problem features	Algorithmic features	Objectives	Data set
Kilby, Prosser, and Shaw [85]	VRP	Periodic reoptimization; Local Search heuristics	Min. distance traveled	Modified CVRP benchmark data sets (up to 385 visits)
Montemanni et al. [105]; Euchi, Yassine, and Chabchoub [39]	VRP	Periodic reoptimization; ACO	Min. distance traveled	Modified CVRP benchmark data sets (see [85])
Gambardella et al. [46]; Rizzoli et al. [129]	VRP; VRPTW	Periodic reoptimization; ACO	Min. n. of routes; Min. distance traveled	Data from real-life applications
Hong [69]	VRPTW	Continuous reoptimization; LNS	Min. n. of routes; Min. distance traveled	Based on Solomon VRPTW benchmark data sets
Angelelli et al. [1]	Multi-period VRP; Postponable requests	Short-term strategies; VNS	Max. n. of served requests; Min. distance traveled	Based on Solomon VRPTW benchmark data sets
Wen et al. [155]	Multi-period VRP; Service frequencies	Three-phase rolling horizon heuristic; VNS	Min. distance traveled and customer waiting; Balance daily workloads	Data from real-life application (15-day planning period; up to 80 orders)
Angelelli, Mansini, and Speranza [2]	PDP; Courier Services; Multiple Shifts	VNS; Insertion and Local Update heuristics	Min. operational costs; Max. value of requests served	Numerical example
Chang, Chen, and Hsueh [24]	VRPTW with PDP	Instant reoptimization; TS	Min. weighted function of traveling and waiting times	Based on Solomon VRPTW benchmark data sets
Beaudry et al. [9]	DARP with various side constraints	TS; Local update heuristics	Min. fleet operating costs and patient inconvenience	Real data from a large German hospital
Attanasio et al. [6]	DARP	Parallel TS	Max. n. of served requests; Min. distance traveled	Real world and randomly generated instances
Du, Wang, and Lu [38]	VRPTW with PDP	Local search and local update heuristics	Min. service penalty and distance traveled	Based on Solomon VRPTW benchmark data sets
Fabri and Recht [40]	PDP with several time windows	TS; A^*-algorithm	Max. n. of served requests; Min. distance traveled	Randomly generated problem instances
Berbeglia, Cordeau, and Laporte [14]	DARP	TS; Constraint Programming	Min. distance traveled	Modified DARP benchmark data sets (see Ropke, Cordeau, and Laporte [130])
Khouadjia et al. [84]	VRP with route duration constraints	Continuous reoptimization; PSO; VNS	Min. distance traveled; Max. n. served requests	Modified CVRP benchmark data sets

11.3.1.3 ▪ Multiple Plan Approaches

Motivated by the local operation of long-distance express courier services, Gendreau et al. [48] proposed a parallel TS method for the dynamic VRPTW. This work assumes that the only information provided to the drivers is their next destination. The proposed approach utilizes an *adaptive memory* that is used to maintain a pool of elite solutions. Whenever an immediate request occurs, insertion procedures are applied to all elite solutions to check whether feasible insertion positions exist. If the request is accepted, then a new starting solution that includes the new request is generated by combining the routes of the elite solutions. The TS method is resumed to improve this new solution and terminates when there is an immediate request or the service of a known request is completed. The solution space is explored on the basis of a cross-exchange neighborhood, while a two-level parallelization scheme is also adopted. Ichoua, Gendreau, and Potvin [75, 76] applied the same algorithm for the dynamic VRP with time-dependent travel times and for the dynamic PDP, respectively. Gendreau et al. [47] proposed a TS heuristic for solving a local express courier problem with pickups and deliveries. The neighborhood structure is based on node ejection chains. The problem of determining the best chain or cycle of ejection/insertion moves (of any length) over the existing set of routes is modeled as a constrained shortest path problem. The latter is solved via an adaptation of the all-pairs Floyd–Warshall algorithm. Furthermore, a master-slave parallelization scheme is employed. Recently, Kergosien et al. [83] have adopted a similar TS method for solving a patient transportation problem related to the dynamic DARP.

Bent and Van Hentenryck [10] generalized the "adaptive memory" framework of Gendreau et al. [48] and proposed a multiple plan approach. The idea is to maintain a set of routing plans that are coherent with the plan being executed, as well as with the current state of the vehicles and customers. At each iteration, a so-called distinguished plan is generated to serve known requests, and this plan is followed until the next event occurs. In an effort to limit the amount of modification to the existing plan, the distinguished plan selected is not necessarily one of minimum cost but one that is most similar to the other plans. When an immediate request comes in, a local update procedure is applied to check whether it can be accommodated without destroying feasibility. If at least one feasible plan is found, the request is accepted and incompatible plans are discarded.

Multiple plan approaches equipped with a parallel Adaptive LNS algorithm have recently been developed by Pillac, Guéret, and Medaglia [113, 114]. The first paper studies the dynamic TRP and examines different objectives. The authors demonstrated that the minimization of the total working time as an objective is not well suited to a dynamic environment. Instead, they suggest that minimizing the total distance leads to solutions that are better both in terms of total distance and duration. The second paper studies a dynamic VRP and introduces the notion of driver inconvenience, indicative of the consistency between an updated routing plan and the initial reference plan handed out to the drivers. To that end, a bi-objective optimization problem is defined that minimizes the cost, while maintaining consistency throughout the day.

The concept of maintaining a repository of feasible as well as compatible solutions according to partial executed routes is also utilized by Coslovich, Pesenti, and Ukovich [37]. They proposed a two-phase insertion algorithm for the DARP. While the vehicle is moving between two successive stops (first phase), a set of feasible neighboring solutions is generated and maintained. Whenever an immediate request occurs, i.e., a trip demand by a person located at a stop, an insertion algorithm is used to see whether it could be inserted into the existing route by checking all solutions in the repository (second phase).

Table 11.2 provides an overview of multiple plan approaches proposed for various dynamic and deterministic problem classes.

Table 11.2. *Overview of multiple plan approaches.*

References	Problem features	Algorithmic features	Objectives	Data set
Gendreau et al. [48]; Gendreau et al. [47]	VRPTW; PDP; Courier Services	Parallel TS	Min. weighted sum of distance traveled, lateness and overtime	Randomly generated instances (33 and 24 average request per hour)
Ichoua, Gendreau, and Potvin [75]	VRP with time-dependent travel times	Parallel TS	Min. weighted sum of distance traveled and lateness	Based on Solomon VRPTW benchmark data sets
Kergosien et al. [83]	DARP	TS	Min. transportation costs and tardiness	Randomly generated instances (130 requests per day)
Bent and van Hentenryck [10]	VRPTW	Scenario-based planning; Local update heuristics; Consensus function	Max. n. of served customers	Based on Solomon VRPTW benchmark data sets
Pillac, Guéret, and Medaglia [113]	TRP	Parallel Adaptive LNS	Min. distance traveled and working time	Based on Solomon VRPTW benchmark data sets
Pillac, Guéret, and Medaglia [114]	Bi-objective VRP	Parallel Adaptive LNS	Min. distance traveled and driver inconvenience	Based on Solomon VRPTW benchmark data sets
Coslovich, Pesenti, and Ukovich [37]	DARP	Two-phase insertion algorithm; Solution repository	Min. distance traveled; Max. no of served customers	Randomly generated instances (up to 50 customers)

11.3.1.4 ▪ Routing Strategies for the Anticipation of Future Requests

In contrast to local update heuristics, look-ahead routing strategies can be applied to smooth future perplexities and enhance the performance of the solution methods by anticipating future events. Mitrović-Minić and Laporte [104] proposed a TS algorithm for the dynamic uncapacitated Pickup-and-Delivery Problem with Time Windows (PDPTW). The objective is to serve all requests, while minimizing the total distance traveled. A cheapest insertion algorithm is used to accommodate immediate requests. In addition, a TS algorithm that is similar to the one proposed earlier by Gendreau et al. [47] is applied for further improvements. Four waiting strategies are examined, namely drive-first, wait-first, dynamic waiting, and advanced dynamic waiting. If the dispatcher is forced to postpone the decision until the next destination (wait-first), this reduces the total detour but increases the number of vehicles. In contrast, if waiting times are used (drive-first), the number of vehicles is reduced, but this comes at the expense of increased traveling distance. Dynamic and advance dynamic waiting strategies produced the best results, which seek to distribute the waiting times along routes. For this purpose, the service area is divided into zones, and waiting time is allocated proportionally to the time needed to serve them.

Branke et al. [21] studied the dynamic VRP and examined alternative waiting strategies. The objective is to maximize the probability that immediate requests can be inserted into the fixed routing plan and to minimize the average distance traveled to visit them without violating time constraints. They developed an evolutionary algorithm for solving the corresponding waiting drivers problem with known request arrival times and performed a comparative analysis for different heuristic waiting strategies. Additionally, optimal waiting strategies are proposed for special variants with limited fleet size. In particular, the optimal policy is not to wait for the single-vehicle case. If two vehicles are available and they travel from opposite directions to the depot, the best waiting positions are about half the total distance they have to travel. The waiting strategies proposed by Branke et al. [21] can be summarized as follows:

– *Depot strategy* that forces a vehicle to wait at the depot as long as possible before visiting any customer. Here, the total available waiting time of a vehicle is spent at the beginning, which is the opposite of the no wait strategy.

– *Current location strategy* where each vehicle is forced to wait at a customer location and the waiting time is equally distributed among the customers

– *Proportional current location strategy* where each vehicle is forced to wait after service completion at customers and the waiting time is distributed to each customer proportionally to the distance traveled. In other words, this strategy assumes that the more distant a customer is, the more a vehicle should wait at this location.

– *Distant customer strategy* where the vehicle waits only at the most distant customer; therefore, it is less costly to service new nearby isolated requests.

– *Variable strategy* where a vehicle waits following the proportional current location strategy but only after visiting a number of customers, and where the residual travel distance to the depot is equal to the total waiting time.

Of the strategies above, the variable strategy proved to be the best. The overall performance of other strategies is consistent with the results reported by Mitrović-Minić and Laporte [104].

Pureza and Laporte [125] described a method for solving the dynamic PDPTW with random travel times using a construction-destruction heuristic. They examined the impact of waiting and request buffering strategies for different problem sizes and degrees of dynamism. The former is based on fastest paths with random travel times, whether these are time-dependent or not. The main effort is to take advantage of faster paths in order to wait at given location and arrive no earlier than, but as close as possible to. the time at which service may start at the next destination. The latter seeks to postpone the insertion of non-urgent requests to a later time. Request buffering strategies are rarely studied in the literature. The key issue is to define rules that can provide a quick and correct assessment of the viability of inserting new requests in future route adjustments.

Instead of waiting at customer locations, more advanced strategies may suggest performing anticipatory moves and waiting at promising areas (e.g., moving to "dummy" customer locations) that exhibit a higher probability for a new request to appear nearby. However, stochastic information about future requests is needed to identify promising areas in this case. This information can be either provided in advance or it can be generated from historical request information without assuming any existing distributions. Relocation strategies that exploit available stochastic information are discussed in Section 11.3.2.1. It is worth mentioning that even if a vehicle is waiting at a promising area, sometimes it is not feasible to visit a new customer due to capacity constraints, even if an extra travel cost has already been paid. Therefore, relocation strategies need careful design and validation.

A proactive reoptimization approach for the dynamic VRP is proposed by Ferrucci, Bock, and Gendreau [41] with an objective of minimizing customer inconvenience. Without assuming any prior information, stochastic knowledge about future events is generated using historical information on the requests. This information is used to coordinate the utilization of vehicles and to guide them into request-likely areas by integrating dummy customers. The process is controlled by a TS algorithm that switches between

Table 11.3. *Overview of routing strategies for anticipating future requests.*

References	Problem features	Algorithmic features	Objectives	Data set
Mitrović-Minić and Laporte [104]	PDPTW (uncapacitated)	TS;Various waiting strategies	Min. distance traveled	Randomly generated instances (10–1000 requests)
Branke et al. [21]	VRP	Evolutionary algorithm; Various waiting strategies	Max. n. of served requests; Min. distance traveled	Based on Beasley's ORLib VRP instances
Pureza and Laporte [125]	PDPTW with random travel times	Insertion heuristics; Buffering strategy; Waiting strategy	Hierarchical; Min. n. of lost requests, n. of routes and distance traveled	Randomly generated instances (up to 100 requests)
Ferrucci, Bock, and Gendreau [41]	VRP with customer inconvenience	Proactive reoptimization; TS; Relocation strategies	Min. customer inconvenience	Random instances based on the road network of Dortmund (150 requests on average)

different stages (depending on previously explored solutions) to control the neighborhood operators. Computational experiments demonstrated that the integration of derived stochastic knowledge may lead to considerable improvements, especially when the request data has high structural diversity.

