Chapter 7

# Pickup-and-Delivery Problems for People Transportation

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## 7.1 ▪ Introduction

Modern societies demand transportation systems that offer high efficiency and comfort but also minimal noise, pollution, cost, and delays. These requirements apply to the transportation of both goods and people. The previous chapter was devoted to vehicle routing problems related to the transportation of goods; this chapter focuses on vehicle routing problems associated with the transportation of people. The wide variety of different applications in this context creates a range of different optimization problems. This chapter starts dealing with one of these problems, the so-called *dial-a-ride problem*, as presented in Section 7.2. This service, provided by public authorities, primarily serves elderly and handicapped people who cannot use regular transit systems. In this context, cost minimization is important, but the quality of the service is the main aim. A mathematical formulation is described in Section 7.3, and prior research approaches in the literature are summarized in Section 7.4. Section 7.5 details other articles dealing with related vehicle routing problems with regard to transporting people.

## 7.2 ▪ Dial-a-Ride Problems

*Dial-a-Ride problem* is a name used to define a transportation system providing multi-occupancy, door-to-door transport service for people. In a sense this transportation mode covers the gap between a rigid bus system and a flexible taxicab system. Ideally a vehicle moves a large number of passengers with personalized service. Although in some applications a customer may be transported by two vehicles, one after the other joined by a transfer point, in most cases only one vehicle is assigned to serve each customer's request. An example applies to elderly or handicapped people. Including customer satisfaction in the planning for this service is the major difference between transporting people (this chapter) and transporting goods (the previous chapter).

*Dial-a-Ride Problem* (DARP) implies that a set of customers makes a request for service. Each one wants to be collected from a given location (origin), wants to be delivered in a different location (destination), and requires some time constraints. Typical time constraints are the so-called *time-window constraints*, which consist of imposing a desired pickup time within an interval and a desired delivery time within an interval for each customer. Extreme values defining these intervals may create infeasible instances, especially in the special cases of time-of-delivery and time-of-pickup constraints, for which a customer specifies a desired maximum delivery time and a desired minimum pickup time. These special cases are also associated with the aim of minimizing the difference between actual and desired times.

The optimization problem consists of assigning a vehicle (or sequence of vehicles) to each customer's request and designing the routes for the vehicles to perform all services within the time constraints, with minimum cost. This cost is a combination of the total travel distance and the inconvenience to customers. Several routes must be determined, and each route must satisfy the following constraints:

(i) Visiting: each customer's request must be served once.

(ii) Depot: each vehicle returns to the depot from which it departed.

(iii) Pairing (coupling): the pickup and the delivery services of a customer's request are served by the same vehicle.

(iv) Precedence: each customer's request must be picked up before being delivered.

(v) Vehicle capacity: the number of customers in a vehicle cannot exceed a given value.

(vi) Time windows: each customer must be picked up and delivered within given time intervals.

(vii) Maximum waiting time: the vehicle serving a customer cannot be waiting in an intermediate stop more than a given value.

(viii) Maximum ride time: the time a customer spends in a vehicle cannot exceed a given value.

This DARP is a multi-vehicle routing problem and is also categorized as a *Vehicle Routing Problem with Pickup and Delivery and Time Windows*. However, the DARP focuses specifically on the operational constraints associated with transporting people, such that high-quality service is mandatory. For example, vehicles should not idle when carrying passengers, which motivates the maximum waiting time constraints (vii). Indeed, the violation of constraints (vi)–(viii) creates the so-called *customer inconveniences*, and the main aim of DARP is to minimize them.

Depending on when the customer's request arrives in the system, the DARP may be static or dynamic. The *static* (also called *advance request*) case occurs when all the customer's requests are precisely known before planning the routes. It does not consider any probabilistic information describing the spatial or time distribution of other further requests. The *dynamic* case arises in two situations: (1) customer's requests are included in the problem when they occur, so a reoptimization procedure is needed, or (2) probabilistic information about customer's requests is considered. The first situation suggests online programming, whereas the second situation is typical in stochastic programming. Within the online situation appears the *immediate request* case, which occurs when customers desire to be collected as soon as possible.

The simplest DARP variant appears in the static situation, with one vehicle and with only precedence constraints (iv). The DARP is then a generalization of the *Precedence-Constraints Traveling Salesman Problem*, also known as *sequential ordering problem*, where each customer's origin must precede that customer's destination on the route. When there is also a vehicle capacity, i.e., constraint (v), then the DARP is called *Capacitated Traveling Salesman Problem with Pickups and Deliveries*. The literature on this problem concerns good transportation and allows a customer request to be temporarily stored in intermediate locations. If the vehicle capacity equals 1 (i.e., the vehicle can only move one customer at a time), then the problem is known as a *stacker-crane problem*. This problem finds most of the application in good transportation, and several studies exist also on the *preemptive* version (where the transported load can be temporally stored in an intermediate location). By adding the time-window constraints (vi), the problem becomes a *vehicle routing problem with pickup and delivery and time windows*. As mentioned before, in all these particular problems, the objective function must include customer inconvenience to properly remain in the DARP area. Therefore, if the time constraints are not included as *hard constraints*, some of them should be considered at least as *soft constraints* through coefficients in the objective function. The issue of whether customer service guarantees (or the corresponding time constraints) should be hard or soft is a debatable one; each approach has advantages and disadvantages. A general DARP may contain all the time constraints in a hard or a soft way. Clearly, other extensions are possible by also considering transit points, heterogeneous fleets of vehicles, different types of customers, probabilistic information, multiple objective functions, and so forth.

