What counts as *math*? Relating conceptions of math with anxiety about math

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Abstract

What do people think of when they think of "math?" We propose that individuals may have very different working definitions of the category of math, and that those with broader *math conceptions* may have less math anxiety. In Study 1, we introduce a method for indexing the "breadth" of individuals' math conceptions, and show that there is an inverse relation between conception breadth and math anxiety. These results suggest that math anxiety is related both to how expansive individuals perceive math to be, and how skillful they feel at the activities they think it could involve. Study 2 attempts an intervention on students' conceptions of math with a sample of middle school students. We find the same inverse relationship in students between math conception breadth and math anxiety as found in adults. We discuss ongoing work that further explores qualitative variation in math conceptions, and the lessons this may hold for intervening on math anxiety.

Keywords: math anxiety, conceptual structure, intervention

Introduction

Recent U.S. initiatives in early science, technology, engineering, and math (STEM) highlight the growing importance of STEM education (e.g., White House Press Briefing, 2016), as well as the need for professionals in those fields to better represent the population. However, multiple barriers to an educated and diverse STEM workforce remain. One such barrier is psychological: an estimated 25–50% of U.S. college students are math anxious (Jones, 2001; Yeager, 2012), with women disproportionately affected (Hembree, 1990). *Math anxiety* refers to the tension or fear associated with the prospect of doing math (Ashcraft, 2002). In addition to being associated with lower math performance, math anxiety causes math-anxious individuals to generally avoid math. Given the national goal of broadening STEM participation, *math avoidance* might be the most devastating byproduct of anxiety about math, as it implies that math-anxious individuals will choose to end their formal math training as soon as possible.

Here, we are interested in how individuals' ideas of what *math* is–i.e., their *math conceptions*—might be a factor in their math anxiety and avoidance. "Math" can be used to refer to a wide range of activities, involving diverse skill sets and forms of reasoning. Individuals may differ in how they implicitly define the category of math, however, and properties of those definitions may be linked to their math anxiety.

Of particular interest in the present studies is what we will call the "breadth" of an individual's math conception. Guided by the idea that category structures can differentially license inferences (e.g., Ross & Murphy, 1999), our studies test the hypothesis that having a working math conception that is narrow (i.e., limited to a few branches of the math taxonomy, like arithmetic operations and numeric notation) might facilitate generalization of negative associations across the category. If this makes individuals confident about disliking *math*, rather than disliking only *arithmetic* or *algebra*, it could make them wary of future topics labeled as "math" that might have otherwise been appealing. In contrast, anxiety about the math category, and any new topics that are labeled as "math," might be harder to maintain if it encompasses many diverse subtopics and skills, ranging from the concrete (e.g., algebraic notation) to the abstract (thinking about infinity). In other words, insofar as math anxiety consists of anxiety generalized across the category of things construed as *math*, having a "broad" math conception may serve as a protective factor against the propagation of math anxiety.

As a first test of these ideas, we explore whether adults and children have different conceptions of what counts as math, and whether individuals with broader math conceptions may be less susceptible to math anxiety, such that math conception breadth and math anxiety will be inversely related.

Origins of Math Anxiety

The origins of math anxiety are unclear. While research on math anxiety is motivated in large part by its impact on math performance, there is evidence suggesting the reverse direction of causation, as well. Much of this evidence comes from longitudinal studies where performance in an earlier year is more strongly correlated with math anxiety in a later year than earlier anxiety is with later performance (see Carey, Hill, Devine, & Szücs, 2016, for a review). The relation between math anxiety and performance might be most accurately described as *reciprocal*, with early math difficulty leading to math anxiety, and math anxiety in turn leading to low performance, via avoidant behavior and increased constraints on processing (Carey et al., 2016).

In thinking about the relation between math conception and anxiety, we have thus far focused on a particular direction of causality, namely that narrow math conceptions might be a risk factor for developing math anxiety. But one could imagine a reciprocal relationship here, too. A child could acquire a math conception that is narrow, maybe via their early schooling, and find that they dislike or struggle with the contents of the category of *math*, leading them to become math anxious. Their math anxiety could in turn lead them to avoid engaging with new aspects of math that they might otherwise like or ex-

cel at, leading them to maintain both their narrow conception of math and their math anxiety.