Table 11.3 provides an overview of selected papers that discuss and demonstrate the application of various routing strategies for anticipating future requests.

Another issue which may arise during the solution update process is *vehicle diversion*. Diversion allows a moving vehicle to change its current destination to serve an immediate request that is geographically close to its location. Using this option increases the flexibility to manipulate the current routing plan to a large extent and may yield considerable improvements. Recent technological advances allow the dispatcher to track vehicle positions and speeds in real time and allow exploiting diversion opportunities. However, the use of vehicle diversion also makes the driver operations more complex (see Berbeglia, Cordeau, and Laporte [13]) and a number of issues must be taken into consideration, especially when applied in highly dynamic environments.

Regan, Mahmassani, and Jaillet [126, 127] were the first to study vehicle diversion issues for dynamic full truckload PDPs. They assessed the benefits of diversion assuming different demand patterns and dispatching rules. Later, Ichoua, Gendreau, and Potvin [74] studied the effect of a broader diversion strategy for a dynamic uncapacitated VRP in the context of long-distance courier services. The authors employ a parallel TS heuristic of Gendreau et al. [48] and allow vehicle redirections between their current and planned destinations. This, however, might result in destinations of one or more vehicles being changed. A critical factor is the amount of time available for reoptimization, since the environment constantly changes and diversion opportunities may no longer be valid. Therefore, it is important to keep a balance between the solution quality (as a result of the amount of time invested for reoptimization) and the response time. Vehicle diversion strategies have been also considered in Angelelli et al. [1], Branchini, Armentato, and Lokketangen [20], Chen and Xu [26], and Lorini, Potvin, and Zufferey [96], among others.

Finally, one alternative approach that allows one to reduce the complexity of reoptimization for dynamic multi-period VRPs is districting, i.e., dividing the service region into smaller districts. In this setting, all customer requests that appear within the district are served by a single vehicle or by a predefined group of vehicles that service this region. Territory shaping and sharing schemes applied to multi-period problems can be found in the works of Haughton [64, 65, 66, 67].

11.3.1.5 ▪ Approaches Based on Dynamic Programming and Integer Programming

This section focuses on exact approaches that are based on adaptations of static algorithms. The literature in this field is scarce, and the existing methods are either based on dynamic programming or linear (mixed) integer programming. Table 11.4 provides an overview of exact approaches proposed for pure dynamic problems.

Table 11.4. *Overview of exact approaches.*

References	Problem features	Algorithmic features	Objectives	Data set
Psaraftis [122]	DARP	DP	Min. time and ride-time	Numerical example
Savelsbergh and Sol [132]	PDP (general)	Branch-and-Price	Min. n. of vehicles; Min. route costs	Random generated data
Yang, Jaillet, and Mahmassani [158]	Multi-vehicle truckload PDP	Mathematical Programming	Max. Net revenue	Random generated data
Chen and Xu [26]	VRPTW	Column generation	Min. distance traveled	Based on Solomon's VRPTW benchmark data sets

An application of *Dynamic Programming* (DP) for static and dynamic variants of the DARP, where a single vehicle serves incoming requests from customers wishing to be picked up from a given origin and be transported to a specified destination, is presented by Psaraftis [122]. The features of this problem are that (i) the vehicle routes should be in the form of open paths (as opposed to tours); (ii) each customer should be picked up before being dropped off; (iii) the vehicle capacity, defined as the total number of passengers that can be transported, should be respected; and (iv) there are special priority constraints preventing a customer's request from being deferred indefinitely. DP recursions, for both the static and the dynamic cases, are described, where the latter is based on the observation that there is no need to reoptimize an existing (optimized) route unless new customer requests are introduced into the problem. If there are updates, then the objective function can be modified by excluding customers that are no longer part of the input.

Linear integer programming stands as another exact solution technique on which solution algorithms for the dynamic VRP are based. One of the earlier approaches developed for a dynamic *General Pickup and Delivery Problem* (GPDP) arising in shipping cargo is presented by Savelsbergh and Sol [132]. The approach is based on repeated solution of the GPDP using a Branch-and-Price algorithm whereby new incoming requests are taken into account through algorithmic adjustments. More specifically, all routes obtained through solving the GPDP are split into two parts, head and tail, where head tours are fixed and implemented as short-term decisions, and tail tours can be changed in the future depending on the new requests. The overall approach has been embedded into a decision support system, and results are reported on randomly generated test problems as well as on a case study which involves simulating the dynamic planning environment with real-life data.

Another mathematical programming–based approach is used to model an offline version of a real-time multi-vehicle truckload PDP in Yang, Jaillet, and Mahmassani [158], where a fleet of trucks is available to serve a number of requests, each identified by its pickup location, the delivery location, the earliest pickup time, and the latest delivery time. A request can be either rejected or accepted, and in the latter case the revenue generated is proportional to the distance between its pickup-and-delivery locations. Each truck can serve only one request at a time. This problem is modeled as a variant of the classical assignment problem, which is used to find a least-cost set of cycles going through all the nodes. In an "online" setting, the model includes the requests that are known at a particular decision epoch which have not yet been served or rejected. Yang, Jaillet, and

Mahmassani [158] use such a model in combination with several real-time policies to solve the problem.

Another application of linear (mixed) integer programming is presented by Chen and Xu [26] to solve the dynamic variant of the VRPTW. This problem is defined on a planning horizon divided into evenly spaced epochs. A set of homogeneous vehicles is located at a depot to serve customers with incoming requests, each with a time window that cannot be violated. Each request has a weight, and the total weight carried on a vehicle cannot exceed the vehicle's capacity. Chen and Xu [26] describe a formulation for the static version of the problem where a binary variable is associated with every possible trip to indicate whether it is used by a vehicle or not. To deal with the dynamic nature of the problem, column generation is used to model incoming requests in future epochs. These requests are incorporated into the existing formulation by either (i) modifying the existing routes by inserting or deleting requests, or (ii) creating new routes to serve new requests. New columns are generated by using fast local-search-based heuristics. The proposed approach allows tackling problems of up to 100 requests with planning horizons of up to 2400 seconds.

11.3.1.6 ▪ Miscellaneous Approaches

An application of genetic programming for a dynamic PDP was described by Benyahia and Potvin [12]. The problem was motivated by a local courier service company that receives calls for the pickup and delivery of express mail in an urban area. The goal was to model and approximate the decision process of an expert vehicle dispatcher via a utility function. Computational results are reported on real data sets, and a comparative analysis is carried out with a neural network model and a simple dispatching policy.

Jemai and Mellouli [80] proposed a neural TS approach for the dynamic VRPTW. The proposed approach is composed of two parts. The first part consists of learning and reproducing previous routing decisions using a feed forward neural network. For this purpose, different problem instances are simulated, and the learning sets are constructed using the exact solver GLPK. The second part consists of a post-optimization TS algorithm that tries to improve the initial solution generated based on the assignment provided earlier by the neural network. Computational results demonstrated better performance compared to well-known local update heuristics.

A disruption recovery model is proposed by Wang and Cao [149] for the VRPBTW with new pickup requests, service cancellations, and increase/decrease in the pickup quantities from the backhaul customers. The objective is to find a recovered solution that minimizes the deviation from the original plan after the occurrence of a disruption. Disruption recovery models are also proposed for the multi-objective VRPTW by Wang, Xu, and Yang [153] and for the VRP with fuzzy time windows by Wang, Zhang, and Yang [154]. A more enhanced combinatorial disruption recovery model for the VRPTW is described by Wang et al. [151] that takes into account multiple types of customer disruption events, such as changes of time windows, changes of customer locations, removal of requests, and combinatorial disruption events. The term combinatorial, in this context, indicates that some disruption events occur simultaneously at one or more customers.

11.3.2 ▪ Dynamic and Stochastic Problems

So far, it was assumed that no exploitable stochastic information is available in advance. However, it is often the case in practice that a dispatcher has some valuable a priori knowledge regarding the demand patterns, not only in terms of time (e.g., "peak" time periods) but also in terms of locations (e.g., intense geographical areas; see Ichoua, Gendreau, and Potvin [77]). This information can be used to anticipate forecasted needs.

A common scenario is that the locations of the customers are known in advance; however, the demand of each customer is a random variable following a known probability distribution and is revealed only after the vehicle visits the customer location. This problem variant is known as the dynamic VRP with *stochastic demand*. Another scenario arises when the only stochastic information available is on the expected number of customer requests. This problem variant is known as the dynamic VRP with *stochastic customers*. Additionally, other forms of stochastic information can also be provided and/or extracted from historical data, for example, service cancellations, absence of customers, and service times.

This section reviews models and algorithms proposed for addressing dynamic vehicle routing and dispatching problems with various forms and combinations of available stochastic information. In particular, Section 11.3.2.1 discusses anticipatory algorithms and predictive routing strategies that exploit stochastic information about customer locations, demand patterns, and/or the occurrence time of immediate requests. More advanced sampling algorithms and multiple scenario approaches are described in Section 11.3.2.2. On the other hand, Section 11.3.2.3 presents analytical studies of various routing policies and problem settings. Finally, stochastic programming as well as other stochastic models and algorithms are described in Section 11.3.2.4 for a wide variety of dynamic problems with stochastic information.

11.3.2.1 ▪ Anticipatory Algorithms and Predictive Routing Strategies

Anticipatory algorithms in the context of vehicle dispatching were first introduced by Powell et al. [119] for long-haul truckload trucking applications. This problem consists of assigning drivers to pickup-and-delivery requests that arise randomly over time within a given time window. A vehicle relocation strategy in anticipation of future demands is introduced. Future demand forecasts are used, and time-space graphs are proposed where nodes correspond to actual and forecasted demands at different regions and time periods. The deterministic cost and the expected cost of forecasted capacity needs of future periods are minimized at each period. Computational results show that it is always advantageous to take forecasted demands into account, as compared to a model that only reacts whenever a new load appears. Section 11.3.2.4 reviews works based on stochastic modeling, including approaches based on *Markov Decision Processes* (MDPs) and *Approximate Dynamic Programming* (ADP) methods, which integrate stochastic information by evaluating expected values to take predictive or preventing actions.

Larsen, Madsen, and Solomon [88] studied the so-called partially dynamic TSP with soft time windows, and they define waiting and relocation points based on a priori information regarding the distribution and clustering of customers. The requests are revealed over time in a number of subregions following a Poisson process with different arrival rates. Starting from the basic latest departure policy, they have developed more enhanced relocation policies that make use of advance information to reposition the vehicle at idle points. The set of idle points are defined as locations with a high probability of generating new requests, and they are selected based on different criteria, i.e, the nearest idle point, the busiest idle point (highest arrival intensity), and the idle point with the highest expected number of new requests. Among them, the policy to reposition the vehicle at the nearest location performed best in terms of average lateness and number of late customers.

An evolutionary algorithm applied over a rolling horizon basis is proposed by van Hemert and La Poutré [146] for a VRP with dynamic pickup loads. The objective is to serve as many customers as possible. They considered that advanced knowledge regarding the probability distributions for the occurrence of a new service request is provided, but in the form of regions. To that end, the concept of fruitful regions is introduced. From the implementation viewpoint, a self-adaptive mechanism is employed that allows the evolutionary algorithm to explore the possibility of vehicles to visit nodes that have not yet requested service. The fitness function is extended with anticipated moves towards these fruitful regions. They also examine the conditions under which such moves improve effectiveness.

Ichoua, Gendreau, and Potvin [76] proposed a threshold-based waiting strategy. The latter exploits probabilistic knowledge about future requests in an effort to better manage the fleet of vehicles and to provide better area coverage. In particular, a vehicle waits at its current location if (a) its next destination is "far enough"; (b) the probability of an immediate request to occur in the vehicle's neighborhood in the near future is "high enough"; and (c) there are not "too many" vehicles in the current zone. The proposed strategy is employed within an adaptive parallel TS algorithm, and significant improvements are observed.

Branchini, Armentano, and Løkketangen [20] proposed an adaptive granular local search heuristic for the dynamic VRPTW. Time windows can be violated by assuming a lateness cost, and the objective is to maximize the expected profit, i.e., the difference between the total revenue and the sum of lateness and travel costs. Initially, a construction heuristic is applied that scatters vehicles over the service area. The number of vehicles deployed is based not only on advanced requests but also on the expected required capacity based on historical data. The local search heuristic employs an adaptive granularity threshold to control the neighborhoods that is relative to the frequency of new customers. Additionally, waiting, repositioning, and vehicle diversion strategies are employed. In particular, a waiting-first scheme is followed. The option of repositioning a vehicle to wait at a strategic location (with a higher probability of new customers to appear) depends on the amount of time needed to reach the location and return before the depot closing time (spare time) and the number of vehicles currently repositioned at each location.

