Chapter 13

Ship Routing and Scheduling in Industrial and Tramp Shipping

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13.1 ▪ Introduction

International trade heavily depends on maritime transportation with more than eight billion tons of goods carried at sea annually (UNCTAD [73]). It is common to distinguish between the following three modes of operation in maritime transportation (Lawrence [52]): liner, industrial, and tramp shipping. Liner vessels follow a fixed route according to a published schedule, trying to maximize profit, similar to a public bus service. An industrial operator owns the cargoes and controls the ships, trying to minimize the cost of transporting its own cargoes, similar to a private fleet. In a tramp operation the vessels follow the available cargoes, with a mix of mandatory contract cargoes and optional spot ones. Since some cargoes are optional, the tramp operator must in addition to the routing and scheduling also determine which spot cargoes to accept/reject, trying to maximize profit. Industrial and tramp shipping usually have several similarities when it comes to operational characteristics.

In this chapter we present models and discuss solution methods for some important routing and scheduling problems in industrial and tramp shipping, which include most of what is referred to as bulk shipping, i.e., transportation of wet (oil, chemicals, and oil products) and dry bulk products (iron ore, grain, coal, bauxite/alumina, and phosphate). These product types constitute more than 60% of the weight transported at sea (UNCTAD [73]). The world fleet consists of more than 7.000 tankers with more than 10.000 deadweight tons for carrying wet bulk products and a similar number of dry bulk carriers.

Given the large amounts of goods transported and the high revenues and costs related to maritime transportation it is easy to imagine that the impact of *Operations Research* (OR) models and methods in planning of ship routes and schedules can be huge, especially given the large concentration of players in this billion dollar industry. As an example, a *Very Large Crude Carrier* (VLCC) with a cargo carrying capacity of about 200.000 tons has a building cost of approximately 100 million *United States Dollars* (USD), a daily time-charter rate of 70.000 USD, a daily fuel cost of 50.000 USD, while the value of one full shipload is 80–100 million USD. (All values depend heavily, of course, on the market and are only intended to give an impression of the order of magnitude of the values related to a typical segment in bulk shipping). Therefore, efficient operation of this fleet is in the best interest of the world economy and vessel operators, and may reduce bunker fuel consumption and environmental impact. In the short to intermediate run, routing and scheduling of this fleet determines the efficiency of its operation. Despite this, there are still relatively few applications of OR in maritime transportation compared to land-based transportation (Christiansen, Fagerholt, and Ronen [24] and Christiansen et al. [25]).

In this chapter, we use the definition from Al-Khayyal and Hwang [3] and distinguish between cargo routing and inventory routing. We use *cargo routing* to denote the planning problem of routing a fleet of ships to service a number of specified cargoes that are given as input to the planning process, in contrast to *inventory routing*, where the cargoes are determined through the planning process itself. In Section 13.2 we present mathematical models for some important cargo routing and scheduling problems in tramp shipping. The industrial shipping counterparts can easily be derived from the corresponding tramp shipping versions of the problems and will be explained. Section 13.3 deals with maritime inventory routing. Illustrative examples are provided in both Sections 13.2 and 13.3. In Section 13.4 we discuss dynamic and stochastic routing in industrial and tramp shipping before we summarize in Section 13.5.

13.2 ▪ Cargo Routing and Scheduling

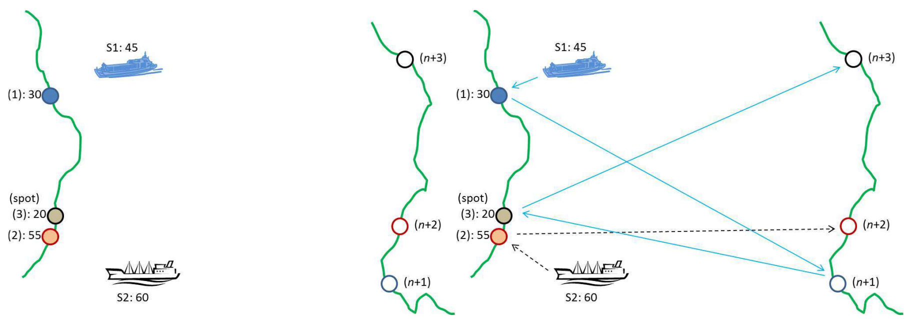
In this section we present mathematical models and discuss solution methods for some important cargo routing and scheduling problems in tramp shipping. We start in Section 13.2.1 by presenting a basic version of the problem, which can be modeled as a maritime pickup-and-delivery problem with time windows. Then in Section 13.2.2 we proceed by presenting and discussing some important extensions of the basic version of the problem, such as flexible cargo sizes, split loads, and variable sailing speeds, amongst others.

13.2.1 ▪ Maritime Pickup-and-Delivery Problem with Time Windows

Cargo routing and scheduling is an important problem arising in industrial and tramp shipping. A cargo consists of a specified amount of product(s) to be picked up at a specified port, transported, and unloaded at a specified delivery port. There is usually a time window during which the loading (pickup) of the cargo must start, and there may also be an unloading (delivery) time window. The operator controls a heterogeneous fleet of ships that are available to transport the cargoes. For various reasons some cargoes may not be compatible with certain ships (e.g., due to loading and/or unloading ports' draft limitations). Generally, the ship capacities and the cargo quantities are such that ships can carry multiple cargoes simultaneously. Whereas for major bulk commodities a cargo is usually a full shipload, for minor bulk commodities and chemicals where the shipments are smaller, the ship capacity may accommodate several cargoes simultaneously. The model that follows reflects this more general case.

A ship operator in industrial shipping must transport all cargoes while minimizing costs, whereas a tramp operator focuses on profit maximization. The tramp operator usually has a set of mandatory contracted cargoes and will try to increase its revenue by transporting optional spot cargoes. The mandatory cargoes come from long-term agreements between the shipping company and the cargo owners. The challenge for the tramp shipping company is to select spot cargoes and construct routes and schedules that maximize profit. Here, the profit is defined as the revenue from all transported cargoes minus the variable sailing costs, which mainly consist of fuel and port/canal costs, and sometimes also costs for spot charters (i.e., chartering in ships from the market to service given cargoes). We focus here on the tramp routing and scheduling problem, as this is the more general case and includes most of the characteristics of the corresponding industrial shipping problem. The problem that we discuss here in detail has similarities with the *multi-vehicle pickup and delivery problem with time windows* described by Desrosiers et al. [28] (see also Chapter 6).

Figure 13.1(a) illustrates a small example with two ships with capacities of 45 and 60 units, respectively. We assume that the ships are empty in the beginning of the planning period, and have an initial position somewhere at sea, as shown in the figure. There are three cargoes with quantities of 30, 55, and 20 units, respectively, where, for example, (1):30 in the figure denotes the pickup port node of cargo 1 of 30 units, while $(n+1)$ denotes the corresponding delivery port node. Cargo 3 is an optional spot cargo. The example reflects a situation from deep sea shipping where the loading and unloading of each cargo take place in different continents (e.g., North America and Europe with the Atlantic Ocean in between). To make the example simple, we do not include time windows.



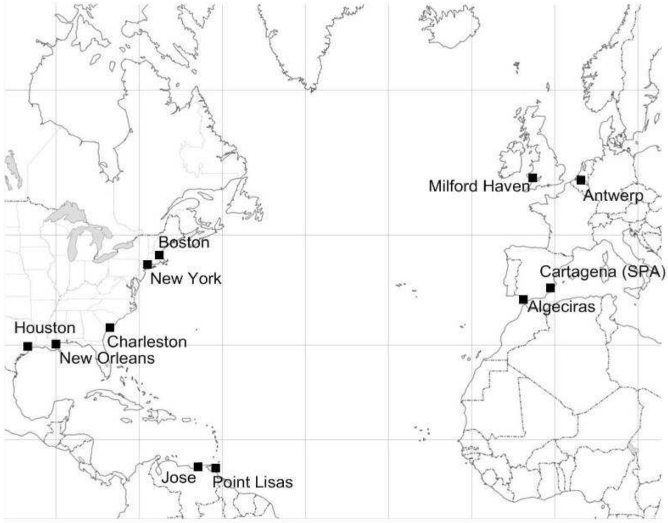
(a) *Example problem* (b) *Solution*

Figure 13.1. *Example of multi-vehicle pickup-and-delivery problem and its solution.*

Figure 13.1(b) shows a possible solution to the problem, where ship 1 first transports cargo 1 and then sails back (e.g., over the Atlantic Ocean) to service spot cargo 3, while ship 2 carries cargo 2. If we had included time windows, it is easy to imagine that ship 1 would have arrived at the pickup port of the optional spot cargo too late, so that one would have had to reject that cargo, thus missing the revenue from transporting it.

An excerpt from a real ship routing and scheduling working sheet used by a shipping company in the planning of ship routes and schedules is provided in Figure 13.2 (ship names and other information have been modified to preserve confidentiality). The pickup ports are divided into four different regions: United Kingdom and Continental Europe (UK/CONT), Mediterranean Sea (MEDIT), Caribbean Sea (CARIB), and United States and Gulf of Mexico (US/GULF). The cargoes are given in the upper row of the table, based on the locations of their pickup ports. Each cargo is specified by its pickup and delivery ports, its weight, its product type (i.e., MTBE or methanol), and the time window during which pickup can start. In the lower row, the available ships based in each region are specified with a time and a port. When planning ship routes and schedules the shipping company uses such a sheet as an aid to assign cargoes to the available ships.

