1. Consider a box that has a 25% chance of giving a 6, a 70% chance of giving an 8, and a 5% chance of giving a 25. What is the probability of having a sum greater than 600 if we draw 80 times from the box with replacement?

   This question is about the distribution of sample sum. So we apply CLT to solve it.

   First check the requirement of CLT: sample is drawn with replacement and sample size 80 is large enough (≥ 30). CLT can be applied here.

   $EV_{mean} = ave(box) = 25\%(6) + 70\%(8) + 5\%(25) = 8.35.$

   $EV_{sum} = n * EV_{mean} = (8.35)(80) = 668.$

   $SD(box) = \sqrt{25\%(6-8.35)^2 + 70\%(8-8.35)^2 + 5\%(25-8.35)^2} = 3.92.$

   $SE_{sum} = SD(box) * \sqrt{n} = (3.92)\sqrt{80} = 35.06.$

   Our z-score is then $\frac{600-668}{35.06} = -1.94$*, and the probability of having a greater z-score is 0.974, according to the z-table.*

   See similar questions in Discussion 6.

2. After making her recovery, Snow White wants to prove that eating apples in her kingdom is not worth it by showing that at least 20% of them have been poisoned by an evildoer. She finds a simple random sample of 150 apples and find that 80 have been poisoned.

   (a) State hypothesis for this test.

   H_0*: 20% of all apples are poisoned.*

   H_A*: More than 20% of all apples are poisoned.*

   (b) Compute the test statistic and p-value. The test statistic should be a z or t test statistic, whichever is appropriate.

   The observed proportion is $\frac{80}{150} = 0.53$*, and the standard error of the proportion is* $\sqrt{\frac{(0.2)(0.8)}{150}} = 0.033$*, so the z test statistic is* $\frac{0.53-0.2}{0.033} = 10.2$*. Our p-value is the probability of finding a greater value, since 10.2 goes past the z-table and its smallest value is 0.0002, we can say the p-value is smaller than 0.0002 (corresponding to 3.49):* $p-value = P(z > 10.2) < P(z > 3.49) = 0.0002.$
   If we use a t-test, the t test statistic is also 10.2 which goes past the t-table. So the p-value is smaller than 0.005.

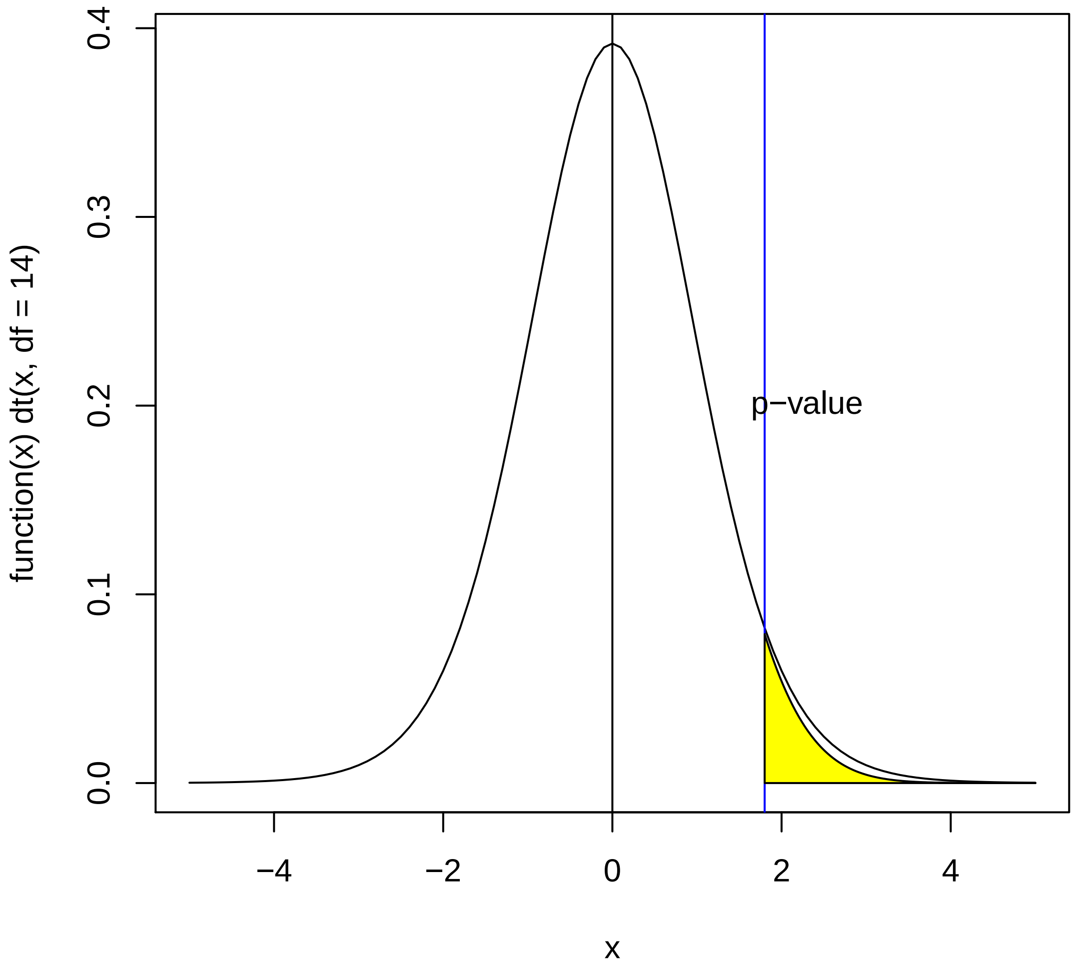
   (c) Reach your conclusions at the 5% significance level.