Repositioning strategies have long been recognized as key elements, especially in the context of emergency response systems. Vehicles' preparation and travel times directly influence the resulting service level. Thus, it is essential to exploit advance information and move idle vehicles currently siting at a low demand areas to cover higher demand areas. Gendreau, Laporte, and Semet [50] studied the ambulance redeployment problem. The objective is to maximize the proportion of the demand covered by at least two vehicles within a predefined radius minus a relocation cost. A parallel TS algorithm is employed to predefined redeployment scenarios in anticipation of future events. More recently, Gendreau, Laporte, and Semet [51] proposed a dynamic relocation strategy that seek to maximize the expected covered demand. Haghani and Yang [63] described an emergency response fleet deployment system having the capability to look ahead for future demands. Several dispatching policies are examined, and a mathematical model that considers vehicle dispatching and relocation decisions jointly, in a way similar to that of Yang, Hamedi, and Haghani [159], is solved in an exact fashion over a rolling horizon.

An agent-based approach for the dynamic scheduling of full truckload transportation orders with time windows has been proposed by Mes et al. [101], where intelligent vehicle agents schedule their own routes. Multi-agent systems consist of groups of agents that interact with each other. Whenever an immediate request occurs, vehicle agents interact with job agents, who seek to minimize transportation costs using a Vickrey auction. For every new request, the bid consists of a price and the expected arrival and departure times. Based on this information, the job agent assigns the request to the highest bidder, but at the price of the second highest bidder. Later, Mes et al. [100] extended this work and presented new look-ahead strategies for the dynamic PDP. They considered the expected profit of new jobs, where in addition to bids, vehicles also decide about insertion positions for the new jobs and waiting locations. A multi-agent approach for the dynamic VRP is also proposed in Barbucha [8].

Below, Table 11.5 provides an overview of selected anticipatory algorithms and predictive routing strategies.

Table 11.5. *Overview of anticipatory algorithms and predictive routing strategies.*

References	Problem setting	Algorithmic features	Objectives	Data set
Larsen, Madsen, and Solomon [88]	Partial dynamic TSP;	Various relocation strategies	Min. lateness	Randomly generated data; Real world data
van Hemert and Poutré [146]	VRP with dynamic pickup loads	Self-adapted Evolutionary algorithm	Max. n. of served customers	Randomly generated data (up to 50 customers and 5 clusters)
Ichoua, Gendreau, and Potvin [76]	VRP with soft time windows;	Parallel TS	Max. no of served customers; Min. distance traveled and lateness	Randomly generated data (up to 204 requests)
Branchini, Armentano, and Løkketangen [20]	VRP with soft time windows	Adaptive granular local search; Waiting, repositioning and diversion strategies	Max. expected profit	Randomly generated data (up to 1101 customers)
Mes et al. [101]	Full truckload trucking with time windows; stochastic customers	Multi-agent approach; Coordination through auctions	Min. transportation costs	Self-generated instances based on real world data

11.3.2.2 ▪ Sampling-Based Anticipatory Algorithms and Multiple Scenario Approaches

In the previous section, various anticipatory strategies are discussed, including, among others, waiting and repositioning heuristics integrated within reoptimization frameworks, which aim to enhance the response to foreseen upcoming events. Another alternative is to evaluate possible decisions at each time step (or decision epoch), and thus to solve (optimize) future scenarios in order to determine the best course of action. On this basis, several sampling-based anticipatory algorithms have been proposed in the literature that seek to make decisions using samples (scenarios or simulations resulting from the sampling of the input distributions) of the future.

For practical dynamic VRP applications with time-critical decisions, the task of evaluating every possible decision at each time step is extremely challenging, since a large number of samples may be required to effectively reflect the reality (see Pillac et al. [111]). Therefore, most authors resort to approximations, such as consensus and regret, and to sampling/scenario–based approaches that will provide similar benefits at a fraction of the computational cost. Interested readers on sampling-based anticipatory algorithms applied in a variety of applications, including vehicle routing, vehicle dispatching, packet scheduling, and reservation systems, as well as on the relevant theoretical analysis, may refer to Van Hentenryck and Bent [147].

A multiple scenario approach for the dynamic VRPTW with stochastic customers is introduced by Bent and Van Hentenryck [10]. The goal is to maximize the number of serviced customers. The proposed approach is highly flexible and generic, and it extends the multiple plan approach described earlier in Section 11.3.1.3. It maintains a pool of routing plans for scenarios that include both known and possible future requests. The realizations of future requests are obtained by sampling their probability distributions. Once a routing plan is obtained for each scenario, future virtual customers are removed from the plan, leaving room for new immediate requests. At each decision epoch, a distinguished plan is chosen according to a consensus function (based on similarity among plans), and the pool is updated to reflect the current state. In particular, obsolete scenarios are removed as new information is disclosed. Experiments illustrate the effectiveness of the multiple scenario approach, which in many cases outperformed the multiple plan approach.

Vehicle relocation and waiting strategies guided by scenarios are proposed by Bent and Van Hentenryck [11]. In particular, vehicles can wait or can be relocated anywhere at any time. Decisions of when and where to relocate or whether to wait are derived systematically, based on the scenario solutions and not on any prior knowledge or distribution. In addition to the decision of moving to a known or an accepted request, a wait action is considered whenever a sampled request is scheduled after an accepted request. As for relocations, this strategy simply considers moving a vehicle to the location of sampled customers which yields the best evaluation. Computational experiments indicate that both strategies are very effective in maximizing the number of serviced customers, especially for highly dynamic problems with many late requests.

Ghiani et al. [54] proposed anticipatory insertion and local search algorithms for the uncapacitated dynamic VRP with pickups and deliveries. Requests arrive according to a given stochastic process, while the objective is to minimize the expected customer inconvenience. Whenever an immediate request occurs, the request arrival process is sampled and potential solutions are evaluated with respect to the expected inconvenience assuming perfect information. A particular feature is that sampling is only performed for a predefined short-term period (empirically correlated to the problem data). As a result, the computational effort is relatively small. On the other hand, a fully sequential indifference zone selection procedure is employed to determine the number of demand samples which allows one to eliminate clearly inferior solutions determined earlier. Compared to the corresponding pure reactive algorithms, significant improvements are observed.

Azi, Gendreau, and Potvin [7] proposed a scenario-based adaptive LNS approach for a dynamic VRP with multiple delivery routes. Customer requests are received according to an independent time-space Poisson process. Based on this information, multiple scenarios for the occurrence in time and space of future requests are generated and maintained, while they are used to decide whether to include an immediate request based on an opportunity value. Whenever a new request occurs, it is checked whether it can be feasibly inserted into the actual as well as into every scenario solution. Subsequently, an opportunity value is determined as the sum of differences in solution prior to and after the insertion of the new request. If the value is positive, the new request is accepted. On the other hand, the adopted and all scenario solutions are improved via the adaptive LNS, while a repair procedure is applied to ensure the consistency of scenarios with respect to the executed plan.

Flatberg et al. [43] presented a scenario-based anticipatory algorithm and described the relevant architecture of a solver for real-life dynamic and stochastic VRPs. Several practical and implementation issues are also discussed, including the representation of statistical knowledge of events and learning event models building upon both domain knowledge and historical data. Furthermore, two real-life application examples are discussed (transportation of goods and transportation of persons) that illustrate how dynamic and stochastic aspects show up and why proper handling of dynamic events is essential for successful operations. More recently, an event-driven framework for the multiple scenario approach that allows high reactivity to changes occurring in highly dynamic environments is described in Pillac, Guéret, and Medaglia [112].

Below, Table 11.6 provides an overview of sampling algorithms and multiple scenario approaches proposed for the dynamic and stochastic VRPs. Compared to the corresponding myopic counterparts of these methods, in all cases it is demonstrated that incorporating stochastic information about future requests can lead to significant improvements.

Table 11.6. *Overview of sampling algorithms and multiple scenario approaches.*

References	Problem features	Algorithmic features	Objectives	Data set
Bent and Van Hentenryck [10, 11]	VRPTW with stochastic customers	Multiple scenario approach; Consensus function	Max. n. of serviced customers	Based on Solomon VRPTW benchmark data sets
Ghiani et al. [54]	VRP with PDP (uncapacitated)	Sampling; Anticipatory insertion and local search heuristics	Min. expected customer inconvenience	Randomly generated data (up to 600 requests)
Azi, Gendreau, and Potvin [7]	VRP with multiple delivery routes	Scenario-based adaptive LNS	Max. profit	Randomly generated data (up to 144 requests)

11.3.2.3 ▪ Analytical Studies

Stochasticity in the dynamic VRP is often related to the nature of information on customer demands, which are usually described by probability distributions. One of the earliest mentions of a dynamic and stochastic routing problem is in Psaraftis [123]. The considered problem consists of a single (uncapacitated) vehicle that seeks to service a number of nodes, where the arrival of demands at each node is described by a Poisson distribution. This problem is termed the dynamic TSP and concerns finding an optimal policy with respect to a number of objectives, such as maximizing the average expected number of demands serviced per unit time or minimizing the average expected time to serve the demands. No analytical results are derived by Psaraftis [123] for the dynamic TSP, but useful insights are provided.

To the best of our knowledge, the first analytical study on the so-called dynamic TRP, a variant of the stochastic and dynamic VRP, is introduced and studied by Bertsimas and Van Ryzin [15]. This problem arises in practical contexts where the wait for deliveries has priority over travel cost; hence the objective of the problem is to minimize the average system time (or to maximize the level of services provided by the vehicle). Such problems arise in route planning for replenishment of stock, a fleet of taxis, emergency services, etc. Bertsimas and Van Ryzin [15] describe a model for the problem, from which a lower bound on the optimal system time is derived. They also propose a number of policies for the problem and show that the one based on locating the vehicle at the median of a given region, serving customers according to the *First-Call First-Served* (FCFS) principle and returning to the median after each service, is optimal for the light traffic case. These results are extended to the case of multiple capacitated vehicles with objective functions involving system time *and* travel cost in Bertsimas and Van Ryzin [16]. Further generalization to the case where assumptions on the distribution of demand locations and on the arrival process are relaxed are considered in [17]. The modeling framework put forward in [15] has been adopted for the single-vehicle dynamic PDP in Swihart and Papastavrou [139]. While in the dynamic TRP the services are characterized by the time spent at respective locations, and in the dynamic PDP the services require that the vehicle change location during service, as each demand must be transported from its pick-up point to its location of delivery. Swihart and Papastavrou [139] derive bounds for cases where the vehicle has capacity for unit or multiple demands and investigate the performance of a number of policies for both cases.

A stochastic and dynamic version of the VRPTW is addressed by Pavone et al. [110], who looked at two variations where (i) for each customer there is a time at which demand is released and a deadline by which service needs to be made, and (ii) the deadline for each customer is modeled by using a random variable denoting the *impatience* time, and any request not met within this time after it being available will expire. The authors derive lower bounds on the minimum number of vehicles for both cases and computationally test several policies for the problem.

11.3.2.4 ▪ Stochastic Programming Models, Markov Decision Processes, and Other Algorithms

This section describes various stochastic models, MDP-based approaches, and other algorithms for solving dynamic vehicle routing and dispatching problems with stochastic information, including problem settings with stochastic customers and/or stochastic demands and/or other combinations.

Thomas and White [142] studied a single (uncapacitated) vehicle dynamic PDP. The vehicle travels from an origin to a known destination point (that must be reached by some fixed time) and services potential customers at known locations along the route. If the vehicle visits a customer after that customer has requested a pickup, the vehicle gets a reward. The probability that a customer will request a pickup before a particular time is known. The objective is to minimize the expected total cost (travel costs minus pickup rewards). The problem is modeled as a finite-horizon MDP. Based on structural results, an optimal policy is derived, and it is compared with a reactive strategy that ignores potential customer requests. Computational experiments on instances derived from a real-world road network demonstrate that the proposed approach obtains good results, especially when customer requests occur late.

A similar problem with a mix of advance and late customer requests from known locations is studied later by Thomas [141]. The author examined policies for selecting the next customer as well as where and how long to wait. The goal is to maximize the expected number of late request customers. The problem is modeled as a finite-horizon MDP, and the optimal policy is derived for the single dynamic customer. Using the structural results, the author developed a real-time heuristic and compared it with five waiting heuristic algorithms, namely the center-of-gravity longest wait heuristic with and without stochastic information, the center-of-gravity closest heuristic, the wait-at-start heuristic, and the distribute-available-waiting-time heuristic. Overall, the heuristic based on structural results performs better for problems with less than 25% late customer requests. Furthermore, the customer location information seems more valuable compared to the likelihood of occurrence. For problems with many late customer requests, the strategy of distributing waiting time across the already routed customers performs better. Note that this result is consistent with those reported earlier in Branke et al. [21] and Mitrović-Minić and Laporte [104].