In the following we present a mathematical model for the maritime pickup-and-delivery problem with time windows in tramp shipping. Let each cargo be represented by an



UK/CONT	MEDIT	CARIB	US/GULF
Milford Haven – New York 40.000 ton MTBE 3/11-07 – 9/11-07 Antwerp – Boston 37.000 ton MTBE 5/11-07 – 10/11-07	Cartagena (SPA) – Houston 28.000 ton Methanol 15/11-07 – 19/11-07 Algeciras – Charleston 16.000 ton Methanol 15/11-07 – 20/11-07	Point Lisas – Houston 17.000 ton Methanol 4/11-07 – 10/11-07 Jose – New Orleans 30.000 ton Methanol 4/11-07 – 10/11-07 Jose – New York 18.000 ton Methanol 10/11-07 – 15/11-07	Houston – New York 18.000 ton MTBE 6/11-07 – 11/11-07
Cecilia Antwerp 2/11-07 Catharina Antwerp 7/11-07		Suzanna Jose 15/10-07	Alberta Boston 4/11-07 Maria Charleston 23/10-07

Figure 13.2. *Excerpt from a real ship routing and scheduling working sheet.*

index i. Associated with cargo i is a loading port node i and a unloading port node $n+i$, where n is the number of cargoes that might be transported during the planning horizon. Note that different nodes may correspond to the same physical port. Let $N^P = 1,2,\ldots,n$ be the set of pickup nodes and $N^D = n+1, n+2, \ldots, 2n$ be the set of delivery nodes. The set of pickup nodes is partitioned into two subsets, N^C and N^O, where N^C is the set of pickup nodes for the mandatory contracted cargoes and N^O is the set of pickup nodes for the optional spot cargoes.

Let K be the set of ships. A network (N_k, A_k) is associated with each ship k. Here, N_k is the set of nodes that can be visited by ship k, including the origin and an artificial destination for ship k, denoted by $o(k)$ and $d(k)$, respectively. Geographically, the origin can be either a port or a point at sea, while the artificial destination is the last planned unloading port for ship k. If the ship is not used, $d(k)$ will represent the same location as $o(k)$. From this, we can extract the sets $N_k^P = N^P \cap N_k$ and $N_k^D = N^D \cap N_k$ consisting of the pickup and delivery nodes that ship k may visit, respectively. The set A_k contains all feasible arcs for ship k, which is a subset of $N_k \times N_k$.

For each ship $k \in K$ and each arc $(i,j) \in A_k$, let T_{ijk}^S be the sailing time from node i to node j, while T_{ik}^P represents the service time in port at node i with ship k. The variable transportation costs C_{ijk} consist of the sum of the sailing costs from node i to node j and the port costs of node i for ship k. In this model, we assume that a (contract) cargo i can be serviced by a ship chartered from the spot market at a given cost, C_i^S. Further, let $[\underline{T}_{ik}, \overline{T}_{ik}]$ denote the time window for ship k associated with node i, where $\underline{T}_{ik}$ is the earliest time for start of service and $\overline{T}_{ik}$ is the latest. Each cargo i has a quantity Q_i and generates a revenue R_i per unit if it is transported. Let Q_k^C be the capacity of ship k.

The binary variable x_{ijk} is assigned the value 1 if ship k sails directly from node i to node j, and 0 otherwise. The variable t_{ik} represents the time for start of service for ship k at node i. The variable l_{ik} is the load (weight) on board ship k when leaving node i. To ease the reading of the model, we assume that each ship is empty when leaving the origin and when arriving at the artificial destination, i.e., $l_{o(k)k} = l_{d(k)k} = 0$. Let z_i be a binary variable that is equal to 1 if cargo i is serviced by a ship from the spot market, and 0 otherwise. Finally, the binary variable y_i is equal to 1 if the optional spot cargo i is transported, and 0 otherwise.

The basic tramp ship routing and scheduling problem can now be formulated as follows:

$$\text{maximize} \sum_{i \in N^C} R_i Q_i + \sum_{i \in N^O} R_i Q_i y_i - \sum_{k \in K} \sum_{(i,j) \in A_k} C_{ijk} x_{ijk} - \sum_{i \in N^C} C_i^S z_i \tag{13.1}$$

$$\text{s.t.} \quad \sum_{k \in K} \sum_{j \in N_k} x_{ijk} + z_i = 1 \qquad \forall\, i \in N^C, \tag{13.2}$$

$$\sum_{k \in K} \sum_{j \in N_k} x_{ijk} - y_i = 0 \qquad \forall\, i \in N^O, \tag{13.3}$$

$$\sum_{j \in N_k} x_{o(k)jk} = 1 \qquad \forall\, k \in K, \tag{13.4}$$

$$\sum_{j \in N_k} x_{ijk} - \sum_{j \in N_k} x_{jik} = 0 \qquad \forall\, k \in K,\ i \in N_k \setminus \{o(k), d(k)\}, \tag{13.5}$$

$$\sum_{i \in N_k} x_{id(k)k} = 1 \qquad \forall\, k \in K, \tag{13.6}$$

$$l_{ik} + Q_j - l_{jk} - Q_k^C(1 - x_{ijk}) \le 0 \qquad \forall\, k \in K,\ (i,j) \in A_k \mid j \in N_k^P, \tag{13.7}$$

$$l_{ik} - Q_j - l_{n+j,k} - Q_k^C(1 - x_{i,n+j,k}) \le 0 \qquad \forall\, k \in K,\ (i,n+j) \in A_k \mid j \in N_k^P, \tag{13.8}$$

$$\sum_{j \in N_k} Q_i x_{ijk} \le l_{ik} \le \sum_{j \in N_k} Q_k^C x_{ijk} \qquad \forall\, k \in K,\ i \in N_k^P, \tag{13.9}$$

$$0 \le l_{n+i,k} \le \sum_{j \in N_k} (Q_k^C - Q_i) x_{n+i,jk} \qquad \forall\, k \in K,\ i \in N_k^P, \tag{13.10}$$

$$t_{ik} + T_{ik}^P + T_{ijk}^S - t_{jk} - M_{ijk}(1 - x_{ijk}) \le 0 \qquad \forall\, k \in K,\ (i,j) \in A_k, \tag{13.11}$$

$$\sum_{j \in N_k} x_{ijk} - \sum_{j \in N_k} x_{n+i,jk} = 0 \qquad \forall\, k \in K,\ i \in N_k^P, \tag{13.12}$$

$$t_{ik} + T_{ik}^P + T_{i,n+i,k}^S - t_{n+i,k} \leq 0 \qquad \forall\, k \in K,\ i \in N_k^P, \tag{13.13}$$

$$\underline{T}_{ik} \leq t_{ik} \leq \overline{T}_{ik} \qquad \forall\, k \in K,\ i \in N_k, \tag{13.14}$$

$$l_{ik} \geq 0 \qquad \forall\, k \in K,\ i \in N_k, \tag{13.15}$$

$$x_{ijk} \in \{0,1\} \qquad \forall\, k \in K,\ (i,j) \in A_k, \tag{13.16}$$

$$y_i \in \{0,1\} \qquad \forall\, i \in N^O, \tag{13.17}$$

$$z_i \in \{0,1\} \qquad \forall\, i \in N^C. \tag{13.18}$$

The objective function (13.1) maximizes the profit from operating the fleet. The four terms are the revenue gained by transporting the mandatory contracted cargoes, the revenue from transporting the optional spot cargoes, the variable sailing costs, and the cost of using spot charters. The fixed revenue for the contracted cargoes can be omitted, but is included here to obtain a more complete picture of the profit. Constraints (13.2) state that all mandatory contract cargoes are transported, either by a ship in the fleet or by a spot charter. The corresponding requirements for the optional spot cargoes are given by constraints (13.3). Constraints (13.4)–(13.6) describe the flow along the sailing route used by ship k. Constraints (13.7) and (13.8) keep track of the load on board at the pickup and delivery nodes, respectively. Constraints (13.9) and (13.10) represent the ship capacity constraints at the loading and discharging nodes, respectively. Constraints (13.11) ensure that the time of starting service at node j must be greater than or equal to the departure time from the previous node i, plus the sailing time between the nodes. The big M coefficient in constraints (13.11) can be calculated as $M_{ijv} = \max(0, \overline{T}_{iv} + T_{iv}^P + T_{ijv}^S - \underline{T}_{jv})$. Constraints (13.12) ensure that the same ship k visits both loading node i and the corresponding discharging node $n+i$. Constraints (13.13) force node i to be visited before node $n+i$, while constraints (13.14) define the time window within which service must start. If ship k is not visiting node i, we will get an artificial starting time within the time windows for that (i,k)-combination. The non-negativity requirements for the load on board the ship are given by constraints (13.15). Constraints (13.16), (13.17), and (13.18) impose the binary requirements on the flow, spot cargo, and spot charter variables, respectively.

In the industrial shipping case, the objective will be to minimize the variable sailing costs, which correspond to the third and fourth terms in objective function (13.1), while constraints (13.3) and variable y_i are no longer required since in industrial shipping all cargoes are mandatory.

