
code

data (pre-computed)

```
In[ ]:= type = "E8";
(*Define basis elements as symbols*)
basisElements = {V0, V1, V2};
(*Define multiplication rules*)
multiplicationRules = {
  V0 * V0  $\Rightarrow$  V0,
  V0 * V1  $\Rightarrow$  V1,
  V0 * V2  $\Rightarrow$  V2,
  V1 * V1  $\Rightarrow$  V0,
  V1 * V2  $\Rightarrow$  V2,
  V2 * V2  $\Rightarrow$  V0 + V1};
KRvariableToVerlindeImage = {
  x1  $\rightarrow$  ringElement[V0 + V1 + V2],
  x2  $\rightarrow$  ringElement[V0],
  x3  $\rightarrow$  ringElement[2 V0 + 3 V1 + 4 V2],
  x4  $\rightarrow$  ringElement[V0],
  x5  $\rightarrow$  ringElement[8 V0 + 8 V1 + 12 V2],
  x6  $\rightarrow$  ringElement[V0],
  x7  $\rightarrow$  ringElement[V0 + 2 V1 + 2 V2],
  x8  $\rightarrow$  ringElement[V0],
  x9  $\rightarrow$  ringElement[4 V0 + 5 V1 + 6 V2],
  x10  $\rightarrow$  ringElement[V0],
  x11  $\rightarrow$  ringElement[2 V0 + 3 V1 + 3 V2],
  x12  $\rightarrow$  ringElement[V0],
  x13  $\rightarrow$  ringElement[V0 + V1 + 2 V2],
  x14  $\rightarrow$  ringElement[V0],
  x15  $\rightarrow$  ringElement[V0 + V2],
  x16  $\rightarrow$  ringElement[V0],
  x17  $\rightarrow$  ringElement[V0 + V2],
  x18  $\rightarrow$  ringElement[V0 + V1 + 2 V2],
  x19  $\rightarrow$  ringElement[2 V0 + 3 V1 + 3 V2],
  x34  $\rightarrow$  ringElement[V0 + 2 V1 + 2 V2],
  x37  $\rightarrow$  ringElement[2 V0 + 3 V1 + 4 V2],
  x44  $\rightarrow$  ringElement[V0 + V1 + V2],
  x113  $\rightarrow$  ringElement[4 V0 + 5 V1 + 6 V2],
  x114  $\rightarrow$  ringElement[8 V0 + 8 V1 + 12 V2]
};
```

import exchange relations

```

In[ ]:= extractxindex[var_] := Module[{numberStr}, Which[
  ListQ[var], extractxindex /@ var,
  True, numberStr = StringDrop[SymbolName[var], 1];
  ToExpression[numberStr]]]
xSort[vars_List] := SortBy[vars, extractxindex]
swapIfNotOrdered[triple_List] := Module[{second = triple[[2]], third = triple[[3]]},
  If[OrderedQ[{second, third}], ReplacePart[triple, {2 → third, 3 → second}], triple]]
filePath = FileNameJoin[{NotebookDirectory[], "exchange_relations_" <> type <> ".txt"}];
fileLines = ReadList[filePath, String];
convertedLines = DeleteDuplicates[StringReplace[fileLines, {"[" → "{", "]" → "}"}]];
exchangerelations = ToExpression /@ convertedLines;
exchangerelations =
  Map[{xSort[#[[1]]], xSort[#[[2]]], xSort[#[[3]]]} &, exchangerelations];
exchangerelations = SortBy[exchangerelations, extractxindex[#[[1]]] &];
exchangerelations = Map[swapIfNotOrdered, exchangerelations];
exchangerelations = DeleteDuplicates[exchangerelations];
(* display sample exchange relations *)
exchangerelations[[1 ;; 10]]
(* display cluster variables *)
clustervariables = Flatten[exchangerelations] // DeleteDuplicates // xSort

Out[ ]:= {{x1, x21}, {x2, x3}, {x4}}, {{x1, x22}, {x2, x7, x9}, {x126}},
  {{x1, x23}, {x2, x7}, {x55}}, {{x1, x24}, {x2, x9}, {x84}},
  {{x1, x27}, {x2, x7, x15}, {x61}}, {{x1, x28}, {x2, x5}, {x29}},
  {{x1, x30}, {x2, x15}, {x35}}, {{x1, x44}, {x37}, {x2}},
  {{x1, x45}, {x2, x52}, {x41}}, {{x1, x49}, {x2, x68}, {x43}}

Out[ ]:= {x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15, x16, x17,
  x18, x19, x20, x21, x22, x23, x24, x25, x26, x27, x28, x29, x30, x31, x32, x33,
  x34, x35, x36, x37, x38, x39, x40, x41, x42, x43, x44, x45, x46, x47, x48, x49,
  x50, x51, x52, x53, x54, x55, x56, x57, x58, x59, x60, x61, x62, x63, x64, x65,
  x66, x67, x68, x69, x70, x71, x72, x73, x74, x75, x76, x77, x78, x79, x80, x81,
  x82, x83, x84, x85, x86, x87, x88, x89, x90, x91, x92, x93, x94, x95, x96, x97,
  x98, x99, x100, x101, x102, x103, x104, x105, x106, x107, x108, x109, x110,
  x111, x112, x113, x114, x115, x116, x117, x118, x119, x120, x121, x122, x123,
  x124, x125, x126, x127, x128, x129, x130, x131, x132, x133, x134, x135, x136}