## 7.3 ▪ Problem Formulation

The DARP is a combination of a vehicle routing problem and a scheduling problem. The first consists of designing the routes for the vehicles, assigning each customer's request to a vehicle, and sequencing the locations to be visited. The second consists of deciding the time at which each vehicle starts the service at each location. Although both problems must be integrated in a single one when searching for an optimal DARP solution, some heuristic approaches support the scheduling subproblem for given vehicle routes. In these approaches, the scheduling subproblem aims to minimize the time duration of each route.

We provide mathematical formulations for each of these problems. First of all we need some notation. We adopt notation previously used in, e.g., Cordeau [12]. The number of customers is represented by $n$. For each customer $i \in \{1,\dots,n\}$, the pickup location (origin) is represented by $i$, and the delivery location (destination) is indicated by $i+n$. The set of all origins is denoted by $P$ and the set of all destinations by $D$. The depot is represented by two locations: $0$ represents the location from where each route starts, and $2n+1$ represents the location where each route ends. The set $P \cup D \cup \{0, 2n+1\}$ is the vertex set $V$ of a directed graph $G=(V,A)$. The arc set $A$ contains all possible pairs of different locations $(i,j)$ except

- $i=0$ and $j=2n+1$;
- $i=0$ and $j \in D$;
- $i \in P$ and $j=2n+1$.

Then, given a set of vertices $S \subset V$, the set $A(S)$ denotes all the arcs with tail and head in $S$, and the set $\delta^+(S)$ denotes all the arcs with tail in $S$ and head in $V \setminus S$. When $S \subseteq P \cup D$, let $S'$ be $(P \cup D) \setminus S$. The time window at a location $i \in P \cup D$ is denoted by $[a_i, b_i]$.

The maximum ride time acceptable by customer $i$ is $R_i$, and the maximum waiting time is denoted by $W_i$. These values are assumed to be given. We also assume that all customer requests consume the same amount of capacity in a vehicle. The vehicle capacity is known and represented by $Q$.

### 7.3.1 ▪ The Whole Problem

Because the DARP is a generalization of the classical VRP, a natural attempt to attain a whole formulation is to adapt a VRP formulation, which is not always easy. Under some conditions it can be done. To exemplify this procedure, consider a static DARP with one depot, a homogeneous fleet of vehicles, and the (hard) constraints (i)–(viii). When there are no soft constraints, the DARP objective functions may coincide with the VRP, which already includes the visiting constraint (i), the depot constraint (ii), and the vehicle capacity constraint (v). Therefore, to ensure the DARP feasibility of a VRP solution we need to eliminate infeasible routes, i.e., routes violating the pairing constraint (iii), the precedence constraint (iv), or the time constraints (vi)–(viii). To this end, suppose one has a model for the VRP on the binary variable $x_a$ ($a \in A$) assuming value 1 if a vehicle travels through arc $a$, and 0 otherwise. The literature has several models based on these variables. One of them is, e.g., the so-called *two-index vehicle flow model* introduced by Laporte and Nobert [41]. As is standard, when $a$ is the arc with tail $i$ and head $j$, we write $x_{i,j}$ instead of $x_a$. Then a valid DARP model can be obtained by extending the VRP model with the inequalities

$$\sum_{a\in R} x_a \leq |R| - 1 \tag{7.1}$$

for each set of arcs $R$ defining a (piece of the) route from 0 to $2n+1$ violating the pairing, precedence, or time constraints of some customer's requests. These *infeasible path inequalities* have been used extensively in prior literature to impose different side constraints on routing problems (see, e.g., Ascheuer, Fischetti, and Grötschel [2, 3]). Although they are linear inequalities on the $x_a$ variables, they are not nice in a mathematical formulation, because

- they do not help close the gap between the integer formulation and its linear programming relaxation;
- it may be easy to find a violated one by an integer VRP solution $x^*$ but very complex when $x^*$ contains fractional numbers.

The first drawback is due to the low number of variables in the left-hand side of the inequality compared to the right-hand value, thus leading to a "weak inequality". The second drawback refers to the "separation problem", which requires an efficient algorithm to deal with the large number of inequalities from equations (7.1). However, in some cases, these drawbacks can be partially eliminated.

One example occurs with the pairing constraint (iii), where an invalid route can be eliminated by the inequalities

$$\sum_{a\in A(\{0,2n+1\}\cup S)} x_a \leq |S| \tag{7.2}$$

for each set of vertices $S \subset P \cup D$ containing the origin $i$ of a request but not the destination $i+n$ of this request. Inequalities from equations (7.2) are stronger than inequalities from equations (7.1), yet at the same time they have a polynomially solvable separation problem even when $x^*$ contains fractional numbers. To show this result let us consider a subset $S \subset P \cup D$. Using the equations

$$\sum_{a\in\delta^+(j)} x_a = 1 \qquad \forall j \in S,$$

the inequalities (7.2) are linearly equivalent to

$$\sum_{a\in\delta^+(S)} x_a \geq \sum_{j\in S} x_{0,j} + \sum_{j\in S} x_{j,2n+1},$$

which is also equivalent to

$$\sum_{i\in S, j\in S'} x_{i,j} + \sum_{j\in S'} x_{0,j} \geq \sum_{j\in P\cup D} x_{0,j}.$$

