1. Trace metals in drinking water affect the flavor and an unusually high concentration can pose a health hazard. Ten pairs of data were taken measuring zinc concentration in bottom water and surface water.

Location:	1	2	3	4	5	6	7	8	9	10
Zinc conc. in bottom	0.43	0.27	0.57	0.53	0.71	0.72	0.65	0.59	0.47	0.72
Zinc conc. in top	0.42	0.24	0.39	0.41	0.61	0.61	0.63	0.52	0.41	0.61
Bottom minus Top	0.01	0.03	0.18	0.12	0.10	0.11	0.02	0.07	0.06	0.11

Does the data suggest that the true average concentration in the bottom water exceeds that of surface water? Carry out the appropriate test.

This is a one-sided paired t-test. It's paired by location and has a small sample size (n=10). We are interested in the hypotheses: $H_0 : \mu_{Bottom-Top} = 0$ vs. $H_1 : \mu_{Bottom-Top} > 0$, where μ_d is the mean difference between the bottom and surface water Bottom-Top. First, we will calulate the set of differences and this will be the sample that we do the test with. We will define these as Diff = Bottom - Top.

The average of the differences is 0.081, and the sample standard deviation of the differences is 0.053. Using this, the test statistic can be computed as follows: $t = \frac{\text{observed mean(difference) - null mean(difference)}}{SE_{\text{mean(difference)}}} = \frac{0.081}{0.053/\sqrt{10}} = 4.83$. The degrees of freedom is equal to $n - 1 = 10 - 1 = 9$. Using this information, we can look up our corresponding p-value, which is $\mathbb{P}(t_9 > 4.83) < 0.0005$. In conclusion, we reject the null hypothesis in favor of the alternative. We have gathered statistically significant evidence in support that the average concentration in the bottom water exceeds that of the surface water.

2. In a typical year, out of 365 days, about 250 days are weekdays, 105 days are weekends, and 10 days are U.S. Federal holidays (if a holiday falls on a weekend, it is 'observed' on a weekday). It is desired to know whether the distribution of births matched the distribution of days in a year. A simple random sample of 400 students at a large university were asked whether they were born on a weekday, weekend, or (observed) holiday.

   (a) State null and alternative hypotheses relevant to the question. *The null is that the distribution of births matches the distribution of days in the year. The alternative is that the distribution of births does not match the distribution of days in the year.*

   (b) Suppose that from the 400 students sampled, 302 were born on a weekday, 92 were born on a weekend, and 6 were born on a holiday. Perform a test of the stated hypotheses at the 5% significance level. Make sure to choose an appropriate test statistic, compute the test statistic and P-value, and make a conclusion in the context of the problem. *If we think of this as a box model, under the null, the box has three kinds of tickets, weekday ($250/365 = 68.5\%$ of the tickets), weekend ($105/365 = 28.8\%$ of the tickets) and holiday ($10/365 = 2.7\%$ of tickets). Using this information, we can check how well our observed data fits these expected values using a χ^2 GOF test. The expected counts are the population percentages times the sample size (400) so they are 274.0, 115.2, and 10.8 for weekday, weekend, and holiday births, respectively. The statistic is $\chi^2 = \frac{(302-274)^2}{274} + \frac{(92-115.2)^2}{115.2} + \frac{(6-10.8)^2}{10.8} = 2.86 + 4.67 + 2.13 = 9.66$, on $3 - 1 = 2$ df. Since the expected counts are all greater than 5, we can compare to a χ^2 table, and the p-value is a little less than 1%. So we reject the null and conclude the the distribution of birth days does not match the distribution in the year. It seems people are more likely to be born on weekdays than expected.*

3. An experiment was conducted to compare three pesticides (call them A, B, and C) for use on alfalfa plants. The pesticides are designed to control aphids. 250 alfalfa plants were randomly assigned to each of the three

treatments. Pesticide A had 94 plants, B had 84, and C had 72. After 8 weeks, the plants were observed, and the aphids on each plant were counted. If a plant had 10 or fewer aphids, it was designated as 'successful control', and if it had more than 10 aphids, it was designated as 'failed control.' It was desired to know if the successful control percentages were equal for the three pesticides.

(a) State null and alternative hypotheses appropriate to the study question. *The null is that the three pesticides work equally well, so that the percentage showing successful control will be the same for all three. The alternative is that at least one of the pesticides works better or worse than the others.*

(b) Suppose that the number of successful control plants for pesticides A, B, and C, were 45, 40, and 38, respectively. Use an appropriate test to make a decision about the hypotheses using a significance level of 1%. Make sure to compute the test statistic and P-value, and make a conclusion in context. *Although it doesn't seem like it at first glance, this is data that is suitable for a* χ^2 *test of independence. The two factors are successful or failed control, and pesticide A, B, or C. The observed counts are as follows:*

	A	B	C	Total
Successful	45	40	38	123
Failed	49	44	34	127
Total	94	84	72	250

And the expected counts are:

	A	B	C	Total
Successful	46.25	41.33	35.42	123
Failed	47.75	42.67	36.58	127
Total	94	84	72	250

The test statistic is $\chi^2 = \frac{(45-46.25)^2}{46.25} + ... + \frac{(34-36.58)^2}{36.58} = 0.034 + 0.043 + 0.188 + 0.033 + 0.041 + 0.182 = 0.521$. *Since all the expected counts are greater than 5, we can compare to a* χ^2 *on* $(2-1)*(3-1) = 2$ *df. The observed statistic falls between 90% and 70%, (a computer gives 77.4%) so the P-value is greater than 1%, and we would not reject. We conclude that we have insufficient evidence that any of the pesticides work better or worse than the others.*