```

code for Verlinde ring

```

In[ ]:= rankofVerlinde = Length[basisElements];

simplifyRingElement[expr_] := Module[{result},
  result = Total[Coefficient[expr, #] * # & /@basisElements];
  result]

(*Define a function for addition of ring elements*)
ringAdd[element1_, element2_] := element1 + element2
(*Define a function for multiplication of ring elements*)
ringMultiply[element1_, element2_] :=
  Expand[Expand[(element1 element2)] /. multiplicationRules]

(*Define the ringElement class*)
ringElement /: Plus[ringElement[e1_], ringElement[e2_]] :=
  ringElement[simplifyRingElement[ringAdd[e1, e2]]];
ringElement /: Times[ringElement[e1_], ringElement[e2_]] :=
  ringElement[simplifyRingElement[ringMultiply[e1, e2]]];
ringElement /: Times[scalar_, ringElement[e_]] /;
  FreeQ[symbol, Alternatives @@ basisElements] :=
  ringElement[simplifyRingElement[symbol * e]];
(*Subtraction rule using addition and scalar multiplication*)
ringElement /: Subtract[ringElement[e1_], ringElement[e2_]] :=
  ringElement[simplifyRingElement[ringAdd[e1, -1 * e2]]];

unknownindex = Complement[Range[1, Length[clustervariables]],
  Map[extractxindex, First /@ KRvariableToVerlindeImage]];

variableToVerlindeImage = Union[
  KRvariableToVerlindeImage,
  Table[ToExpression["x" <> ToString[index]] ->
    ringElement[Sum[c[index, k] * Evaluate[Symbol["V" <> ToString[k]]],
      {k, 0, rankofVerlinde - 1}]], {index, unknownindex}];
unknowns = Flatten@Table[Table[c[index, k], {k, 0, rankofVerlinde - 1}],
  {index, unknownindex}];

(* convert expression in cluster variables to Verlinde element *)
convertToRingElements[expr_] := expr /. variableToVerlindeImage
(*Function to transform {{x1,x17},x2 x3+x4} to x1*x17-(x2*x3+x4)*)
exchangeVerlinde[{{a_, b_}, X_, Y_}] :=
  convertToRingElements[a] * convertToRingElements[b] -
  (Times @@ convertToRingElements[X]) - (Times @@ convertToRingElements[Y])
exponents[expr_] := Module[{terms, totalDegree},
  terms = If[Head[Expand[expr]] === Plus, List @@ Expand[expr], {Expand[expr]}];
  totalDegree = Map[Total[Cases[Factor[#], c[_], _]^n_. -> n, {0, 1}]] &, terms];
  totalDegree
]
extractCoefficients[ringElement[expr_]] := Coefficient[expr, #] & /@basisElements