Based on a real-world case, Hvattum, Løkkentangen, and Laporte [72] proposed a so-called dynamic sample scenario hedge heuristic for solving a dynamic VRP with pickup and delivery orders and rich stochastic information. The problem is formulated as a multi-stage stochastic programming model with recourse. Customer locations and demands are stochastic variables, and they are revealed at the time of call in, while historical data are used to determine probability distributions of the order attributes. Sample scenarios are generated and used to guide a heuristic method that builds a plan for each predefined time interval. Each sample scenario is solved at each period as a static VRP. When all scenarios are solved, the most common features (based on the assignment frequency) are used to build the routing plan for the upcoming period. This gradual solution construction scheme based on the sample solutions can be seen as progressive hedging. Computational experiments illustrates the effectiveness of the proposed method compared to a pure dynamic heuristic method. Later, Hvattum, Løkkentangen, and Laporte [73] extended the problem and considered the possibility of stochastic demands, while a more enhanced sampling-based Branch-and-Regret heuristic is proposed.

MDP-based approaches have been proposed by Secomandi [133], Secomandi and Margot [134], and Novoa and Storer [108] for the single-vehicle dynamic VRP with stochastic demands. Compared to traditional two-stage approaches with recourse that seek to minimize expected routing costs (including the cost of recourse actions at the second stage), routing decisions are made whenever a vehicle arrives at a customer location and demand is revealed. The idea is to generate an initial solution, and adapt it at each decision point so it always terminates from the state the system is currently in. Secomandi [133] formulated the problem as a stochastic shortest path problem and proposed a sequential consistent roll-out algorithm based on a cycling heuristic. Later, Secomandi and Margot [134] provided a partial characterization of the optimal policy and proposed two partial reoptimization heuristics. The idea is to take into account only a subset of all the possible states and compute an optimal policy on this restricted set of states. Computational experiments showed that the partial reoptimization heuristics are highly effective.

Novoa and Storer [108] described a two-stage roll-out algorithm for the single-vehicle dynamic VRP with stochastic demand using ADP approaches (see Powell [117]). As proposed in [133], the problem is formulated as a stochastic shortest path problem; however, a Monte Carlo simulation is used to compute the expected cost-to-go. At each stage, decisions are estimated based on the current and expected future costs. The proposed two-step roll-out algorithm sequentially improves a single or multiple a priori suboptimal base solution, while improved pruning schemes and look-ahead strategies are also introduced. The authors presented results of computational experiments on randomly generated instances, similar to those in [133], and provided solutions close to those obtained with perfect information.

Goodson, Ohlmann, and Thomas [58] propose a family of roll-out policies based on fixed routes to obtain dynamic solutions for the multi-vehicle VRP with stochastic demand and duration limits. Initially, a traditional one-step algorithm is proposed that yields promising results for up to 50 customers. Next, a post-decision roll-out is proposed that looks ahead from each post-decision state that fixes vehicle destinations, capacities, and arrival times at the next decision epoch (but does not observe demand as in one-step roll-out). To that end, a hybrid policy is also introduced that limits the number of post-decision states. In an effort to enhance computational tractability, a dynamic decomposition scheme (i.e., repartition of customers at each decision epoch) is adopted that allows applying roll-out policies to single-vehicle problems and using the resulting policies to select actions for multiple vehicles. Compared to the static version, the proposed dynamic decomposition-based roll-out policy produces much better results.

ADP and other approaches have been successfully applied for a wide range of fleet management problems, including, among others, dynamic vehicle allocation and dynamic resource allocation problems (see Powell and Topaloglu [121], Spivey and Powell [137], Topaloglu [143], and Topaloglu and Powell [144]). In an earlier work Powell [116] studied a long-haul truckload trucking problem and proposed models for static and dynamic cases where the demand could be deterministic or stochastic. One of the ways in which this paper models the stochastic and dynamic assignment problem is using a conditional recourse function, which is approximated through several methods such as scenario aggregation, stochastic gradient methods, and a successive linear approximation procedure. Computational testing showed that the dynamic model outperforms a more standard myopic one. It also provides evidence that the value of having advance information on shipper demands is minimal. A later work by Powell, Snow, and Cheung [120] described two heuristics for the solution of the dynamic problem using static snapshots of data, where the focus is on routing a driver through a sequence of more than one request. Spivey and Powell [137] studied a more general class of dynamic assignment problems and tested the effect of advance information on myopic and nonmyopic models. The tests showed that the latter outperformed the former only when insufficient advance information is available, but the situation is the opposite when sufficient advance information is at hand.

A tabulated summary of algorithms based on stochastic programming models and MDPs is provided in Table 11.7.

Table 11.7. *Overview of algorithms based on stochastic programming models and MDPs.*

References	Problem features	Algorithmic features	Objectives	Data set
Thomas and White [142]; Thomas [141]	single vehicle (uncapacitated) PDP	Finite-horizon MDP; Optimal policies; Reactive strategies; Waiting Strategies	Min. expected travel cost minus pickup rewards; Max. n. of late request customers served	Randomly generated data based on real world road network
Hvattum, Løkkentangen, and Laporte [72]	VRP with PDP	Dynamic sample scenario hedge heuristic	Min. unserved customers, vehicles, travel time	Real and random data (up to 200 orders)
Hvattum, Løkkentangen, and Laporte [73]	VRP with PDP and stochastic demands	Branch-and-Regret heuristic	Min. unserved customers, vehicles, travel time	Real and random data (up to 200 orders)
Secomandi [133]; Secomandi and Margot [134]	single-vehicle VRP with stochastic demands	Roll-out algorithms; Partial reoptimization	Min. travel cost	Randomly generated instances (up to 60 customers)
Novoa and Storer [108]	single-vehicle VRP with stochastic demands	ADP; Two-stage roll-out algorithm	Min. travel cost	Similar to Secomandi [133]
Goodson, Ohlmann, and Thomas [58]	VRP with stochastic demands and duration limits	One-step, Post-decision, hybrid and decomposition-based roll-out policies	Min. travel cost	Based on Solomon VRPTW benchmark data sets

11.4 ▪ Dynamic and Time-Dependent Travel Times

This section discusses treatment and solution approaches for problem variants with dynamic and time-dependent travel times.

Dynamic VRPs have mainly been characterized through arrival process of customer requests over time not known a priori, but assuming that travel times between pairs of customers are fixed based on forecasted or historical information. However, travel times might change over the course of time (i.e., time-dependent) or be dynamic in nature themselves, only revealed in real time as the vehicles travel. Vehicle routing planning that takes into account real-time information on the travel times might not only have benefits to reap in the total cost of travel but also on the reliability of arrivals at customers, particularly when there are time-window restrictions in place. Advantages of such an approach over assuming fixed (forecasted) travel times are shown by Taniguchi and Shimamoto [140], who show through numerical experiments on a test network of 25 nodes and 80 links that incorporating real-time information enables reducing total costs by 1.5–3.7% and penalties for early or delayed arrivals at customers by 22.2–100%. The authors also state that such an approach would also alleviate traffic congestion by reducing the operating time of vehicles.

A more general VRPTW where *both* service requests and travel times are assumed to be dynamic is studied by Haghani and Jung [61], where requests at customers can be either pickup or delivery, assuming no priority order on requests. Travel time on a given arc is modeled as a continuous function changing over time, as was proposed in the same authors' earlier work (see Jung and Haghani [81]), where a formulation and a genetic algorithm to solve the problem are also described. The approach described in [61] adjusts vehicle routes at given points in time within a planning period as new information becomes available. The adjustments are then communicated to the vehicles from a centralized control center. Results of computational experiments on randomly generated problems with up to 10 nodes comparing the genetic algorithm with the exact solution of the formulation solved by CPLEX as well as a lower-bounding solution procedure confirm that the genetic algorithm is able to produce solutions within 7% from the optimal value. The genetic algorithm is then tested on a larger case study network with 382 nodes and 1398 links. The results reveal that route adjustments are worthwhile when travel times fluctuate significantly over a planning horizon even when requests are static. When requests themselves also change with time a static routing strategy is seen to be between 1.43 and 2.21 times more costly than a dynamic strategy, being particularly effective as travel time uncertainty increases.

Potvin, Xu, and Benyahia [115] look at a similar problem with uncapacitated vehicles but with the additional constraint that each customer i and the depot ($i = 0$) be associated with a predefined time window $[a_i, b_i]$ in which arrivals should be made. A further assumption is that updates to travel information can be conveyed to the vehicles only at customer locations. The objective of the problem is to minimize an objective function comprising travel time, late arrivals at customer locations as calculated by $(\tau_i - b_i)^+ = \max\{0, \tau_i - b_i\}$ for each customer i where τ_i is the planned arrival time, and late arrival to the depot. More specifically, if the sequence of customers visited by vehicle $k = 1, \ldots, |K|$ is given by $\{i_0^k, i_1^k, \ldots, i_{m_k}^k\}$ with $i_0^k = i_{m_k}^k = 0$, the objective can be expressed as follows:

$$\sum_{k=1}^{|K|} \left(\alpha_1 \sum_{p=1}^{m_k} t(i_{p-1}^k, i_p^k) + \alpha_2 \sum_{p=1}^{m_k-1} (\tau_{i_p^k} - b_{i_p^k})^+ + \alpha_3 (\tau_0^k - b_0)^+ \right), \tag{11.4}$$

with α_1, α_2, and α_3 weights and τ_0^k being the time by which vehicle k should return to the depot. One contribution of Potvin, Xu, and Benyahia [115] is the introduction of *tolerance* for delayed arrival times at each customer, which also serves as a threshold value at which "reactive" changes are introduced to existing routes. The authors describe a dispatching algorithm based on local search and a discrete-event simulation scheme to handle the dynamic aspect of the problem, and results on Solomon's 100-node benchmark problems indicate that allowing for a small but positive, rather than zero, tolerance significantly improves the objective function value.

Lorini, Potvin, and Zufferey [96] later extend the approach of Potvin, Xu, and Benyahia [115] to take into account a version of the problem where it is assumed that communications between drivers and the dispatch office are possible even when the vehicles are en-route. This variant of the problem allows for diversion of vehicles from their planned destination and opens up opportunities to serve new requests. Computational results on benchmark instances reported in [96] confirm that allowing for diversion yields savings in the objective function, which increase with the amount of allowable tolerance in the maximum acceptable delay of a vehicle's planned arrival time to a given destination.

As mentioned above, Haghani and Jung [61] do not consider time-window constraints within the context of the dynamic VRP, whereas [115] does not consider capacity constraints on the vehicles. The consideration of both constraints under the time-dependent nature of requests and travel times can be seen in the work by Chen, Hsueh, and Chang [25], who look at the problem with aiming at minimizing an objective function comprising a weighted function of travel times on each arc, as well as pre- and post-service waiting times under time-window and vehicle capacity constraints. Time-dependent travel times for each arc of the network are modeled as a step-wise function, where a different constant travel time is prescribed for each interval of a planning horizon. The authors suggest that the idle waiting after service (i.e., a positive difference between service completion time at and departure time from a customer) allows for flexibility, as such a vehicle is able to receive and respond to new route instructions under real-time dispatching. Chen, Hsueh, and Chang [25] describe a mathematical model and a heuristic procedure for the solution of the problem. The authors also present computational results on Solomon's benchmark problems and a real-life logistics network using the heuristic procedure, which indicate that, in comparison to a method which does not account for time-dependent travel times, the proposed approach not only improves the objective function value from 1% to 22%, but it also reduces the probability of service requests rejected.

Hsueh, Chen, and Chou [70] studied a dynamic VRP for relief logistics in natural disasters and proposed a local search reoptimization approach. They considered in their model dynamic pickup-and-delivery demands for relief goods as well as dynamic travel times. The objective was to minimize the traveling time and the total penalty due to late arrivals. An anticipatory algorithm of a routing and scheduling problem for forwarding agencies handling less-than-truckload freight in disaster reliefs is studied by Wohlgemuth, Oloruntoba, and Clausen [156]. The problem involves dynamic pickup-and-delivery requests that may both receive and send goods and varying travel times. The objective is to avoid delays (minimize travel distance) and increase utilization (minimize the number of vehicles). A TS approach is employed that benefits from the anticipation of the availability of connections and the integration of possible demand regions.