Brown, Graves, and Ronen [18], Fisher and Rosenwein [33], Kim and Lee [48], and Bausch, Brown, and Ronen [8] were among the first to study different versions of the maritime pickup-and-delivery problem with time windows. Their solution methods are based on set-covering path flow formulations where all feasible ship routes are generated a priori. Later, the problem modeled by (13.1) - (13.18) was studied by Brønmo et al. [14], who suggest a multi-start local search heuristic to solve the problem. The study shows that optimal or near-optimal solutions are obtained within reasonable time on eight real-life planning problems from four different shipping companies. Korsvik, Fagerholt, and Laporte [50] propose a unified tabu search heuristic, which is shown to perform better than the heuristic by Brønmo et al. [14], especially for large and tightly constrained problems. In a recent paper, Malliappi, Bennell, and Potts [56] present a variable neighborhood search heuristic for the same problem. Their method produced good results on modified benchmark data originating from benchmark instances for the pickup-and-delivery problem with time windows.

Jetlund and Karimi [45] present a different formulation for a real-life tramp routing and scheduling problem for a shipping company engaged in shipping bulk liquid chemicals in the Asia Pacific region. Lin and Liu [54] also consider a real tramp ship routing and scheduling problem for a shipping company operating seven handy-max dry bulk vessels for transportation of various types of dry cargoes in simple packaging (e.g., steel coils, wood pulp, or stone). They suggest a genetic algorithm to solve the problem.

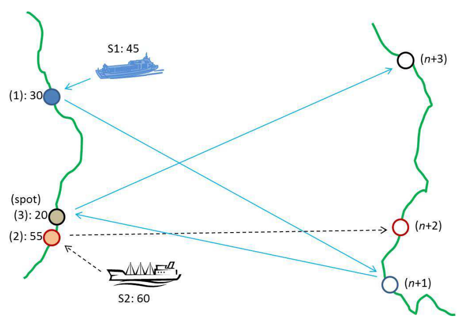
13.2.2 ▪ Application-Driven Modeling Extensions

Real-life problems often impose additional complexities than considered by the mathematical model from Section 13.2.1. These additional complexities may sometimes be viewed as opportunities. In this section we discuss and present the modeling extensions for some important opportunities that arise in several real-life problems.

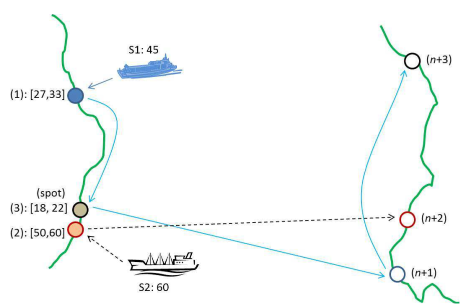
13.2.2.1 ▪ Flexible Cargo Sizes

Bulk cargoes are frequently shipped on a recurrent basis (e.g., under contracts of affreightment). In such cases the exact cargo size is not that important and the ship operator has some flexibility in the size of the cargo. Normally there is a target cargo size with allowed variability around it (e.g., 20.000 tons +/— 10%). This is also known as a *More Or Less Owner's Option* (MOLOO) contract. Under such a contract the ship operator is paid per unit delivered. Such a contract provides the operator with additional flexibility in assigning cargoes to vessels and in utilizing the vessel capacity. Then, the tramp ship routing and scheduling problem also includes determining the optimal size of each cargo to transport (within its interval).

Figure 13.3 illustrates the potential benefit of introducing flexibility in cargo sizes. Here, the same example as in Section 13.2.1 is used, except that flexibility in the cargo sizes is introduced. For example, in Figure 13.3(b), (1):[27,33] denotes that the size of cargo 1 is flexible within the interval 27 and 33 units. In the solution with flexible sizes, ship 1 carries the minimum sizes of cargoes 1 and 3, while ship 2 can carry 60 units of cargo 2. By utilizing this flexibility, it can be noted that the total sailing distance of the ships, and hence the fuel costs, are significantly reduced.



(a) *Without flexible cargo sizes*



(b) *With flexible cargo sizes*

Figure 13.3. *Example solution without and with flexibility in cargo sizes.*

In order to represent flexible cargo sizes the mathematical formulation from Section 13.2.1 has to be modified as follows. Instead of specifying the size of cargo i as Q_i, the quantity that can be transported is now flexible within the interval $[\underline{Q}_i, \overline{Q}_i]$, where $\underline{Q}_i$ and $\overline{Q}_i$ are the minimum and maximum quantities that must be transported (if serviced at all), respectively. Let T^Q_{ik} be the time required to load or discharge one unit of cargo i

with ship k. We now need an additional set of variables, q_{ik}, representing the quantity of cargo i that is transported by ship k.

To include the flexible cargo sizes, we need the following adjustments of the model for the basic tramp ship routing and scheduling problem:

$$\text{maximize} \sum_{k\in K}\sum_{i\in N^C\cup N^O} R_i q_{ik} - \sum_{k\in K}\sum_{(i,j)\in A_k} C_{ijk}x_{ijk} - \sum_{i\in N^C} C_i^S z_i \tag{13.19}$$

s.t. (13.2)—(13.6), (13.12), (13.14)—(13.18), and

$$l_{ik} + q_{jk} - l_{jk} - Q_k^C(1-x_{ijk}) \leq 0 \qquad \forall\, k\in K,\ (i,j)\in A_k \mid j\in N_k^P, \tag{13.20}$$

$$l_{ik} - q_{jk} - l_{n+j,k} - Q_k^C(1-x_{i,n+j,k}) \leq 0 \qquad \forall\, k\in K,\ (i,n+j)\in A_k \mid j\in N_k^P, \tag{13.21}$$

$$q_{ik} \leq l_{ik} \leq \sum_{j\in N_k} Q_k^C x_{ijk} \qquad \forall\, k\in K,\ i\in N_k^P, \tag{13.22}$$

$$0 \leq l_{n+i,k} \leq \sum_{j\in N_k} Q_k^C x_{n+i,jk} - q_{ik} \qquad \forall\, k\in K,\ i\in N_k^P, \tag{13.23}$$

$$t_{ik} + T_{ik}^Q q_{ik} + T_{ijk}^S - t_{jk} - M_{ijk}(1-x_{ijk}) \leq 0 \qquad \forall\, k\in K,\ (i,j)\in A_k, \tag{13.24}$$

$$t_{ik} + T_{ik}^Q q_{ik} + T_{i,n+i,k}^S - t_{n+i,k} \leq 0 \qquad \forall\, k\in K,\ i\in N_k^P, \tag{13.25}$$

$$\sum_{j\in N_k} \underline{Q}_i x_{ijk} \leq q_{ik} \leq \sum_{j\in N_k} \overline{Q}_i x_{ijk} \qquad \forall\, k\in K,\ i\in N_k^P. \tag{13.26}$$

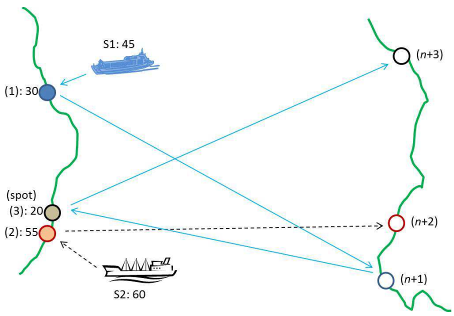
The change in the objective function (13.19) compared with the original one (13.1) is that the revenue now depends on the quantity transported of each cargo. The only difference between the new constraints (13.20)–(13.23) and the original constraints (13.7)–(13.10) is that the cargo size parameter Q_i has been replaced by the variable q_{iv}. Time constraints (13.24) and (13.25) are similar to the original constraints (13.11) and (13.13), except for that the service time in port now depends on the quantity loaded or discharged. Constraints (13.26) are new compared to the original formulation and specify the upper and lower bounds for the new quantity variables.

Tramp ship routing and scheduling problems with flexible cargo sizes have been studied by Brønmo, Christiansen, and Nygreen [15], Brønmo, Nygreen, and Lysgard [16], and Korsvik and Fagerholt [49]. These studies show that this flexibility can be utilized to significantly improve the profit (e.g., reducing the size of some cargoes in order to free enough ship capacity to carry additional spot cargoes by the controlled fleet). Brønmo, Christiansen, and Nygreen [15] solve a path flow formulation where all feasible ship routes are generated a priori and optimized with respect to the cargo sizes. Eight small- to medium-sized problem instances from two different shipping companies are solved. For larger instances this method becomes intractable due to the exponential increase in the number of feasible ship routes and hence variables. Therefore, Brønmo, Nygreen, and Lysgard [16] suggest a dynamic column generation scheme where ship routes are generated as needed. However, they discretize the cargo quantities, which turns the solution method into a heuristic column generation approach. Korsvik and Fagerholt [49] develop a tabu search algorithm for the same problem and show that very good solutions are obtained with their method within reasonable time.