```

exemple of exchange relations and corresponding

equations for Verlinde coefficients

```
In[ ]:= unknowns // Length
```

```
Out[ ]:= 336
```

```
In[ ]:= exchangerelations[[1]]
```

```
Out[ ]:= {{x1, x21}, {x2, x3}, {x4}}
```

```
In[ ]:= Map[# == 0 &, extractCoefficients@exchangeVerlinde[{{x1, x21}, {x2, x3}, {x4}}]] //  
TableForm
```

```
Out[ ]//TableForm=
```

```
- 3 + c[21, 0] + c[21, 1] + c[21, 2] == 0  
- 3 + c[21, 0] + c[21, 1] + c[21, 2] == 0  
- 4 + c[21, 0] + c[21, 1] + 2 c[21, 2] == 0
```

```
In[ ]:= Flatten[Table[extractCoefficients@exchangeVerlinde[exchangerelations[[index]],  
{index, 10}]] // TableForm
```

```
Out[ ]//TableForm=
```

```
- 3 + c[21, 0] + c[21, 1] + c[21, 2]  
- 3 + c[21, 0] + c[21, 1] + c[21, 2]  
- 4 + c[21, 0] + c[21, 1] + 2 c[21, 2]  
- 26 + c[22, 0] + c[22, 1] + c[22, 2] - c[126, 0]  
- 25 + c[22, 0] + c[22, 1] + c[22, 2] - c[126, 1]  
- 36 + c[22, 0] + c[22, 1] + 2 c[22, 2] - c[126, 2]  
- 1 + c[23, 0] + c[23, 1] + c[23, 2] - c[55, 0]  
- 2 + c[23, 0] + c[23, 1] + c[23, 2] - c[55, 1]  
- 2 + c[23, 0] + c[23, 1] + 2 c[23, 2] - c[55, 2]  
- 4 + c[24, 0] + c[24, 1] + c[24, 2] - c[84, 0]  
- 5 + c[24, 0] + c[24, 1] + c[24, 2] - c[84, 1]  
- 6 + c[24, 0] + c[24, 1] + 2 c[24, 2] - c[84, 2]  
- 3 + c[27, 0] + c[27, 1] + c[27, 2] - c[61, 0]  
- 4 + c[27, 0] + c[27, 1] + c[27, 2] - c[61, 1]  
- 5 + c[27, 0] + c[27, 1] + 2 c[27, 2] - c[61, 2]  
- 8 + c[28, 0] + c[28, 1] + c[28, 2] - c[29, 0]  
- 8 + c[28, 0] + c[28, 1] + c[28, 2] - c[29, 1]  
- 12 + c[28, 0] + c[28, 1] + 2 c[28, 2] - c[29, 2]  
- 1 + c[30, 0] + c[30, 1] + c[30, 2] - c[35, 0]  
c[30, 0] + c[30, 1] + c[30, 2] - c[35, 1]  
- 1 + c[30, 0] + c[30, 1] + 2 c[30, 2] - c[35, 2]  
0  
0  
0  
- c[41, 0] + c[45, 0] + c[45, 1] + c[45, 2] - c[52, 0]  
- c[41, 1] + c[45, 0] + c[45, 1] + c[45, 2] - c[52, 1]  
- c[41, 2] + c[45, 0] + c[45, 1] + 2 c[45, 2] - c[52, 2]  
- c[43, 0] + c[49, 0] + c[49, 1] + c[49, 2] - c[68, 0]  
- c[43, 1] + c[49, 0] + c[49, 1] + c[49, 2] - c[68, 1]  
- c[43, 2] + c[49, 0] + c[49, 1] + 2 c[49, 2] - c[68, 2]
```