Wang and Zhu [148] proposed an ACO algorithm for the VRP with dynamic requests and travel times. The objective is to minimize the weighted sum of expected travel times, expected waiting times, and expected penalties. Finally, a GA-based reoptimization procedure coupled with effective local update heuristics has been developed by Cheung et al. [27] for the dynamic PDP with capacity constraints, time windows, and dynamic travel times. On the other hand, Haghani, Tian, and Hu [62] developed a simulation model to evaluate vehicle dispatching strategies for emergency response with the objective of minimizing average response times associated with different types of accidents. Various strategies were analyzed, namely the FCFS strategy, the nearest origin assignment strategy, and the flexible assignment strategy, for assigning and dispatching response vehicles under various circumstances (i.e., different accident occurrence rates, route change strategies, and dynamic travel time information).

Cortés et al. [36] modeled the capacitated dynamic PDP as a *Hybrid Adaptive Predictive Control* (HAPC) problem based on state space variables. The system state is defined in terms of departure times and vehicle loads at stops. The inputs are routing decisions, and the outputs are departure times. The demand requests are modeled as disturbances. The proposed model is discrete and considers a variable step size that is equal to the time between successive calls. The objective function takes into account future requests via probabilities computed from historical data, and accounts for both the total expected waiting and travel time for passengers and the idle travel time. Furthermore, it is considered that potential rerouting of vehicles could affect the current dispatch decisions through the extra cost of inserting immediate requests into the current routing plan. A clustering technique (classic zoning) is used to estimate the spatial-temporal probabilities to forecast future request points. The HAPC problem is solved by means of a PSO method. Computational experiments indicated savings in actual waiting times as compared to myopic dispatch decision models. Saez, Cortés, and Nunez [131] proposed a similar HAPC approach, where fuzzy zoning is used to compute probabilities and trip patterns from historical data and a GA is used to solve the HAPC problem. Finally, Cortés, Nunez, and Saez [35] extended the HAPC scheme proposed in [131] to consider network traffic conditions and other types of uncertainties (e.g., accidents).

Attanasio et al. [5] presented a real-time fleet management system for a same-day courier service application. The problem addressed can be seen as a dynamic uncapacitated PDP. An allocation procedure is proposed to assign customer requests to available couriers, and to reposition idle couriers to high demand zones. The latter employs fast insertion heuristics, feasibility procedures, and a TS method. Furthermore, a forecasting module is utilized to provide reliable near future predictions of customer demands via a service territory zoning method and forecasted travel times to the allocation procedure. Travel time forecasts took into account both traffic and real-time information, and they are adjusted using artificial neural networks. Computational experiments based on real-world data are reported.

Finally, another alternative to capture the uncertain nature of travel times is to consider the combination of stochastic and time dependency. The latter is known as stochastic time-dependent networks, and travel times are random variables with time-dependent distributions. We are not aware of any published work addressing stochastic time-dependent travel times for dynamic VRPs; however, interested readers may refer to the works of Fu [45], Kim, Lewis, and White [86], and Lecluyse, Van Wossel, and Peremans [91].

11.5 ▪ Dynamic Vehicle Availability

Besides more or less expected events, for example dynamic customer requests, in practice it is also likely to experience unforeseen disruption events, such as vehicle breakdowns and accidents, traffic congestions, delayed departures from the depot, and so on. Among others, dynamic vehicle availability events have a major impact, since the original or the currently executed routing plan becomes in most cases infeasible, and thus complex recovery actions are needed to restore it. As mentioned earlier, in the context of disruption management the goal is to revise the plan such that the negative effects are minimized and their impact is alleviated. However, it is often the case that conflicting objectives need to be considered (see Mu and Eglese [106]).

So far, few studies have addressed real-time rerouting and rescheduling problems with dynamic vehicle availabilities. Li, Mirchandani, and Borenstein [94] introduced the so-called Real-time Vehicle Rescheduling problem. Both pickup and delivery service functions are examined, while time-window and vehicle capacity constraints are considered. Whenever a vehicle disruption event occurs, the fleet plan needs to be adjusted in real time depending on the current state. A set-covering model is developed for the rerouting problem. The objective is to minimize the weighted sum of operation, service cancellation, and route disruption costs. A Lagrangian relaxation approach is developed that employs an insertion heuristic to obtain a feasible solution for the primal problem, while the shortest path problems with resource constraints are solved via a DP algorithm.

Earlier, Li, Mirchandani, and Borenstein [92, 93] studied the single-depot Vehicle Rescheduling Problem. The problem consists of assigning vehicles to a set of predetermined trips with fixed starting and ending times with an objective of minimizing capital and operating costs. The problem is partially modeled as a sequence of static vehicle scheduling problems, and a pseudo-polynomially auction algorithm is used to solve it. Two assumptions are made, i.e., the scheduled trips, except the disrupted trip, cannot suffer delays, while there are no restrictions on the number of trips that may be reassigned. In [94] this model is extended by taking into account both schedule disruptions and delays of multiple trips. Another related work is that of Huisman, Freling, and Wagelmans [71]. They proposed a cluster-reschedule heuristic for the dynamic multi-depot vehicle scheduling problem to avoid trips starting late in environments characterized by significant traffic jams. The proposed approach consists of solving a series of optimization problems, where different scenarios for travel times are considered. For this purpose, the cluster-reschedule heuristic first assigns trips to depots by solving the corresponding static problem and then dynamically reschedules the trips per depot.

Mu et al. [107] studied the so-called Disrupted CVRP with Vehicle Breakdown. An extra (backup) vehicle with full capacity is assumed available at the depot, and the objective is to minimize the total number of vehicles used and the total travel distance, such that all customers (including those of the immobilized vehicle) are served. Vehicle diversion is not allowed. Two TS algorithms are developed for solving the problem that differ in the neighborhood structure and the neighborhood selection procedure. Both approaches are assessed in relation to an exact algorithm, and they seem adequate to respond under tight time constraints when disruption occurs. Later, Mu and Eglese [106] studied the so-called Disrupted CVRP with Order Release Delay. The disruption event is that the supplies do not arrive at the depot on time; therefore some vehicles cannot be loaded and begin their delivery schedules. Therefore, new plans must be generated to reduce the impact of delays.

More recently, Minis, Mamasis, and Zeimpekis [102] studied a problem in which a vehicle breaks down and the remaining vehicles are rerouted to serve some or all customers of the disrupted route. The problem is modeled as a Capacitated Team Orienteering Problem with vehicle breakdown. Compared to Mu et al. [107] and Li, Mirchandani, and Borenstein [94], they considered that each vehicle can serve only its own customers and the customers of the failed vehicle, while no extra vehicles are available other than those already routed and products cannot be exchanged between customers (as in the case of Mu et al. [107]). The objective is to maximize the profit. A heuristic and a genetic algorithm are proposed.

Zhang and Tang [161] presented a rescheduling model for the VRPTW with vehicle disruptions. The objective is to revise the vehicle routing plan such that the weighted sum of traveling distance and deviations from time windows is minimized. A hybrid metaheuristic approach is proposed. On the other hand, Wang et al. [150] developed a disruption recovery model for the VRPTW with vehicle breakdowns, and proposed two rescue strategies, namely adding vehicles and neighboring rescue. A similar vehicle breakdown problem is also studied by Wang, Wu, and Hu [152].

Finally, Xiang, Chu, and Chen [157] addressed a dynamic DARP on a time-dependent network with many different types of dynamic events, including traveling and service time fluctuations, new requests, no-shows, vehicle breakdowns, cancellations, and traffic jams. Each event has its own priority, and it is placed within an event queue that is sorted in ascending order of priority (a first-come first-served policy is applied for events with the same priority). Furthermore, a prescribed reaction and/or rescuing decision is proposed for each event type. A local search procedure based on inter-trip moves is applied for the insertion of new requests. Additionally, a diversification strategy is adopted that utilizes a secondary objective measuring the idle time of the vehicles.

11.6 ▪ Performance Measurements and Evaluation of Solution Approaches

A well-known method for measuring the performance of algorithms applied to dynamic problems is the so-called *competitive analysis* (see Borodin and El-Yaniv [19] and Sleator and Tarjan [135]). Let I denote an instance of the set of all instances $\mathscr{I}$ of a minimization problem. The competitive ratio can be defined as $c_r = \sup_I \frac{z_A(I)}{z^*(I_s)}$, where $z_A(I)$ is the cost (objective function value) of the solution obtained by an algorithm A for the problem instance I, and $z^*(I_s)$ is the optimal cost found by an offline algorithm that had access to all input data of instance I beforehand. To that end, the competitive ratio can be used to measure the worst-case performance of a routing policy. However, in principle the competitive analysis requires examining all instances of a given problem and to obtain the optimal solution for the corresponding offline (static) instances, which is in most cases a complex task even for deriving tight lower bounds.

The analytical studies discussed in Section 11.3.2.3 used the competitive analysis framework and produced important analytical results and insights for a number of routing policies. Another more flexible alternative is to measure the so-called *value of information* V_A. In this case, the algorithm is applied on both the dynamic instance I (via some form of discrete-event simulation) and the corresponding static instance I_s. To that end, the value of information for an algorithm A can be defined as $V_A = \frac{z_A(I_s) - z_A(I)}{z_A(I_s)}$. Although this is an empirical study that provides only an estimate of the performance of the algorithm, in most cases it captures the impact of the dynamism on the solution yielded by the examined algorithm (assuming that the arrival order of the inputs is not too significant).

Several benchmark data sets appear in the literature that can be used for the evaluation of different solution approaches. Unfortunately, publicly available data sets are scarce even for well-studied dynamic VRPs, while in many cases the performance of different algorithms is assessed in the restricted scope of the problem variant examined at each paper. Furthermore, it is difficult to find a common ground for comparisons due to the wide variety of objective functions. Many authors base their computational studies on adaptations of well-known data sets used for static VRPs, including those of Solomon [136] and Christofides, Mingozzi, and Toth [30]. Interested readers may refer to the procedure proposed by Bent and Van Hentenryck [10] for the modification of Solomon's data sets [136]. A repository of data sets is also maintained by Pankratz and Krypczyk in [109].

11.7 ▪ Conclusions and Future Research Directions

This chapter has presented a review of dynamic vehicle routing and dispatching problems where information evolves over time and decisions must be made in a continuously changing environment. The review has shown that many variants of such problems have been looked at, in varying levels of detail, with the VRP (with or without time windows) and the PDP being the most prominent. The review has indicated that the nature of the solution algorithms developed for dynamic VRPs primarily relies on either (i) testing a variety of policies or (ii) various heuristic or metaheuristic algorithms, some of them making use of parallel implementations. Exact algorithms that have been proposed for this class of problems are highly limited in number, with the most recent reference reviewed in this chapter dating back to 2006, even though interesting ideas have been put forward. Another interesting observation on the existing literature is on the imbalance between the number of studies concerning the deterministic and stochastic versions of the problem, where the amount of research on the former is seen to be considerably greater than that of the latter. One reason behind this discrepancy is the inherent difficulty of the stochastic version.

Although the broader research field has arguably reached some level of maturity, many challenges still stand and new ones emerge. Furthermore, many important problem variants and application areas remain open. Research is still needed for theory building, development of solution methods for moderately and strongly dynamic problem variants, and the study and adaptation of existing approaches for real-life applications. Below is a list of suggestions for further research in this area:

- *Development of Taxonomies and Classification Schemes.* The domain of dynamic VRPs itself is very wide. Although various taxonomies and classification schemes have been proposed in earlier survey papers, the boundaries and similarities among different problem variants as well as links with particular applications need to be clearly defined. As mentioned in Pillac et al. [111] this will foster the development of more generic solution frameworks.

- *Integrating and Exploiting Knowledge about Future Uncertain Events.* Strategies that anticipate future events have been receiving an increasing interest and are incorporated into the recently developed solution methods. However, there is still a lack of formal and general mathematical modeling frameworks and analytical studies. Furthermore, the study of more advanced location analysis and forecasting techniques may yield considerable improvements.

- *Handling Less Predictable Events.* Unforeseen disruption events, such as vehicle failures, accidents, congestions, and others, need further investigation both in terms of robustness considerations as well as novel strategies to alleviate their impact. In the context of emergency services, disaster events may create a highly dynamic problem environment characterized by total uncertainty as well as frequent changes to the input data. Thus, special solution methodologies must be developed to address these challenging problems.

- *Degrees of Freedom, Real-Time Performance Measurements, and Benchmark Data Sets.* There is an evident need to design more enhanced and encompassing measures that will determine the level of dynamism for dynamic vehicle routing and dispatching problems with different sources of uncertainties. The ability to monitor, in real time, the impact of decisions already made or those of alternative future actions is essential for the evaluation of the services provided. Furthermore, in many cases there is a lack of publicly available reference benchmark data sets, even for well-studied dynamic VRP variants.

