

Foreword

Statistical analysis carried out were done using R version 3.3.1 (2016-06-21), which is freely available from CRAN. You may find convenient to run R through RStudio. RStudio offers really great support for editing and running R scripts. You can even organize your work into a project, with version control and automatic reporting built on the fly. I personally choose to work like in the 80s—although I was just a kid at that time—with a simple text editor and an interactive shell available within few key presses. This is possible thanks to Emacs and the brilliant ESS mode.

I replicated all SAS code using SAS University Edition. It can run locally on your computer (using, e.g., Virtual Box) or directly in the cloud. I do not hold a personal licence for SAS (although I could get one from several universities where I am teaching) and so I found this solution particularly handy to compare SAS and R output.

In addition, I provide Stata code to replicate most if not all analyses described in this document. The code has been tested with Stata 13 but should work on any version > 10.

1 Analysis of Clinical Trials using SAS

The following analyses are based on Dmitrienko et al. [2005], with data available online at Analysis of Clinical Trials Using SAS: A Practical Guide.

1.1 The HAMD17 study

Context. This is a multicenter clinical trial comparing experimental drug vs. placebo in patients with major depression disorder. The outcome is the change from baseline after 9 weeks of acute treatment, and efficacy is measured using the total score of the Hamilton depression rating scale (17 items).

This is a classical application of unbalanced design and potential heterogeneity between clinical centres, where there is an unequal number of observations per treatment (here, drug by center).

Here is one of many ways to get data right into R:

```
raw <- textConnection("
100 P 18 100 P 14 100 D 23 100 D 18 100 P 10 100 P 17 100 D 18 100 D 22
100 P 13 100 P 12 100 D 28 100 D 21 100 P 11 100 P 6 100 D 11 100 D 25
100 P 7 100 P 10 100 D 29 100 P 12 100 P 12 100 P 10 100 D 18 100 D 14
101 P 18 101 P 15 101 D 12 101 D 17 101 P 17 101 P 13 101 D 14 101 D 7
101 P 18 101 P 19 101 D 11 101 D 9 101 P 12 101 D 11 102 P 18 102 P 15
102 P 12 102 P 18 102 D 20 102 D 18 102 P 14 102 P 12 102 D 23 102 D 19
102 P 11 102 P 10 102 D 22 102 D 22 102 P 19 102 P 13 102 D 18 102 D 24
102 P 13 102 P 6 102 D 18 102 D 26 102 P 11 102 P 16 102 D 16 102 D 17
102 D 7 102 D 19 102 D 23 102 D 12 103 P 16 103 P 11 103 D 11 103 D 25
103 P 8 103 P 15 103 D 28 103 D 22 103 P 16 103 P 17 103 D 23 103 D 18
103 P 11 103 P -2 103 D 15 103 D 28 103 P 19 103 P 21 103 D 17 104 D 13
104 P 12 104 P 6 104 D 19 104 D 23 104 P 11 104 P 20 104 D 21 104 D 25
104 P 9 104 P 4 104 D 25 104 D 19
")
d <- scan(raw, what = "character")
rm(raw)
d <- as.data.frame(matrix(d, ncol = 3, byrow = TRUE))
names(d) <- c("center", "drug", "change")
d$change <- as.numeric(as.character(d$change))
d$drug <- relevel(d$drug, ref = "P")
```