Solving equations for Verlinde coefficients

set of all equations among coefficients $\{c[j,k]\}$

```

In[ ]:= coeffList = DeleteDuplicates@
  Flatten[Table[(extractCoefficients@exchangeVerlinde[exchangerelations[[index]]],
    {index, 1, Length[exchangerelations]}]];
coeffList = SortBy[coeffList, Max[exponents[#]] &];
coeffList[[1 ;; 50]]

Out[ ]:= {0, -28 + 8 c[20, 0] + 8 c[20, 1] + 12 c[20, 2], -40 + 12 c[20, 0] + 12 c[20, 1] + 16 c[20, 2],
  1 - c[21, 0], 1 - c[21, 1], 1 - c[21, 2], -3 + c[21, 0] + c[21, 1] + c[21, 2],
  -4 + c[21, 0] + c[21, 1] + 2 c[21, 2], -9 + 3 c[21, 0] + 2 c[21, 1] + 4 c[21, 2],
  -9 + 2 c[21, 0] + 3 c[21, 1] + 4 c[21, 2], -13 + 4 c[21, 0] + 4 c[21, 1] + 5 c[21, 2],
  -c[20, 1] - 25 c[21, 0] - 26 c[21, 1] - 36 c[21, 2] + 3 c[22, 0] + 2 c[22, 1] + 4 c[22, 2],
  -c[20, 0] - 26 c[21, 0] - 25 c[21, 1] - 36 c[21, 2] + 2 c[22, 0] + 3 c[22, 1] + 4 c[22, 2],
  -c[20, 2] - 36 c[21, 0] - 36 c[21, 1] - 51 c[21, 2] + 4 c[22, 0] + 4 c[22, 1] + 5 c[22, 2],
  1 - c[23, 0], -c[23, 1], 1 - c[23, 2], -3 - c[22, 1] + 5 c[23, 0] + 4 c[23, 1] + 6 c[23, 2],
  -2 - c[22, 0] + 4 c[23, 0] + 5 c[23, 1] + 6 c[23, 2],
  -3 - c[22, 2] + 6 c[23, 0] + 6 c[23, 1] + 9 c[23, 2], 1 - c[24, 0],
  2 - c[24, 1], 2 - c[24, 2], -c[22, 1] + 2 c[24, 0] + c[24, 1] + 2 c[24, 2],
  -1 - c[22, 0] + c[24, 0] + 2 c[24, 1] + 2 c[24, 2],
  -c[22, 2] + 2 c[24, 0] + 2 c[24, 1] + 3 c[24, 2], 1 - c[25, 0], -c[25, 1],
  1 - c[25, 2], -2 + c[25, 0] + c[25, 2], -1 + c[25, 1] + c[25, 2],
  -2 + c[25, 0] + c[25, 1] + c[25, 2], -3 + c[25, 0] + c[25, 1] + 2 c[25, 2],
  -4 + 2 c[25, 0] + 2 c[25, 1] + 2 c[25, 2], 1 - c[26, 0], 1 - c[26, 1], 2 - c[26, 2],
  -1 - 2 c[25, 0] - 3 c[25, 1] - 3 c[25, 2] + c[26, 0] + c[26, 1] + 2 c[26, 2],
  -3 c[25, 0] - 2 c[25, 1] - 3 c[25, 2] + c[26, 0] + c[26, 1] + 2 c[26, 2],
  -3 c[25, 0] - 3 c[25, 1] - 5 c[25, 2] + 2 c[26, 0] + 2 c[26, 1] + 2 c[26, 2],
  -5 c[25, 0] - 4 c[25, 1] - 6 c[25, 2] + 3 c[26, 0] + 2 c[26, 1] + 3 c[26, 2],
  -1 - 4 c[25, 0] - 5 c[25, 1] - 6 c[25, 2] + 2 c[26, 0] + 3 c[26, 1] + 3 c[26, 2],
  -1 - 6 c[25, 0] - 6 c[25, 1] - 9 c[25, 2] + 3 c[26, 0] + 3 c[26, 1] + 5 c[26, 2],
  -1 + c[23, 0] + c[23, 2] - c[27, 0], c[23, 1] + c[23, 2] - c[27, 1],
  c[23, 0] + c[23, 1] + c[23, 2] - c[27, 2],
  -c[22, 1] - c[22, 2] - c[26, 1] + 5 c[27, 0] + 4 c[27, 1] + 6 c[27, 2],
  -c[22, 0] - c[22, 2] - c[26, 0] + 4 c[27, 0] + 5 c[27, 1] + 6 c[27, 2],
  -c[22, 0] - c[22, 1] - c[22, 2] - c[26, 2] + 6 c[27, 0] + 6 c[27, 1] + 9 c[27, 2], 2 - c[28, 0]}