- *Parallel Algorithms.* As mentioned in Ichoua, Gendreau, and Potvin [77], time pressure remains as the major impediment for the adaptation of well-performing solution approaches proposed for the corresponding static problem instances. The recent emergence of multi-core processors, graphical processing units, and the ever growing computing power of workstations and parallel implementation techniques can help in solving increasingly larger and more realistic problems, even for the time-demanding multi-plan and multi-scenario sampling approaches.

– *Emergency Services*. The field of emergency services, including allocation and relocation problems for emergency vehicles, warrants more research efforts. The seeming randomness of impacts, the uniqueness of emergency incidents, and the uncertain nature of emergency services make them ideally suitable for real-time optimization solution frameworks. The main idea behind integration of real-time information flows within the optimization process is to enable governmental or other organizations to make decisions in real time, to draw policies, and to react to emergency events under tight time constraints.

– *Links with Other Decision Areas.* As highlighted by Larsen, Madsen, and Solomon [90], an important yet unexplored area of research is to study the interface between dynamic vehicle routing and dispatching problems and other parts of the supply chain in which these problems arise, such as warehousing and manufacturing. This is particularly relevant in the context of just-in-time global supply chains.

– *Dynamic Arc Routing Problems.* Modeling and design of algorithms for dynamic arc routing problems is an important research direction that has only received little attention. We refer the reader to Ghiani et al. [53] for a review of applications related to road gritting and snow collection.

For many of the extensions listed above, there is a need to design solution algorithms that can generate high-quality solutions in short time scales to guarantee that new information is reacted to in a timely fashion. Such algorithms should be coupled with look-ahead capabilities that exploit special problem structures and advance knowledge to anticipate future uncertain events and to be able to go beyond the current state of the art and open new scholarly and technological horizons. Given the recent innovations in vehicle tracking technologies, which are able to provide information in real time, the advances in the emerging fields of big data and predictive analytics might further stimulate algorithmic developments in the area of dynamic VRPs. In our opinion, this research topic is still open to a new generation of solution methods that will not only allocate resources optimally but will also integrate real-time decision making, react to expected or unexpected events, anticipate the future, and learn from the past to produce robust solutions.

Acknowledgments

Thanks are due to two anonymous referees for their comments and suggestions on an earlier version of this chapter.

Bibliography

[1] E. ANGELELLI, N. BIANCHESSI, R. MANSINI, AND M. G. SPERANZA, *Short term strategies for a dynamic multi-period routing problem*, Transportation Research Part C: Emerging Technologies, 17 (2009), pp. 106–119.

[2] E. ANGELELLI, R. MANSINI, AND M. G. SPERANZA, *A real-time vehicle routing model for a courier service problem*, in Distribution Logistics - Advanced Solutions to Practical Problems, Lecture Notes in Economics and Mathematical Systems, vol. 544, Springer-Verlag, Berlin, Heidelberg, 2005, pp. 87–104.

[3] E. ANGELELLI, M. W. P. SAVELSBERGH, AND M. G. SPERANZA, *Competitive analysis of a dispatch policy for a dynamic multi-period routing problem*, Operations Research Letters, 35 (2007), pp. 713–721.

[4] N. ASCHEUER, S. O. KRUMKE, AND J. RAMBAU, *Online dial-a-ride problems: Minimizing the completion time*, in Lecture Notes in Computer Science, vol. 1770, Springer-Verlag, Berlin, Heidelberg, 2004, pp. 639–650.

[5] A. ATTANASIO, J. BREGMAN, G. GHIANI, AND E. MANNI, *Real-time fleet management at eCourier Ltd*, in Dynamic Fleet Management: Concepts, Systems, Algorithms and Case Studies, V. Zeimpekis, C. D. Tarantilis, G. M. Giaglis, and I. Minis, eds., vol. 38 of Operations Research/Computer Science Interfaces Series, Springer-Verlag, New York, 2007, pp. 219–238.

[6] A. ATTANASIO, J.-F. CORDEAU, G. GHIANI, AND G. LAPORTE, *Parallel tabu search heuristics for the dynamic multi-vehicle dial-a-ride problem*, Parallel Computing, 30 (2004), pp. 377–387.

[7] N. AZI, M. GENDREAU, AND J.-Y. POTVIN, *A dynamic vehicle routing problem with multiple delivery routes*, Annals of Operations Research, 199 (2012), pp. 103–112.

[8] D. BARBUCHA, *Simulating activities of the transportation company through multi-agent system solving the dynamic vehicle routing problem*, in Lecture Notes in Artificial Intelligence, vol. 5559, Springer-Verlag, Berlin, Heidelberg, 2009, pp. 773–782.

[9] A. BEAUDRY, G. LAPORTE, T. MELO, AND S. NICKEL, *Dynamic transportation of patients in hospitals*, OR Spectrum, 32 (2010), pp. 77–107.

[10] R. W. BENT AND P. VAN HENTENRYCK, *Scenario-based planning for partially dynamic vehicle routing with stochastic customers*, Operations Research, 52 (2004), pp. 977–987.

[11] ——, *Waiting and relocation strategies in online stochastic vehicle routing*, in Proceedings of the Twentieth International Joint Conference on Artificial Intelligence, 2007, pp. 1816–1821.

[12] I. BENYAHIA AND J.-Y. POTVIN, *Decision support for vehicle dispatching using genetic programming*, IEEE Transactions on Systems Man and Cybernetics Part A - Systems and Humans, 28 (1998), pp. 306–314.

[13] G. BERBEGLIA, J.-F. CORDEAU, AND G. LAPORTE, *Dynamic pickup and delivery problems*, European Journal of Operational Research, 202(1) (2010), pp. 8–15.

[14] ——, *A hybrid tabu search and constraint programming algorithm for the dynamic dial-a-ride problem*, INFORMS Journal on Computing, 24 (2012), pp. 343–355.

[15] D. J. BERTSIMAS AND G. VAN RYZIN, *A stochastic and dynamic vehicle routing problem in the Euclidean plane*, Operations Research, 39 (1991), pp. 601–615.

[16] ——, *Stochastic and dynamic vehicle routing in the Euclidean plane with multiple capacitated vehicles*, Operations Research, 41 (1993), pp. 60–76.

[17] ——, *Stochastic and dynamic vehicle routing with general demand and interarrival time distributions*, Advances in Applied Probability, 25 (1993), pp. 947–978.

[18] L. BIANCHI, *Notes on dynamic vehicle routing - the state of the art*, 2000. Technical Report IDSIA-05-01, Instituto Dalle Molle Di Studi Sull Intelligenza Artificiale, Manno, Switzerland.

[19] A. BORODIN AND R. EL-YANIV, *Online Computation and Competitive Analysis*, Cambridge University Press, Cambridge, UK, 2005.

[20] R. M. BRANCHINI, V. A. ARMENTATO, AND A. LOKKETANGEN, *Adaptive granular local search heuristic for a dynamic vehicle routing problem*, Computers & Operations Research, 36 (2009), pp. 2955–2968.

[21] J. BRANKE, M. MIDDENDORF, G. NOETH, AND M. DESSOUKY, *Waiting strategies for dynamic vehicle routing*, Transportation Science, 39(3) (2005), pp. 298–312.

[22] L. BROTCORNE, G. LAPORTE, AND F. SEMET, *Ambulance location and relocation models*, European Journal of Operational Research, 147 (2003), pp. 451–463.

[23] M. CARAMIA, G. ITALIANO, G. ORIOLO, A. PACIFICI, AND A. PERUGIA, *Routing a fleet of vehicles for dynamic combined pick-up and deliveries services*, in Proceedings of the Symposium on Operations Research 2001, Duisburg, Germany, 2001, pp. 3–5.

[24] M.-S. CHANG, S.-R. CHEN, AND C.-F. HSUEH, *Real-time vehicle routing problem with time windows and simultaneous delivery/pickup demands*, Journal of the Eastern Asia Society for Transportation Studies, 5 (2003), pp. 2273–2286.

[25] H.-K. CHEN, C.-F. HSUEH, AND M.-S. CHANG, *The real-time time-dependent vehicle routing problem*, Transportation Research Part E: Logistics and Transportation Review, 42 (2006), pp. 383–408.

[26] Z.-L. CHEN AND H. XU, *Dynamic column generation for dynamic vehicle routing with time windows*, Transportation Science, 40 (2006), pp. 74–88.

[27] B. K.-S. CHEUNG, K. L. CHOY, C.-L. LI, W. SHI, AND J. TANG, *Dynamic routing model and solution methods for fleet management with mobile technologies*, International Journal of Production Economics, 113 (2008), pp. 694–705.

[28] R. K. CHEUNG, D. XU, AND Y. GUAN, *A solution method for a two-dispatch delivery problem with stochastic customers*, Journal of Mathematical Modelling and Algorithms, 6 (2007), pp. 87–107.

[29] C. H. CHRISTIANSEN AND J. LYSGAARD, *A branch-and-price algorithm for the capacitated vehicle routing problem with stochastic demands*, Operations Research Letters, 35 (2007), pp. 773–781.

[30] N. CHRISTOFIDES, A. MINGOZZI, AND P. TOTH, *The vehicle routing problem*, in Combinatorial Optimization, N. Christofides, A. Mingozzi, P. Toth, and C. Sandi, eds., Wiley, Chichester, UK, 1979, pp. 315–338.

[31] J.-F. CORDEAU AND G. LAPORTE, *A tabu search for the static multi-vehicle dial-a-ride problem*, Transportation Research Part B: Methodological, 37 (2003), pp. 579–594.

[32] ——, *The dial-a-ride problem: Models and algorithms*, Annals of Operations Research, 153 (2007), pp. 29–46.

[33] J.-F. CORDEAU, G. LAPORTE, J.-Y. POTVIN, AND M. W. P. SAVELSBERGH, *Transportation on demand*, in Transportation, C. Barnhart and G. Laporte, eds., vol. 14 of Handbooks in Operations Research and Management Science, Elsevier, Amsterdam, 2007, pp. 429–466.

[34] J.-F. CORDEAU, G. LAPORTE, M. W. P. SAVELSBERGH, AND D. VIGO, *Vehicle routing*, in Transportation, C. Barnhart and G. Laporte, eds., vol. 14 of Handbooks in Operations Research and Management Science, Elsevier, Amsterdam, 2007, pp. 367–428.

[35] C. E. CORTÉS, A. NUNEZ, AND D. SAEZ, *Hybrid adaptive predictive control for a dynamic pick-up and delivery problem including traffic congestion*, International Journal of Adaptive Control and Signal Processing, 22 (2008), pp. 103–123.

[36] C. E. CORTÉS, D. SAEZ, A. NUNEZ, AND D. MUNOZ-CARPINTERO, *Hybrid adaptive predictive control for a dynamic pick-up and delivery problem*, Transportation Science, 43 (2009), pp. 27–42.

[37] L. COSLOVICH, R. PESENTI, AND W. UKOVICH, *A two-phase insertion technique of unexpected customers for a dynamic dial-a-ride problem*, European Journal of Operational Research, 175 (2006), pp. 1605–1615.

[38] T. DU, F. K. WANG, AND P.-Y. LU, *A real-time vehicle-dispatching system for consolidating milk runs*, Transportation Research Part E: Logistics and Transportation Review, 45 (2007), pp. 565–577.

[39] J. EUCHI, A. YASSINE, AND H. CHABCHOUB, *Solving the dynamic vehicle routing problem by means of artificial ant colony*, in Proceedings of the International Conference on Metaheuristics and Nature Inspired Computing (META'2010), Tunisia, 2010.

[40] A. FABRI AND P. RECHT, *On dynamic pickup and delivery vehicle routing with several time windows and waiting times*, Transportation Research Part B: Methodological, 40 (2006), pp. 335–350.

[41] F. FERRUCCI, S. BOCK, AND M. GENDREAU, *A pro-active real-time control approach for dynamic vehicle routing problems dealing with the delivery of urgent goods*, European Journal of Operational Research, 225 (2013), pp. 130–141.

[42] E. FEUERSTEIN AND L. STOUGIE, *On-line single-server dial-a-ride problems*, Theoretical Computer Science, 268 (2001), pp. 91–105.

[43] T. FLATBERG, G. HASLE, O. KLOSTER, E. J. NILSSEN, AND E. RIISE, *Dynamic and stochastic vehicle routing in practice*, in Dynamic Fleet Management: Concepts, Systems, Algorithms and Case Studies, V. Zeimpekis, C. D. Tarantilis, G. M. Giaglis, and I. Minis, eds., vol. 38 of Operations Research/Computer Science Interfaces Series, Springer-Verlag, New York, 2007, pp. 42–63.

[44] B. FLEISCHMANN, S. GNUTZMANN, AND E. SANDVOSS, *Dynamic vehicle routing based on online traffic information*, Transportation Science, 38 (2004), pp. 420–433.