In light of recent findings that math anxiety can be transmitted between generations, it is just as important to alleviate math anxiety in adults (i.e., so that they don't transmit it to children) as it is to intervene directly in children. Prior work has found that teachers' math anxiety may "spread" to their students (Beilock, Gunderson, Ramirez, & Levine, 2010). This is especially problematic because aspiring teachers with math anxiety tend to gravitate toward teaching earlier grades, where they will be able to engage less with math (see Hadley & Dorward, 2011), but where they will also be interacting with students in the early school years, when children are most impressionable. Parents' math anxiety can also affect their children. In one study, children of math anxious parents learned less during the school year than did children of non–math-anxious parents—but only if these parents frequently gave their children homework help (Maloney, Ramirez, Gunderson, Levine, & Beilock, 2015). Given this evidence for the intergenerational transmission of math anxiety, our studies focus both on adults and children.

Relating Math Conceptions and Math Anxiety

In principle, individuals could have 'math conceptions' that range from narrow (*Math is the symbolic operations one learns in school*) to broad (*Math relies on logic, spatial reasoning, and pattern recognition*).

Here, we develop a new measure to characterize the breadth of math conceptions. This measure presents participants with a diverse list of activities or topics, ranging from "sewing" to "playing soccer" or "physics." Participants are asked to indicate whether each item "could involve math," and, in some cases, to explain why. The idea is that when asked to answer whether a given activity "could involve math," individuals will be encouraged to come up with some rationale for how it could or could not involve math, and that their flexibility in categorizing activities as "math" will depend on the breadth of their (implicit) definition of the category. The point here is not that individuals typically construe an activity like "playing soccer" as involving math. Instead, our interest is in whether individuals vary in how flexible they are in categorizing activities that are not conventionally thought of as "math" as involving math. We present a diversity of activities to math, that can be related to math via diverse aspects of mathematical reasoning or subtopics, thus revealing the capacity and/or bounds of an individual's math conception. If an individual's conception of math is itself broad and diverse, we expect that it will be able to support explanations for the math-involvement of a wide range of activities. We thus operationalize breadth of math conception in the following studies as the number of activities that individuals say "could involve math." In Study 1, we also ask participants to rate their own skill at these same activities.

We hypothesize that broader math conceptions will relate to lower math anxiety in that they will afford individuals with more opportunities to recognize their own math engagement or expertise, and dilute the negative impact of components of math that individuals have negative associations with. Related to this, we expect self-assessed skill with activities classified as involving "math" to mediate the proposed relation between conception breadth and math anxiety. Study 1 examines the relation between math conception and math anxiety in adults, taking subjective skill into account. Study 2 tests for the same relation in middle-school children, within the context of a intervention study that tests the effect of broadening math conceptions.

Study 1: Adult Math Conception & Anxiety

Study 1 examined the relation between math conception and math anxiety in adults via an online survey composed of seven counterbalanced blocks probing participants' math attitudes and associations.

Stimuli & Methods

Participants A total of 62 U.S. adults were recruited via Amazon's Mechanical Turk (31 female, 19–74 years, $M = 33.24$, $SD = 10.25$). Participants were compensated for their participation, and the study took approximately 15 minutes to complete.

Math Conception In one block, participants saw a randomized list of topics and activities (e.g., "architecture," "cooking," "exercising"). Participants were asked to *indicate whether... each activity or topic listed involves math or does not involve math.* They responded by dragging each item into one of three boxes, labeled "Math," "Not Math," and "Not Sure." The more items categorized as involving math, the broader we considered their math conception to be (see above). We included the item "Math" as a control.

Activity Skill In another block, participants saw the same items in a new randomized order, and rated their skill at each item (*How good would you say you are at each of these things?*). They responded on a five-point Likert scale from 'Not at all good' to 'Very good.' We included a control item (*For this question, respond 'Good'*), as well as an opt-out scale option ('NA') for participants who had no experience with the item.

Math Anxiety We assessed participants' math anxiety using the single item math anxiety scale (SIMA; Núñez-Peña, Guilera, & Suárez-Pellicioni, 2013). This measure asks simply, *On a scale from 1 to 10, how math anxious are you?* The SIMA has been validated on a large sample of U.S. college students. It shows the expected negative correlation with math achievement measures, high test-retest reliability, and is consistent with lengthier, established measures of math anxiety, like the Shortened Math Anxiety Rating Scale (sMARS; Alexander & Martray, 1989, $r = .77$).

