

1. A potato chip producer has just received a truckload of potatoes from its main supplier. If the producer finds convincing evidence that more than 8% of the potatoes in the shipment have blemishes, the truck will be sent away to get another load from the supplier. A supervisor selects a simple random sample of 500 potatoes from the truck. An inspection reveals that 47 of the potatoes have blemishes.

- (a) State null and alternative hypotheses relevant to determining whether the shipment should be returned.

The null is that the true percentage of blemished potatoes in the truckload is equal or less than 8%. The alternative is that the true percentage of blemished potatoes in the truckload is greater than 8%.

It is fine if you write the null hypothesis as: the true percentage of blemished potatoes in the truckload is equal to 8%. However, for this problem, it is better to write it as "less or equal" because we only care about if it is more than 8% or not.

- (b) Perform a test of the stated hypotheses at the 5% significance level. Make sure to compute the test statistic and P-value and make a conclusion in the context of the problem. *The observed percentage is $47/500 = 9.4\%$. Under the null, the EV of the percentage is 8%. Under the null, the SD of the box is $\sqrt{0.08 * 0.92} = 0.27$, so the SE of the percentage is $\frac{0.27}{\sqrt{500}} = 1.2\%$. Since the sample size is large, the approximate distribution of the sample percentage is normal, by the CLT. So we can do a z -test. The test statistic is $z = \frac{9.4\% - 8.0\%}{1.2\%} = 1.17$. Since the alternative is greater than, the P-value is the chance of falling to the right of this value under a standard normal curve. From the table, the chance of falling to the left of 1.17 is 0.8790, so the chance of falling to the right is $1 - 0.8790 = 0.121$, or 12.1%. Because the P-value is greater than 5%, we fail to reject the null. There is not convincing evidence that the shipment contains more than 8% blemished potatoes.*

The significance level 5% is used in many statistical tests. However, the significance level can be arbitrarily chosen and should be field-specific based on Type I error tolerance. You cannot rely on significance 5% too much.

- (c) What type of error *could* we have made given our conclusions in (b)? Describe the implications of that error in the context of the question. *We decided to not reject the null. The null is either true or false. If the null is true, we made the correct conclusion by not rejecting it. If the null is false, we should have rejected, so we made the wrong conclusion, and that would be a Type II error. If we decided the shipment was ok, but in reality, more than 8% of the potatoes were blemished, we'd end up with some low-quality potato chips.*

In general, we want to control Type II error because when this thing happens, it can be unacceptable to us. As the same reason, we often put the event that is expensive to reject in the null hypothesis and the event that is cheap in the alternative hypothesis.

2. At the Hawaii Pineapple Company, managers are interested in the sizes of the pineapples grown in the company's fields. Last year, the mean weight of the pineapples harvested from one large field was 31 oz. A different irrigation system was installed in this field after the growing season and managers want to know if there is convincing evidence that the mean weight of pineapples produced in the field is different this year. A sample of 50 pineapples from this year's crop has a sample mean weight of 31.9 oz and sample SD of 2.4 oz. You can assume the weights of all pineapples is approximately Normally distributed.

- (a) State null and alternative hypotheses appropriate to the company's question. *The null is that the mean weight of all pineapples grown in the field this year is equal to 31 oz. The alternative is that the mean*

weight of all pineapples grown in the field this year is not equal to 31 oz.

- (b) Use a z-test to make a decision about the hypotheses using a significance level of 1%. Make sure to compute the test statistic and p-value, and make a conclusion in context. *The sample mean was 31.9 oz and the sample SD was 2.4 oz. The EV of the average under the null is the average of the box, or 31 oz. The SD of the box is estimated by the SD of the sample, or 2.4 oz, so the SE of the average is $\frac{2.4}{\sqrt{50}} = 0.34$. Since sample size is large, the distribution of the sample average is approximately normal by the CLT. The test statistic is $z = \frac{31.9-31.0}{0.34} = 2.65$. This is a two-sided alternative, so the p-value is the chance that you fall to the right of 2.65 or to the left of -2.65 under a standard normal curve. From the table, the area to the left of 2.65 is 0.9960, so the area to the right is $1 - 0.9960 = 0.004$, or 0.4%, and this is also the area to the left of -2.65 , so the p-value is $2 * 0.4\% = 0.8\%$. This is less than our significance level of 1%, so we would reject the null, and conclude that the mean weight seems different this year, specifically, that it is a bit higher than last year.*

Notice that you need to determine the significance level before derivation of p-value. Otherwise, it is a p-value cheat.

- (c) Can we conclude that the new irrigation system *caused* a change in the mean weight of pineapples produced? Explain. *We cannot infer causation here, since there was no control field observed using the old system at the same time. It is possible that other things besides the irrigation system changed from last year to this year. For example, maybe the weather was better for pineapples this year.*

3. Indicate whether the questions below are True or False, and give a brief explanation.

- (a) The p-value is the probability that the null hypothesis is true. *False. The null is either true or it isn't. The p-value is the probability of a test statistic as or more extreme than what was observed, assuming the null is true.*
- (b) Smaller p-values are more evidence for the null hypothesis. *False. Smaller p-values are more evidence against the null, since a small p-value means the data we observed was unlikely if the null was true.*
- (c) If we do not find sufficient evidence against the null, we say that we accept the null. *False. We assume the null from the start, so if we do not have evidence against the null, this is not the same as finding sufficient evidence for the null. We can say that we fail to reject the null.*
- (d) The observed significance level will change depending on the particular sample taken. *True. The observed significance level is the same as the p-value, and the p-value is computed based on the data from the sample, so different samples will give different observed significance levels.*