preparation (evaluate this cell to initialize)

general definition

definition for E_6

norcharC[6,λ] : computation of $C_\lambda^{(6)}$

norcharC[3,λ] : computation of $C_\lambda^{(3)}$ for $\lambda \in \{0, \lambda_1, \lambda_3, \lambda_4\}$

norcharD[3,λ] : computation of $D_\lambda^{(3)}$ for $\lambda \in \{0, \lambda_1, \lambda_3, \lambda_4\}$

proof of $\sum_{w \in W_\lambda} (-1)^{l(w)} e^{w(\rho)} E_{w;\lambda} = 0$ for $\lambda \in \{\lambda_2, \lambda_3, \lambda_4\}$

$\lambda = \lambda_2$

sum over $W_{1,3,5,6}$

```
Block[{ty = E, rk = 6, summand, targetweight, subgroup, sum},
 targetweight = λ[2];
 summand[w_] :=
  (-1)^Length[w] exp[WeylR[ty, rk][w][rho]] * Factor[pfE[w, targetweight]];
 subgroup = isotropysubgp[ty, rk][targetweight];
 Print[{"size of the isotropy subgroup : ", subgroup // Length}];
 sum = Total@Map[summand, subgroup];
 Print[{"time elapsed", "sum"}];
 Timing[Simplify[sum]]
]
```

```
{size of the isotropy subgroup : , 24}
```

```
{time elapsed, sum}
```

```
{0., 0}
```

sum over $W_{1,5,6}$

```
Block[{ty = E, rk = 6, summand, targetweight, subgroup, sum},
 targetweight = λ[2];
 summand[w_] :=
  (-1)^Length[w] exp[WeylR[ty, rk][w][rho]] * Factor[pfE[w, targetweight]];
 subgroup = isotropysubgp[ty, rk][wt[0, 1, 1, 1, 0, 0]];
 Print[{"size of the isotropy subgroup : ", subgroup // Length}];
 sum = Total@Map[summand, subgroup];
 Print[{"time elapsed", "sum"}];
 Timing[Simplify[sum]]
]
```

```
{size of the isotropy subgroup : , 8}
```

```
{time elapsed, sum}
```

```
{0., 0}
```

$\lambda = \lambda_3$

sum over $W_{1,2,4,5,6}$

```
Block[{ty = E, rk = 6, summand, targetweight, subgroup, sum},
 targetweight = λ[3];
 summand[w_] :=
  (-1)^Length[w] exp[WeylR[ty, rk][w][rho]] * Factor[pfE[w, targetweight]];
 subgroup = isotropysubgp[ty, rk][targetweight];
 Print[{"size of the isotropy subgroup : ", subgroup // Length}];
 sum = Total@Map[summand, subgroup];
 Print[{"time elapsed", "sum"}];
 Timing[Simplify[sum]]
]
```

```
{size of the isotropy subgroup : , 72}
```

```
{time elapsed, sum}
```

```
{0., 0}
```

sum over $W_{1,4,5,6}$

```
Block[{ty = E, rk = 6, summand, targetweight, subgroup, sum},
 targetweight = λ[3];
 summand[w_] :=
  (-1)^Length[w] exp[WeylR[ty, rk][w][rho]] * Factor[pfE[w, targetweight]];
 subgroup = isotropysubgp[ty, rk][wt[0, 1, 1, 0, 0, 0]];
 Print[{"size of the isotropy subgroup : ", subgroup // Length}];
 sum = Total@Map[summand, subgroup];
 Print[{"time elapsed", "sum"}];
 Timing[Simplify[sum]]
]
```

```
{size of the isotropy subgroup : , 24}
```

```
{time elapsed, sum}
```

```
{0., 0}
```

$\lambda = \lambda_4$

```
Block[{ty = E, rk = 6, summand, targetweight, subgroup, sum},
 targetweight = λ[4];
 summand[w_] :=
  (-1)^Length[w] exp[WeylR[ty, rk][w][rho]] * Factor[pfE[w, targetweight]];
 subgroup = isotropysubgp[ty, rk][targetweight];
 Print[{"size of the isotropy subgroup : ", subgroup // Length}];
 sum = Total@Map[summand, subgroup];
 Print[{"time elapsed", "sum"}];
 Timing[Simplify[sum]]
]
```

```
{size of the isotropy subgroup : , 720}
```

```
{time elapsed, sum}
```

```
{70.9375, 0}
```