Other Measures We collected several other measures of participants' attitudes toward and history with math. One block assessed participants' "math mindset:" an analogy

to intelligence mindsets made specific to math (Yeager & Dweck, 2012). Five items probed participants' beliefs about the fixedness of math ability (e.g., *Math is a gift: you either have it or you don't.*), which they responded to using a five-point Likert scale of agreement. Two blocks consisted of a single, open-ended question, one asking participants for an informal definition of math (*Please describe what you think math is in the space below*), and one eliciting their personal math history (*Please write a brief summary of your experience with math from childhood until now*). In the final block, we collected demographic information, including the number of semesters of college they had completed, and a list of all math classes they had taken.

Results & Discussion

Qualtitative Variation in Math Conceptions There was substantial variation in the activities that participants categorized as involving math (Figure 1). All participants appropriately responded that "Math" involved math, which we took as confirmation of their attention to the task. Items obviously involving math were categorized as such by the vast majority of participants (e.g., finance), while those representing related disciplines (e.g., biology), daily activities (e.g., cooking), and abstract, creative and language-related tasks (e.g., composing music, reading) received the fewest math-categorizations. In a separate study, we elicited explanations for participants' categorizations of a similar list of items. In that study, both adults and children frequently used contrast categories (e.g., "No, that's *music*!"), often from the humanities, to explain why items could not involve math. This type of explanation implies that participants perceived the categories of music, art, and even science as exclusive with math. Such a picture of what math is (and isn't) is consistent with the idea of a narrow math conception, and echoes what mathematician Paul Lockhart famously lamented as the sorry byproduct of American math education:

> The first thing to understand is that mathematics is an art. The difference between math and the other arts, such as music and painting, is that our culture does not recognize it as such. [...] Nevertheless, the fact is that there is nothing as dreamy and poetic, nothing as radical, subversive, and psychedelic as mathematics. It is every bit as mind-blowing as cosmology or physics (mathematicians conceived of black holes long before astronomers actually found any), and allows for more freedom of expression than poetry, art, or music (which depend heavily on properties of the physical universe). Mathematics is the purest of arts as well as the most misunderstood. (Lockhart, 2009).

Math Conception & Anxiety To answer whether breadth of math conception and math anxiety are related, we conducted a linear regression on individuals' math anxiety and the number of items they categorized as math, controlling for the number of semesters of college they had completed.

Table 1: Descriptive statistics for four blocks in Study 1. 'Items Categorized as Math' is out of a total of 32, and was analyzed as a proxy for the breadth of participants' math conceptions. 'Math Anxiety' is on a 10-point self-report scale. 'Self-Assessed Skill' represents the mean skill rating on a 5-point Likert scale, across all items for all participants. 'Math Mindset' is coded to be on a 5-point scale indexing how fixed individuals believe math ability to be, with larger values indicating more fixed mindsets.

Variable	*M*	*SD*
Items Categorized as Math	13.10	5.35
Math Anxiety	4.44	3.04
Self-Assessed Skill	3.28	0.44
Math Mindset	2.13	0.99

In accordance with our predictions, math anxiety was negatively related to the number of items participants categorized as math, even controlling for education ($F(1,61) = 6.44$ $p < .05$ with an R^2 of .082; see Figure 2). This supports the idea that individuals with broader math conceptions are less likely to experience math anxiety, and that this relation may not be attributable to exposure to topics in math alone.

To address whether the relation between math conception and anxiety is due in part to individuals' perception of their own skill at things they think might involve "math," we analyzed self-assessed skill and anxiety. For each individual, we took the mean skill of the items they had categorized as involving math and those they had categorized as not involving math. We dropped items for which participants reported having had no experience. A linear regression on self-reported skill and math anxiety revealed a significant negative correlation between math anxiety and mean self-assessed skill for items the individual was able to relate to math ($\beta = -1.98$, $SE = 0.60$, $t = -3.29$, $p < .01$), but no correlation between math anxiety and self-assessed skill for items judged to not involve math ($\beta = 0.11$, $SE = 0.69$, $t = 0.154$, $p = .88$). This asymmetry is important because it suggests that it is not just individuals who are less confident overall who suffer from math anxiety—if this were the case, we would have expected to find that lower skill related to higher anxiety for both items judged to involve math and items judged to not involve math.

