

A fan of the Price is Right is trying to determine what prize he should try for if he is called onstage for his favorite game. In this game, contestants choose what they want to try to win and then spin a large, elevated wheel 10 times. Each spin of the wheel is independent. The "wheel" is really a 20-sided object, with 10 of its sides labeled \$20, 6 of its sides labeled \$50, 3 of its sides labeled \$100, and the remaining side labeled \$500. Each of the sides are equally likely to stop under the arrow-and therefore be the selection- on each spin.

1. One of the prizes the contestant can choose to win is the sum of the 10 rolls.

(a) Make a box model that will help you make computations about the sum of the 10 rolls.

*Box should have 20 tickets: 10 labeled 20, 6 labeled 50, 3 labeled 100, and 1 labeled 500.*

(b) What are the minimum and maximum values that any contestant who chooses this option can win?

*We will always draw 10 balls, and it is with replacement. so the min is 10 \* the smallest value and max is similar Min:  $10 * 20 = 200$ , Max:  $10 * 500 = 5000$*

(c) What is the expected value for the sum of the draws?

*Remember here we look at the number of rolls and the average value of a draw  $EV_{sum} = (Box.Avg)(n.draws)$ .*

$$Box.Avg = \frac{10*20 + 6*50 + 3*100 + 1*500}{20} = \frac{1300}{20} = 65.$$

$$So, EV_{sum} = 65 * 10 = 650.$$

(d) How much will the winnings typically vary with this outcome? ie, what is the standard error of the sum?

*A helpful thing to do first is consider the difference between Standard deviation and the Standard Error of the mean. The standard deviation describes how variable a single observation is. The standard error of the mean describes how variable a sample mean is in regard to the population mean.  $SE_{sum} = \sqrt{n.draws} * SD.Box$ .*

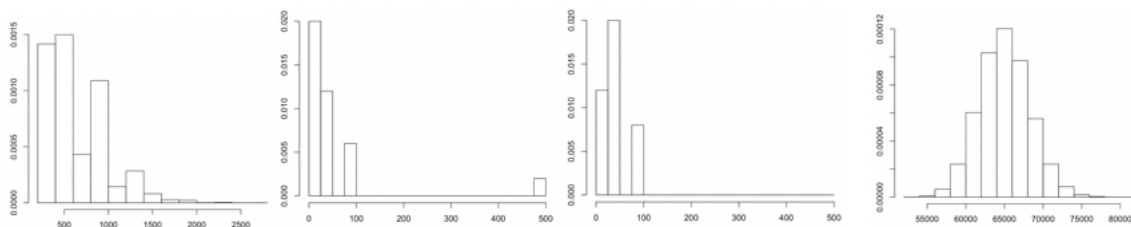
$$SD.Box = \sqrt{\frac{10*(20-65)^2 + 6*(50-65)^2 + 3*(100-65)^2 + 1*(500-65)^2}{20}} = 103.56$$

*Make sure to note why we're dividing by 20. We "know" the population. We know the values of all of the outcomes and how often each outcome comes up- so we divide by n.*

$$SE_{sum} = \sqrt{10} * 103.56 = 327.49$$

*This means we should expect to win about \$650 give or take \$327.*

(e) Below there are 4 graphs. (i) the probability histogram of the population of values (ii) the empirical histogram of the values obtained in one set of 10 spins (iii) the sampling distribution for the sum of sets of 10 spins, and (iv) the sampling distribution for the sum of sets of 100 spins. Specify which is which and how you know.



*A couple things to consider before you begin. What is the sample size, or is this a population? Will CLT start to work on a given sample size. The first graph is (iii) the sampling distribution for the sum of sets of 10 spins - we notice it is right skewed. The sample size of 10 is not large enough for the distribution of sums to be normally distributed. The second graph is (i) the probability histogram of the population of values as we see the prizes in the same relative density as on the wheel. The third graph is (ii) the empirical histogram of the values obtained in one set of 10 spins, as there are the same*

*outcomes that are available on the wheel, but not in the same relative density as on the wheel. The last graph is (iv) the sampling distribution for the sum of sets of 100 spins. 100 is a large enough sample size, that the sampling distribution of the sums is approximately normal*

2. Another prize the contestant can choose to win is a mystery prize for each time they roll a \$100 or \$500. They will again have the chance to spin the wheel 10 times.

- (a) Make a box model that will help you make computations about the number of mystery prizes under this scheme.

*Here we will consider this a binary problem. You win the special magic or you don't. So here our outcomes are not necessarily quantitative. Our box should have 20 tickets: 4 labeled 1 (for success - 100s or 500) 16 labeled 0 (for failure - 20s or 50s).*

- (b) What is the expected value for the number of mystery prizes in 10 spins of the wheel under this scheme?

$$EV_{sum} = (Box.Avg)(n.draws).$$

$$Box.Avg = \frac{4*1 + 16*0}{20} = 0.2.$$

$$So, EV_{sum} = 0.2 * 10 = 2.$$

- (c) How much will the number of mystery prizes won vary between contestants who choose this scheme? ie, what is the standard error of the count of wins?

*To solve this problem, think of the probability of receiving magic prize. A binary framework would help when setting up these equations where p is the 4/20.  $SE_{sum} = \sqrt{n.draws} * SD.Box$ .*

$$SD.Box = (1 - 0) \sqrt{\frac{4}{20} \frac{16}{20}} = 0.4$$

*Notice we are using the shortcut for when we only have 2 outcomes*

$$SE_{sum} = \sqrt{10} * 0.4 = 1.26$$

*This means we should expect to win about 2 mystery prizes give or take 1.26 for 10 spins of the wheel.*