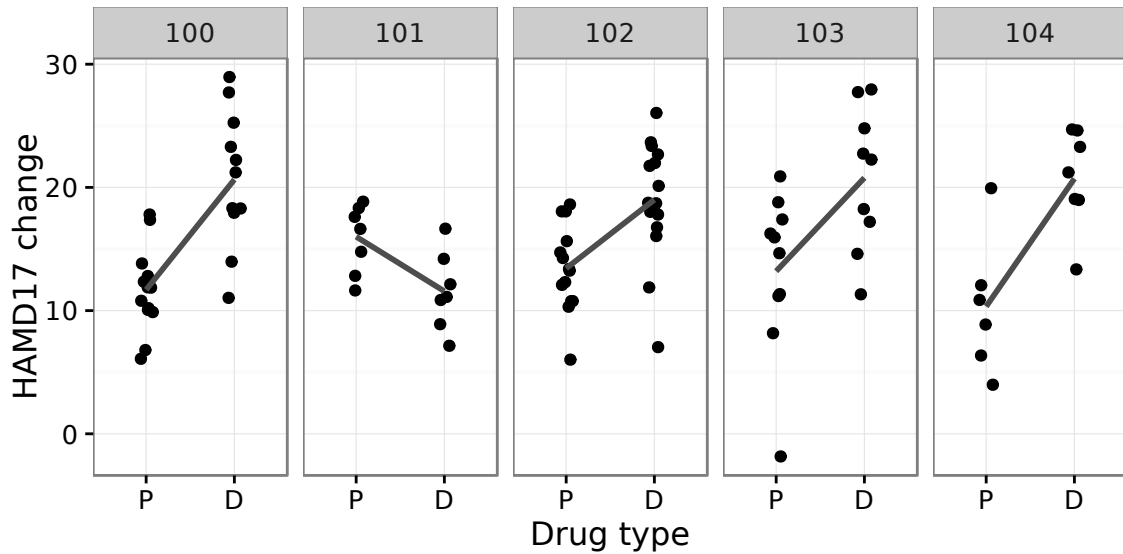


Figure 1: Distribution of change scores in each centre

Briefly, the idea is to copy and paste the SAS DATALINES instruction as raw text and to scan the flow of characters. The next bit of code uses matrix to arrange the data into a tabular dataset with 3 columns corresponding to center, drug and change score. When transforming this table to a data frame, center and drug will be converted to factors but we need to handle the proper conversion of change to numerical values. Also, note that we set the reference category to the Placebo group to simplify things a bit.

Some basic exploratory graphical analysis follows. In the next chunk, we display the raw data for each centre and highlight the difference between drug and placebo using a trend line (Figure 1). Note the use of `aes(group = 1)` when calling `geom_smooth` as there is no real grouping variable in the data structure other than the ones that are already used (drug on the x-axis and center for facetting).

```
p <- ggplot(data = d, aes(x = drug, y = change))
p <- p + geom_jitter(width = .2)
p <- p + geom_smooth(aes(group = 1), method = "lm", se = FALSE, colour = "grey30")
p + facet_grid(~ center) + labs(x = "Drug type", y = "HAMD17 change")
```

Using Hmisc package, we can easily build a Table of summary statistics by drug and center. For simplicity, we will limit the display to the first 3 centers in Table 1.

```
fm <- change ~ drug + center
s <- summary(fm, data = subset(d, center %in% c("100", "101", "102")),
             method = "cross", fun = smean.sd)
```

Table 1: Mean HAMD17 change by drug, center

drug	100			101			102			Total		
	N	Mean	SD	N	Mean	SD	N	Mean	SD	N	Mean	SD
P	13	12	3.4	7	16	2.7	14	13	3.6	34	13	3.6
D	11	21	5.6	7	12	3.3	16	19	4.7	34	18	5.7
Total	24	16	6.3	14	14	3.7	30	16	5.0	68	16	5.3

Only 3 out of 5 centres are shown.

Now, let's consider average change scores by center, which are displayed in Figure 2. First, we need to compute the average score in each group, and then compute the difference between the