$\lambda = \lambda_4$: more efficient check

```
Block[{ty = E, rk = 6, summand, WA5, WA2A2, sumoverWA2A2, sum, WA5WA2Acosets},
 summand[w_] := (-1)^Length[w] exp[WeylR[ty, rk][w][rho]] * pfE[w, λ[4]];
 WA5 = isotropysubgp[E, 6][wt[0, 0, 0, 0, 0, 1]];
 Print[{"size of the isotropy subgroup W' : ", WA5 // Length}];
 WA2A2 = Select[WA5, FreeQ[#, 3] &];
 Print[{"size of a subgroup W'' of W' : ", WA2A2 // Length}];
 sumoverWA2A2 = Total@Map[summand, WA2A2] // Simplify;
 WA5WA2Acosets = cosetReps[A, 5][{1, 2, 4, 5}];
 Print[{"size of W'/W''  : ", WA5WA2Acosets // Length}];
 Print[{"coset reps:", WA5WA2Acosets}];
 WA5WA2Acosets = {{}, {3, 2, 1, 4, 3, 2, 5, 4, 3}, {3}, {2, 1, 4, 3, 2, 5, 4, 3}, {2, 3},
   {1, 4, 3, 2, 5, 4, 3}, {4, 3}, {2, 1, 3, 2, 5, 4, 3}, {1, 2, 3}, {4, 3, 2, 5, 4, 3},
   {2, 4, 3}, {1, 3, 2, 5, 4, 3}, {5, 4, 3}, {2, 1, 3, 2, 4, 3}, {3, 2, 4, 3},
   {1, 2, 5, 4, 3}, {1, 2, 4, 3}, {3, 2, 5, 4, 3}, {2, 5, 4, 3}, {1, 3, 2, 4, 3}};
 Print[{"coset reps reordered:", WA5WA2Acosets}];
 sum = 0;
 Do[
  sum = Simplify[sum + (-1)^Length[ww] (sumoverWA2A2 /. weyltorule[ty, rk][ww])];
  (* Print[sum]; *)
  , {ww, WA5WA2Acosets}
 ];
 sum
]
```

```
{size of the isotropy subgroup W' : , 720}

{size of a subgroup W'' of W' : , 36}

{size of W'/W''  : , 20}

{coset reps:, {{}, {3}, {2, 3}, {4, 3}, {1, 2, 3}, {2, 4, 3}, {5, 4, 3}, {3, 2, 4, 3},
  {1, 2, 4, 3}, {2, 5, 4, 3}, {3, 2, 5, 4, 3}, {1, 3, 2, 4, 3}, {1, 2, 5, 4, 3},
  {4, 3, 2, 5, 4, 3}, {1, 3, 2, 5, 4, 3}, {2, 1, 3, 2, 4, 3}, {1, 4, 3, 2, 5, 4, 3},
  {2, 1, 3, 2, 5, 4, 3}, {2, 1, 4, 3, 2, 5, 4, 3}, {3, 2, 1, 4, 3, 2, 5, 4, 3}}}

{coset reps reordered:, {{}, {3, 2, 1, 4, 3, 2, 5, 4, 3}, {3}, {2, 1, 4, 3, 2, 5, 4, 3}, {2, 3},
  {1, 4, 3, 2, 5, 4, 3}, {4, 3}, {2, 1, 3, 2, 5, 4, 3}, {1, 2, 3}, {4, 3, 2, 5, 4, 3},
  {2, 4, 3}, {1, 3, 2, 5, 4, 3}, {5, 4, 3}, {2, 1, 3, 2, 4, 3}, {3, 2, 4, 3},
  {1, 2, 5, 4, 3}, {1, 2, 4, 3}, {3, 2, 5, 4, 3}, {2, 5, 4, 3}, {1, 3, 2, 4, 3}}}

0
```

proof of $C_\lambda^{(3)} = D_\lambda^{(3)}$ for $\lambda \in \{\lambda_1, \lambda_3, \lambda_4\}$

$\lambda = \lambda_1$

```
Block[{C3, D3, la, cdlist, sum, timestart},
 Print["process initiated!"];
 timestart = Floor[AbsoluteTime[]];
 la = λ[1];
 Print["list of summands for Cλ : "];
 Print[norcharC3C6List[la]];
 C3 = (Weyldenom * norcharCList[3, la]);
 D3 = (-Weyldenom * norcharDList[3, la]);
 cdlist = Join @@ {C3, D3};
 sum = Factor[Total[cdlist]];
 Print[{"time elapsed: ", Floor[AbsoluteTime[]] - timestart}];
 sum
]
```

process initiated!

list of summands for C_λ :

$$\left\{-\frac{c[6, wt[0, 0, 0, 0, 0, 1]]\, c[6, wt[1, 0, 0, 0, 1, -1]]\, \left(-1 + x[2]\, x[3]^2\, x[4]\, x[6]^2\right)^2}{x[2]\, x[3]^2\, x[4]\, x[6]^2},\right.$$

$$-\frac{c[6, wt[0, 0, 1, 0, 0, -1]]\, c[6, wt[1, 0, -1, 0, 1, 1]]\, \left(-1 + x[2]\, x[3]^2\, x[4]\right)^2}{x[2]\, x[3]^2\, x[4]},$$

$$-\frac{c[6, wt[0, 1, -1, 1, 0, 0]]\, c[6, wt[1, -1, 1, -1, 1, 0]]\, (-1 + x[2]\, x[4])^2}{x[2]\, x[4]},$$

$$\left.-\frac{c[6, wt[0, 1, 0, -1, 1, 0]]\, c[6, wt[1, -1, 0, 1, 0, 0]]\, (x[2] - x[4])^2}{x[2]\, x[4]}\right\}$$

```
{time elapsed: , 100}

0
```