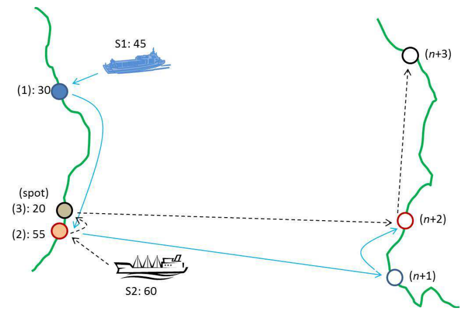
13.2.2.2 ▪ Split Loads

In the previous models a cargo cannot be transported by more than one ship. By introducing split loads this restriction is relaxed and a cargo may be split among several ships.

Figure 13.4 shows the example considered in Figure 13.1, now illustrating the potential benefit from load splitting. Figure 13.4(a) illustrates the solution without load splitting, while Figure 13.4(b) shows a solution where load splitting is utilized (though without flexible cargo sizes). Here, ship 1 services cargo 1 and uses the remaining capacity to take 15 units of cargo 2, while ship 2 transports the remaining 40 units of cargo 2 together with spot cargo 3. It is easy to imagine the solution in Figure 13.4(b) as better than the no-split solution in Figure 13.4(a), as this solution requires only two sailing legs between the continents. Furthermore, as previously mentioned, time windows could have restricted cargo 3 to be picked up after delivering cargo 1, like shown in Figure 13.4(a). Note that the spot cargo was not split due to the location at the unloading side.



(a) *Without splitting*



(b) *With splitting*

Figure 13.4. *Example solution without and with load splitting.*

Also here (same as for the former extension for flexible cargo sizes) we need quantity variables, q_{ik}, specifying how much of cargo i ship k is carrying. In addition we need the following adjustments to the original model (13.1)–(13.18):

$$\text{maximize } (13.19)$$

$$\text{s.t.} \quad (13.4)-(13.6),\ (13.11)-(13.18),\ (13.20)-(13.23), \text{ and}$$

$$\sum_{k\in K}\sum_{j\in N_k} x_{ijk} + z_i \geq 1 \qquad \forall\, i \in N^C, \tag{13.27}$$

$$\sum_{k\in K}\sum_{j\in N_k} x_{ijk} - y_i \geq 0 \qquad \forall\, i \in N^O, \tag{13.28}$$

$$l_{ik} + q_{jk} - l_{jk} - Q_k^C(1 - x_{ijk}) \leq 0 \qquad \forall\, k \in K,\ (i,j) \in A_k,\ | j \in N_k^P, \tag{13.29}$$

$$l_{ik} - q_{jk} - l_{n+j,k} - Q_k^C(1 - x_{i,n+j,k}) \leq 0 \qquad \forall\, k \in K,\ (i,n+j) \in A_k,\ | j \in N_k^P, \tag{13.30}$$

$$q_{ik} \leq l_{ik} \leq \sum_{j\in N_k} Q_k^C x_{ijk} \qquad \forall\, k \in K,\ i \in N_k^P, \tag{13.31}$$

$$0 \leq l_{n+i,k} \leq \sum_{j\in N_k} Q_k^C x_{n+i,jk} - q_{ik} \qquad \forall\, k \in K,\ i \in N_k^P, \tag{13.32}$$

$$(13.33) \qquad \sum_{k\in K} q_{ik} + Q_i z_i = Q_i \qquad \forall\, i \in N^C,$$

$$(13.34) \qquad \sum_{k\in K} q_{ik} - Q_i y_i = 0 \qquad \forall\, i \in N^O,$$

$$(13.35) \qquad q_{ik} \geq 0 \qquad \forall\, k \in K,\ i \in N_k^P.$$

Constraints (13.27) replace the original constraints (13.2) and state that all mandatory contract cargoes are transported. Each of these cargoes can be transported by one or several ships since it is now possible to split each cargo. The corresponding requirements for the optional cargoes are given by constraints (13.28). Constraints (13.29)–(13.32) are similar to the original constraints (13.7)–(13.10), except that the cargo quantity parameter has been replaced with the quantity variable q_{ik}. It can be noted that constraints (13.29)–(13.32) are similar to constraints (13.20)–(13.23) for the flexible cargo case. Constraints (13.33)–(13.35) are new and specific for the split problem. Constraints (13.33) ensure that the total quantity of contracted cargo i is lifted by one or several ships in the fleet, or by a spot charter. A similar requirement for optional cargoes is given by constraints (13.34), while constraints (13.35) impose non-negativity requirements on the load variables.

This tramp ship routing and scheduling problem with split loads was recently studied by Andersson, Christiansen, and Fagerholt [5], Korsvik, Fagerholt, and Laporte [51], and Stålhane et al. [71]. Andersson, Christiansen, and Fagerholt [5] suggest a solution method based on a priori generation of single ship routes and two alternative path flow models that deal with the selection of ship schedules and assignment of cargo quantities to the schedules. Computational results show that the solution method can provide optimal solutions only to realistic problems of small sizes. In order to overcome this limitation Stålhane et al. [71] propose a Branch-and-Price approach, while Korsvik, Fagerholt, and Laporte [51] suggest a large neighborhood search heuristic for the same problem. Both are able to find optimal solutions to the same instances as Andersson, Christiansen, and Fagerholt [5] within a short time, as well as solving larger problems. All three papers show that utilizing split loads can result in improved utilization of the fleet and hence significantly increased profit.

A segment in maritime transportation where splitting cargoes can be important is crude oil tanker routing and scheduling. Hennig et al. [41] present an extensive mathematical formulation to illustrate various aspects of that problem. One characteristic is that in contrast to the problem modeled above there are no predefined cargoes. There is rather a quantity requirement for each crude grade that must be picked up at each loading port within some (rather wide) time windows. This quantity may be picked up by several tankers. Similarly, there are quantity requirements also for the delivery ports, which can also be satisfied by deliveries from more than one tanker (or more than one source). These pickup-and-delivery requirements are not paired into cargoes beforehand, which results in a problem with split pickups and split deliveries. Logically, this problem lies between inventory routing (see Section 13.3) and cargo routing and scheduling. Hennig et al. [40] propose a path flow model with a priori route generation for the problem in Hennig et al. [41], though omitting some of the complicating aspects. They introduce continuous variables to distribute the cargo among the different routes. It is demonstrated that small realistic instances can be solved to optimality.

It should be mentioned that even though the advantages of load splitting can be significant, as demonstrated by the small example in Figure 13.1, there may also be some drawbacks. Load splitting gives an increased number of port calls. In the above example both the pickup-and-delivery ports of cargo 2 will be visited twice. However, the resulting increase in port costs can be more than outweighed by reduced sailing cost, especially in deep sea shipping with long sailing distances and high fuel costs compared to port costs. It may still be considered as reduced service quality for customers that may have to manage the service of more than one ship. It may also lead to increased operational complexity for the planners.

13.2.2.3 ▪ Variable Sailing Speed

Oil price increases during recent years have brought the issue of optimizing sailing speed to the forefront. Bunker fuel cost is a major component of the variable operating cost of a ship, and when fuel prices are high it may amount to the majority of the operating costs. The bunker fuel consumption of a cargo ship per time unit is often estimated to be proportional to the third power of its speed (within normal operating speeds) (Ronen [64]). Bunker fuel consumption per unit of distance is thus proportional to the second power of the speed. Thus, reducing the sailing speed by 10% will reduce the bunker fuel consumption for a given sailing leg by close to 20%. Figure 13.5 shows an example of the fuel consumption per nautical mile as a function of sailing speed over the speed interval that is feasible in practice for a *Liquefied Natural Gas* (LNG) carrier. It should be evident that reducing sailing speed provides a significant potential for cost savings. On the contrary, speeding up can sometimes result in that a vessel is able to arrive at a port in time to service an additional spot cargo. Determining optimal speeds within the planning of ship routes and schedules can therefore obviously influence both the fuel costs and revenues.

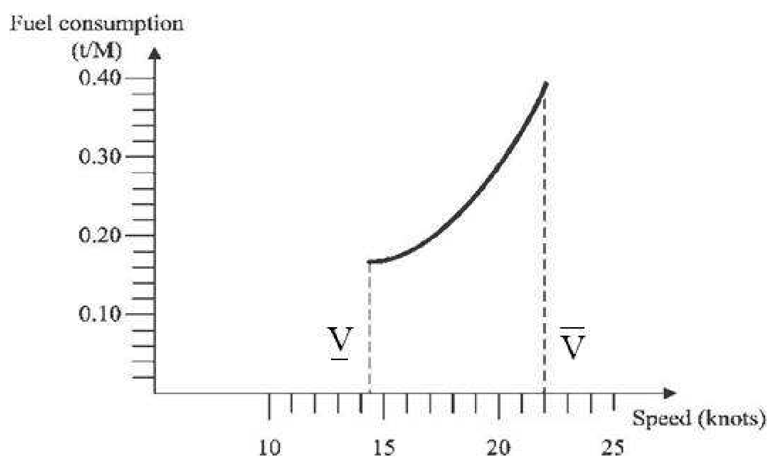


Figure 13.5. *Fuel consumption [ton/nautical mile] as a function of speed for an LNG carrier.*

The following amendments to the basic model presented in Section 13.2.1 are necessary in order to incorporate sailing speed optimization. Let D_{ij} be the sailing distance from node i to node j. The variable s_{ijk} defines the speed of travel from node i to node j with ship k. The time it takes to sail along arc (i,j) is D_{ij}/s_{ijk}. The non-linear function $C_k(s)$, defined on the speed interval $[\underline{V}_k, \overline{V}_k]$, represents the sailing costs per unit of distance for ship k sailing at speed s. The cost of sailing an arc (i,j) with ship k at speed s_{ijk} is then $D_{ij}C_k(s_{ijk})$.