Given a customer $i$, checking whether a given solution $x^*$ satisfies this inequality for each $S \subset P \cup D$ with $i \in S$ and $i+n \in S'$ can be done by solving a min-cut problem. The capacitated directed network $G^*$ for this min-cut problem can be built from $G$, removing vertex $2n+1$ and its incident arcs. The capacity of the arc $(0,i)$ is $\sum_{j\in P\cup D} x^*_{0,j}$ units, to force that $0$ and $i$ are on the same side of an optimal min-cut solution leading to a violated inequality. The capacity of another arc $a$ in the network $G^*$ is $x^*_a$. The min-cut problem to be solved has $0$ as the source and $n+i$ as the sink. When the capacity of an optimal min-cut solution is smaller than $\sum_{j\in P\cup D} x^*_{0,j}$, a violated inequality from equations (7.2) has been identified. In conclusion, the separation of these inequalities can be done in polynomial time.

Another example occurs with the precedence constraint (iv). To force that the pickup location $i$ of a customer's request is visited before the delivery location $i+n$, we can impose the inequalities

$$\sum_{a\in A(\{0\}\cup S)} x_a \leq |S| - 1 \tag{7.3}$$

for each set of vertices $S \subset P \cup V$ containing $i+n$ and not containing $i$. A procedure to find inequalities (7.3) violated by a (fractional) solution $x^*$, if any exists, is similar to the one described for the previous family. To see it, observe that inequalities (7.3) can be rewritten as

$$\sum_{a\in\delta^+(S)} x_a \geq 1 + \sum_{j\in S} x_{0,j},$$

which is also equivalent to

$$\sum_{i\in S, j\in S'} x_{i,j} + \sum_{j\in S'} x_{0,j} + \sum_{j\in S} x_{j,2n+1} \geq 1 + \sum_{j\in P\cup D} x_{0,j}.$$

Again, one min-cut problem must be solved for each customer $i$ on a network $G^*$. Now the vertex $2n+1$ and its incident arcs in $G$ should also be considered in $G^*$. The capacity of the arcs $(0,i)$ and $(n+i, 2n+i)$ is $1+\sum_{j\in P\cup D} x^*_{0,j}$ units to force that, in an optimal min-cut solution, the vertices $0$ and $i$ are on one side and the vertices $2n+1$ and $n+i$ are on the other side. The capacity of another arc $a \in A$ is $x^*_a$. The min-cut problem to be solved has 0 as the source and $2n+1$ as the sink. When the capacity of an optimal min-cut solution is smaller than $1+\sum_{j\in P\cup D} x^*_{0,j}$, then a violated inequality from equations (7.3) has been identified. Again, the separation problem of these inequalities is polynomially solvable.

Strengthening and separating inequalities (7.1) associated with other constraints may be very complicated. In these situations, it is of interest to have an efficient algorithm to detect an inequality from equations (7.1), violated by a given integer VRP solution, or to prove that one does not exist. This implies checking the DARP validity of a VRP solution and providing a dynamic procedure to approach the DARP. The procedure can be seen as a kind of Benders' decomposition technique, where the MP designs an integer VRP solution and the slave problem (also called subproblem) checks the DARP validity of the (integer) VRP solution. The major difference of this procedure with respect to the classical Benders' decomposition technique is that, depending on the DARP constraints, the slave problem may not be modelled as a linear program. Inequalities (7.1) cannot be determined by dual variables but rather are of a combinatorial type. For that reason, they are named *combinatorial Benders' cuts*, according to Codato and Fischetti [10]. The efficient algorithm mentioned before solves the slave problem, if it is available. The next section addresses this problem, called *the scheduling problem* in the DARP literature.

### 7.3.2 ▪ The Scheduling Problem

Let us consider a given route starting at 0, ending at $2n+1$, and visiting $q$ locations from $P\cup D$ in between. It is easy to check whether these locations correspond to the pickups and the deliveries of a subset of customer's requests, with each pickup location visited before its associated delivery location, which is feasible to be performed by a capacitated vehicle. This feasibility can be checked in $O(q)$. Assume that the result of this check on the given route is positive.

The scheduling problem consists of determining the time to start the route from the depot and the time to start serving each customer's request at the pickup location. The time-window constraint (vi), the maximum waiting time constraint (vii), and the maximum ride time constraint (viii) are part of the scheduling problem. The aim is to minimize the route duration. It is a timing problem (see, e.g., Vidal et al. [78]). A mathematical formulation for this optimization problem is as follows.

To simplify the notation, let $i+1$ represent the location in the route after $i$ and $i-1$ the location in the route before $i$. The route can then be seen as the sequence $0,1,\dots,h,2n+1$. Let $T_i$ be the time at which service begins at location $i$ for each of the $h$ locations in the route. Let $T_0$ be the time when the vehicle leaves location 0 and $T_{h+1}$ the time when the vehicle enters location $2n+1$:

$$
\begin{array}{lll}
 & \text{minimize } T_{h+1} & \\
\text{s.t.} & T_i + t_{i,i+1} \le T_{i+1} & \forall\, i\in P\cup D,\\
 & a_i \le T_i \le b_i & \forall\, i\in P\cup D,\\
 & T_i - a_i \le W_i & \forall\, i\in P,\\
 & (T_{(i+n)-1} + t_{(i+n)-1,i+n}) - T_i \le R_i & \forall\, i \text{ to } i+n,
\end{array}
$$

where, as defined above, $W_i$ is the maximum waiting time acceptable by customer $i$, and $R_i$ is the maximum ride time.

