

1. Anne reads that the average price of regular gas in her state is \$4.06 per gallon. She thinks it's lower in her city. To test this, she selects an SRS of 8 gas stations and records the price per gallon for regular gas at each station. The data are below (in dollars):

4.13, 4.01, 4.09, 4.05, 3.97, 3.99, 4.05, 3.98

- (a) State null and alternative hypotheses appropriate to Anne's question. *$H_0 : \text{avgprice} = \4.06 The null is that the average gas price in her city is equal to \$4.06. $H_a : \text{avgprice} < \4.06 The alternative is that the average gas price in her city is less than \$4.06.*
- (b) Perform a test of the hypotheses you stated in part (a) at a significance level of 10%. You may assume that the distribution of gas prices in the city is approximately normal. Make sure to compute the test statistic and P-value, and make a conclusion in context. *Before we begin, let us consider when and where to use Z vs T tests. Here are a few guidelines. If your sample is less than 30 or the population variance is unknown, use a T test. If your sample is greater than 30, you can use a Z test. Now let us get into solving the problem! The sample average price is \$4.03, and the sample SD is \$0.06. The EV of the average is the average of the box under the null, which is \$4.06. The SD of the box can be estimated by the sample SD, or \$0.06, so the SE of the average is $\frac{\$0.06}{\sqrt{8}} = \0.02 . Since the sample size is small and the population is normal, we should use a T-test. The test statistic is $t = \frac{\$4.03 - \$4.06}{\$0.02} = -1.5$. The alternative is less than, so we need the chance of falling to the left of that value under a T distribution with $8 - 1 = 7$ df. This calculation is again $n-1$. 1.5 falls between 10% and 5%, so that is the P-value. At a significance level of 10%, we would reject the null, and conclude that the gas does seem cheaper in her city. A way to think about this is that we reject at any p value of 10 percent or less. We know the pvalue is between 10 and 5 percent. So we know to reject the null hypothesis.*
- (c) Anne sees an article in the paper that a census of all of the gas stations in Anne's city found an average price of \$4.02 per gallon. What error, if any, occurred in Anne's hypothesis test? *Even though the estimate of \$4.03 missed the truth by a little bit, Anne still correctly rejected the null. So no mistake was made. We know if a mistake was made by checking to see if the population parameter, the "truth" matches what our test determined.*
2. A study was conducted to determine whether smartphone use either positively or negatively affects the number of hours of sleep. A group of 300 volunteers was recruited, all of which owned smartphones. Of the 300, 100 were randomly assigned to have a device installed on their phone which limited the daily use time to a maximum of 2 hours. Call this the 'limited' group. The remaining 200 were allowed to use their phones as usual. Call this the 'normal' group. For two weeks, the volunteers in both groups were asked to keep a log of the number of hours of sleep they got per day. In the limited group, the sample average number of hours of sleep per day was 7.5 hrs, with a sample SD of 1.1 hrs. In the normal group, the sample average number of hours of sleep per day was 6.9 hrs, with a sample SD of 1.8 hrs.
- (a) State null and alternative hypotheses appropriate to the study question. *The null is that the average number of hours of sleep per day is the same in both groups. Since the study is interested in either positive or negative effects, they should use a two-sided alternative, that the average number of hours of sleep per day is not equal for the two groups.*

- (b) Use an appropriate test to make a decision about the hypotheses using a significance level of 1%. Make sure to compute the test statistic and p-value, and make a conclusion in context. *Since this is a randomized experiment, and the number of people in both groups is large enough for the CLT, we can use a two-sample Z-test. The observed difference in averages is $7.5 - 6.9 = 0.6$ hrs. This is the crucial problem here. We need to know how to find the SE for two groups with different SDs. We will use the square root law from the textbook! The SE of the average for the limited group is $\frac{1.1}{\sqrt{100}} = 0.11$, and the SE of the average for the normal group is $\frac{1.8}{\sqrt{200}} = 0.13$, so by the square root law, the SE of the difference of the average is $\sqrt{0.11^2 + 0.13^2} = 0.17$. The expected difference in the averages under the null is 0, so the test statistic is $z = \frac{0.6 - 0.0}{0.17} = 3.53$. This is a two-sided alternative, so the p-value is the chance that you fall to the right of 3.53 or to the left of -3.53 under a standard normal curve. From the table, the area to the left of 3.53 is greater than 0.9998, so the area to the right is less than $1 - 0.9998 = 0.0002$, or 0.02%, and this is also the area to the left of -3.53 , so the p-value is less than $2 * 0.02\% = 0.04\%$. This is much less than our significance level of 1%, so we would reject the null, and conclude that the average number of hours of sleep is not the same for the two groups. Specifically, the people who had their smartphone use limited had a greater number of hours of sleep per day.*