$\lambda = \lambda_3$

```
Block[{C3, D3, la, cdlist, sum, timestart},
 Print["process initiated!"];
 timestart = Floor[AbsoluteTime[]];
 la = λ[3];
 Print["list of summands for Cλ : "];
 Print[norcharC3C6List[la]];
 C3 = (Weyldenom * norcharCList[3, la]);
 D3 = (-Weyldenom * norcharDList[3, la]);
 cdlist = Join @@ {C3, D3};
 sum = Factor[Total[cdlist]];
 Print[{"time elapsed: ", Floor[AbsoluteTime[]] - timestart}];
 sum
]
```

```
process initiated!
```

```
list of summands for Cλ :
```

$$\left\{-\frac{c[6, wt[0, 0, 0, 0, 0, 1]]\, c[6, wt[0, 0, 1, 0, 0, -1]]\, (-1 + x[6])^2}{x[6]}\right\}$$

```
{time elapsed: , 2}
```

```
0
```

$\lambda = \lambda_4$ (this uses the saved list of functions; put the data file in the same folder with this notebook)

```
Block[{summandlist, timestart, out, toexp},
 SetDirectory[FileNameJoin[{NotebookDirectory[]}]];
 summandlist = Last /@
   SortBy[ReadList[FileNameJoin[{Directory[], "E6_3_la4_poly_46.txt"}]], First];
 toexp[pairs_] := Times @@ Map[#[[1]]^#[[2]] &, pairs];
 timestart = Floor[AbsoluteTime[]];
 out = Expand[Total[toexp /@ summandlist]];
 Print[{"result: ", out}];
 Print[{"time elapsed: ", Floor[AbsoluteTime[]] - timestart}];
]
```

numerical check for $C_\lambda^{(3)} = D_\lambda^{(3)}$ for $\lambda \in \{\lambda_0\}$

$\lambda = \lambda_0$

```
Block[{C3, D3, la, rule, rk = 6},
 la = λ[0];
 rule = Inner[Rule, Array[x, rk], RandomInteger[{1, 1000}, rk], List];
 norcharC3C6List[la] // Print;
 C3 = norcharC[3, la] /. rule;
 D3 = norcharD[3, la] /. rule;
 C3 - D3
]
```

$$\Big\{-\big(\big(c[6, wt[0, 0, 0, 0, 0, -1]]\, c[6, wt[0, 0, 0, 0, 0, 1]]\, \big(-1 + x[1]\, x[2]^2\, x[3]^3\, x[4]^2\, x[5]\, x[6]^2\big)^2 \big(1 + x[1]\, x[2]^2\, x[3]^3\, x[4]^2\, x[5]\, x[6]^2\big)^2\big) / \big(x[1]^2\, x[2]^4\, x[3]^6\, x[4]^4\, x[5]^2\, x[6]^4\big)\big),$$

$$-\big(\big(c[6, wt[0, 0, -1, 0, 0, 1]]\, c[6, wt[0, 0, 1, 0, 0, -1]]\, \big(-1 + x[1]\, x[2]^2\, x[3]^3\, x[4]^2\, x[5]\, x[6]\big)^2 \big(1 + x[1]\, x[2]^2\, x[3]^3\, x[4]^2\, x[5]\, x[6]\big)^2\big) / \big(x[1]^2\, x[2]^4\, x[3]^6\, x[4]^4\, x[5]^2\, x[6]^2\big)\big),$$

$$-\big(\big(c[6, wt[0, -1, 1, -1, 0, 0]]\, c[6, wt[0, 1, -1, 1, 0, 0]]\, \big(-1 + x[1]\, x[2]^2\, x[3]^2\, x[4]^2\, x[5]\, x[6]\big)^2 \big(1 + x[1]\, x[2]^2\, x[3]^2\, x[4]^2\, x[5]\, x[6]\big)^2\big) / \big(x[1]^2\, x[2]^4\, x[3]^4\, x[4]^4\, x[5]^2\, x[6]^2\big)\big),$$

$$-\big(\big(c[6, wt[-1, 1, 0, -1, 0, 0]]\, c[6, wt[1, -1, 0, 1, 0, 0]]\, \big(-1 + x[1]\, x[2]\, x[3]^2\, x[4]^2\, x[5]\, x[6]\big)^2 \big(1 + x[1]\, x[2]\, x[3]^2\, x[4]^2\, x[5]\, x[6]\big)^2\big) / \big(x[1]^2\, x[2]^2\, x[3]^4\, x[4]^4\, x[5]^2\, x[6]^2\big)\big),$$

$$-\big(\big(c[6, wt[0, -1, 0, 1, -1, 0]]\, c[6, wt[0, 1, 0, -1, 1, 0]]\, \big(-1 + x[1]\, x[2]^2\, x[3]^2\, x[4]\, x[5]\, x[6]\big)^2 \big(1 + x[1]\, x[2]^2\, x[3]^2\, x[4]\, x[5]\, x[6]\big)^2\big) / \big(x[1]^2\, x[2]^4\, x[3]^4\, x[4]^2\, x[5]^2\, x[6]^2\big)\big),$$

$$-\big(\big(c[6, wt[-1, 0, 0, 1, 0, 0]]\, c[6, wt[1, 0, 0, -1, 0, 0]]\, \big(-1 + x[2]\, x[3]^2\, x[4]^2\, x[5]\, x[6]\big)^2 \big(1 + x[2]\, x[3]^2\, x[4]^2\, x[5]\, x[6]\big)^2\big) / \big(x[2]^2\, x[3]^4\, x[4]^4\, x[5]^2\, x[6]^2\big)\big),$$