The model for the basic tramp ship routing and scheduling problem (13.1)–(13.18) can now be adjusted as follows:

$$\text{(13.36)} \quad \text{maximize} \sum_{i\in N^C} R_i Q_i + \sum_{i\in N^O} R_i Q_i y_i - \sum_{k\in K}\sum_{(i,j)\in A_k} D_{ij} C_k(s_{ijk}) x_{ijk} - \sum_{i\in N^C} C_i^S z_i$$

$$\text{s.t.} \quad (13.2)-(13.10),\ (13.12),\ (13.14)-(13.18), \text{ and}$$

$$\text{(13.37)} \quad t_{ik} + T_{ik}^P + \frac{D_{ij}}{s_{ijk}} - t_{jk} - M_{ijk}(1 - x_{ijk}) \le 0 \qquad \forall\, k \in K,\ (i,j)\in A_k,$$

$$\text{(13.38)} \quad t_{ik} + T_{ik}^P + D_{i,n+i}/s_{i,n+i,k} - t_{n+i,k} \le 0 \qquad \forall\, k \in K,\ i \in N_k^P,$$

$$\text{(13.39)} \quad \underline{V}_k \le s_{ijk} \le \overline{V}_k \qquad \forall\, k \in K,\ (i,j)\in A_k.$$

The objective function (13.36) has now become a non-linear function because of the non-linear relationships between fuel consumption and speed. Constraints (13.37) and (13.38) correspond to constraints (13.11) and (13.13) in the original formulation. These constraints are also non-linear because the sailing time depends on the speed variable. The new constraints (13.39) define the lower and upper bounds for the speed variables.

Psaraftis and Kontovas [61] provide an excellent taxonomy and survey of speed models in maritime transportation. Fagerholt, Laporte, and Norstad [31] present some alternative mathematical models for the speed optimization problem for a given route. Norstad, Fagerholt, and Laporte [57] develop a local search heuristic, including a specialized algorithm for determining optimal speeds for given ship routes, to solve the combined tramp ship routing and speed optimization problem discussed above. It is demonstrated that incorporating sailing speeds as decision variables when planning vessel routes significantly improves fleet utilization and profit. Gatica and Miranda [36] deal with minimization of the cost of serving a set of mandatory single trip cargoes while determining the speed for each trip. The port time windows were discretized, which facilitated the use of a network model.

13.2.2.4 ▪ Other Extensions

In many industrial and tramp shipping problems, the time window for when a cargo must be picked up or delivered is somewhat negotiable, especially when planning some time in advance. This gives rise to soft time windows, which is yet another such model extension that increases solution space and can give improved solutions. Fagerholt [30] shows that huge cost reductions were achieved on real-life industrial shipping problems by introducing soft time windows with a maximum allowed violation of the target time windows.

In contrast to the above extensions, which can all be viewed as opportunities since the solution space increases, there may also exist important additional limiting constraints that must be considered in real-life ship routing and scheduling problems. One such example is stowage on board the ships. In many maritime bulk shipping operations the ships can have tens of tanks and carry several cargoes of different products simultaneously, and one must decide which tanks should be used for each cargo. When allocating loads to the different tanks on board the ship, numerous constraints must be satisfied, such as capacity, stability, and hazardous material regulation constraints. Stowage problems in bulk shipping have been studied by Vouros, Panayiotopoulos, and Spyropoulos [76] and Hvattum, Fagerholt, and Armentano [42].

Sometimes one must also consider bunkering/refueling decisions when planning shipping routes. Besbes and Savin [10] and Kim et al. [47] study the handling of refueling decisions for a single vessel taking into account varying fuel prices between ports. This problem is also closely related to the speed optimization problem.

Other examples of additional practical constraints that sometimes must be considered are draft limits in ports (Rakke et al. [62]), time slot assignments in berths to avoid clashes (Pang, Xu, and Li [58]), and restricted port opening hours (Christiansen and Fagerholt [21]).

13.3 ▪ Maritime Inventory Routing

In this section we present mathematical models and refer to solution methods for the *Maritime Inventory Routing Problem* (MIRP). This problem can be defined as a planning problem where an actor has the responsibility for both the inventory management at one or both ends of the maritime transportation legs and for the ships' routing and scheduling. This actor is most often operating in the industrial bulk shipping mode, and the problem appears both in deep and short sea shipping. We start in Section 13.3.1 by presenting a basic version of the problem including a simple example. Then, in Section 13.3.2, we describe a mathematical model for the basic MIRP, and related research on models and solution methods is briefly reviewed. Finally, application-driven extensions are discussed in Section 13.3.3.

13.3.1 ▪ Problem Description and Example

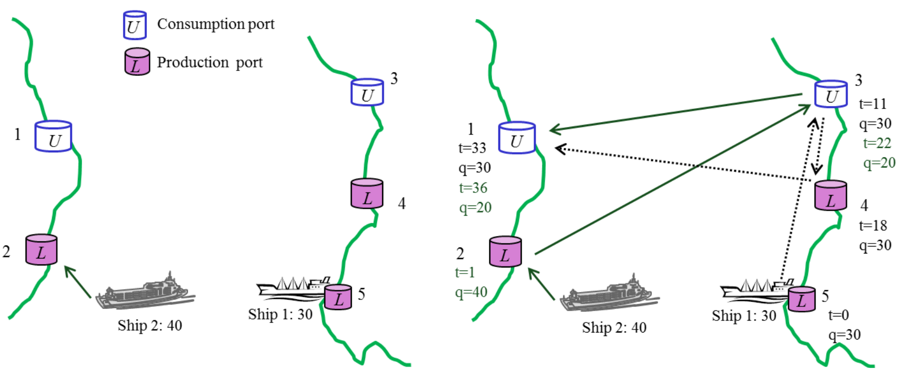
Vital management issues in maritime supply chains are often inventory control at processing facilities and consumption at the customer sites as well as the routing and scheduling of ships. MIRPs often occur in maritime supply chain management when the inventory management and the routing and scheduling of the ship fleet have to be coordinated simultaneously. By coordinating these planning challenges it is possible to achieve monetary benefits, flexibility in services, and improved robustness. This has resulted in increased attention towards MIRP both in the OR community and the industry. These activities have formed the basis of the following surveys in the last decade: Christiansen and Fagerholt [22] and Andersson et al. [6].

Much of the research described in the literature on MIRPs is developed based on real planning problems, particularly from the chemical, petroleum, and pulp industries. In recent years, we have seen the beginning of successful implementation stories of *Decision Support Systems* (DSSs) for MIRPs. Dauzère-Pérès et al. [27] report the development of a DSS for Omya Hustadmarmor transporting calcium carbonate sloppy, which is a product used in paper manufacturing. Furman et al. [35] describe a DSS for feedstock routing in the ExxonMobil downstream sector. A LNG inventory routing model developed by Fodstad et al. [34] is used by Statoil and GDF SUEZ.

According to Christiansen and Fagerholt [22], the basic MIRP concerns the transportation of a single product. The product is stored in inventories at or close to loading (production) and unloading (consumption) ports. Inventory storage capacities are defined in all ports. Further, we assume that the production and consumption rates are given in all ports and are constant during the planning horizon. A heterogeneous fleet of ships is used to transport the product. Each ship has a given capacity and sailing speed. The ships can wait outside a port before entering for (un)loading. A ship can both load and unload at multiple ports in succession. The initial position and load on board each ship are known at the beginning of the planning horizon. The sailing and port costs are all ship-dependent. Inventory carrying costs are not considered because the inventories at both ends of the shipping legs usually belong to the same company. The objective of the MIRP is to design routes and schedules for the fleet that minimize the total transportation cost and determine the (un)loading quantity at each port visit without violating the inventory limits. A typical planning horizon may span from one week up to several months depending on the shipping segment.

In contrast to the cargo routing problem, the MIRP has no predetermined number of calls at a given port during the planning horizon and no predetermined quantity to be (un)loaded at each port call. There are also no predefined pickup and delivery pairs in the basic MIRP, as opposed to cargo routing, where each cargo has a specified pickup and delivery port. Normally, no time windows are given.

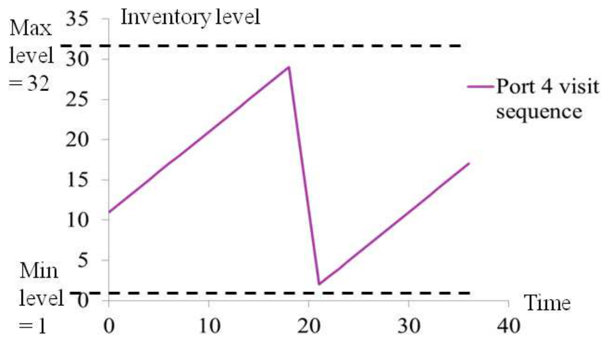
Figure 13.6(a) illustrates a small example with two ships with capacities of 30 and 40 units, respectively. For simplicity, we assume that the ships are empty in the beginning of the planning period. Ship 1 is positioned in port 5, while ship 2 has an initial position at sea. The time horizon is 36 days. There are five ports with storages placed close to the ports. At ports 1 and 3, the product is consumed and the ports are defined as consumption (delivery or unloading) ports. Likewise, ports 2, 4, and 5 are production ports and the product is picked up or loaded in these ports. Each of the storages at the ports has defined lower and upper inventory limits and a specific production or consumption rate. For instance, at port 4 the production rate is one unit per day and the lower and upper storage limits are 1 and 32 units, respectively. The corresponding numbers for consumption port 3 are −2, 1, and 28. See Figures 13.7(a) and 13.7(b).