```

first set of linear equations

```

In[ ]:= Block[{knownsols, newSols, numEquations, equations, solution, currentIndex, batchSize},
  knownsols = {};
  Print["number of known variables before:", Length@knownsols];
  equations = coeffList;
  equations = equations /. knownsols;
  equations = DeleteDuplicates@ Select[equations, Max[exponents[#]] == 1 &];
  Print["number of equations used:", Length[equations]];
  equations = Map[# == 0 &, equations];
  equations = Join[equations, knownsols /. Rule -> Equal];
  solution = Solve[equations, unknowns];
  If[Length[solution] == 1,
    newSols = Select[solution[[1]], NumericQ[#[[2]]] &];
    knownsols = Union[knownsols, newSols];
    Print["number of known variables after: ", {Length[knownsols], #} &@knownsols];
  ]
]

```

number of known variables before:0

number of equations used:998

 **Solve:** Equations may not give solutions for all "solve" variables.

number of known variables after:

```
{216, {c[20, 0] → 1, c[20, 1] → 1, c[20, 2] → 1, c[21, 0] → 1, c[21, 1] → 1, c[21, 2] → 1,
c[22, 0] → 8, c[22, 1] → 8, c[22, 2] → 12, c[23, 0] → 1, c[23, 1] → 0, c[23, 2] → 1,
c[24, 0] → 1, c[24, 1] → 2, c[24, 2] → 2, c[25, 0] → 1, c[25, 1] → 0, c[25, 2] → 1, c[26, 0] → 1,
c[26, 1] → 1, c[26, 2] → 2, c[27, 0] → 1, c[27, 1] → 1, c[27, 2] → 2, c[28, 0] → 2, c[28, 1] → 3,
c[28, 2] → 4, c[29, 0] → 1, c[29, 1] → 1, c[29, 2] → 1, c[30, 0] → 1, c[30, 1] → 0, c[30, 2] → 1,
c[31, 0] → 2, c[31, 1] → 3, c[31, 2] → 3, c[32, 0] → 1, c[32, 1] → 0, c[32, 2] → 1, c[33, 0] → 1,
c[33, 1] → 1, c[33, 2] → 2, c[35, 0] → 1, c[35, 1] → 2, c[35, 2] → 2, c[36, 0] → 4, c[36, 1] → 5,
c[36, 2] → 6, c[38, 0] → 2, c[38, 1] → 3, c[38, 2] → 3, c[39, 0] → 1, c[39, 1] → 0, c[39, 2] → 1,
c[40, 0] → 1, c[40, 1] → 1, c[40, 2] → 2, c[41, 0] → 8, c[41, 1] → 8, c[41, 2] → 12,
c[42, 0] → 1, c[42, 1] → 2, c[42, 2] → 2, c[43, 0] → 4, c[43, 1] → 5, c[43, 2] → 6, c[45, 0] → 2,
c[45, 1] → 3, c[45, 2] → 4, c[46, 0] → 2, c[46, 1] → 3, c[46, 2] → 3, c[47, 0] → 1, c[47, 1] → 0,