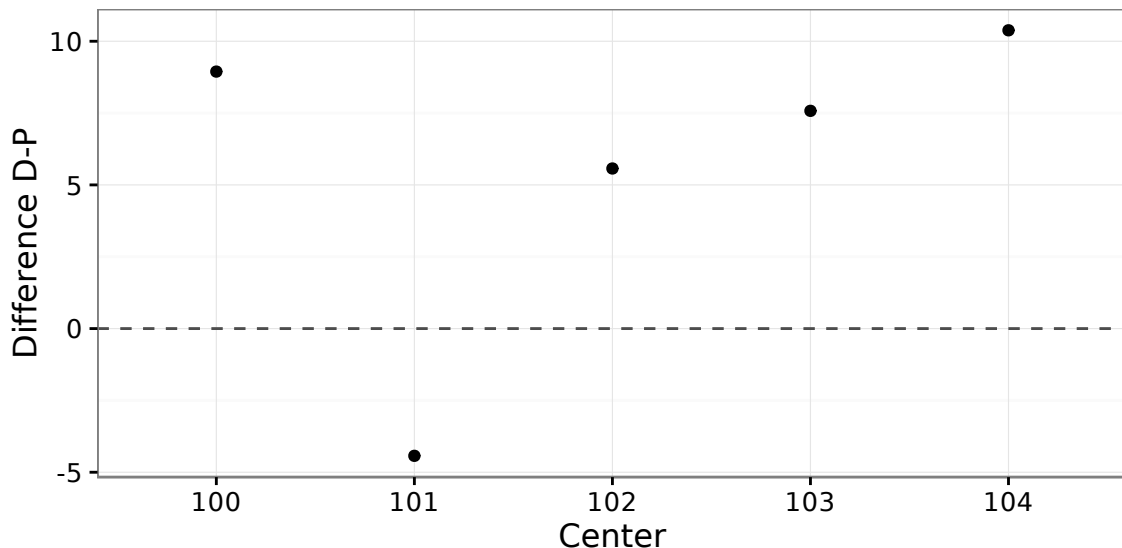


Figure 2: Average difference between drug and placebo in each centre

two (called delta). This could be done with Hmisc summarize command, but we will rely on the plyr package and its ddply command. What is important is that the results are returned as a data frame to facilitate the use of ggplot data structure in turn.

```
r <- ddply(d, "center", summarize,
           delta = mean(change[drug == "D"]) - mean(change[drug == "P"]))
p <- ggplot(data = r, aes(x = center, y = delta))
p <- p + geom_point() + geom_hline(yintercept = 0, linetype = 2, colour = "grey30")
p + labs(x = "Center", y = "Difference D-P")
```

Now comes the modeling stage. First, we will analyse the primary endpoint using fixed-effect models. Dmitrienko et al. [2005] provide all the maths that are necessary to understand how to derive various types of sum of squares, and this is further addressed in, e.g., REF, or on Stack Exchange.

Let us first update the formula we used for producing Table 1 to incorporate an interaction term, drug:center (in R, drug * center will expand to drug + center + drug:center):

```
fm <- change ~ drug * center

replications(change ~ drug:center, data = d)

## $`drug:center`
##   center
## drug 100 101 102 103 104
##   P   13   7  14  10   6
##   D   11   7  16   9   7
```

As can be seen, data are slightly imbalanced for all but centre 101.

By default, R computes so-called “sequential” Type I sum of squares (SS), and here is what we get when using a standard combination of lm (to compute parameter estimates) and anova (to build the ANOVA table for the regression model):

```
options(contrasts = c("contr.sum", "contr.poly"))
m <- lm(fm, data = d)
anova(m)

## Analysis of Variance Table
##
## Response: change
##           Df Sum Sq Mean Sq F value    Pr(>F)
```

```
## drug      1  888.04  888.04 40.0745 9.365e-09 ***
## center    4   87.14   21.78  0.9831 0.4209278
## drug:center 4  507.45  126.86  5.7249 0.0003761 ***
## Residuals 90 1994.38   22.16
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The car package allows to work with Type II and Type III SS. Type III SSs, also called partial or Yates' weighted squares of means are the default in Stata, SPSS or SAS. Stata does not even offer Type II SS. So, if we are interested in computing Type II sum of squares in R, we could call Anova like this:

```
car::Anova(m, type = "II")

## Anova Table (Type II tests)
##
## Response: change
##           Sum Sq Df F value    Pr(>F)
## drug      889.78  1 40.1528 9.109e-09 ***
## center    87.14   4  0.9831 0.4209278
## drug:center 507.45  4  5.7249 0.0003761 ***
## Residuals 1994.38 90
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Type III analysis is readily obtained by replacing type = "II" with type = "III" as shown in the next code block. It should be noted that without altering the default contrast treatment that are used by R, as we did in the above chunk, we would not get the correct results for the Type III analysis.

```
car::Anova(m, type = "III")

## Anova Table (Type III tests)
##
## Response: change
##           Sum Sq Df F value    Pr(>F)
## (Intercept) 22344.6  1 1008.3442 < 2.2e-16 ***
## drug        709.8   1  32.0320 1.783e-07 ***
## center      91.5    4  1.0318 0.3953130
## drug:center  507.4   4   5.7249 0.0003761 ***
## Residuals   1994.4 90
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Note that in the case of Type III SS, we can also use the base command drop1 and we will get similar results:

```
drop1(m, scope = ~ ., test = "F")

## Single term deletions
##
## Model:
## change ~ drug * center
##           Df Sum of Sq  RSS    AIC F value    Pr(>F)
## <none>                 1994.4 319.29
## drug      1    709.82 2704.2 347.74 32.0320 1.783e-07 ***
## center    4     91.46 2085.8 315.78  1.0318 0.3953130
## drug:center 4    507.45 2501.8 333.96  5.7249 0.0003761 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

To sum up, the results from the different approaches are exposed in Table 2.