[45] L. FU, *Scheduling dial-a-ride paratransit under time-varying, stochastic congestion*, Transportation Research Part B: Methodological, 36 (2004), pp. 485–506.

[46] L. GAMBARDELLA, A. RIZZOLI, F. OLIVERIO, N. CASAGRANDE, A. DONATI, R. MONTEMANNI, AND E. LUCIBELLO, *Ant colony optimization for vehicle routing in advanced logistics systems*, in Proceedings of the International Workshop on Modelling and Applied Simulation (MAS 2003), 2003, pp. 3–9.

[47] M. GENDREAU, F. GUERTIN, J.-Y. POTVIN, AND R. SEGUIN, *Neighborhood search heuristics for a dynamic vehicle dispatching problem with pick-ups and deliveries*, Transportation Research Part C: Emerging Technologies, 14 (2006), pp. 157–174.

[48] M. GENDREAU, F. GUERTIN, J.-Y. POTVIN, AND É. D. TAILLARD, *Parallel tabu search for real-time vehicle routing and dispatching*, Transportation Science, 33 (1999), pp. 381–390.

[49] M. GENDREAU, G. LAPORTE, AND R. SÉGUIN, *Stochastic vehicle routing*, European Journal of Operational Research, 88 (1996), pp. 3–12.

[50] M. GENDREAU, G. LAPORTE, AND F. SEMET, *A dynamic model and parallel tabu search heuristic for real-time ambulance relocation*, Parallel Computing, 27 (2001), pp. 1641–1653.

[51] ———, *The maximal expected coverage relocation problem for emergency vehicles*, Journal of the Operational Research Society, 57 (2006), pp. 22–28.

[52] M. GENDREAU AND J.-Y. POTVIN, *Dynamic vehicle routing and dispatching*, in Fleet Management and Logistics, T. G. Crainic and G. Laporte, eds., Kluwer, Boston, 1998, pp. 115–126.

[53] G. GHIANI, F. GUERRIERO, G. LAPORTE, AND R. MUSMANNO, *Real-time vehicle routing: Solution concepts, algorithms and parallel computing strategies*, European Journal of Operational Research, 151 (2003), pp. 1–11.

[54] G. GHIANI, E. MANNI, A. QUARANTA, AND C. TRIKI, *Anticipatory algorithms for same-day courier dispatching*, Transportation Research Part E: Logistics and Transportation Review, 45 (2009), pp. 96–106.

[55] G. M. GIAGLIS, I. MINIS, A. TATARAKIS, AND V. ZEIMPEKIS, *Minimizing logistics risk through real time vehicle routing and mobile technologies*, International Journal of Physical Distribution and Logistics Management, 34 (2004), pp. 749–764.

[56] A. GOEL, *Fleet telematics: Real-time management and planning of commercial vehicle operations*, vol. 40 of Operations Research and Computer Science Interfaces Series, Springer-Verlag, New York, 2008.

[57] A. GOEL AND V. GRUHN, *A general vehicle routing problem*, European Journal of Operational Research, 191 (2008), pp. 650–660.

[58] J. GOODSON, J. OHLMANN, AND B. THOMAS, *Rollout policies for dynamic solutions to the multivehicle routing problem with stochastic demand and duration limits*, Operations Research, 61 (2013), pp. 138–154.

[59] C. E. GOUNARIS, W. WIESEMANN, AND C. A. FLOUDAS, *The robust capacitated vehicle routing problem under demand uncertainty*, Operations Research, 61 (2013), pp. 677–693.

[60] G. E. GOUNARIS, P. P. REPOUSSIS, C. D. TARANTILIS, W. WIESEMANN, AND C. A. FLOUDAS, *An adaptive memory programming framework for the robust capacitated vehicle routing problem*, Transportation Science, forthcoming.

[61] A. HAGHANI AND S. JUNG, *A dynamic vehicle routing problem with time-dependent travel times*, Computers & Operations Research, 32 (2005), pp. 2959–2986.

[62] A. HAGHANI, Q. TIAN, AND H. HU, *A simulation model for real-time emergency vehicle dispatching and routing*, in Proceedings of the Transportation Research Board Annual Meeting, 2003.

[63] A. HAGHANI AND S. YANG, *Real-time emergency fleet deployment*, in Dynamic Fleet Management: Concepts, Systems, Algorithms and Case Studies, V. Zeimpekis, C. D. Tarantilis, G. M. Giaglis, and I. Minis, eds., vol. 38 of Operations Research/Computer Science Interfaces Series, Springer, New York, 2007, pp. 133–162.

[64] M. A. HAUGHTON, *Quantifying the benefits of route reoptimisation under stochastic customer demands*, Journal of the Operational Research Society, 51 (2000), pp. 320–332.

[65] ——, *Measuring and managing the learning requirements of route reoptimization on delivery vehicle drivers*, Journal of Business Logistics, 23 (2002), pp. 45–66.

[66] ——, *Assigning delivery routes to drivers under variable customer demands*, Transportation Research Part E: Logistics and Transportation Review, 43 (2007), pp. 157–172.

[67] ——, *The efficacy of exclusive territory assignments to delivery vehicle drivers*, European Journal of Operational Research, 184 (2008), pp. 24–38.

[68] D. HAUPTMEIER, S. O. KRUMKE, AND J. RAMBAU, *The online dial-a-ride problem under reasonable load*, in Lecture Notes in Computer Science, vol. 1767, Springer-Verlag, Berlin, Heidelberg, 2000, pp. 125–136.

[69] L. HONG, *An improved LNS algorithm for real-time vehicle routing problem with time windows*, Computers & Operations Research, 39 (2012), pp. 151–163.

[70] C.-F. HSUEH, H.-K. CHEN, AND H.-W. CHOU, *Dynamic vehicle routing for relief logistics in natural disasters*, in Vehicle Routing Problem, T. Caric and H. Gold, eds., I-Tech, Vienna, Austria, 2008, pp. 71–84.

[71] D. HUISMAN, R. FRELING, AND A. P. M. WAGELMANS, *A robust solution approach to the dynamic vehicle scheduling problem*, Transportation Science, 38 (2004), pp. 447–458.

[72] L. M. HVATTUM, A. LØKKETANGEN, AND G. LAPORTE, *Solving a dynamic and stochastic vehicle routing problem with a sample scenario hedging heuristic*, Transportation Science, 40 (2006), pp. 421–438.

[73] ——, *A branch-and-regret heuristic for stochastic and dynamic vehicle routing problems*, Networks, 49 (2007), pp. 330–340.

[74] S. ICHOUA, M. GENDREAU, AND J.-Y. POTVIN, *Diversion issues in real-time vehicle dispatching*, Transportation Science, 34 (2000), pp. 426–438.

[75] ——, *Vehicle dispatching with time-dependent travel times*, European Journal of Operational Research, 114 (2003), pp. 379–396.

[76] ——, *Exploiting knowledge about future demands for real-time vehicle dispatching*, Transportation Science, 40 (2006), pp. 211–225.

[77] ——, *Planned route approaches for real-time vehicle routing*, in Dynamic Fleet Management: Concepts, Systems, Algorithms and Case Studies, V. Zeimpekis, C. D. Tarantilis, G. M. Giaglis, and I. Minis, eds., vol. 38 of Operations Research/Computer Science Interfaces Series, Springer, New York, 2007, pp. 1–18.

[78] P. JAILLET AND M. R. WAGNER, *Online routing problems: Value of advanced information as improved competitive ratios*, Transportation Science, 40 (2006), pp. 200–210.

[79] ——, *Online vehicle routing problems: A survey*, in The Vehicle Routing Problem: Latest Advances and New Challenges, B. L. Golden, S. Raghavan, and E. A. Wasil, eds., vol. 43 of Operations Research/Computer Science Interfaces Series, Springer, New York, 2008, pp. 221–237.

[80] J. JEMAI AND K. MELLOULI, *A neural-tabu search heuristic for the real time vehicle routing problem*, Journal of Mathematical Modelling and Algorithms, 7 (2008), pp. 161–176.

[81] S. JUNG AND A. HAGHANI, *A genetic algorithm for the time dependent vehicle routing problem*, Transportation Research Record: Journal of the Transportation Research Board, 1771 (2001), pp. 161–171.

[82] A. S. KENYON AND D. P. MORTON, *Stochastic vehicle routing with random travel times*, Transportation Science, 37 (2003), pp. 69–82.

[83] Y. KERGOSIEN, C. LENTE, L. PITON, AND J.-C. BILLAUT, *A tabu search heuristic for the dynamic transportation of patients between care units*, European Journal of Operational Research, 214 (2011), pp. 442–452.

[84] M. R. KHOUADJIA, B. SARASOLA, E. ALBA, L. JOURDAN, AND E. TALBI, *A comparative study between dynamic adapted PSO and VNS for the vehicle routing problem with dynamic requests*, Applied Soft Computing, 12 (2012), pp. 1426–1439.

[85] P. KILBY, P. PROSSER, AND P. SHAW, *Dynamic VRPS: A study of scenarios*, APES-06-1998, University of Strathclyde, Glasgow, Scotland, 1998.

[86] S. KIM, M. LEWIS, AND C. C. WHITE III, *Optimal vehicle routing with real-time traffic information*, IEEE Transactions on Intelligent Transportation Systems, 6 (2005), pp. 178–188.

[87] A. LARSEN, O. B. G. MADSEN, AND M. M. SOLOMON, *Partially dynamic vehicle routing—models and algorithms*, Journal of the Operational Research Society, 53 (2002), pp. 637–646.

[88] ——, *The a-priori dynamic travelling salesman problem with time windows*, Transportation Science, 38 (2004), pp. 459–472.

[89] ——, *Classification of dynamic vehicle routing systems*, in Dynamic Fleet Management: Concepts, Systems, Algorithms and Case Studies, V. Zeimpekis, C. D. Tarantilis, G. M. Giaglis, and I. Minis, eds., vol. 38 of Operations Research/Computer Science Interfaces Series, Springer, New York, 2007, pp. 19–40.

[90] ——, *Recent developments in dynamic vehicle routing systems*, in The Vehicle Routing Problem: Latest Advances and New Challenges, B. L. Golden, S. Raghavan, and E. A. Wasil, eds., vol. 43 of Operations Research/Computer Science Interfaces Series, Springer, New York, 2008, pp. 199–218.

[91] C. Lecluyse, T. Van Wossel, and H. Peremans, *Vehicle routing with stochastic time-dependent travel times*, 4OR, 7 (2009), pp. 363–377.

[92] J.-Q. Li, P. B. Mirchandani, and D. Borenstein, *A decision support system for the single-depot vehicle rescheduling problem*, Computers & Operations Research, 34 (2007), pp. 1008–1032.

[93] ——, *Vehicle rescheduling problem: Model and algorithms*, Networks, 50 (2007), pp. 211–229.

[94] ——, *Real-time vehicle rerouting problems with time windows*, European Journal of Operational Research, 194 (2009), pp. 711–727.

[95] M. Lipmann, X. Lu, W. E. Paepe, R. A. Sitters, and L. Stougie, *Online dial-a-ride problems under restricted information model*, Algorithmica, 40 (2004), pp. 319–329.

[96] S. Lorini, J.-Y. Potvin, and N. Zufferey, *Online vehicle routing and scheduling with dynamic travel times*, Computers & Operations Research, 38 (2011), pp. 1086–1090.

[97] K. Lund, O. B. G. Madsen, and J. M. Rygaard, *Vehicle routing problems with varying degrees of dynamism*, Technical Report IMM, Institute of Mathematical Modelling, Technical University of Denmark, Lyngby, Denmark, 1996.

[98] O. B. G. Madsen, H. F. Ravn, and J. M. Rygaard, *A heuristic algorithm for a dial-a-ride problem with time windows, multiple capacities and multiple objectives*, Annals of Operations Research, 60 (1995), pp. 193–208.

[99] O. B. G. Madsen, K. Tosti, and J. Voelds, *A heuristic method for dispatching repair men*, Annals of Operations Research, 61 (1995), pp. 213–226.

[100] M. Mes, M. Van Der Heijden, and P. Schuur, *Look-ahead strategies for dynamic pickup and delivery problems*, OR Spectrum, 32 (2010), pp. 395–421.

[101] M. Mes, M. Van der Heijden, and A. Van Harten, *Comparison of agent-based scheduling to look-ahead heuristics for real-time transportation problems*, European Journal of Operational Research, 181 (2007), pp. 59–75.

[102] I. Minis, K. Mamasis, and V. Zeimpekis, *Real-time management of vehicle breakdowns in urban freight distribution*, Journal of Heuristics, 18 (2012), pp. 375–400.