c[47, 2] → 1, c[48, 0] → 1, c[48, 1] → 1, c[48, 2] → 2, c[49, 0] → 1, c[49, 1] → 2, c[49, 2] → 2,
c[50, 0] → 8, c[50, 1] → 8, c[50, 2] → 12, c[51, 0] → 4, c[51, 1] → 5, c[51, 2] → 6,
c[52, 0] → 1, c[52, 1] → 1, c[52, 2] → 1, c[53, 0] → 2, c[53, 1] → 3, c[53, 2] → 4, c[54, 0] → 2,
c[54, 1] → 3, c[54, 2] → 3, c[55, 0] → 1, c[55, 1] → 0, c[55, 2] → 1, c[56, 0] → 1, c[56, 1] → 1,
c[56, 2] → 2, c[57, 0] → 1, c[57, 1] → 2, c[57, 2] → 2, c[58, 0] → 1, c[58, 1] → 1, c[58, 2] → 1,
c[59, 0] → 1, c[59, 1] → 1, c[59, 2] → 1, c[60, 0] → 1, c[60, 1] → 0, c[60, 2] → 1, c[61, 0] → 1,
c[61, 1] → 0, c[61, 2] → 1, c[64, 0] → 1, c[64, 1] → 1, c[64, 2] → 2, c[65, 0] → 1, c[65, 1] → 1,
c[65, 2] → 1, c[66, 0] → 1, c[66, 1] → 1, c[66, 2] → 2, c[67, 0] → 1, c[67, 1] → 0, c[67, 2] → 1,
c[68, 0] → 1, c[68, 1] → 0, c[68, 2] → 1, c[71, 0] → 1, c[71, 1] → 0, c[71, 2] → 1, c[72, 0] → 1,
c[72, 1] → 1, c[72, 2] → 1, c[75, 0] → 1, c[75, 1] → 1, c[75, 2] → 1, c[78, 0] → 1, c[78, 1] → 0,
c[78, 2] → 1, c[79, 0] → 1, c[79, 1] → 1, c[79, 2] → 2, c[80, 0] → 1, c[80, 1] → 1, c[80, 2] → 1,
c[81, 0] → 1, c[81, 1] → 1, c[81, 2] → 1, c[84, 0] → 1, c[84, 1] → 0, c[84, 2] → 1,
c[86, 0] → 1, c[86, 1] → 2, c[86, 2] → 2, c[87, 0] → 1, c[87, 1] → 1, c[87, 2] → 1,
c[88, 0] → 1, c[88, 1] → 1, c[88, 2] → 1, c[94, 0] → 1, c[94, 1] → 1, c[94, 2] → 2,
c[97, 0] → 1, c[97, 1] → 1, c[97, 2] → 1, c[102, 0] → 1, c[102, 1] → 2, c[102, 2] → 2,
c[108, 0] → 1, c[108, 1] → 1, c[108, 2] → 2, c[111, 0] → 2, c[111, 1] → 3, c[111, 2] → 3,
c[112, 0] → 1, c[112, 1] → 1, c[112, 2] → 2, c[116, 0] → 1, c[116, 1] → 1, c[116, 2] → 2,
c[119, 0] → 1, c[119, 1] → 2, c[119, 2] → 2, c[120, 0] → 2, c[120, 1] → 3, c[120, 2] → 4,
c[123, 0] → 1, c[123, 1] → 2, c[123, 2] → 2, c[124, 0] → 2, c[124, 1] → 3, c[124, 2] → 4,
c[126, 0] → 2, c[126, 1] → 3, c[126, 2] → 4, c[127, 0] → 4, c[127, 1] → 5, c[127, 2] → 6,
c[128, 0] → 2, c[128, 1] → 3, c[128, 2] → 3, c[129, 0] → 2, c[129, 1] → 3, c[129, 2] → 4,
c[133, 0] → 8, c[133, 1] → 8, c[133, 2] → 12, c[135, 0] → 4, c[135, 1] → 5, c[135, 2] → 6}}
```