Firat and Woeginger [28] describe an algorithm with run time $O(q)$ to check the feasibility of this problem and to find a solution when it is feasible. The algorithm is based in expressing the feasibility problem as a shortest path problem in a vertex-weighted interval graph. Starting from the seminar articles by Savelsbergh [67, 68], other articles on this timing problem are Hunsaker and Savelsbergh [36], Haugland and Ho [32], and Tang et al. [75].

## 7.4 ▪ Solution Methods for Dial-a-Ride Problems

The literature on the DARP is vast, including several surveys. A recent survey was published by Cordeau and Laporte [14]. Another survey with two parts on pickup and delivery problems was published by Parragh, Doerner, and Hartl [53, 54]. This section offers a different classification in heuristic, metaheuristic, and exact methods of the major contributions, with an update of the existing literature on the DARP in chronological order.

### 7.4.1 ▪ Exact Methods

1980: Psaraftis [59] developed a dynamic programming algorithm to solve the single-vehicle DARP problem without (hard) time constraints. The algorithm was adapted to consider time-window constraints in Psaraftis [60]. Both these articles address the static and online dynamic variants. The objective functions include the customer's dissatisfaction while waiting for service. They are weighted combinations of the time needed to service all customers and the sum of a customer's waiting and riding times. The algorithms run in $O(n^2 3^n)$ time and were able to deal with instances with no more than 10 customer requests.

1986: Desrosiers, Dumas, and Soumis [18] describe a dynamic programming algorithm for the static single-vehicle DARP with time-window, vehicle capacity, and precedence constraints. Note that for a single vehicle, the pairing constraints are automatically satisfied, and the efficiency of a dynamic programming algorithm relies on the use of criteria for the elimination of states which do not satisfy the time-window, vehicle capacity, and precedence constraints. They solved instances with up to 40 customer requests.

1991: An exact approach for the multi-vehicle variant of this problem is described in Dumas, Desrosiers, and Soumis [24]. This approach solves a set-partitioning problem where each row is a customer request and each column is a route for a vehicle. The routes are generated by a subproblem that finds the shortest path on a network with side constraints (pairing, precedence, capacity, and time-window) by a dynamic programming procedure. The algorithm was capable of solving instances to optimality with up to 55 customer requests.

2006: Cordeau [12] presents a mixed integer linear programming formulation for the static multi-vehicle DARP with time constraints. A Branch-and-Cut implementation based on this model solved instances with up to 36 customer requests.

2007: Ropke, Cordeau, and Laporte [66] propose a pure integer linear programming formulation for the same problem. A Branch-and-Cut implementation based on this model solved instances with up to 96 customer requests.

2009: When there are no time windows, the DARP is called a multi-commodity one-to-one pickup-and-delivery routing problem. An exact method based on Benders' decomposition for a single vehicle is described in Hernández-Pérez and Salazar-González [35]. This problem can be solved as a one-commodity pickup-and-delivery routing problem with precedence constraints. A Branch-and-Cut algorithm for this one-commodity problem is described in Hernández-Pérez and Salazar-González [34].

2011: A DARP with a different objective was studied by Garaix et al. [30]. The objective is to maximize the passenger occupancy rate. The problem is motivated from a real-world situation in France where the objective of encouraging people meetings is pursued. They use a column generation approach to solving the problem.

Heilporn, Cordeau, and Laporte [33] developed an integer L-shaped method for the DARP with stochastic customer delays. Here the problem characteristic is that customers may have stochastic delays at their pickup location. When the customer is absent the request is fulfilled by an alternative service. This leads to the effect that the corresponding delivery location has not been serviced. The aim of the problem is to determine an a priori tour minimizing the expected cost of the solution.

### 7.4.2 ▪ Heuristic Methods

In the past several decades many heuristic and metaheuristic algorithms have been proposed for the static DARP and also for its dynamic and stochastic variants. This section presents an overview of heuristic solution methods, reported in chronological order.

1978: Early heuristic algorithms for the DARP are discussed in Daganzo [17], focusing mainly on a dynamic version of the DARP. This early account analyzed three insertion algorithms: visiting the closest stop next, visiting the closest origin or the closest destination in alternating order, and allowing the insertion of delivery locations only after a fixed number of passengers have been picked up. The second algorithm results in higher waiting to ride time ratios than the first algorithm. However, it also tends to generate lower waiting times for equal ride times when compared with the third algorithm.

1981: One of the first heuristic solution procedures for static multi-vehicle DARP is discussed in Cullen, Jarvis, and Ratliff [16]. They develop an interactive algorithm that follows the "cluster-first route-second" approach. It is based on a set partitioning formulation, solved by means of column generation. The subproblem is an location-allocation problem where clusters must be selected (location) and the customers' requests must be assigned to selected clusters (allocation). This subproblem can only be solved approximately. However, user-related constraints or objectives are not explicitly considered.

1985: Sexton and Bodin [71, 72] developed a heuristic routing and scheduling algorithm for the single-vehicle DARP using Benders' decomposition. The scheduling problem can be solved optimally; the routing problem is solved with a heuristic algorithm.

1986: Jaw et al. [38] propose a sequential insertion procedure. First, customers were ordered by their increasing earliest time for pickup. Second, they could be inserted according to the cheapest feasible insertion criterion. They use the notion of active vehicle periods. In contrast to Sexton and Bodin [71, 72], they consider already multiple vehicles.