   The p-value is much smaller than 5%, so we reject the null. We have evidence that the proportion of poisoned apples in the kingdom is greater than 20%.

3. When running a t-test for means, you find a test statistic of 1.8. Your sample size is 15. What is the p-value if the alternative hypothesis is...

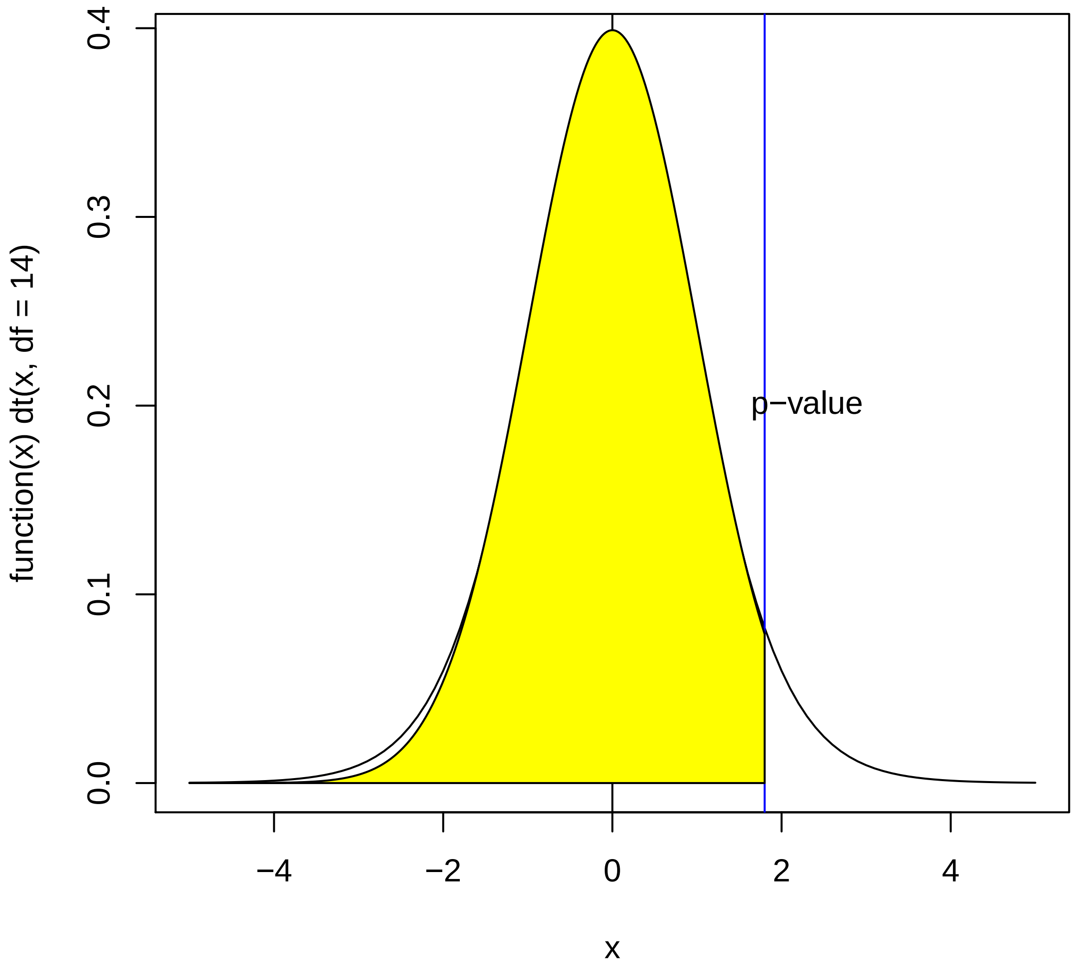
   (a) Greater than?

   Our sample size is 15, so we have 14 degrees of freedom. 1.8 is between 1.761 and 2.145, so the one-sided p-value when the direction of the test statistic and alternative match is between 0.025 and 0.05.