In Study 1, both the number of items construed as involving math and participants' perceived skill at those items were related to math anxiety. As discussed above, one of the most dangerous features of math anxiety is its tendency to make individuals avoid math and thus fail to take advantage of opportunities to discover new aspects of mathematics they might excel at or appreciate. The fact that mean self-assessed skill at activities categorized as involving math was negatively related to math anxiety lends support to the idea that broad conceptions may be a protective factor in math anxiety, attenuating the impact of negative associations that individuals might have with activities they think could involve 'math.' Having a

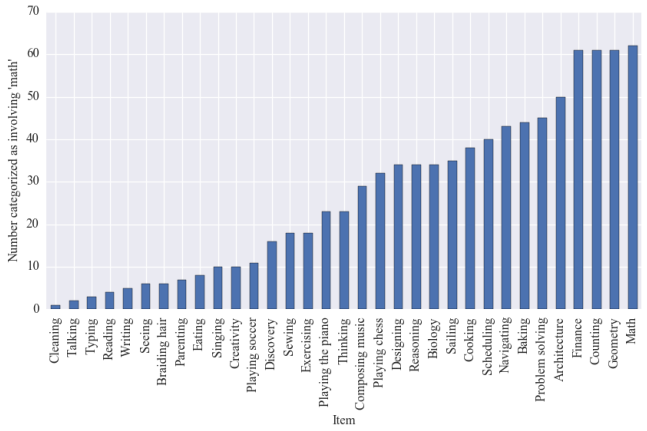

Figure 1: Number of participants in Study 1 who labeled each activity as involving math.

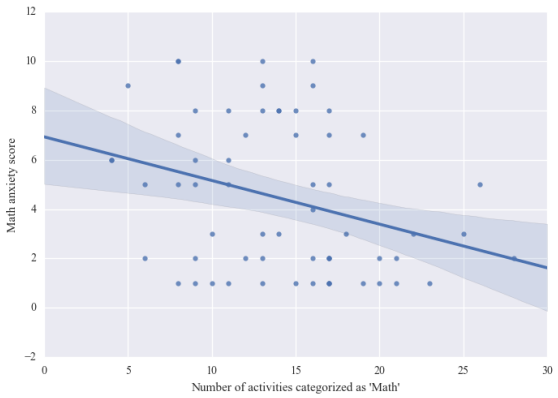

Figure 2: Plot of linear regression line showing relationship between breadth of conception and math anxiety in Study 1, controlling for education ($\alpha = 6.93$, $\beta = -0.18$, $p < .05$).

broad math conception does not mean that an individual has to feel confident and have positive associations with all activities that they think involves math, but it could mean that negative associations with specific topics (like geometry or algebraic notation) will have less of an impact on their associations with the category as a whole.

Study 2: Middle School Intervention

We were interested in whether students would exhibit the same qualitative variation in math conceptions and link between breadth and anxiety that we had seen with adults in Study 2. Additionally, as a first pass at investigating the causal relation between math conception and anxiety, and potential educational implications, we designed a brief intervention intended to broaden students' math conceptions.

Stimuli & Methods

Study 2 consisted of an interactive origami activity followed by four measures administered to participants in two between-subjects conditions, BASELINE and BROAD. Only participants in the BROAD condition received an explanation for the ways in which the activity had involved math before completing the other assessments.

Participants A total of 80 6th, 7th, and 8th grade students at a school in Gujarat, India participated (33 6th-graders, 7 girls; 21 7th-graders, 9 girls; 26 8th-graders, 9 girls). All 6th-grade participants were excluded for sharing answers ($n = 33$), leaving 47 7th-8th grade students in our sample. Participants were tested in groups of 10–15 assigned to the BASELINE or BROAD conditions in a classroom at their school.

Origami Activity Students sat in a circle on the floor around two experimenters who guided them through folding an origami crane. A third experimenter circulated to answer any questions, and students could also refer to printed, diagrammatic instructions distributed before the activity. All experimenters avoided using explicit math language during the folding instruction (e.g., reference to "angles," "half," "diagonal"), opting instead for generically narrated demonstration (e.g., "fold the paper like this"). Each student folded a paper crane, which they got to take home.