1988: Desrosiers, Dumas, and Soumis [19] and Dumas, Desrosiers, and Soumis [23] use the dynamic programming algorithm described in Section 7.4.1 in a column generation approach for heuristically solving the multi-vehicle DARP with time-window constraints. The basic concept is a "mini-cluster", which is a set of geographically and temporally cohesive customer requests that can be served by the same vehicle. A heuristic algorithm groups together nearby customers into a mini-cluster (route segment) that can be served by a single vehicle. A vehicle is empty when it enters and when it leaves a mini-cluster, but it is never empty in between. Then a column generation algorithm constructs vehicle routes by stringing together these mini-clusters. The solved instances with up to 200 customer requests are grouped into 85 mini-clusters.

1995: Ioachim et al. [37] developed an optimization-based mini-clustering algorithm. Here a column generation approach is used to obtain mini-clusters and an enhanced initialization procedure to decrease processing times.

By its very nature, the problem has multiple objectives. In addition to the traditional cost objectives, it is necessary to address also client-centered objectives (such as the minimization of ride times). A heuristic for the multi-objective variant of the DARP is presented by Madsen, Ravn, and Rygaard [42]. They discuss an insertion-based algorithm called REBUS. The objectives considered are the total driving time, the number of vehicles, the total waiting time, the deviation from promised service times, and the cost.

Dial [20] studied the dynamic case of the multi-vehicle DARP. In this paper new transportation requests appear dynamically. Thus, new transportation requests are assigned to clusters according to lowest cost insertion. The routes could be optimized then by using dynamic programming. These authors report the results for a real-life problem instance.

Larger instances are heuristically solved in Ioachim et al. [37]. This approach was based on the "cluster-first route-second" strategy, but where part of the routing problem is used to solve the clustering problem. Mini-clusters are generated by applying dynamic programming to solve shortest path problems with pickups and deliveries and then selected by solving a set-partitioning problem. The heuristic approach was run on instances with up to 250 requests and on real data with 2545 requests.

2004: Diana and Dessouky [22] developed a regret insertion algorithm for the static DARP. In this case, all requests are ranked first according to ascending pickup times. Some swaps in this order are allowed, giving preference to requests that might be difficult to insert later on, because of their spatial location. The first $m$ requests are used as seed customers, with $m$ being the number of vehicles. All the remaining requests are inserted following a regret insertion strategy (see Potvin and Rousseau [58]). The regret insertion–based process is also subject to analysis by Diana [21] in an effort to determine why the performance of this heuristic is superior to that of other insertion rules.

### 7.4.3 ▪ Metaheuristic Methods

This section presents an overview of metaheuristic solution methods, reported in chronological order.

1996: A first metaheuristic was published by Toth and Vigo [77]. They developed a local search–based metaheuristic, namely a tabu thresholding algorithm, for the static multi-vehicle DARP. Initial solutions were constructed using parallel insertion and that employed the neighborhoods described in Toth and Vigo [76].

2003: Another tabu search algorithm was published in Cordeau and Laporte [13]. The neighborhood used is defined by moving one request to another route. The best possible move serves to generate a new incumbent solution. Reverse moves are declared tabu. However, an aspiration criterion is defined, such that tabu moves that provide a better solution, with respect to all other solutions already constructed by the same move, can constitute a new incumbent solution. This algorithm developed for the static version has been used to solve the dynamic case. The tabu search also has been adapted to the dynamic DARP through parallelization by Attanasio et al. [4].

Starting from now the problems became richer, often focused on the transportation of patients. Melachrinoudis, Ilhan, and Min [46] developed a double request DARP model with soft time windows and its application in health care. The objective of the proposed model was to minimize a weighted sum of total vehicle transportation costs and total clients' inconvenience time. The latter consists of excess riding time, early/late delivery time before service, and late pickup time after service. As a solution technique, this approach used a tabu search technique.

2009: Two years later, another approach is developed and applied to a real-world patient transportation problem. Beaudry et al. [6] considered a patient transportation problem arising in large hospitals. Therefore this study extended the classical DARP by several complicating constraints that were specific to a hospital context and by dynamic requests. The study provided a detailed description of the problem and proposed a two-phase heuristic procedure capable of handling the different features. In the first phase, a simple insertion scheme generated a feasible solution, improved in the second phase with a tabu search algorithm. The same algorithm is used for the decision support system developed by Hanne, Melo, and Nickel [31] as well.

2010: Parragh, Doerner, and Hartl [55] developed solution techniques also for the patient transportation problem in Austria. They started to develop a variable neighborhood search metaheuristic for the classical DARP. The neighborhood concepts are based on swap and ejection chains. As in the real world, besides the classical cost-objective, also the patient related objective is important; this solution concept is adapted to solve the bi-objective case (besides the cost objective, the maximum ride time of the patients will be minimized). For this variant a two-phase method is presented in Parragh et al. [52]: in the first phase a variable neighborhood search is used, and in the second phase path relinking is applied to generate Pareto efficient solutions.