$$-\big(\big(c[6, wt[-1, 1, -1, 1, -1, 0]]\, c[6, wt[1, -1, 1, -1, 1, 0]]\, \big(-1 + x[1]\, x[2]\, x[3]^2\, x[4]\, x[5]\, x[6]\big)^2 \big(1 + x[1]\, x[2]\, x[3]^2\, x[4]\, x[5]\, x[6]\big)^2\big) / \big(x[1]^2\, x[2]^2\, x[3]^4\, x[4]^2\, x[5]^2\, x[6]^2\big)\big),$$

$$-\big(\big(c[6, wt[0, -1, 0, 0, 1, 0]]\, c[6, wt[0, 1, 0, 0, -1, 0]]\, \big(-1 + x[1]\, x[2]^2\, x[3]^2\, x[4]\, x[6]\big)^2 \big(1 + x[1]\, x[2]^2\, x[3]^2\, x[4]\, x[6]\big)^2\big) / \big(x[1]^2\, x[2]^4\, x[3]^4\, x[4]^2\, x[6]^2\big)\big),$$

$$-\big(\big(c[6, wt[-1, 0, 1, 0, -1, -1]]\, c[6, wt[1, 0, -1, 0, 1, 1]]\, (-1 + x[1]\, x[2]\, x[3]\, x[4]\, x[5]\, x[6])^2 (1 + x[1]\, x[2]\, x[3]\, x[4]\, x[5]\, x[6])^2\big) / \big(x[1]^2\, x[2]^2\, x[3]^2\, x[4]^2\, x[5]^2\, x[6]^2\big)\big),$$

$$-\big(\big(c[6, wt[-1, 0, 1, -1, 1, 0]]\, c[6, wt[1, 0, -1, 1, -1, 0]]\, \big(-1 + x[2]\, x[3]^2\, x[4]\, x[5]\, x[6]\big)^2 \big(1 + x[2]\, x[3]^2\, x[4]\, x[5]\, x[6]\big)^2\big) / \big(x[2]^2\, x[3]^4\, x[4]^2\, x[5]^2\, x[6]^2\big)\big),$$

$$-\big(\big(c[6, wt[-1, 1, -1, 0, 1, 0]]\, c[6, wt[1, -1, 1, 0, -1, 0]]\, \big(-1 + x[1]\, x[2]\, x[3]^2\, x[4]\, x[6]\big)^2 \big(1 + x[1]\, x[2]\, x[3]^2\, x[4]\, x[6]\big)^2\big) / \big(x[1]^2\, x[2]^2\, x[3]^4\, x[4]^2\, x[6]^2\big)\big),$$

$$-\big(\big(c[6, wt[-1, 1, -1, 0, 1, 1]]\, c[6, wt[1, -1, 1, 0, -1, -1]]\, (-1 + x[2]\, x[3]\, x[4]\, x[5]\, x[6])^2 (1 + x[2]\, x[3]\, x[4]\, x[5]\, x[6])^2\big) / \big(x[2]^2\, x[3]^2\, x[4]^2\, x[5]^2\, x[6]^2\big)\big),$$

$$-\big(\big(c[6, wt[-1, 0, 1, -1, 1, -1]]\, c[6, wt[1, 0, -1, 1, -1, 1]]\, (-1 + x[1]\, x[2]\, x[3]\, x[4]\, x[6])^2 (1 + x[1]\, x[2]\, x[3]\, x[4]\, x[6])^2\big) / \big(x[1]^2\, x[2]^2\, x[3]^2\, x[4]^2\, x[6]^2\big)\big),$$

$$-\big(\big(c[6, wt[-1, 0, 1, 0, -1, 0]]\, c[6, wt[1, 0, -1, 0, 1, 0]]\, \big(-1 + x[2]\, x[3]^2\, x[4]\, x[6]\big)^2 \big(1 + x[2]\, x[3]^2\, x[4]\, x[6]\big)^2\big) / \big(x[2]^2\, x[3]^4\, x[4]^2\, x[6]^2\big)\big),$$

$$-\big(\big(c[6, wt[-1, 0, 0, 0, -1, 1]]\, c[6, wt[1, 0, 0, 0, 1, -1]]\, (-1 + x[1]\, x[2]\, x[3]\, x[4]\, x[5])^2 (1 + x[1]\, x[2]\, x[3]\, x[4]\, x[5])^2\big) / \big(x[1]^2\, x[2]^2\, x[3]^2\, x[4]^2\, x[5]^2\big)\big),$$

$$-\big(\big(c[6, wt[0, -1, 0, 0, 1, 1]]\, c[6, wt[0, 1, 0, 0, -1, -1]]\, (-1 + x[3]\, x[4]\, x[5]\, x[6])^2$$