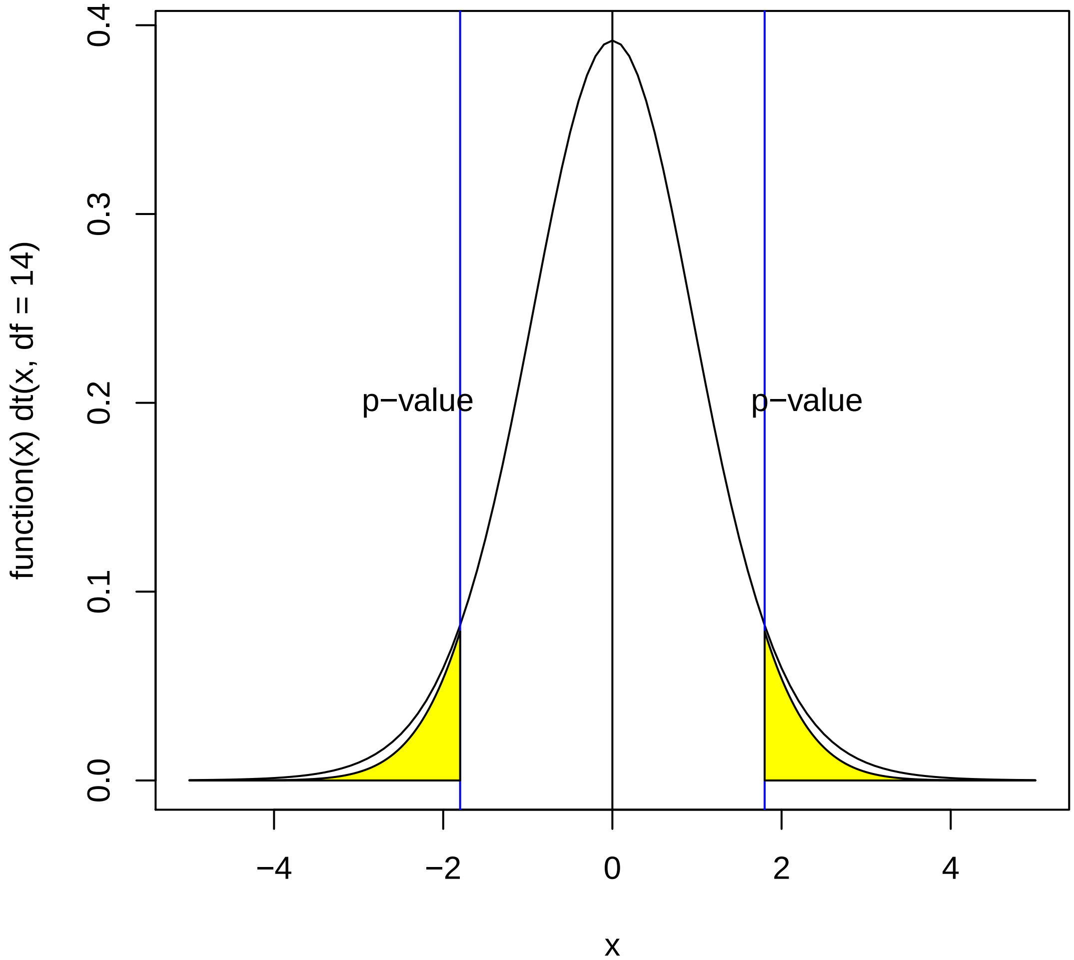
(b) Less than?

Here the alternative hypothesis are in the opposite direction, so we need to find complement. The p-value is between 0.975 and 0.95.



(c) Not equal to?

We need to double the one-tailed p-value here, this will give a p-value between 0.05 and 0.1.



4. Being a foresighted Viking chief, Thorgest would like to know how many coins he should expect from raiding one village. He chooses 87 villages as an SRS from all villages with populations under 1000 people. Each time he raids a village, he records the number of coins he found. The mean of his sample is 178 coins, and the SD is 46 coins. Create and interpret 80% confidence interval for the population mean number of coins per village.

The study was performed using a SRS and the number of draws 87 is large enough to apply the CLT, so the CI is given by:

$$Point\ estimate \pm standard\ error * confidence\ multiplier.$$

$Point\ estimate = sample\ mean = 178.$
$standard\ error = SE_{mean} = \frac{SD}{\sqrt{n}} = \frac{46}{\sqrt{87}} = 4.93.$
$confidence\ multiplier = Z_{90} = 1.29$
$4.93 * 1.29 = 6.31$
So the 80% confidence interval for the population mean number of coins per village is $[178-6.31, 178+6.31] =$

$[171.69, 184.31]$. *This means that we are 80% confident that the interval from 171.69 to 184.31 covers the true population mean number of coins per village.*
More questions about the calculation and computation of confidence interval are ins Discussion 7.