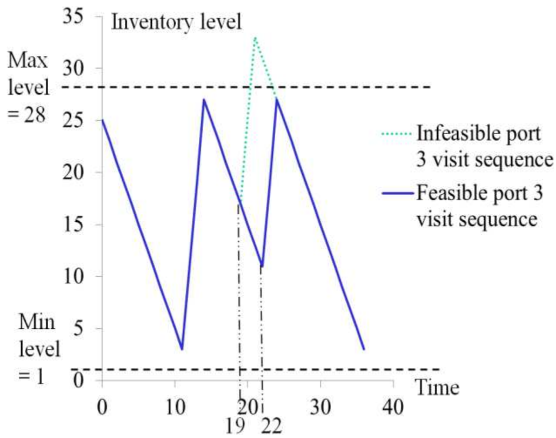
(a) *Example MIRP* (b) *Example solution*

Figure 13.6. *Example MIRP and its solution with routes, start times (t), and (un)load quantities (q).*

In this example, the berth capacity in all ports is one, which means that just one ship can load or unload at the same time in a port. The (un)loading rate of both ships is 10 units per day. Figure 13.6(b) indicates a solution to the problem where the routes and schedules satisfy the inventory and berth constraints. In port 5, ship 1 loads up to the capacity of the ship before sailing to port 3, where it unloads this quantity. Ship 1 starts loading at port 5 in the beginning of the planning period, and the loading operation lasts for 3 days, while the sailing takes 8 days. Without any waiting in port 3, the unloading starts at day 12 and lasts for 3 days. The inventory limits are tight and the consumption rate big, so the port needs to be visited often. The unloaded quantity is 30 in total, and this quantity is larger than the storage capacity. However, the consumption rate is two units per day, so 30 units can be unloaded during the 3 days. The ship continues on a



(a) *Inventory level at port* 4



(b) *Inventory level at port* 3

Figure 13.7. *Inventory levels at port* 4 *and port* 3 *during the planning horizon.*

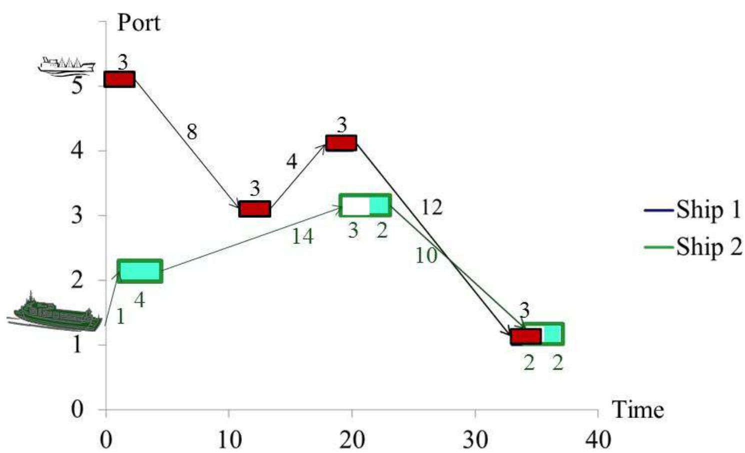


Figure 13.8. *Routes and schedules including time consumption of the various tasks.*

4-day sailing leg to port 4 before loading to its capacity for 3 days. Finally, the ship leaves port 4 at day 22 and sails toward port 1. Here it starts unloading at day 34 and unloads for 3 days. See Figure 13.8 for an illustration of the ship routes and schedules including the time consumption of each task.

Ship 2 starts at sea and reaches port 2 after 1 day. Here ship 2 loads to its capacity before sailing toward port 3. The loading at port 2 lasts for 4 days, while the sailing leg takes 14 days. This means that the unloading can start after 19 days. To unload half the ship's capacity, the ship has to wait until day 23 (after 22 days) before unloading because of the tight inventory constraints; see Figure 13.7(b). Ship 2 continues to port 1 and unloads the rest of the quantity on board. Also here, the ship has to wait because ship 1 is unloading when ship 2 arrives and the berth capacity is one. Ship 2 calls two consumption ports in succession.

13.3.2 ▪ Mathematical Model and Solution Methods

In order to present a mathematical model for the MIRP, most of the mathematical notations from Section 13.2 have to be modified. In addition, some new notation will be defined. Each port is represented by an index i, and the set of ports is given by N. Still let K be the set of available ships and N_k the set of ports (including $o(k)$ and $d(k)$) that can be visited by ship k. Each port can be visited several times during the planning horizon, and M_i is the set of possible calls at port i, while M_{ik} is the set of calls at i that can be made by ship k. The port call number is represented by an index m, and $|M_i|$ is the last possible call at port i.

The set of nodes in the flow network represents the set of port calls, and each port call is specified by (i,m), $i \in N$, $m \in M_i$. In addition, we specify flow networks for each ship k with nodes (i,m), $i \in N_k$, $m \in M_{ik}$. Finally, A_k contains all feasible arcs for ship k, which is a subset of $\{i \in N_k, m \in M_{ik}\} \times \{i \in N_k, m \in M_{ik}\}$.

Some of the parameters remain the same as defined in Section 13.2, such as Q_k^C for the ship capacity, T_{ik}^Q representing the unit loading time, the sailing time T_{ijk}^S, and, finally, the sailing cost C_{ijk}. The berth capacities in MIRPs are often limited, and just one ship can call a port at a time. Due to small port areas or narrow channels from the port to the pilot station a minimum time, T_i^B, between a departure of one ship and the arrival of the next ship might be given. Let T denote the length of the planning horizon.

The levels of the inventory (or stock) have to be within a given interval at each port $[\underline{S}_i, \overline{S}_i]$. The production rate R_i is positive if port i is producing the product and negative if port i is consuming the product. Further, constant I_i is equal to 1 if i is a loading port, -1 if i is an unloading port, and 0 if i is $o(k)$ or $d(k)$.

In the mathematical formulation many of the variables are similar to the ones defined in Section 13.2 but are modified. We use the following types of variables: the binary flow variable x_{imjnk} equals 1 if ship k sails from node (i,m) directly to node (j,n), and 0 otherwise, and the slack variable w_{im} is equal to 1 if no ship takes port call (i,m), and 0 otherwise. The time variable t_{im} represents the time at which service begins at node (i,m). Variable l_{imk} gives the total load on board ship k just after the service is completed at node (i,m), while variable q_{imk} represents the quantity loaded or unloaded at port call (i,m) when ship k calls (i,m). Finally, s_{im} represents the inventory (or stock) level when service starts at port call (i,m). It is assumed that, for each ship k, nothing is loaded or unloaded at the artificial origin $o(k)$, $q_{o(k)1k} = 0$ and that the ship starts at $o(k)$ in the beginning of the planning horizon, $t_{o(k)1}$. At this time, the ships may have cargo on board, L_k^0; $l_{o(k)1k} = L_k^0$, and the stock level at each port i is S_i^0. These fixed variables are omitted in the presentation of the model.

The arc flow formulation of the basic MIRP is as follows:

$$\text{maximize} \sum_{k \in K} \sum_{(i,m,j,n) \in A_k} C_{ijk} x_{imjnk} \tag{13.40}$$

$$\text{s.t.} \quad \sum_{k\in K}\sum_{j\in N_k}\sum_{n\in M_{jk}} x_{imjnk} + w_{im} = 1 \qquad \forall\, i\in N,\ m\in M_i, \tag{13.41}$$

$$\sum_{j\in N_k}\sum_{n\in M_{jk}} x_{o(k)1jnk} = 1 \qquad \forall\, k\in K, \tag{13.42}$$

$$\sum_{i\in N_k}\sum_{m\in M_{ik}} x_{imjnk} - \sum_{i\in N_k}\sum_{m\in M_{ik}} x_{jnimk} = 0 \qquad \forall k\in K,\ j\in N_k\setminus\{o(k),d(k)\},\ n\in M_{jk}, \tag{13.43}$$

$$\sum_{i\in N_k}\sum_{m\in M_{ik}} x_{imd(k)1k} = 1 \qquad \forall\, k\in K, \tag{13.44}$$

$$w_{im} - w_{i(m-1)} \geq 0 \qquad \forall\, i\in N,\ m\in M_i, \tag{13.45}$$

$$l_{imk} + I_j q_{jnk} - l_{jnk} - Q_k^C(1-x_{imjnk}) \leq 0 \qquad \forall\, k\in K,\ (i,m,j,n)\in A_k \mid j\neq d(k), \tag{13.46}$$

$$l_{imk} + I_j q_{jnk} - l_{jnk} + Q_k^C(1-x_{imjnk}) \geq 0 \qquad \forall\, k\in K,\ (i,m,j,n)\in A_k \mid j\neq d(k), \tag{13.47}$$