Sidenote. Here is how we could compute the parameter estimates and the SS corresponding to the drug effect in the case of a Type III analysis. The code follows that posted on Stack Exchange, with minor adaptation. How this works is quite simple: We first get the design matrix stored in our model `m` and then solve the “normal equations” $(X'X)\hat{\beta} = X'y$ in order to get $\hat{\beta} = (X'X)^{-1}X'y$.

```
D <- model.matrix(m)                                ## design matrix
bhat <- solve(t(D) %*% D) %*% t(D) %*% d$change     ## beta parameters
get.ss <- function(C) {
  require(MASS)
  teta <- C %*% bhat
  M <- C %*% ginv(t(D) %*% D) %*% t(C)
  SSH <- t(teta) %*% ginv(M) %*% teta
  return(as.numeric(SSH))
}
## SS(drug|center, drug:center)
get.ss(matrix(c(0,1,0,0,0,0,0,0,0,0), nrow = 1, ncol = 10))
## [1] 709.8196
```

Table 2: Overview of fixed-effects analysis for the HAMD17 study

(a) Type I SS						(b) Type II SS					(c) Type III SS				
	Df	Sum Sq	Mean Sq	F value	Pr(>F)		Sum Sq	Df	F value	Pr(>F)		Sum Sq	Df	F value	Pr(>F)
drug	1	888.04	888.04	40.07	0.0000	drug	889.78	1	40.15	0.0000	drug	709.82	1	32.03	0.0000
center	4	87.14	21.78	0.98	0.4209	center	87.14	4	0.98	0.4209	center	91.46	4	1.03	0.3953
drug:center	4	507.45	126.86	5.72	0.0004	drug:center	507.45	4	5.72	0.0004	drug:center	507.45	4	5.72	0.0004
Residuals	90	1994.38	22.16			Residuals	1994.38	90			Residuals	1994.38	90		

The authors later used the Gail-Simon test[Gail and Simon, 1985] to test for qualitative interaction between treatment and strata. The corresponding two-tailed Likelihood ratio test is implemented in the QualInt package.

```
library(QualInt)
with(d, qualint(change, drug, center, test = "LRT"))

##
## Call:
## qualint(y = change, trtment = drug, subgrp = center, test = "LRT")
##
## Type:
## continuous
##
## Estimating Results for Mean Difference:
##      Estimate Std. Error Lower CI Upper CI
## 100    -8.944      1.922  -12.711  -5.177
## 101     4.429      1.601   1.290   7.567
## 102    -5.571      1.517  -8.544  -2.599
## 103    -7.578      2.877 -13.217  -1.939
## 104   -10.381      2.793 -15.856  -4.906
##
## Test:
## LRT
##
## p-value:
## 0.02968
##
## Power:
## 0.5797
##
## Alpha:
## 0.05
```

This R package even provides a graphical method when specifying options `test = "IBGA"` and

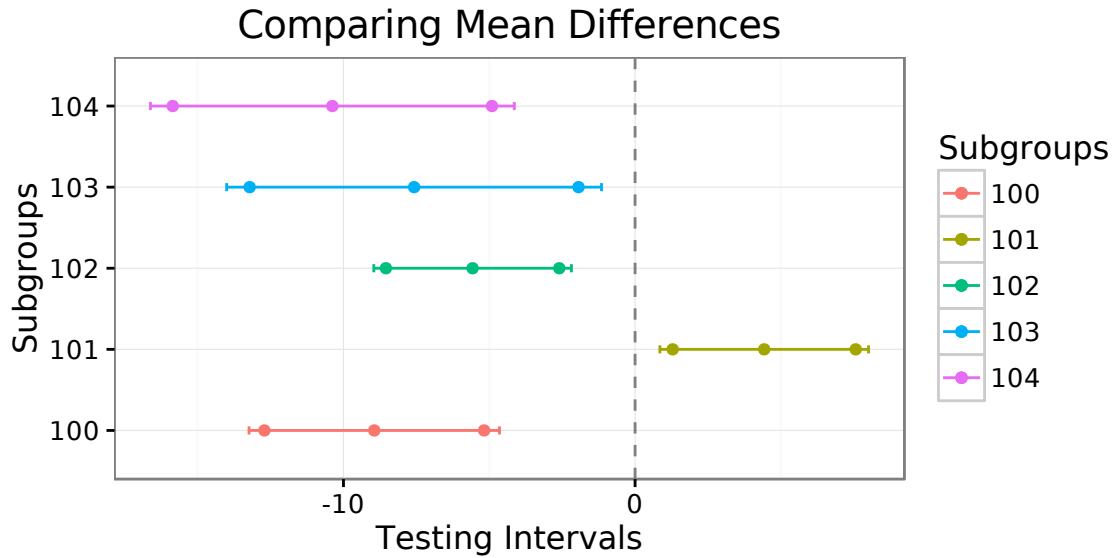


Figure 3: Average differences between drug and placebo stratified by centres

plotout = TRUE (Figure 3). The IBGA method relies on simultaneous 95% confidence intervals as described in Pan and Wolfe [1997].