[103] S. Mitrović-Minić, R. Krishnamurti, and G. Laporte, *Double-horizon based heuristics for the dynamic pickup and delivery problem with time windows*, Transportation Research Part B: Methodological, 38 (2004), pp. 669–685.

[104] S. MITROVIĆ-MINIĆ AND G. LAPORTE, *Waiting strategies for the dynamic pickup and delivery problem with time windows*, Transportation Research Part B: Methodological, 38 (2004), pp. 635–655.

[105] R. MONTEMANNI, L. M. GAMBARDELLA, A. E. RIZZOLI, AND A. V. DONATI, *A new algorithm for a dynamic vehicle routing problem based on ant colony system*, Journal of Combinatorial Optimization, 10 (2005), pp. 327–343.

[106] Q. MU AND R. EGLESE, *Disrupted capacitated vehicle routing problem with order release delay*, Annals of Operations Research, 207 (2013), pp. 201–216.

[107] Q. MU, Z. FU, J. LYSGAARD, AND R. EGLESE, *Disruption management of the vehicle routing problem with vehicle breakdown*, Journal of the Operational Research Society, 62 (2010), pp. 742–749.

[108] C. NOVOA AND R. STORER, *An approximate dynamic programming approach for the vehicle routing problem with stochastic demands*, European Journal of Operational Research, 196 (2009), pp. 509–515.

[109] G. PANKRATZ AND V. KRYPCZYK, *Benchmark data sets for dynamic vehicle routing problems*, 2009. http://www.fernuni-hagen.de/WINF/inhfrm/benchmark_data.htm.

[110] M. PAVONE, N. BISNIK, E. FRAZZOLI, AND V. ISLER, *A stochastic and dynamic vehicle routing problem with time windows and customer impatience*, Mobile Network Applications, 14 (2009), pp. 350–364.

[111] V. PILLAC, M. GENDREAU, C. GUÉRET, AND A. L. MEDAGLIA, *A review of dynamic vehicle routing problems*, European Journal of Operational Research, 225 (2013), pp. 1–11.

[112] V. PILLAC, C. GUÉRET, AND A. L. MEDAGLIA, *An event-driven optimization framework for dynamic vehicle routing*, Technical Report 11/2/AUTO, École des Mines de Nantes, France, 2011.

[113] ——, *On the dynamic technician routing and scheduling problem*, Technical Report 12/5/AUTO, École des Mines de Nantes, France, 2012.

[114] ——, *A fast re-optimization approach for dynamic vehicle routing*, Technical Report 12/6/AUTO, École des Mines de Nantes, France, 2012.

[115] J.-Y. POTVIN, Y. XU, AND I. BENYAHIA, *Vehicle routing and scheduling with dynamic travel times*, Computers & Operations Research, 33 (2006), pp. 1129–1137.

[116] W. B. POWELL, *A stochastic formulation of the dynamic assignment problem, with an application to truckload motor carriers*, Transportation Science, 30 (1996), pp. 195–219.

[117] ——, *What you should know about approximate dynamic programming*, Naval Research Logistics, 56 (2009), pp. 239–249.

[118] W. B. POWELL, P. JAILLET, AND A. ODONI, *Stochastic and dynamic networks and routing*, in Network Routing, M. O. Ball, T. L. Magnanti, C. L. Monma, and G. L. Nemhauser, eds., vol. 8 of Handbooks Operations Research and Management Science, Elsevier Science, Amsterdam, 1995, pp. 141–295.

[119] W. B. POWELL, Y. SHEFFI, K. S. NICKERSON, K. BUTTERBAUGH, AND S. ATHERTON, *Maximizing profits for North American Van Lines' truckload division: A new framework for pricing and operations*, Interfaces, 18 (1988), pp. 21–41.

[120] W. B. POWELL, W. SNOW, AND R. K. CHEUNG, *Adaptive labeling algorithms for the dynamic assignment problem*, Transportation Science, 34 (2000), pp. 50–66.

[121] W. B. POWELL AND H. TOPALOGLU, *Stochastic programming in transportation and logistics*, in Stochastic Programming, Handbooks Operations Research and Management Science, Elsevier Science, Amsterdam, 2003, pp. 555–635.

[122] H. N. PSARAFTIS, *A dynamic programming solution to the single vehicle many-to-many immediate request dial-a-ride problem*, Transportation Science, 14 (1980), pp. 130–154.

[123] ——, *Dynamic vehicle routing problems*, in Vehicle Routing: Methods and Studies, B. L. Golden and A. A. Assad, eds., North–Holland, Amsterdam, 1988, pp. 223–248.

[124] ——, *Dynamic vehicle routing: Status and prospects*, Annals of Operations Research, 61 (1995), pp. 143–164.

[125] V. PUREZA AND G. LAPORTE, *Waiting and buffering strategies for the dynamic pickup and delivery problem with time windows*, INFOR, 46 (2008), pp. 165–175.

[126] A. C. REGAN, H. S. MAHMASSANI, AND P. JAILLET, *Improving efficiency of commercial vehicle operations using real time information: Potential uses and assignment strategies*, Transportation Research Record: Journal of the Transportation Research Board, 1493 (1995), pp. 188–197.

[127] ——, *Evaluation of dynamic fleet management systems*, Transportation Research Record: Journal of the Transportation Research Board, 1645 (1998), pp. 176–184.

[128] U. RITZINGER, J. PUCHINGER, AND R. HARTL, *A survey on dynamic and stochastic vehicle routing problems*, Technical Report, 2012. Available at: http://www.jakobpuchinger.com/publications.

[129] A. RIZZOLI, R. MONTEMANNI, E. LUCIBELLO, AND L. GAMBARDELLA, *Ant colony optimization for real-world vehicle routing problems*, Swarm Intelligence, 1 (2007), pp. 135–151.

[130] S. ROPKE, J.-F. CORDEAU, AND G. LAPORTE, *Models and branch-and-cut algorithms for pickup and delivery with time windows*, Networks, 49 (2007), pp. 258–272.

[131] D. SAEZ, C. E. CORTÉS, AND A. NUNEZ, *Hybrid adaptive predictive control for the multi-vehicle dynamic pick-up and delivery problem based on genetic algorithms and fuzzy clustering*, Computers & Operations Research, 35 (2008), pp. 3412–3438.

[132] M. W. P. SAVELSBERGH AND M. SOL, *DRIVE: Dynamic routing of independent vehicles*, Operations Research, 46 (1998), pp. 474–490.

[133] N. SECOMANDI, *Comparing neuro-dynamic programming algorithms for the vehicle routing problem with stochastic demands*, Computers & Operations Research, 27 (2000), pp. 1201–1225.

[134] N. SECOMANDI AND F. MARGOT, *Reoptimization approaches for the vehicle routing problem with stochastic demands*, Operations Research, 57 (2009), pp. 214–230.

[135] D. SLEATOR AND R. E. TARJAN, *Amortized efficiency of list update and paging rules*, Communications of the ACM, 28 (1985), pp. 202–208.

[136] M. M. SOLOMON, *Algorithms for the vehicle routing and scheduling problems with time window constraints*, Operations Research, 35 (1987), pp. 254–265.

[137] M. Z. SPIVEY AND W. B. POWELL, *The dynamic assignment problem*, Transportation Science, 38 (2004), pp. 399–419.

[138] I. SUNGUR, F. ORDONEZ, AND M. M. DESSOUKY, *A robust optimization approach for the capacitated vehicle routing problem with demand uncertainty*, IIE Transactions, 40 (2008), pp. 509–523.

[139] M. R. SWIHART AND J. D. PAPASTAVROU, *A stochastic and dynamic model for the single-vehicle pick-up and delivery problem*, European Journal of Operational Research, 114 (1999), pp. 447–464.

[140] E. TANIGUCHI AND H. SHIMAMOTO, *Intelligent transportation system based dynamic vehicle routing and scheduling with variable travel times*, Transportation Research Part C: Emerging Technologies, 12 (2004), pp. 235–250.

[141] B. W. THOMAS, *Waiting strategies for anticipating service requests from known customer locations*, Transportation Science, 41 (2007), pp. 319–331.

[142] B. W. THOMAS AND C. C. WHITE III, *Anticipatory route selection*, Transportation Science, 38 (2004), pp. 473–487.

[143] H. TOPALOGLU, *A parallelizable and approximate dynamic programming-based dynamic fleet management model with random travel times and multiple vehicle types*, in Dynamic Fleet Management: Concepts, Systems, Algorithms and Case Studies, V. Zeimpekis, C. Tarantilis, G. Giaglis, and I. Minis, eds., vol. 38 of Operations Research/Computer Science Interfaces Series, Springer, New York, 2007, pp. 65–93.

[144] H. TOPALOGLU AND W. P. POWELL, *Dynamic-programming approximations for stochastic time-staged integer multicommodity-flow problems*, INFORMS Journal on Computing, 18 (2006), pp. 31–42.

[145] P. TOTH AND D. VIGO, *The Vehicle Routing Problem*, SIAM, Philadelphia, 2002.

[146] J. I. VAN HEMERT AND J. A. LA POUTRÉ, *Dynamic routing problems with fruitful regions: Models and evolutionary computation*, in Lecture Notes in Computer Science, vol. 3242, X. Yao, E. Burke, and J. A. Lozano, eds., Springer-Verlag, Berlin, Heidelberg, 2004, pp. 692–701.

[147] P. VAN HENTENRYCK AND R. BENT, *Online Stochastic Combinatorial Optimization*, MIT Press, Cambridge, MA, 2006.

[148] J. WANG AND R. ZHU, *Efficient intelligent optimized algorithm for dynamic vehicle routing problem*, Journal of Software, 6 (2011), pp. 2201–2208.

[149] X. WANG AND H. CAO, *A dynamic vehicle routing problem with backhaul and time windows*, in Proceedings of the IEEE International Conference on Service Operations and Logistics and Informatics IEEE/SOLI 2008, 2008, pp. 1256–1261.

[150] X. WANG, J. NIU, X. HU, AND C. XU, *Rescue strategies of the VRPTW disruption*, Systems Engineering-Theory & Practice, 27 (2007), pp. 104–111.

[151] X. WANG, J. RUAN, H. SHANG, AND C. MA, *A combinational disruption recovery model for vehicle routing problem with time windows*, in Intelligent Decision Technologies, SIST 10, J. Watada, G. Phillips-Wren, L. C. Jain, and R. J. Howlett, eds., Springer-Verlag, Berlin, Heidelberg, 2011, pp. 3–12.

[152] X. WANG, X. WU, AND X. HU, *A study of urgency vehicle routing disruption management problem*, Journal of Networks, 5 (2010), pp. 1426–1433.

[153] X. WANG, C. XU, AND D. YANG, *Disruption management for vehicle routing problem with the request changes of customers*, International Journal of Innovative Computing, Information and Control, 5 (2009), pp. 2427–2438.

[154] X. WANG, K. ZHANG, AND D. YANG, *Disruption management of urgency vehicle routing problem with fuzzy time window*, ICIC Express Letters, 3 (2009), pp. 883–890.

[155] M. WEN, J.-Y. CORDEAU, G. LAPORTE, AND J. LARSEN, *The dynamic multi-period vehicle routing problem*, Computers & Operations Research, 37 (2010), pp. 1615–1623.

[156] S. WOHLGEMUTH, R. OLORUNTOBA, AND U. CLAUSEN, *Dynamic vehicle routing with anticipation in disaster relief*, Socio-Economic Planning Services, 46 (2012), pp. 261–271.

[157] Z. XIANG, C. CHU, AND H. CHEN, *The study of a dynamic dial-a-ride problem under time-dependent and stochastic environments*, European Journal of Operational Research, 185 (2008), pp. 534–51.

[158] J. YANG, P. JAILLET, AND H. S. MAHMASSANI, *Real-time multivehicle truckload pickup and delivery problems*, Transportation Science, 38 (2004), pp. 135–148.

[159] S. YANG, M. HAMEDI, AND A. HAGHANI, *Online dispatching and routing model for emergency vehicles with area coverage constraints.*, Transportation Research Record: Journal of the Transportation Research Board, 1923 (2006), pp. 1–9.

[160] V. ZEIMPEKIS, C. D. TARANTILIS, G. M. GIAGLIS, AND I. MINIS, *Dynamic fleet management*, vol. 38 of Operations Research and Computer Science Interfaces Series, Springer-Verlag, New York, 2007.

[161] X. ZHANG AND L. TANG, *Disruption management for the vehicle routing problem with time windows*, in Advanced Intelligent Computing Theories and Applications: With Aspects of Contemporary Intelligent Computing Techniques, vol. 2 of Communications in Computer and Information Science, D. Huang, L. Heutte, and M. Loog, eds., Springer-Verlag, Berlin, Heidelberg, 2007, pp. 225–234.