$$q_{imk} \leq l_{imk} \leq \sum_{j\in N_k}\sum_{n\in M_{jk}} Q_k^C x_{imjnk} \qquad \forall\, k\in K,\ i\in N_k,\ m\in M_{ik} \mid I_i = 1, \tag{13.48}$$

$$0 \leq l_{imk} \leq \sum_{j\in N_k}\sum_{n\in M_{jk}} Q_k^C x_{imjnk} - q_{imk} \qquad \forall\, k\in K,\ i\in N_k,\ m\in M_{ik} \mid I_i = -1, \tag{13.49}$$

$$t_{im} + T_{ik}^Q q_{imk} + T_{ijk}^S - t_{jn} - T(1-x_{imjnk}) \leq 0 \qquad \forall\, k\in K,\ (i,m,j,n)\in A_k \mid j\neq d(k), \tag{13.50}$$

$$t_{im} - t_{i(m-1)} - \sum_{k\in K} T_{ik}^Q q_{i(m-1)k} + T_i^B w_{im} \geq T_i^B \qquad \forall\, i\in N, m\in M_i\setminus\{1\}, \tag{13.51}$$

$$s_{i1} - R_i t_{i1} = S_i^0 \qquad \forall\, i\in N, \tag{13.52}$$

$$s_{i(m-1)} - \sum_{k\in K} I_i q_{i(m-1)k} + R_i(t_{im} - t_{i(m-1)}) - s_{im} = 0 \qquad \forall\, i\in N, m\in M_i\setminus\{1\}, \tag{13.53}$$

$$\underline{S}_i \leq s_{im} \leq \overline{S}_i \qquad \forall\, i\in N, m\in M_i, \tag{13.54}$$

$$\underline{S}_i \leq s_{im} - \sum_{k\in K} I_i q_{imk} + R_i(T - t_{im}) \leq \overline{S}_i \qquad \forall\, i\in N, m=|M_i|, \tag{13.55}$$

$$t_{im} \geq 0 \qquad \forall\, (i\in N, m\in M_i)\cup(i\in o(k),\ k, m=1), \tag{13.56}$$

$$q_{imk} \geq 0 \qquad \forall\, k\in K,\ i\in N_k\setminus\{d(k)\}, m\in M_{ik}, \tag{13.57}$$

$$x_{imjnk} \in \{0,1\} \qquad \forall\, k\in K,\ (i,m,j,n)\in A_k, \tag{13.58}$$

$$w_{im} \in \{0,1\} \qquad \forall\, i\in N,\ m\in M_i. \tag{13.59}$$

The objective function (13.40) minimizes the total costs. Constraints (13.41) ensure that each port call is visited at most once. Constraints (13.42)–(13.44) describe the flow on the sailing route used by ship k. One or several of the calls in a specified port can be made by a dummy ship, and the highest call numbers will be assigned to dummy ships in constraints (13.45). For the calls made by a dummy ship, we get artificial starting times and artificial inventory levels within their limits. Constraints (13.46)–(13.49) keep track of the load on board. The relationship between the binary flow variables and the ship load at each port call is given by constraints (13.46)–(13.47). We need both (13.46) and (13.47) to ensure the balance for the load on board after each port visit. This is in contrast to the problems presented in Section 13.2 where just the first type of constraints is necessary. There the quantity loaded is the same as the quantity unloaded for a particular cargo with known port pair. For the MIRP the quantity loaded in one port might be unloaded in several ports. Constraints (13.48) and (13.49) give the ship capacity intervals at the port calls for loading and unloading ports, respectively. The scheduling of the route is taken into account in constraints (13.50). Constraints (13.51) prevent service overlap in the ports and ensure the order of real calls in the same port. A ship must complete its service before the next ship starts its service in the same port. Most of the constraints (13.41)–(13.50) are similar to the constraints for the tramp ship routing and scheduling problem with flexible cargo sizes given in Section 13.2. In addition, we have some inventory constraints for this problem. The inventory level at the first call in each port is calculated in constraints (13.52). From constraints (13.53), we find the inventory level at any port call (i,m) from the inventory level upon arrival at the port in the previous call $(i,m-1)$, adjusted for the loaded/unloaded quantity at the port call and the production/consumption between the two visits. The general inventory limit constraints at each port call are given in (13.54). Constraints (13.55) ensure that the inventory level at the end of the planning horizon is within its limits. The non-negativity requirements for the time for start of service and the quantity loaded or unloaded are given by (13.56) and (13.57), respectively. Finally, the formulation involves binary requirements (13.58) and (13.59) on the flow variables and port call slack variables, respectively.

The model could easily be extended by quantity intervals for minimum and maximum quantities to be (un)loaded at a port call given that the port is called. It is also possible to derive time windows for start of service based on data in the model, such as the inventory conditions. Finally, it is possible to calculate the minimum number of visits to each port based on the data, and the dummy ship variable should then only be defined for the optional visits.

A real planning problem similar to this basic MIRP is studied by Christiansen [20]. There, a company produces and consumes ammonia in its factories worldwide. The planners at the company are responsible for keeping the inventory levels within their limits at all its factories. The factories are placed close to ports, and the ammonia is transported by ships from production ports to consumption ports. In addition to the inventory management, the planners have to design routes and schedules for their fleet of ships. Many of the single product MIRPs described in the literature are studied for transportation of petroleum products. For example, Furman et al. [35] present an industrial MIRP for the transportation of vacuum gas oil. LNG is usually considered a single product for distribution planning; see, for instance, Grønhaug and Christiansen [37]. However, the structure of the particular MIRPs deviates in some respects from the model (13.40)–(13.59).

The combination of inventory management and ship routing and scheduling makes the MIRP a very complex problem to solve. Many of the studies in the literature formulate a MIP model and use this model in various solution approaches. We can divide the models into two main classes of formulations: arc-flow models and path-flow models.

Model (13.40)–(13.59) is an arc-flow model where each sailing leg for each ship is defined as a binary variable. Both Christiansen [20] and Al-Khayyal and Hwang [3] present MIRP arc-flow models. These models (like (13.40)–(13.59)) have a weak linear programming relaxation, so often different types of valid inequalities are developed to strengthen the formulation. Agra, Christiansen, and Delgado [1] consider a short sea fuel oil MIRP where an arc-flow model is improved by tightening bounds, using extended formulations and including valid inequalities. Song and Furman [70] solve subproblems that are restricted versions of their MIRP problem by Branch-and-Cut. These subproblems are solved iteratively in a large neighborhood search heuristic. Arc-flow models are also solved by metaheuristic-based algorithms as in Dauzère-Pérès et al. [27]. Siswanto, Essam, and Sarker [69] combine an arc-flow model with a heuristic. Bredström, Carlsson, and Rönnqvist [13] develop a hybrid algorithm based on a genetic algorithm and linear programming. Finally, Sherali and Al-Yakoob [68] use a rolling horizon heuristic based on an arc-flow model. Path-flow models make the other class of MIRP formulations, and here each path is most often defined as a binary variable. The path is described by the route and/or schedule, and sometimes also the quantities loaded or unloaded on the route are included. Path-flow models are used in Branch-and-Price methods (see, e.g., Christiansen [20], Persson and Göthe-Lundgren [59], Grønhaug et al. [38], Andersson [4], and Engineer et al. [29]), rolling horizon heuristics (Rakke et al. [63]), and various fix and relax heuristics (Gunnarsson, Rönnqvist, and Carlsson [39] and Bilgen [11]). Pure heuristics are also used to solve complicated MIRPs; see, for example, Christiansen et al. [23], who develop a construction heuristic that was embedded in a genetic algorithmic framework.

13.3.3 ▪ Application-Driven Extensions for the MIRP

Many real applications of MIRPs have a more complex structure than the basic MIRP presented in Sections 13.3.1 and 13.3.2. In this section we discuss some of the extensions that are described in the literature. In many real-life applications, several of the extensions are combined.

13.3.3.1 ▪ Multiple Products and Allocation of Products to Compartments

In the basic MIRP several cargoes may be transported simultaneously in one ship, but the product is assumed to be the same. This means that the product does not need to be transported in separate compartments on board the ship or stored in separate storages at the ports. When we move from the single-product MIRP to a multi-product MIRP, we need to keep track of the storages of each product in each port and the amount of each product on board each ship during the entire planning horizon.

Most often the allocation of products to compartments is not considered, and it is assumed that the allocation can be solved by the people responsible for stowage, or as a separate planning problem. Christiansen et al. [23], Persson and Göthe-Lundgren [59], and Siswanto, Essam, and Sarker [69] present multi-product MIRPs without taking the allocation of products to compartments into account. In other studies it is assumed that the ship compartments are dedicated to specific products. Both Al-Khayyal and Hwang [3] and Li, Karimi, and Srinivasan [53] assume that compartments are dedicated to specific products. Recently, Agra, Christiansen, and Delgado [1] considered both the case without any allocation of different fuel products into different cargo tanks as well as the case where there are dedicated tanks for families of products. See Section 13.2.2.4 for additional comments regarding stowage on board the ships.