2011: As the real-world patient transportation problem is dynamic and stochastic by nature, Schilde, Doerner, and Hartl [69] developed solution techniques for the dynamic stochastic version of the DARP. The aim is to design vehicle routes to serve partially dynamic transportation requests using a fixed vehicle fleet. In patient transportation each request requires transportation from a patient's home to a hospital (outbound request) or back home from the hospital (inbound request). Some of these requests are known in advance. Some requests are dynamic in the sense that they appear during the day without any prior information. Finally, some inbound requests are stochastic. With a certain probability, each outbound request causes a corresponding inbound request on the same day. Some stochastic information about these return transports is available from historical data. This study investigated whether using this information to design the routes had a significant positive effect on the solution quality. The problem can be modeled as a dynamic stochastic DARP with expected return transports. As solution techniques, a stochastic variable neighborhood search and different variants of the multiple plan and multiple scenario approach appear viable.

A hybrid tabu search with an exact constraint programming developed by Berbeglia, Cordeau, and Laporte [7] is presented to solve the dynamic DARP. An important component of the tabu search consists of three scheduling procedures which are executed sequentially. The constraint programming algorithm is used to accept or reject incoming requests.

2012: In a recent work by Masson, Lehuédé, and Péton [45] the model of the pickup-and-delivery problem is extended by transfer points. The corresponding problem is called the pickup-and-delivery problem with transfers. The solution technique relies on adaptive large neighborhood search. The authors evaluate the method on generated instances and apply it to the transportation of people with disabilities. On these real-life instances they show that the introduction of transfer points can bring significant improvements (up to 9%) to the value of the objective function.

Beyond the multiple objectives and dynamic or stochastic aspects, also driver-related constraints are important for the patient transportation problem, which requires extending the classical DARP by driver-related constraints and different modes of transportation. In Parragh et al. [51] patients may request to be transported either seated, lying in a bed, or in a wheelchair. The driver-related constraints are expressed in terms of maximum route duration limits and mandatory lunch breaks. In this paper a three-index and a set-partitioning formulation of the problem is introduced. The linear programming relaxation of the latter is solved by a column generation algorithm. A variable neighborhood search heuristic and a hybrid method combining column generation are proposed to generate upper bounds. The different transportation modes are introduced by Parragh [50], where also a Branch-and-Cut algorithm is described.

A related problem to the DARP was considered by Kergosien et al. [39]. The problem stems from a real application to the transportation of patients in a large hospital complex. Patients have to be transported between care units and treatment rooms. In large hospital compounds vehicles are required. The problem is dynamic by nature, as not all the transportation requests are known a priori. They consider a heterogeneous fleet, and a vehicle can transport only one patient at a time. Also disinfection operations after the transport of specific patients have to be considered. The problem is solved by using tabu search, and the method is applied to real-world data of a French hospital.

As mentioned before, the DARP without time windows is called a multi-commodity one-to-one pickup-and-delivery routing problem. A metaheuristic algorithm for the single vehicle variant of this problem is described in Rodríguez-Martín and Salazar-González [65].

2013: Parragh and Schmid [56] developed further an innovative hybrid method based on column generation and large neighborhood search for the classical DARP.

In the paper Schmid and Doerner [70] the patient transportation problem is further extended by the scheduling of the patients in the treatment rooms. Hospitalized patients typically undergo several examinations before their actual surgery. Transportation between service units is provided by trained personnel who escort patients. However, valuable resources may be under-utilized if patients arrive too late for scheduled appointments, whereas on the other side, many patients have to wait for a long time before being picked up for, or after, their actual appointment. To improve those deficits, this study adopts on both resource- and client-centered perspectives. In this paper the authors present an integrative combinatorial optimization model combining both scheduling- and routing-related aspects. The problem is solved by using a cooperative hybrid metaheuristic. Traditionally, both underlying subproblems—the scheduling of patients and the transportation of patients—would be solved independently, but the cooperative approach yields substantial advantages over decoupled hierarchical optimization processes.

## 7.5 ▪ Other Problems Concerning Pickup and Delivery of People

Many situations in practice have motivated the study of several pickup-and-delivery problems related with transporting people. The variety of applications is so extended that it makes it too complex to summarize all of them in a single chapter. To illustrate the current status of the research in the area we have selected two specific topics. One is related to logistic problems arising in school bus routing. The other is related to car pooling, where employees are organized to share some vehicles.

### 7.5.1 ▪ School Bus Routing

The school bus routing problem is related to the DARP, such that each user (student) must be transported from a given pickup location (home) to a given delivery location (school). For one of the two locations, a time window may be specified. In addition, maximum user ride time limits are usually considered. In contrast with standard DARP situations, the passengers' destinations often coincide (i.e., many children attend the same school), and a hard time-window constraint is always associated with the delivery location (the school). Similar to the classical DARP though, maximum ride time limits can implicitly be considered by artificially constructing a time window at each bus stop.

The *School Bus Routing Problem* (SBRP) consists of routing a given number of buses, such that all children in a certain (rural) area are picked up from the bus stop closest to their homes and delivered to their respective schools. This problem has multiple objectives: on the one hand, the bus company aims to minimize its operating costs, while on the other hand, the children transported must arrive at their schools in time (but not too early with respect to the beginning of the first class). Furthermore, they should spend as little time on the bus as possible. Obviously, these three objectives are conflicting in many cases, and it is difficult to assign a certain weight to any of them a priori. Cost minimization is linked to the total time the drivers spend driving. Combining several transportation requests usually decreases the total route length, but it increases the individual ride times of the children. Arriving exactly at the time school starts instead entails longer ride times for children picked up early and may also increase total route length as well.