$$
\begin{aligned}
&\quad (1+x[3]\,x[4]\,x[5]\,x[6])^2\big)\big/\big(x[3]^2\,x[4]^2\,x[5]^2\,x[6]^2\big)\big),\\
&-\big(\big(c[6,\mathrm{wt}[-1,0,0,1,0,-1]]\,c[6,\mathrm{wt}[1,0,0,-1,0,1]]\,(-1+x[1]\,x[2]\,x[3]\,x[6])^2\\
&\quad (1+x[1]\,x[2]\,x[3]\,x[6])^2\big)\big/\big(x[1]^2\,x[2]^2\,x[3]^2\,x[6]^2\big)\big),\\
&-\big(\big(c[6,\mathrm{wt}[-1,1,-1,1,-1,1]]\,c[6,\mathrm{wt}[1,-1,1,-1,1,-1]]\,(-1+x[2]\,x[3]\,x[4]\,x[6])^2\\
&\quad (1+x[2]\,x[3]\,x[4]\,x[6])^2\big)\big/\big(x[2]^2\,x[3]^2\,x[4]^2\,x[6]^2\big)\big),\\
&-\big(\big(c[6,\mathrm{wt}[-1,1,0,0,1,-1]]\,c[6,\mathrm{wt}[1,-1,0,0,-1,1]]\,(-1+x[2]\,x[3]\,x[4]\,x[5])^2\\
&\quad (1+x[2]\,x[3]\,x[4]\,x[5])^2\big)\big/\big(x[2]^2\,x[3]^2\,x[4]^2\,x[5]^2\big)\big),\\
&-\big(\big(c[6,\mathrm{wt}[-1,0,0,-1,1,1]]\,c[6,\mathrm{wt}[1,0,0,1,-1,-1]]\,(-1+x[1]\,x[2]\,x[3]\,x[4])^2\\
&\quad (1+x[1]\,x[2]\,x[3]\,x[4])^2\big)\big/\big(x[1]^2\,x[2]^2\,x[3]^2\,x[4]^2\big)\big),\\
&-\big(\big(c[6,\mathrm{wt}[-1,1,0,-1,0,1]]\,c[6,\mathrm{wt}[1,-1,0,1,0,-1]]\,(-1+x[2]\,x[3]\,x[6])^2\\
&\quad (1+x[2]\,x[3]\,x[6])^2\big)\big/\big(x[2]^2\,x[3]^2\,x[6]^2\big)\big),\\
&-\big(\big(c[6,\mathrm{wt}[0,-1,0,1,-1,1]]\,c[6,\mathrm{wt}[0,1,0,-1,1,-1]]\,(-1+x[3]\,x[4]\,x[6])^2\\
&\quad (1+x[3]\,x[4]\,x[6])^2\big)\big/\big(x[3]^2\,x[4]^2\,x[6]^2\big)\big),\\
&-\big(\big(c[6,\mathrm{wt}[0,-1,1,0,1,-1]]\,c[6,\mathrm{wt}[0,1,-1,0,-1,1]]\,(-1+x[3]\,x[4]\,x[5])^2\\
&\quad (1+x[3]\,x[4]\,x[5])^2\big)\big/\big(x[3]^2\,x[4]^2\,x[5]^2\big)\big),\\
&-\big(\big(c[6,\mathrm{wt}[-1,0,-1,1,0,1]]\,c[6,\mathrm{wt}[1,0,1,-1,0,-1]]\,(-1+x[1]\,x[2]\,x[3])^2\\
&\quad (1+x[1]\,x[2]\,x[3])^2\big)\big/\big(x[1]^2\,x[2]^2\,x[3]^2\big)\big),\\
&-\big(\big(c[6,\mathrm{wt}[-1,1,0,1,-1,-1]]\,c[6,\mathrm{wt}[1,-1,0,-1,1,1]]\,(-1+x[2]\,x[3]\,x[4])^2\\
&\quad (1+x[2]\,x[3]\,x[4])^2\big)\big/\big(x[2]^2\,x[3]^2\,x[4]^2\big)\big),\\
&-\frac{1}{x[4]^2\,x[5]^2}c[6,\mathrm{wt}[0,0,-1,1,1,0]]\,c[6,\mathrm{wt}[0,0,1,-1,-1,0]]\\
&\quad (-1+x[4]\,x[5])^2\,(1+x[4]\,x[5])^2,\\
&-\frac{1}{x[1]^2\,x[2]^2}c[6,\mathrm{wt}[-1,-1,1,0,0,0]]\,c[6,\mathrm{wt}[1,1,-1,0,0,0]]\\
&\quad (-1+x[1]\,x[2])^2\,(1+x[1]\,x[2])^2,\\
&-\frac{1}{x[3]^2\,x[6]^2}c[6,\mathrm{wt}[0,-1,1,-1,0,1]]\,c[6,\mathrm{wt}[0,1,-1,1,0,-1]]\\
&\quad (-1+x[3]\,x[6])^2\,(1+x[3]\,x[6])^2,\\
&-\frac{1}{x[2]^2\,x[3]^2}c[6,\mathrm{wt}[-1,1,1,-1,0,-1]]\,c[6,\mathrm{wt}[1,-1,-1,1,0,1]]\\
&\quad (-1+x[2]\,x[3])^2\,(1+x[2]\,x[3])^2,\\
&-\frac{1}{x[3]^2\,x[4]^2}c[6,\mathrm{wt}[0,-1,1,1,-1,-1]]\,c[6,\mathrm{wt}[0,1,-1,-1,1,1]]\\
&\quad (-1+x[3]\,x[4])^2\,(1+x[3]\,x[4])^2,\\
&-\frac{1}{x[1]^2}c[6,\mathrm{wt}[-2,1,0,0,0,0]]\,c[6,\mathrm{wt}[2,-1,0,0,0,0]]\,(-1+x[1])^2\,(1+x[1])^2,\\
&-\frac{1}{x[6]^2}c[6,\mathrm{wt}[0,0,-1,0,0,2]]\,c[6,\mathrm{wt}[0,0,1,0,0,-2]]\,(-1+x[6])^2\,(1+x[6])^2,\\
&-\frac{1}{x[2]^2}c[6,\mathrm{wt}[-1,2,-1,0,0,0]]\,c[6,\mathrm{wt}[1,-2,1,0,0,0]]\,(-1+x[2])^2\,(1+x[2])^2,\\
&-\frac{1}{x[5]^2}c[6,\mathrm{wt}[0,0,0,-1,2,0]]\,c[6,\mathrm{wt}[0,0,0,1,-2,0]]\,(-1+x[5])^2\,(1+x[5])^2,\\
&-\frac{1}{x[4]^2}c[6,\mathrm{wt}[0,0,-1,2,-1,0]]\,c[6,\mathrm{wt}[0,0,1,-2,1,0]]\,(-1+x[4])^2\,(1+x[4])^2,\\
&-\frac{1}{x[3]^2}c[6,\mathrm{wt}[0,-1,2,-1,0,-1]]\,c[6,\mathrm{wt}[0,1,-2,1,0,1]]\,(-1+x[3])^2\,(1+x[3])^2\Big\}
\end{aligned}
$$