Construal Following the origami activity, students in both conditions answered whether the activity they just did *could involve math* (Yes/No/Not Sure), and to explain why. In addition, they rated how *enjoyable* and *difficult* they had found the activity, on a five-point Likert scale (from 'Not at all—' to 'Extremely—').

Intervention In the BROAD condition–but not in the BASELINE condition–an experimenter then gave a brief explanation of how the origami activity involved and related to math (e.g., *. . . you have to think about spatial relations, and things like measurements of the different sides and angles. When designing new pieces of origami, you have to think creatively and flexibly, and use what you already know to come to new conclusions, like you have to do in math*).

Avoidance The next measure participants completed was intended to indirectly access their math avoidance. The survey consisted of 6 items, each asking about a different school subject (e.g., *How excited are you to learn a new topic in [math/Hindi] class?*). Participants responded on a 5-point scale (from 'Not at all excited' to 'Extremely excited').[1]

Math Anxiety We administered a child math anxiety questionnaire adapted from Ramirez, Gunderson, Levine, and Beilock (2013) by Barner et al. (2016), for use in India. The questionnaire consisted of 16 questions regarding students' experiences with math, which students responded to using a 5-point face scale (from 'Not nervous at all' to 'Very, very nervous'). The experimenter explained the scale and completed three warm-up questions with the students beforehand

[1] Because participants were on average enthusiastic to learn new topics in math ($M = 4.29$, $SD = 0.94$), more so even than other topics, we did not further analyze the results of this measure.

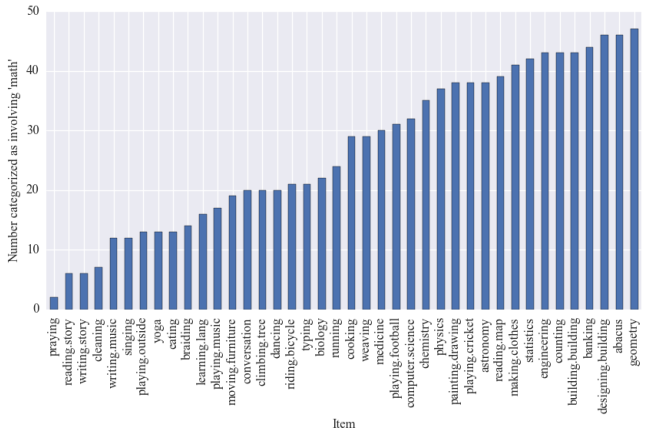

Figure 3: Number of participants in Study 2 who answered "yes" when asked whether each item *could involve math*.

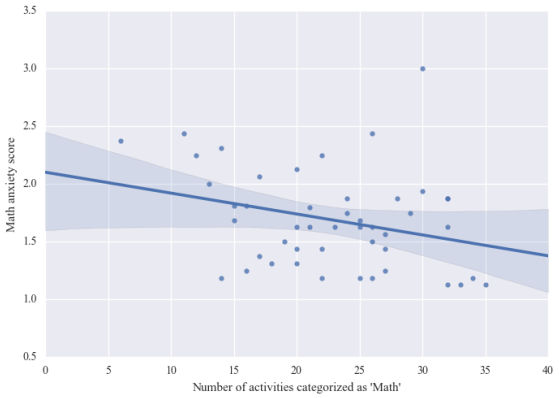

Figure 4: Linear regression showing relationship between breadth of conception and math anxiety in Study 2 ($\alpha = 2.10$, $\beta = -0.02$, $p < .05$).

Table 2: Means and standard deviations for each condition.

	Concept		Anxiety	
Condition	*M*	*SD*	*M*	*SD*
BASELINE	20.96	7.24	1.78	0.47
BROAD	24.61	5.96	1.60	0.36

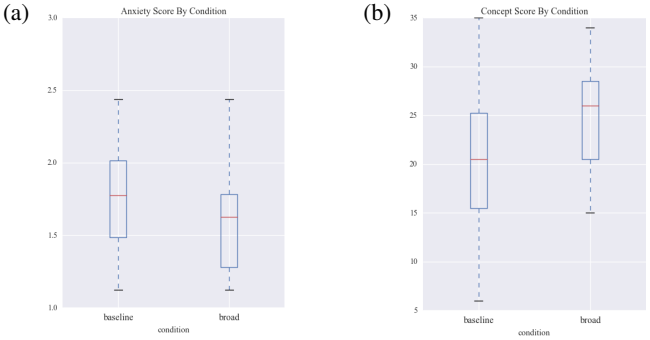

Figure 5: (a) Boxplot of anxiety scores by condition. (b) Boxplot of conception score by condition.

to ensure understanding of the measure.