Recently we have witnessed an increasing attention to multi-product MIRPs relative to single product MIRPs, and these problems are frequently encountered by chemical and petroleum transport companies. Al-Khayyal and Hwang [3] consider the transportation of petrochemicals, Persson and Göthe-Lundgren [59] deal with bitumen products, and Ronen [65] considers refinery products, while Li, Karimi, and Srinivasan [53] study different types of chemicals. However, we can also find studies of multi-product MIRPs in the cement (Christiansen et al. [23]), wheat (Bilgen and Ozkarahan [12]), pulp (Andersson [4]), and calcium carbonate slurry (Dauzère-Pérès et al. [27]) industries.

13.3.3.2 ▪ Particular Network Structures

In model (13.40)–(13.59) we describe the basic MIRP including a network structure consisting of many production and consumption ports. However, some real MIRPs concern inventory constraints at just one of the port types, either in the production or consumption ports. In addition, there might be just one central producer or one central customer. Such networks correspond to a classical vehicle routing structure with one depot and a set of customers. Sherali and Al-Yakoob [67] handle the problem of determining the optimal ship fleet mix and schedules for a problem with a single source and destination, while in Sherali and Al-Yakoob [68] they extend the problem to multiple sources and destinations.

Several real studies with a particular network structure can be found in the literature. Rakke et al. [63] present a real LNG MIRP for one central producer with inventory considerations and many customers with contract requirements instead of inventory. Similarly, the transportation of calcium carbonate slurry products by Dauzère-Pérès et al. [27] starts at a central producer, but in contrast to Rakke et al. [63] the inventories are managed at the unloading ports.

In real life MIRPs we find many unique network structures and constraints regarding an acceptable route structure for each ship. For instance, several studies present problems with a limited number of loadings or unloadings in succession. Bilgen and Ozkarahan [12] define paths with at most two loading ports and one unloading port. On the contrary, in the LNG business considered by Grønhaug and Christiansen [37] it is assumed that an LNG ship is always loaded to its capacity but may unload at two regasification terminals in succession. At each unloading port the unloaded quantity is equal to the capacity of a number of cargo tanks, such that a tank is either empty or fully loaded. This loading and unloading policy is a result of avoiding active movements within the cargo tanks (sloshing) on board the LNG ships.

13.3.3.3 ▪ Time Varying Production and Consumption Rates

In the basic MIRP, the production and consumption rates, R_i, are assumed fixed and constant during the planning horizon for all port storages. This property of the production and consumption rates allows for a mathematical formulation based on continuous time variables; see Christiansen [20], Al-Khayyal and Hwang [3], and Siswanto, Essam, and Sarker [69]. For many real planning problems this assumption is too coarse, and varying production and consumption must be taken into account in the modeling. When the production and/or consumption rate is varying during the planning horizon, the production or consumption rate can, for instance, be given for each day t, by R_{it}, and a discrete time model is applied (see, e.g., Persson and Göthe-Lundgren [59], Sherali and Al-Yakoob [67, 68], Bilgen and Ozkarahan [12], Song and Furman [70], Andersson [4], and Furman et al. [35]).

Finally, the production or consumption in a port might not be known and given to the model. In Grønhaug et al. [38] decision variables are defined for the production and consumption of LNG in each port and each time period.

13.3.3.4 ▪ Other Extensions

Often, the companies facing a MIRP trade cargoes with other operators in order to better utilize the fleet and to ensure the product balance at their own factories. These traded volumes are determined by negotiations. The transporter undertakes to load or unload cargoes with defined pickup and delivery ports and determined quantity intervals and to arrive at a particular port within a given time window. For these cargoes, no inventory management problem exists. This is an example of a shipping company moving from industrial shipping towards tramp shipping. In the real problem described by Christiansen [20], the transporter trades ammonia with other operators. On the contrary, some tramp shipping companies have for a while considered the possibility of introducing a *Vendor Managed Inventory* (VMI) service for some of their customers. This service may replace the more traditional *Contracts of Affreightment* (COAs), which have been the standard agreement between a tramp shipping company and a charterer.

The basic MIRP concerns the sea transportation and the inventory management at both ends of the sailing leg. In many real planning situations, it is sensible to include supply chain activities beyond the MIRP. For instance, Persson and Göthe-Lundgren [59] include the process scheduling at the oil refineries (production ports), while Bilgen and Ozkarahan [12] address bulk grain blending. Bredström, Carlsson, and Rönnqvist [13], Fodstad et al. [34], and Andersson [4] extend the supply chain at the customer side.

13.4 ▪ Dynamic and Stochastic Ship Routing

As explained by Psaraftis [60], a problem is dynamic if some of the problem inputs are not known beforehand but are revealed as time goes by. When probabilistic information concerning the unknown inputs is available, one also faces a stochastic optimization problem. Recent contributions on land-based routing and scheduling problems that are treated as both dynamic and stochastic exist; see, for example, the work of Hvattum, Løkketangen, and Laporte [43], Van Hentenryck, Bent, and Upfal [74], and Schilde, Doerner, and Hartl [66]. We also refer the reader to Chapters 8 and 11 in this book for further references.

Even though most planning of ship routes and schedules in maritime transportation is still performed manually, it is to an increasing degree performed with assistance from optimization-based decision support systems; see, e.g., Fagerholt and Lindstad [32] and Kang et al. [46]. However, most algorithms for ship routing and scheduling solve static and deterministic versions of the problem. In land-based transportation, previous research on dynamic and stochastic routing has indicated that the inclusion of stochastic information within a dynamic planning process is valuable; see, for example, the work of Bent and Van Hentenryck [9] and Hvattum, Løkketangen, and Laporte [43].

Industrial and tramp ship routing and scheduling problems are most often dynamic and stochastic in nature. Sailing times represent one type of problem input that is highly stochastic due to varying environmental conditions. Lo and McCord [55] minimized the expected fuel consumption for a given sailing leg by exploiting uncertain ocean currents, while Azaron and Kianfar [7] represented weather conditions in a stochastic dynamic network. Considering decisions on the tactical level, Cheng and Duran [19] developed a decision support system for a crude oil transportation and inventory problem that takes into account uncertainty in demand. The problem is formulated as a discrete time Markov decision process and solved heuristically.

Recently, Agra et al. [2] presented a robust solution approach to a full load (meaning that only one cargo could be on board a ship at any time) ship routing and scheduling problem where the sailing times are uncertain. The aim is to find robust solutions, where routes are feasible for all travel times defined by a predetermined uncertainty set.

Hwang, Visoldilokpun, and Rosenberg [44] considered the risk associated with fluctuating spot rates and sought to maximize the profit while constraining the variance. The problem is relevant in both tramp and industrial shipping, as optional cargoes are included. Two different formulations were presented, and one of them was solved using a Branch-and-Cut-and-Price method. Only full load instances were considered, and the problem was considered as a static problem. Yet another problem input that can be considered as stochastic in maritime routing problems is the occurrence of future cargoes to be transported. Tirado et al. [72] consider such a problem in industrial shipping. They present three different heuristics for the problem, two of which use stochastic information represented by scenarios. These two heuristics were adapted from the algorithms for land-based routing and scheduling by Bent and Van Hentenryck [9] and Hvattum, Løkketangen, and Laporte [43], respectively. Computational experiments show that the use of stochastic information within the proposed solution methods yields an average cost saving of 2.5% on a set of realistic test instances.

Almost no studies are concerned with the uncertainty in the parameters of the MIRP. However, Rakke et al. [63] and Sherali and Al-Yaakob [67, 68] introduce penalty functions for deviating from the customer contracts and the storage limits, respectively. Christiansen and Nygreen [26] introduce soft inventory levels to handle uncertainties in sailing time and time in port, and these levels are transformed into soft time windows.

Even though there are few research studies on stochastic and dynamic routing and scheduling in the maritime sector, as shown above, we expect an increased interest for this topic in the future. This research area is still in its infancy, and we believe significant improvements in planning, as suggested by Tirado et al. [72], can be made by treating maritime routing and scheduling problems as both dynamic and stochastic.

13.5 ▪ Conclusions and Future Research Directions

The size and importance of the shipping market, which is tightly connected to the world's financial health, create a competitive environment where mainly the skillful actors succeed in the long run. A key to success partly relies on the shipping companies' ability to optimize fleet utilization. Furthermore, considering the focus on environmental emissions and the fact that maritime transportation accounts for 5% of total global CO_2 emissions, which is twice as much as the emissions from airlines (Vidal [75]), it is also easy to recognize the environmental benefits from good planning.

This chapter presents mathematical models and briefly discusses solution methods for some important routing and scheduling problems in industrial and tramp shipping. We hope this chapter will stimulate researchers and students to an increased interest in maritime routing problems. Even though research on ship routing and scheduling has blossomed during the last decade, there are still a number of topics which remain to be properly addressed. As an example, we have observed that some of the recent research is addressing problems that are less grounded in real operations but rather focuses more on theoretical contributions. This trend creates a need for benchmark data sets for the different types of industrial and tramp shipping problems, like those available in land-based transportation as well as the recent ones for network design in liner shipping (Brouer et al. [17]). Such datasets accommodate comparisons among competing solution approaches and would probably attract even more research interest to maritime transportation problems. Furthermore, in maritime transportation there is significant uncertainty in sailing and port times as well as in demand, freight rates, and other inputs. So far just a few studies have explicitly considered uncertainty.

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