1.2 The Urinary incontinence trial

Context. This is a subset of data collected in an RCT on urinary incontinence where the primary endpoint was the percent change from baseline of number of incontinence episodes per week over an 8-week period. Patients were initially randomized into one of three strata depending on the baseline frequency of incontinence episodes.

This is an example of the use of stratified non-parametric analysis.

This time, we managed to get data in the right format using this little R script: `urininc.R`. Assuming it is located in the current working directory, we can source it into R and we will get a data frame named `d`.

```
source("./urininc.R")
str(d)

## 'data.frame': 200 obs. of 3 variables:
## $ group : Factor w/ 2 levels "Placebo","Drug": 1 1 1 1 1 1 1 1 1 ...
## $ strata: Factor w/ 3 levels "1","2","3": 1 1 1 1 1 1 1 1 1 ...
## $ change: num -86 -38 43 -100 289 0 -78 38 -80 -25 ...
```

To summarize the data, we can again make use of `Hmisc` summary for “crossed” data.

```
s <- summary(change ~ group + strata, data = d, method = "cross", overall = FALSE)
```

Table 3: Mean change in number of incontinence episodes by drug, strata

group	1			2			3		
	N	Missing		N	Missing		N	Missing	
Placebo	40	0	-29.0	32	8	-28.7	20	0	-11.7
Drug	39	1	-24.2	33	7	-53.8	19	1	-47.7

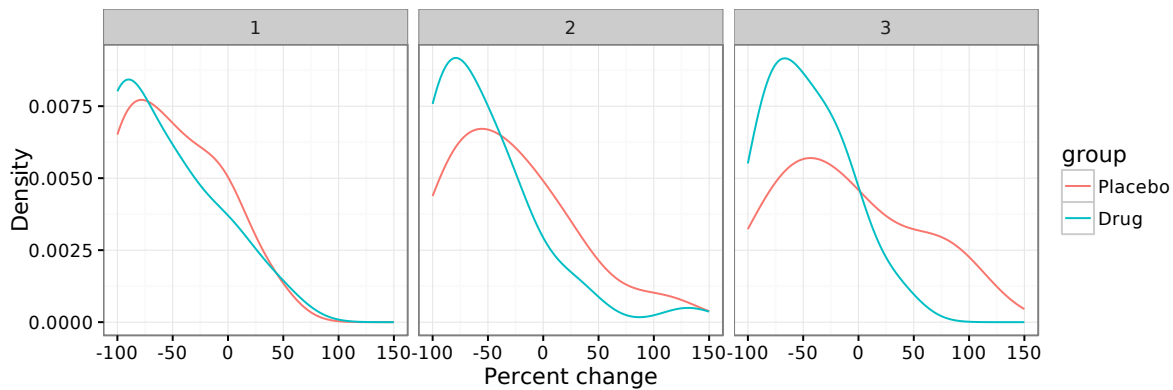


Figure 4: Density estimates for the percent change in frequency of incontinence episodes

As can be seen, there is a higher number of missing values in strata 2 (around 20% in both groups) and larger variations on average between the two group in the third strata. Next, we displayed the distribution of the percent change in frequency of incontinence episodes as density curves in Figure 4. Instead of relying on `geom_density`, we use the rather generic `geom_lines` with an extra `stat=` parameter.

```
p <- ggplot(data = d, aes(x = change, colour = group))
p <- p + geom_line(stat = "density", adjust = 1.2) + facet_grid(~ strata)
p + scale_x_continuous(limits = c(-100, 150)) + labs(x = "Percent change", y = "Density")
```