Math Conception The math conception measure was a variant of the one used in Study 1. We included 40 age- and place-appropriate items, and adjusted the wording used in the prompt from Study 1. Here, participants answered ***Could** this activity involve math?* (Yes/No/Not Sure), which we anticipated would encourage flexible thinking about the items and about math.

Results & Discussion

Qualitative Variation in Math Conceptions Participants indicated that an average of 22.74 out of 40 items *could involve math*. As in Study 1, there was considerable variation across items in the proportion of participants who judged them as involving math (Figure 3).

Math Conception & Anxiety Participants received an average math anxiety score of 1.69 (out of 5). We were interested again in whether math anxiety scores were related to conception breadth, which we examined in our total sample, collapsing across condition. In middle-schoolers, as with adults, math anxiety was negatively related to the number of activities students categorized as *math*, ($F(1,44) = 4.15$, $p < .05$ with an adjusted R^2 of 0.07; see Figure 4).

Conception Intervention We next analyzed math conception and math anxiety for our two conditions separately. If such a brief intervention were successful, we should expect conception scores to be higher in the BROAD condition, and anxiety scores to be lower. While conception and anxiety might be slightly different in the anticipated direction between the two conditions (Table 2), the differences between group means is not significant (as determined by one-way ANOVAs for math conception: $F(1,45) = 3.54$, $p = .066$, and anxiety: $F(1,45) = 2.31$, $p = .14$). The trend for math conceptions in particular is promising (see Figure 5): in the BASELINE condition, there was more spread in the magnitude of participants' conception scores, while those in the BROAD condition had generally 'broader' conceptions. Thus, it may be that with a different or merely more sustained intervention, students' math conceptions could be broadened.

Influence of Construal Out of the 47 participants analyzed, 36 said that the origami activity could involve math. Participants on average enjoyed the activity ($M = 4.43$, $SD = 0.62$) and did not find it difficult ($M = 2.28$, $SD = 0.71$). This raises the possibility that we may not have found a robust intervention effect because our elicitation of construals of the origami activity as *math* itself served as an intervention on breadth of conception. In particular, given that all participants—including those in the BASELINE condition—

were asked to consider whether an enjoyable and easy activity could involve math before completing any of the surveys, they may have been primed to think more broadly and favorably of math.

Discussion & Future Directions

The above studies offer preliminary evidence for the intuition that individuals may have substantially different ideas of what constitutes *math*. Here, we have introduced the idea of *math conceptions* to describe these qualitatively different definitions of the category of math, and focused especially on their "breadth" to explain why certain types of math conceptions might make math anxiety more or less likely. Strikingly, the measure we introduced as a proxy for the breadth of individuals' math conceptions showed the hypothesized inverse link to math anxiety, in both adults and children, though it should be noted that the samples for Studies 1–2 differed in more than age. We see the remarkable dissimilarity of the two populations and contexts as adding strength to our results.

While this link between our measure of math conception breadth and math anxiety is promising, we imagine there is a great deal of additional variation among math conceptions that could be captured in future studies. Eliciting and analyzing participants' explanations for their categorization decisions may be one especially fruitful way to access other qualitative dimensions of math conceptions, alongside canonical methods to access category structure, like primed similarity judgments.

Without robust evidence for the efficacy of our intervention (Study 2), we cannot speak to the potential directionality of the math conception-anxiety relationship. Our ongoing work is exploring this question through an interactive intervention on adults' math conceptions, as well as an adaptation of the math conception measure for use with young children prior to being formally educated in math. Exploring math conceptions in young children, as well as directly assessing math skill in future studies with adults, will also address the heretofore unconsidered possibility that a third variable (like actual proficiency in math) is responsible for both responses on our current conception measure and levels of math anxiety. The ultimate goal of these lines of research is to understand and describe the character of individuals' implicit math categories, and leverage this knowledge to inform interventions aimed at reducing math anxiety in adults and children.

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