0

save list of rational functions for $C_\lambda^{(3)} = D_\lambda^{(3)}$ for $\lambda \in \{\lambda_4, \lambda_0\}$

functions for adding factored rational functions (run this cell for the following computation)

$\lambda = \lambda_4$

list of 802 rational functions (E6_3_la4_ratio_802.txt; ~900KB)

```
Block[{C3, D3, la, rk = 6, summand, C3firstelt, cdlist, file, stream, val,
  timestart, numtogo, commonfacC, commonfacD, commonfac, output = {}},
 la = λ[4];
 Print["C3 = list of summands for C_λ^(3), obtained from the following
    by replacing c[6,λ]=C_λ^(6) by the corresponding summands : "];
 (* C3=norcharCList[3,la]; *)
 C3 = Flatten[norcharC3C6List[la] /.
    {c[6, wt[0, 0, 0, 0, 0, 0]] → norcharCList[6, wt[0, 0, 0, 0, 0, 0]], c -> norcharC}];
 C3 = Factor[C3];
 Print["length of C3 : ", Length[C3]];
 Print["D3 = list of summands for D_λ^(3)"];
 D3 = norcharDList[3, la];
 Print["length of D3 : ", Length[D3]];
 (* make polynomials from rational functions by multiplying a comman factor *)
 commonfac = Weyldenom;
 C3 = commonfac * C3;
 D3 = commonfac * D3;
 cdlist = Join @@ {C3, -D3};
 file = FileNameJoin[{Directory[], "E6_3_la4_ratio_802.txt"}];
 timestart = AbsoluteTime[];
 numtogo = Length[cdlist];
 (* Print[{"step number","time elapsed","size of the expression"}]; *)
 Print["saving file..."];
 Do[
  val = {ss, FactorList@Factor[cdlist[[ss]]]};
  stream = OpenAppend[file];
  Write[stream, val];
  Close[stream];
  AppendTo[output, val];
  ,
  {ss, 1, numtogo}
 ];
 Print[{"time elapsed", Floor[AbsoluteTime[] - timestart]}];
]
```

C3 = list of summands for $C_\lambda^{(3)}$, obtained from the
  following by replacing c[6,λ]=$C_\lambda^{(6)}$ by the corresponding summands :

length of C3 : 82

D3 = list of summands for $D_\lambda^{(3)}$

length of D3 : 720

saving file...