The authors used the van Elteren test [van Elteren, 1960], which can be regarded as an extension of the Wilcoxon rank sum test for stratified data where larger weights are assigned to rank sums from smaller strata. An alternative is the “aligned rank test” proposed by Hodges and Lehman [1962] as discussed by Mehrotra et al. [2010]. In R, there is an old version that is mentioned on the R listserve (August 2005), but for now we will use the `coin` package as shown below:

```
library(coin)
dc <- subset(d, complete.cases(d))
independence_test(change ~ group | strata, data = dc,
  ytrafo = function(data) trafo(data, numeric_trafo = rank,
    block = dc$strata),
  teststat = "quad")

##
## Asymptotic General Independence Test
##
## data: change by
## group (Placebo, Drug)
## stratified by strata
## chi-squared = 4.7446, df = 1, p-value = 0.02939
```

Although we get different results from the authors, we would reach the same conclusion, namely that there is an effect of the treatment on the outcome after adjusting for the centre effect. We will get, however, closer results ($p = 0.02369$ for the row mean squares test statistic) if we simply remove the `scores=` option when calling SAS PROC FREQ [Stokes et al., 2012]:

```
PROC FREQ;
  TABLES strata*group*change / noprint cmh2;
RUN;
```

In comparison, as noted by the authors, a Type III ANOVA would yield non-significant result about the effect of drug on change scores.

```
m <- lm(change ~ group + strata, data = d)
car::Anova(m, type = "III")
```

```
## Anova Table (Type III tests)
##
## Response: change
##           Sum Sq Df F value    Pr(>F)
## (Intercept) 176982  1 25.7904 9.499e-07 ***
## group        9602  1  1.3993  0.2384
## strata       8094  2  0.5898  0.5555
## Residuals   1228358 179
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

1.3 The Severe sepsis trial

Context. This is a placebo-controlled RCT examining the effect of an experimental drug on 28-day all-cause mortality in patients with severe sepsis. Patients were allocated to one of four strata depending on their APACHE II score [Knaus et al., 1985].

This is a classical application of stratified analysis of a binary outcome (dead/alive).

To enter the data in R, we will input individual values of the three-way Table of events as an array. Note that it would also be possible to create two matrix objects and then bind into to a 3-dimensional table. In what follows, we write data for the treated group first. Note that when using array, data should be entered column-wise (there is no byrow = option as in matrix).

```
varnames <- list(strata = 1:4,
                 status = c("Dead", "Alive", "Total"),
                 group = c("Experimental", "Placebo"))

d <- array(c(33,49,48,80,185,169,156,130,218,218,204,210,
            26,57,58,118,189,165,104,123,215,222,162,241),
          dim = c(4,3,2), dimnames = varnames)
```

Note also that the third column ("Total") can be safely omitted as margins can be computed automatically with R, e.g.:

```
addmargins(d[, -3, ], c(1,2))
```

```
d <- d[, -3, ]
dim(d)
## [1] 4 2 2
```

An alternative representation of this array-based Table is provided by R's flat tables (ftable), in long or wide format; see Table 4 for the wide format using ftable(d, row.vars = 1, col.vars = c(3,2)):

```
ftable(d)

##           group Experimental Placebo
## strata status
## 1      Dead           33          26
##       Alive          185         189
## 2      Dead           49          57
##       Alive          169         165
## 3      Dead           48          58
##       Alive          156         104
## 4      Dead           80         118
##       Alive          130         123
```

The following code is used to depict the situation in graphical terms:

```
dd <- as.data.frame(ftable(d))
r <- ddply(dd, c("strata", "group"), mutate, prop = Freq/sum(Freq))
```


strata	group:	Experimental		Placebo	
	status:	Dead	Alive	Dead	Alive
1		33	185	26	189
2		49	169	57	165
3		48	156	58	104
4		80	130	118	123

Table 4: 28-day mortality data from the 1690-patient sepsis study

```
p <- ggplot(subset(r, status == "Dead"), aes(x = prop, y = group))
p <- p + geom_point() + facet_wrap(~ strata, nrow = 2)
p + scale_x_continuous(limits = c(0,0.5)) + labs(x = "Proportion deads", y = "")
```

```
library(vcd)
cotabplot(d, 1)
```

Based on a logistic regression model, the authors presented a summary of a Type III analysis of effects. Here is what can be done in R. First, we will slightly re arrange the data table so that we have a working data frame with total counts for success (here, dead patients) and failure (here, patients still alive) in separate columns, together with columns describing strata and treatment levels.

```
n <- rbind(d[,1:2,1], d[,1:2,2])
rownames(n) <- NULL
n <- as.data.frame(n)
n$strata <- gl(4, 1)
n$group <- gl(2, 4, labels = c("Experimental", "Placebo"))
n$group <- relevel(n$group, ref = "Placebo")
```

Then, since we are working with grouped or aggregated data, we will use the cbind() option to R's glm, as shown below. Note that we also ask to use SAS treatment contrast for the strata factor, in order to ensure that the fourth level is used as the reference category. Type III analysis is readily available within the car package.