This class of problem consists of different subproblems involving bus stop selection, bus route generation, school bell time adjustment, and bus scheduling. A survey by Park and Kim [48] has summarized the various assumptions, constraints, and solution methods used in prior literature on SBRP. The first ideas and concepts of school bus routing solution techniques were published by Bodin and Berman [8] and Swersey and Ballard [74].

Spada, Bierlaire, and Liebling [73] propose a modeling framework where the focus is on optimizing the level of service for a given number of buses, using an automatic procedure to generate a solution to the problem. The procedure first builds a feasible solution, which then can be improved using a heuristic. A simulated annealing technique explores the infeasible solutions.

Fügenschuh [29] presents an integer programming model pursuing the integrated coordination of the school starting times and the public bus services. This approach considered preprocessing techniques, model reformulations, and cutting planes that could be incorporated into a Branch-and-Cut algorithm. Computational results show that fewer buses would be sufficient if the schools started at different times.

Perugia et al. [57] present a model and an algorithm for the design of a home-to-work bus service in a metropolitan area. This type of service must ensure an equilibrium between conflicting criteria such as efficiency, effectiveness, and equity. The authors introduce a multi-objective model in which, among other aspects, equity depends on time windows pertaining to the arrival time of a bus at a stop. Time windows can have other uses too, such as guaranteeing the synchronization of the service with other transportation modes. This "cluster-first route-second" approach models both bus stop location and routing in urban road networks where turn restrictions exist. The resulting multi-objective location-routing model is solved by a tabu search algorithm. A different multi-objective formulation is in Corberán et al. [11].

Martinez and Viegas [44] present an integrated procedure based on traditional formulations of the SBRP. The problem is decomposed: a first step identifies the most suitable concentration points of students, and a second step computes the optimal routes serving those stops. The solution concept is applied to data from Lisbon.

Kim, Kim, and Park [40] introduce a school bus scheduling problem wherein trips for each school are given. Each school has its fixed time window within which trips should be completed. A school bus can serve multiple trips for multiple schools. The school bus scheduling problem seeks to optimize bus schedules to complete all the trips, considering the school time windows. The problem can be modeled as a vehicle routing problem with time windows by treating each trip as a virtual stop. Two exact approaches refer to special cases, and a heuristic algorithm applies to a more general case. This problem also can be extended to mixed loads; see Park, Tae, and Kim [49]. When mixed loads are allowed, students from different schools can get on the same bus at the same time. That study thereby presented an improvement algorithm.

The school bus problem addressing also the issue of the bus stop selection for each child is studied in Riera-Ledesma and Salazar-González [63, 64]. These authors describe a Branch-and-Cut algorithm and a column generation procedure to solve instances with up to 100 children and up to 100 potential stops to optimality.

Fügenschuh [29] examines a pickup-and-delivery problem where the students can be transported by at most two buses. To this end, some locations are considered as transfer points where a passenger can go from one vehicle to another. In this article, the vehicle routes are fixed and the aim is to find the minimum number of buses to cover all the services. Designing the vehicle routes has been addressed by other authors, and the optimization problem is called *Pickup-and-Delivery Problem with Transfers* (PDPT). A mathematical formulation and a Branch-and-Cut algorithm for a general variant have been addressed by Cortés, Matamala, and Contardo in [15], solving instances with up to 6 requests and 2 vehicles to optimality. Mitrović-Minić and Laporte [47] describe a local search approach for an uncapacitated PDPT, solving instances with up to 100 requests. More sophisticated approaches were proposed by Masson, Lehuédé, and Péton [45] and by Qu and Bard [62]. When solving a PDPT, the numbers of requests and vehicles are important, but there are other more crucial features. For example, the problem is simpler when the set of transfer points is much smaller than the set of pickup-and-delivery locations. In some applications the pickup locations may all be different (they are the students' homes), but the delivery locations are very few points (they are the schools). In some cases only the delivery points may offer the service to allow transfers. In other applications there is only one specific location where passengers can be transferred from one vehicle to another. The transfer aspect imposes a synchronization between vehicle routes that is better afforded under the presence of tight time-window constraints.

In the very recent paper of Boegl, Doerner, and Parragh [9] the school bus routing and scheduling problem with transfers is studied. It deals with the planning of the transportation of students from home to their school before it starts under the consideration that they can change buses. Allowing transfers has multiple different consequences. On the one hand, costs can be reduced significantly. On the other hand, transfers clearly have an impact on the service level, i.e., transfers lower the service level, but they may also reduce riding times. The authors develop a heuristic solution framework to solve this problem and compare the results with two standard solution techniques for the DARP and for the open VRP. The main objective is the minimization of costs. The implications on the service level of the pupils, like time loss and number of transfers, are analyzed.

## 7.5.2 ▪ Car Pooling

A problem class related to the DARP is the car pooling problem, which consists of finding subsets of employees who can share a car, determining the path the driver should follow, and identifying who should be the driver. In contrast with the DARP, either the origin or the destination are the same for all users, depending on whether the trip is from home to the office or back. Two variants can be investigated, either one car pool for both ways or differing to-work and from-work problems.

The client-centered goals (maximum driving/travel time, maximum amount of detours) and the economical goals both have to be considered. In prior literature, several variants of the problem have been distinguished, such as the daily car pooling problem, for which drivers/cars are predefined and the problem consists of assigning passengers to cars; the long-term car-pooling problem, in which each user can act as a driver or a passenger (and drivers/passengers have to be determined); or the problem of defining client pools that share a vehicle (the definition of the driver is left to the passengers) (see Baldacci, Maniezzo, and Mingozzi [5], Wolfler Calvo et al. [79], and Yan and Chen [80]).