{time elapsed, 23}

802 rational functions -> 46 rational functions (E6_3_la4_ratio_46.txt; ~600KB)

```
Block[{v3la4ratio802, file, stream, line, goodpartitions802, timestart},
  v3la4ratio802 =
   Last /@ ReadList[FileNameJoin[{Directory[], "E6_3_la4_ratio_802.txt"}]];
  goodpartitions802 = {{1, 72}, {2, 71}, {3, 70}, {4, 68},
    {5, 69}, {6, 65}, {7, 66}, {8, 67}, {9, 64}, {10, 62},
    {11, 63}, {12, 60}, {13, 61}, {14, 59}, {15, 58}, {16, 56},
    {17, 57}, {18, 55}, {19, 53}, {20, 54},
    {21, 51}, {22, 52}, {23, 49}, {24, 50}, {25, 48}, {26, 46},
    {27, 47}, {28, 45}, {29, 43}, {30, 44},
    {31, 39}, {32, 41}, {33, 38}, {34, 42}, {35, 40}, {36, 37},
    {73, 83, 84, 85, 87, 88, 89, 91, 92, 93, 95, 96, 101, 102, 103, 105, 106, 111, 112, 114,
     115, 120, 121, 130, 133, 134, 139, 140, 149, 155, 156, 165, 185, 186, 195, 219, 260,
     625, 666, 690, 699, 700, 720, 729, 730, 736, 745, 746, 751, 752, 755, 764,
     765, 770, 771, 773, 774, 779, 780, 782, 783, 784, 789, 790, 792, 793, 794, 796,
     797, 798, 800, 801, 802}, {74, 86, 90, 94, 99, 100, 104, 109, 110, 113, 118,
     119, 129, 131, 132, 137, 138, 148, 150, 153, 154, 164, 166, 179, 183, 184, 194,
     196, 209, 218, 220, 233, 259, 261, 274, 305, 354, 531, 580, 611, 624, 626, 652,
     665, 667, 676, 689, 691, 701, 702, 706, 719, 721, 731, 732, 735, 737, 747,
     748, 753, 754, 756, 766, 767, 772, 775, 776, 781, 785, 786, 791, 795, 799},
    {75, 97, 107, 116, 124, 128, 135, 143, 147, 151, 159, 163, 177, 180, 181,
     189, 193, 207, 210, 213, 217, 231, 234, 250, 254, 258, 272, 275, 291,
     303, 306, 322, 352, 355, 371, 406, 428, 458, 480, 515, 532, 535, 564, 581,
     584, 595, 612, 615, 628, 630, 636, 653, 656, 669, 671, 677, 680, 693, 695,
     703, 707, 710, 723, 725, 733, 739, 741, 749, 758, 760, 768, 777, 787},
    {76, 98, 108, 117, 125, 127, 136, 144, 146, 152, 160, 162, 175, 178, 182,
     190, 192, 205, 208, 214, 216, 229, 232, 249, 255, 257, 270, 273, 290,
     301, 304, 321, 350, 353, 370, 405, 427, 457, 479, 514, 530, 533, 563, 579,
     582, 594, 610, 613, 627, 631, 635, 651, 654, 668, 672, 675, 678, 692, 696,
     704, 705, 708, 722, 726, 734, 738, 742, 750, 757, 761, 769, 778, 788},
    {77, 122, 141, 157, 168, 176, 187, 198, 206, 211, 222, 230, 247, 251, 252,
     263, 271, 288, 292, 294, 302, 319, 323, 330, 341, 343, 351, 368, 372,
     379, 390, 403, 407, 414, 425, 432, 433, 455, 459, 466, 477, 484, 485, 501,
     512, 519, 520, 536, 537, 550, 561, 568, 569, 585, 586, 599, 600, 616, 617,
     629, 640, 641, 657, 658, 670, 681, 682, 694, 711, 712, 724, 740, 759},
    {78, 123, 142, 158, 171, 172, 188, 201, 202, 212, 225, 226, 246, 248, 253,
     266, 267, 287, 289, 297, 298, 318, 320, 325, 340, 346, 347, 367, 369,
     374, 389, 402, 404, 409, 424, 429, 431, 454, 456, 461, 476, 481, 483, 496,
     511, 516, 518, 538, 539, 545, 560, 565, 567, 587, 588, 596, 598, 618, 619,
```

```
      632, 637, 639, 659, 660, 673, 683, 684, 697, 713, 714, 727, 743, 762},
     {79, 126, 145, 161, 173, 174, 191, 203, 204, 215, 227, 228, 244, 245, 256,
      268, 269, 285, 286, 299, 300, 316, 317, 324, 335, 348, 349, 365, 366,
      373, 384, 400, 401, 408, 419, 426, 430, 452, 453, 460, 471, 478, 482, 495,
      506, 513, 517, 534, 542, 544, 555, 562, 566, 583, 591, 593, 597, 614, 622,
      633, 634, 638, 655, 663, 674, 679, 687, 698, 709, 717, 728, 744, 763},
     {80, 169, 199, 223, 236, 238, 241, 264, 277, 279, 282, 295, 308, 310, 313,
      326, 332, 333, 339, 344, 357, 359, 362, 375, 381, 382, 388, 392, 394,
      397, 410, 416, 417, 423, 436, 439, 441, 444, 446, 449, 462, 468, 469, 475,
      488, 491, 493, 497, 503, 504, 510, 523, 526, 528, 541, 546, 552, 553, 559,
      572, 575, 577, 590, 603, 606, 608, 621, 644, 647, 649, 662, 686, 716},
     {81, 167, 197, 221, 237, 239, 242, 262, 278, 280, 283, 293, 309, 311, 314,
      329, 331, 337, 338, 342, 358, 360, 363, 378, 380, 386, 387, 393, 395,
      398, 413, 415, 421, 422, 434, 437, 442, 445, 447, 450, 465, 467, 473, 474,
      486, 489, 494, 500, 502, 508, 509, 521, 524, 529, 540, 549, 551, 557, 558,
      570, 573, 578, 589, 601, 604, 609, 620, 642, 645, 650, 661, 685, 715},
     {82, 170, 200, 224, 235, 240, 243, 265, 276, 281, 284, 296, 307, 312, 315,
      327, 328, 334, 336, 345, 356, 361, 364, 376, 377, 383, 385, 391, 396,
      399, 411, 412, 418, 420, 435, 438, 440, 443, 448, 451, 463, 464, 470, 472,
      487, 490, 492, 498, 499, 505, 507, 522, 525, 527, 543, 547, 548, 554, 556,
      571, 574, 576, 592, 602, 605, 607, 623, 643, 646, 648, 664, 688, 718}
    };
   file = "E6_3_la4_ratio_46.txt";
   timestart = Floor[AbsoluteTime[]];
   Do[
    (* line = {idx,
       sumofratiolist[v3la4ratio802,randompermute[goodpartitions802[[idx]]]]}; *)
    line = {idx, sumoftworationalfunc[v3la4ratio802[[goodpartitions802[[idx]][[1]]]],
       v3la4ratio802[[goodpartitions802[[idx]][[2]]]]};
    stream = OpenAppend[file];
    Write[stream, line];
    Close[stream],
    {idx, 1, 36}
   ];
   ParallelDo[
    Print[
     {"working on  :", idx, "time elapsed : ", Floor[AbsoluteTime[]] - timestart}];
    (* line = {idx,sumofratiolist[v3la4ratio802,
        randompermute[goodpartitions802[[idx]]]]}; *)
    line = {idx, FactorList@Total[toexp /@ Part[v3la4ratio802,
          goodpartitions802[[idx]]]]};
    stream = OpenAppend[file];
    Write[stream, line];
    Close[stream],
    {idx, 37, 46}
   ];
  ];
{working on  :, 37, time elapsed : , 20}
{working on  :, 39, time elapsed : , 20}
```

```
{working on  :, 41, time elapsed : , 20}
{working on  :, 42, time elapsed : , 20}
{working on  :, 43, time elapsed : , 94}
{working on  :, 40, time elapsed : , 94}
{working on  :, 38, time elapsed : , 97}
{working on  :, 44, time elapsed : , 145}
{working on  :, 45, time elapsed : , 176}
{working on  :, 46, time elapsed : , 177}
```