```
m <- glm(cbind(Dead,Alive) ~ group + strata, data = n, family = binomial,
         contrasts = list(strata = "contr.SAS"))
car::Anova(m, type = "III")

## Analysis of Deviance Table (Type III tests)
##
## Response: cbind(Dead, Alive)
##          LR Chisq Df Pr(>Chisq)
## group      6.989  1  0.008201 **
## strata 105.609  3  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Finally, profile likelihood 95% confidence intervals are simply obtained using confint() which will call the appropriate profile method depending on the kind of model at hand.

```
exp(confint(m))

##              2.5 %      97.5 %
## (Intercept) 0.6412481 0.9318192
## group1      1.0392135 1.2965299
## strata1      0.1443251 0.2807120
## strata2      0.3048534 0.5420573
## strata3      0.3970567 0.7146984
```

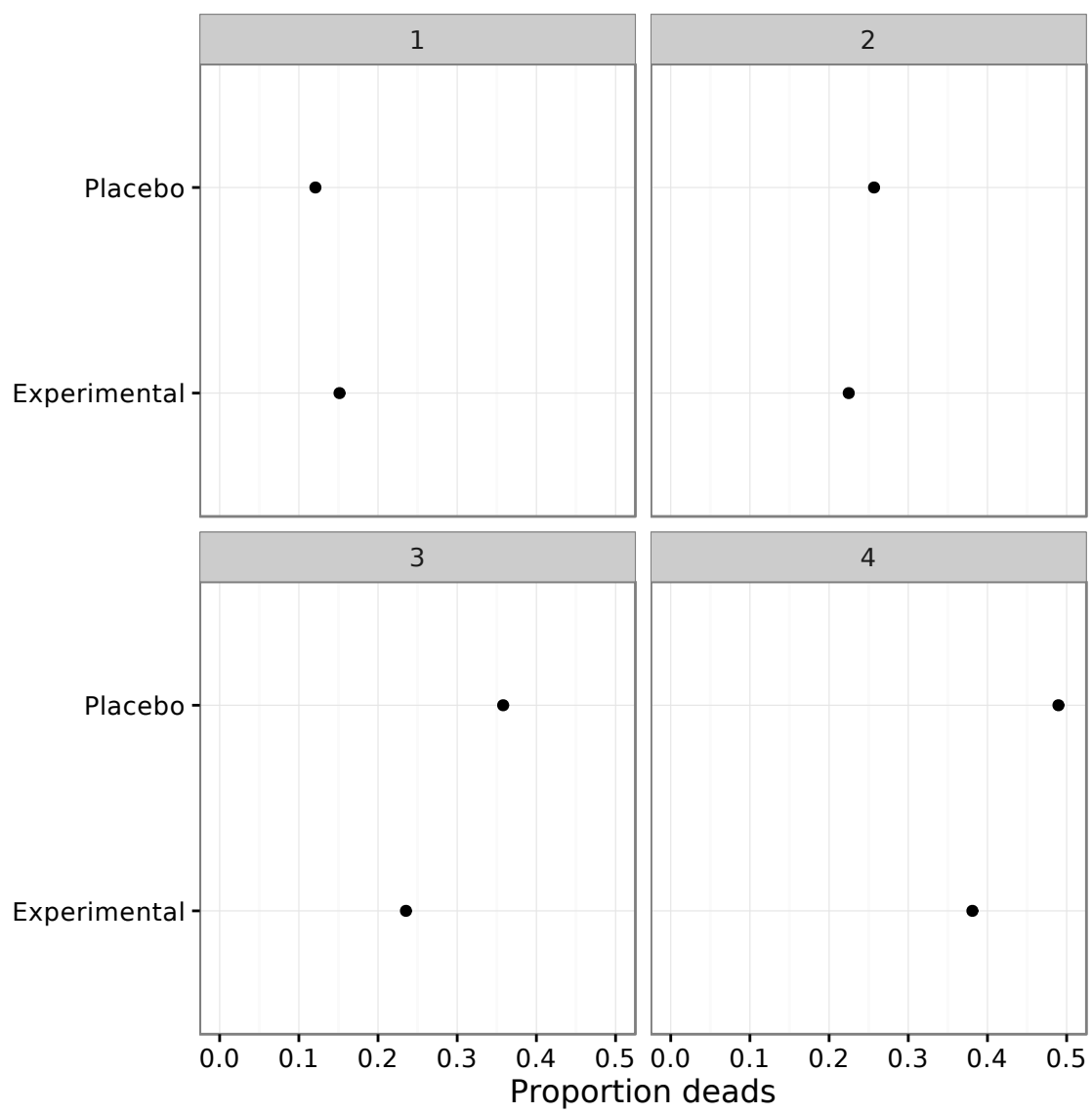


Figure 5: Proportion of patients who died by the end of the study

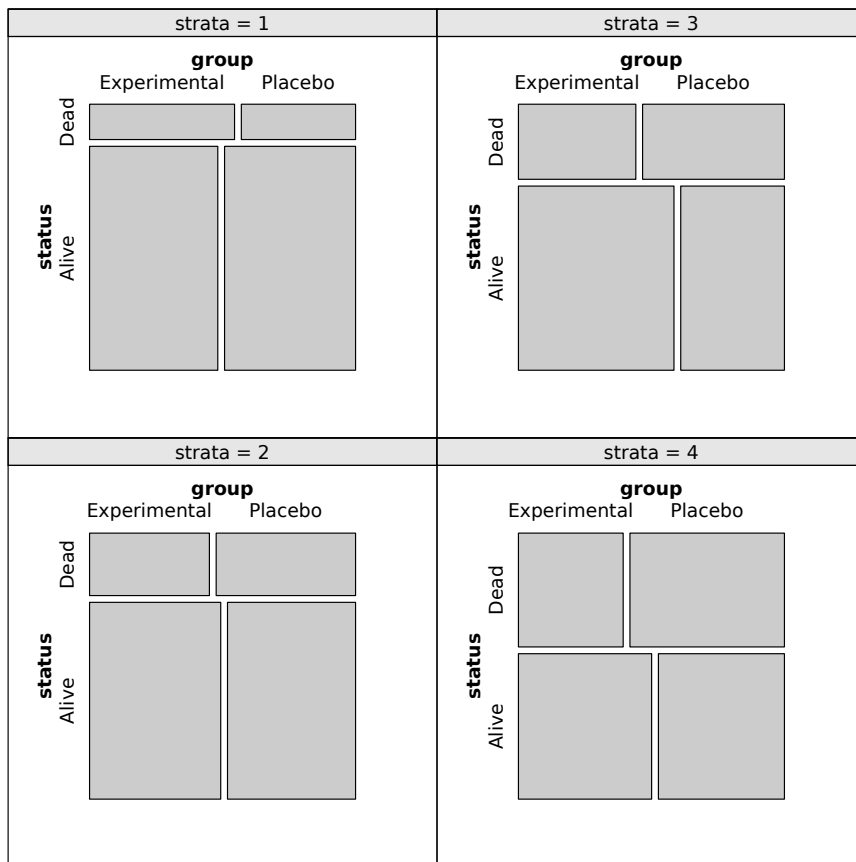


Figure 6: Conditional association plot

1.4 The dose-finding hypertension trial

Context. This trial aimed to compare low, medium and high doses of a new antihypertensive drug to a placebo. The primary efficacy variable that is being considered in this study is diastolic blood pressure.

This example is used to illustrate various methods to deal with multiple testing issues. In what follows we will work with p-values (raw data are not available) estimated when comparing all four groups (P, placebo vs. L, M, and H, the low, medium and high dose groups).

	L vs. P	M vs. P	H vs. P
Scenario 1	0.047	0.0167	0.015
Scenario 2	0.047	0.027	0.015
Scenario 3	0.053	0.026	0.017

Table 5: P-values obtained from different approaches

The `p.adjust()` command can be used to compute various “adjusted” p-values, the default being the step-down method proposed by Holm [1979].

```
pvals <- c(0.047, 0.0167, 0.015)
p.adjust(pvals, method = "bonferroni")
## [1] 0.1410 0.0501 0.0450
```

The Šidák method is not available in `p.adjust()` but it is not difficult to implement a custom function to perform this correction which amounts to update nominal α level with $1 - (1 - \alpha)^{1/n}$, that is:

```
f <- function(x) (1 - (1 - x)^(length(x)))
f(pvals)
## [1] 0.13447682 0.04926799 0.04432838
```

Alternatively, one can dig into the `multtest` package (<http://www.bioconductor.org>) which features the `mt.rawp2adjp()` command.

References

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