

1. You have a large toy box. Inside, there are 30 action figures, 14 cars, and 10 stuffed animals that are all the same size. Answer the questions below.

- (a) Create a box model to represent the situation of picking one toy at random from the box assuming each toy is equally likely to be selected. *Each ticket is equally sized and will list a toy. You would have one ticket for each toy, so there would be 54 tickets, 30 with action figures, 14 with cars, and 10 with stuffed animals. That's your box model.*

This box model is a way of generalizing the situation into its key elements. By using a Box model, you can model many different situations, not just this one.

- (b) Using your box model from (a), what is the chance of choosing an action figure if you select one toy at random (assuming each toy is equally likely to be selected)? *There are 30 action figure tickets, and 54 total tickets, so the chance is $\frac{30}{54} = \frac{15}{27} = 55.6\%$.*
- (c) Suppose you draw two toys with replacement. What is the chance you get any two stuffed animals in a row (assuming each toy is equally likely to be selected)? *Since we're drawing with replacement, the draws are independent. In other words, the chance of drawing a stuffed animal on the first draw is exactly the same as on the second. So we can multiply the probabilities together to get $(10/54) * (10/54) = 3.43\%$.*
- (d) Suppose you draw three toys without replacement. What is the chance you get a stuffed animal, then an action figure, then another stuffed animal (assuming each toy is equally likely to be selected)? *Since we're drawing without replacement, the draws are not independent. We need to multiply the conditional chances, $(10/54) * (30/53) * (9/52) = 0.01814 = 1.8\%$. Note that the denominator reduces by one each time since a toy is taken out of the box. Also, notice how the number of possible stuffed animals is reduced by 1, from 10 to 9, on the third draw.*

2. A large airport services only two airlines, A and B. They have kept records of flight delays for a long period of time. Based on past data, if a flight is chosen at random from the flights in a given day, the probabilities of some events are below.

Event	Probability
The flight is from Airline A	40%
The flight is from Airline A and is delayed	20%
The flight is from Airline B and is not delayed	35%
The flight is delayed	45%

Suppose you select one flight at random from the flights for a day. Answer the questions below.

- (a) Create a two way table to represent the situation.

	A	B	Total
Delayed	20	25	45
Not Delayed	20	35	55
Total	40	60	100

Notice how we make everything out of 100? That makes the math easier. We know from the first row of the given table that there's a 40% chance the flight is from airline A, so we know that the sum of the first column is 40. Using the second piece of information, we can fill a 20 in for the first row and the first column, and then subtract from 40 to get 20 in the second row, first column.

Also, note that since we knew that there are only two airlines, knowing the probability a flight was on airline A, we know the probability a flight is on airline B is 60%. Using our third line in the given table makes us realize that we know the second row, second column entry is 35. So as it turns out, we didn't even need the last entry in the given table, since we can figure out from subtraction that the first entry in the second column is 25. Having said that, we can use the last entry to check our work and realize that the sum of the first row (which is the probability of a delay) is indeed 45%. A good thing to do in these problems is to make sure all of the entries (excluding the Total rows and columns, which should each add up to 100% as well) in the table add up to 100%. If it doesn't, go back and check your work!

- (b) What is the probability the flight is from Airline B? *If the flight is from Airline B, it is not from A, and since there are only 2 possible airlines, the probability is $100\% - 40\% = 60\%$. This matches what we get from the 2-way table, or $60/100$.*
- (c) If you know the flight is from Airline A, what is the probability it is delayed? *This is conditioning on Airline A. The advantage of creating a two-way table is that you can simply read down the first column. So it's the chance of being A and delayed, over the chance of being A, or $\frac{20}{40} = 50\%$.*
- (d) If you know the flight is from Airline B, what is the probability it is delayed? *Compute the probability that the flight is delayed given Airline B, which is the chance of Airline B and delayed over Airline B, which is $\frac{25}{60} = 0.4166667 = 41.7\%$.*
- (e) If you know the flight is delayed, what is the probability that it is from Airline B? that it is from Airline A? *Compute the probability that the flight is from Airline B given that it is delayed: $25/45 = 0.5555556 = 55.6\%$. Compute the probability that the flight is from Airline A given its is delayed: $20/45 = 0.4444444 = 44.4\%$ Note that we could have also done $100 - 55.6 = 44.4$ since these two add up to be the total percent of flights delayed.*
- (f) If you were flying out of this airport and it was important that your flight was not delayed, which airport should you choose? Why? *The chance of delay given Airline A was computed in (c) to be 50%. The chance of delay given Airline B was computed in (d) to be 41.7%, so Airline B gives a smaller chance of delay (a smaller percent of its flights are delayed), and therefore a larger chance of an on-time flight. You should choose Airline B.*
- (g) Are airline and delay status independent? Why? *They would be independent if the chance of being A and delayed was the product of the separate probabilities. But $0.40 * 0.45 = 18\%$, which is not the same as the stated value of 20%, so they are not independent. You could also check whether the conditional probability of delay = the unconditional probability of delay. We see that the chance of delay: $P(\text{Delay}) = 45\%$ and the chance of delay given Airline A was computed in (c) to be 50%. Since these values are not equal, delay status and airline are not independent. Suppose i and j are two events, to check independency of i and j , we need to check $P(i,j) = P(i)P(j)$.*

3. A die is rolled 10 times. Find the chance of:

- (a) getting 10 sixes $(1/6)^{10} = 1/60466176 \approx 0$

We know that it's hard to get 10 sixes in a row, so before calculating anything, we should expect this probability to be close to 0. We also know that dice rolls are independent, so we can simply multiply the chance of rolling 1 six (which is $1/6$), together 10 times to get the answer.

- (b) not getting 10 sixes $1 - 1/60466176 \approx 1$

Remember that in probability, if we know the chance of something happening, we already know the chance of it not happening because the two sum to 1. Thus, we can subtract our answer from a from 1 to get a number very close to 1.

- (c) all the rolls showing 5 or fewer spots $(5/6)^{10} \approx 0.16$, or 16%

This follows similar logic to our answer from 3a. The chance of any one roll showing a 1-5 is $5/6$, so as in 3a (since the rolls are independent), we raise $(5/6)$ to the power of 10 to arrive at our answer.