Car pooling normally leads to fewer total vehicle kilometers traveled (despite short detours) and is therefore an interesting concept to make transportation activities more environmentally friendly. To increase the number of shared car trips, lanes on many freeway systems in the United States are reserved for vehicles with a defined minimum amount of passengers. Further, car pooling can help guarantee mobility in rural areas, considering that the rise of individual mobility is connected to a weakening demand for public means of transport, which leads to declining offers of public transport activities, especially in rural regions. Despite all these advantages, car-pooling concepts are rarely applied in practice, due to the difficulties of the planning stage (see, e.g., Wolfler Calvo et al. [79]). Literature has mainly focused on car-pooling concepts where either the origin or the destination are the same for all users (many-to-one or one-to-many problems) and only private cars are involved in the planning and solution approaches.

Baldacci, Maniezzo, and Mingozzi [5] propose an exact and a heuristic procedure for this problem. A real-life application is reported in Wolfler Calvo et al. [79]. Maniezzo, Carbonaro, and Hildmann [43] describe an ant colony optimization algorithm which also might resolve the long-term problem.

The problem has only been extended by Yan and Chen [80] to a many-origins-to-many-destinations car pooling problem with multiple vehicles and person types, for which vehicles and persons can be divided into different groups according to gender and smoking status. The idea was to consider the individual characteristics and preferences of passengers and drivers when solving the problem (e.g., non-smokers want to share a car with other non-smokers). The problem can be formulated as an integer multiple commodity network flow problem. An algorithm based on Lagrangian relaxation, a subgradient method, and a heuristic for the upper bound have been developed to solve this problem.

A related problem with the car-pooling problem is the dynamic ride-sharing problem. This problem was recently introduced by Agatz et al. [1]. The difference from the car-pooling problem is the planning horizon. In this problem transportation requests of people with similar itineraries and time schedules to share rides on short notice are considered. This is a new problem which emerged through the increased use of smartphones. Now it is possible to share rides on short notice. Different strategies are tested by simulations.

### 7.5.3 ▪ Demand Responsive Transportation and Others

Closely related problems to the DARP are the demand responsive transportation problems. In Errico et al. [25] the different approaches of solving demand responsive transportation problems are surveyed. When demand for people transportation is low, e.g., in rural areas, traditional transit cannot provide an efficient and good-quality level of service, due to the fixed structure. Therefore, public transportation is evolving towards some degree of flexibility. The extension of DARP systems to general public transportation fulfills the requirement of adaptability. New transportation alternatives combining characteristics from both the traditional transit flexible DARP systems start to be introduced. These types of problems are usually called semi-flexible. These new problem characteristics require complex planning activities and a formalization of the decisions. The paper of Errico et al. provides a systematic treatment of the field of semi-flexible systems.

Another related topic is the transportation of people with aircrafts, jets, or helicopters. As an example the following applications will be reported. In Qian et al. [61] employees are transported to and from the offshore installations in the offshore petroleum industry by helicopter. The paper analyzes how to improve transportation safety by solving the helicopter routing problem with a risk objective expressed in terms of expected number of fatalities. A mathematical model is proposed, and a tabu search heuristic is applied to this problem: different objectives, travel time, a passenger risk, and a combined passenger and pilot risk objective. One result show that passenger transportation risk can be reduced by increasing travel time at the expense of pilot risk.

Another application of air transportation is the on-demand air transportation services in which travelers call a few days in advance to schedule a flight. A successful on-demand air transportation service requires an effective scheduling system to construct minimum-cost pilot and jet itineraries for a set of accepted transportation requests. In the paper by Espinoza et al. [26] an integer multicommodity network flow model with side constraints for such dial-a-flight problems is presented. In Espinoza et al. [27] the authors describe how this core optimization technology is embedded in a parallel, large neighborhood, local search scheme to produce high-quality solutions efficiently for large-scale real-life instances.

## 7.6 ▪ Conclusions and Future Research Directions

This chapter has introduced the basic constraints of a vehicle routing problem dealing with moving people, in which minimizing the customer's dissatisfactions is fundamental. The most studied variant is the Dial-a-Ride Problem (DARP), leading to a wide literature including exact and heuristic approaches. Methodologically speaking, the approaches used to solve problems moving people are not really different from those used to solve problems moving goods. Indeed, most of the formulations are commodity-flow formulations or set-packing models, motivating the implementation of Branch-and-Cut and column generation algorithms as exact methods. However, when dealing with people, there is a deeper multi-objective nature in the problem, as the customer's dissatisfaction does not always fit properly into a numerical expression. Also, these problems are of a high stochastic nature as well: the customer's demand may be known while the vehicles are moving. In some applications there are special features (like, for example, the possibility of transit points) that make the optimization problem more complex to be solved. For that reason, and also for the size of the instances in real cases, there has been an explosion of exact and heuristic approaches trying to generate good solutions. The developed hybrid algorithm of Parragh and Schmid [56] currently provides the best results on the standard DARP instances. In addition, the today telecommunication technology provides lot of online information to both customers and managers, and thus the society is demanding solution approaches for larger and more complex vehicle routing problems. In the next years intermodal aspects will require new solution concepts. Especially the combination of public transportation, demand responsive transportation, and car or bike sharing is of interest for public transportation planners in rural areas.

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