46 rational functions -> 46 polynomials (E6_3_la4_poly_46.txt; ~600KB)

```
Block[{v3la4ratio46, v3la4poly46 , denoms,
   commonfac, summandlist, file, stream, line, timestart},
  timestart = AbsoluteTime[];
  v3la4ratio46 = Last /@
    SortBy[ReadList[FileNameJoin[{Directory[], "E6_3_la4_ratio_46.txt"}]], First];
  denoms = Table[toexp@ ({#[[1]], -#[[2]]} & /@
       Select[v3la4ratio46[[jj]], #[[2]] < 0 &]), {jj, 1, Length[v3la4ratio46]}];
  commonfac = JoinFactorList[{
     Table[{x[jj], -Min[exponent[#, x[jj]] & /@ v3la4ratio46]}, {jj, 1, 6}],
     FactorList@( PolynomialLCM @@ denoms)
    }];
  v3la4poly46 = Table[JoinFactorList[{commonfac, ss}], {ss, v3la4ratio46}];
  summandlist = v3la4poly46;
  file = "E6_3_la4_poly_46.txt";
  Do[
   line = {idx, summandlist[[idx]]};
   stream = OpenAppend[file]; Write[stream, line]; Close[stream],
   {idx, 1, Length[summandlist]}
  ];
  Print[{"time elapsed:", Floor[AbsoluteTime[] - timestart]}];
 ];
{time elapsed:, 1}
```

$\lambda = \lambda_0$

list of 36+51480 rational functions (E6_3_la0_ratio.txt; ~ 12MB)

```
Block[{C3, D3, la, rk = 6, summand, C3firstelt, cdlist, file, stream, val,
  timestart, numtogo, commonfacC, commonfacD, commonfac, output = {}},
 la = λ[0];
 Print["C3 = list of summands for C_λ^(3), obtained from the following
    by replacing c[6,λ]=C_λ^(6) by the corresponding summands : "];
 C3 = norcharCList[3, la];
 Print["length of C3 : ", Length[C3]];
 Print["D3 = list of summands for D_λ^(3) "];
 D3 = norcharDList[3, la];
 Print["length of D3 : ", Length[D3]];
 commonfac = Weyldenom;
 C3 = commonfac * C3;
 D3 = commonfac * D3;
 cdlist = Join @@ {C3, -D3};
 file = FileNameJoin[{Directory[], "E6_3_la0_ratio.txt"}];
 stream = OpenAppend[file];
 timestart = AbsoluteTime[];
 numtogo = Length[cdlist];
 (* Print[
   {"step number","time elapsed","size of the expression","polynomial in x_i?"}]; *)
 Print["saving file..."];
 Do[
  val = {ss, FactorList[cdlist[[ss]]]};
  Write[stream, val];
  ,
  {ss, 1, numtogo}
 ];
 Close[stream];
 Print[{"time elapsed:", Floor[AbsoluteTime[] - timestart]}];
]
```

```
C3 = list of summands for C_λ^(3), obtained from the
  following by replacing c[6,λ]=C_λ^(6) by the corresponding summands :
length of C3 : 36
D3 = list of summands for D_λ^(3)
length of D3 : 51840
saving file...
{time elapsed:, 146}
```

numerical check for saved polynomials

```
Block[{v3la0ratio, rule, rk = 6},
 v3la0ratio =
  Last /@ SortBy[ReadList[FileNameJoin[{Directory[], "E6_3_la0_ratio.txt"}]], First];
 rule = Inner[Rule, Array[x, rk], RandomInteger[{1, 1000}, rk], List];
 Total[(toexp /@ v3la0ratio) /. rule]
]
```

```
0
```