second set of linear equations


```

In[ ]:= Block[{ knownsols, newSols, numEquations,
  equations, solution, currentIndex, batchSize},
knownsols = {c[20, 0] → 1, c[20, 1] → 1, c[20, 2] → 1, c[21, 0] → 1, c[21, 1] → 1,
  c[21, 2] → 1, c[22, 0] → 8, c[22, 1] → 8, c[22, 2] → 12, c[23, 0] → 1, c[23, 1] → 0,
  c[23, 2] → 1, c[24, 0] → 1, c[24, 1] → 2, c[24, 2] → 2, c[25, 0] → 1, c[25, 1] → 0,
  c[25, 2] → 1, c[26, 0] → 1, c[26, 1] → 1, c[26, 2] → 2, c[27, 0] → 1, c[27, 1] → 1,
  c[27, 2] → 2, c[28, 0] → 2, c[28, 1] → 3, c[28, 2] → 4, c[29, 0] → 1, c[29, 1] → 1,
  c[29, 2] → 1, c[30, 0] → 1, c[30, 1] → 0, c[30, 2] → 1, c[31, 0] → 2, c[31, 1] → 3,
  c[31, 2] → 3, c[32, 0] → 1, c[32, 1] → 0, c[32, 2] → 1, c[33, 0] → 1, c[33, 1] → 1,
  c[33, 2] → 2, c[35, 0] → 1, c[35, 1] → 2, c[35, 2] → 2, c[36, 0] → 4, c[36, 1] → 5,
  c[36, 2] → 6, c[38, 0] → 2, c[38, 1] → 3, c[38, 2] → 3, c[39, 0] → 1, c[39, 1] → 0,
  c[39, 2] → 1, c[40, 0] → 1, c[40, 1] → 1, c[40, 2] → 2, c[41, 0] → 8, c[41, 1] → 8,
  c[41, 2] → 12, c[42, 0] → 1, c[42, 1] → 2, c[42, 2] → 2, c[43, 0] → 4, c[43, 1] → 5,
  c[43, 2] → 6, c[45, 0] → 2, c[45, 1] → 3, c[45, 2] → 4, c[46, 0] → 2, c[46, 1] → 3,
  c[46, 2] → 3, c[47, 0] → 1, c[47, 1] → 0, c[47, 2] → 1, c[48, 0] → 1, c[48, 1] → 1,
  c[48, 2] → 2, c[49, 0] → 1, c[49, 1] → 2, c[49, 2] → 2, c[50, 0] → 8, c[50, 1] → 8,
  c[50, 2] → 12, c[51, 0] → 4, c[51, 1] → 5, c[51, 2] → 6, c[52, 0] → 1, c[52, 1] → 1,
  c[52, 2] → 1, c[53, 0] → 2, c[53, 1] → 3, c[53, 2] → 4, c[54, 0] → 2, c[54, 1] → 3,
  c[54, 2] → 3, c[55, 0] → 1, c[55, 1] → 0, c[55, 2] → 1, c[56, 0] → 1, c[56, 1] → 1,
  c[56, 2] → 2, c[57, 0] → 1, c[57, 1] → 2, c[57, 2] → 2, c[58, 0] → 1, c[58, 1] → 1,
  c[58, 2] → 1, c[59, 0] → 1, c[59, 1] → 1, c[59, 2] → 1, c[60, 0] → 1, c[60, 1] → 0,
  c[60, 2] → 1, c[61, 0] → 1, c[61, 1] → 0, c[61, 2] → 1, c[64, 0] → 1, c[64, 1] → 1,
  c[64, 2] → 2, c[65, 0] → 1, c[65, 1] → 1, c[65, 2] → 1, c[66, 0] → 1, c[66, 1] → 1,
  c[66, 2] → 2, c[67, 0] → 1, c[67, 1] → 0, c[67, 2] → 1, c[68, 0] → 1, c[68, 1] → 0,
  c[68, 2] → 1, c[71, 0] → 1, c[71, 1] → 0, c[71, 2] → 1, c[72, 0] → 1, c[72, 1] → 1,
  c[72, 2] → 1, c[75, 0] → 1, c[75, 1] → 1, c[75, 2] → 1, c[78, 0] → 1, c[78, 1] → 0,
  c[78, 2] → 1, c[79, 0] → 1, c[79, 1] → 1, c[79, 2] → 2, c[80, 0] → 1, c[80, 1] → 1,
  c[80, 2] → 1, c[81, 0] → 1, c[81, 1] → 1, c[81, 2] → 1, c[84, 0] → 1, c[84, 1] → 0,
  c[84, 2] → 1, c[86, 0] → 1, c[86, 1] → 2, c[86, 2] → 2, c[87, 0] → 1, c[87, 1] → 1,
  c[87, 2] → 1, c[88, 0] → 1, c[88, 1] → 1, c[88, 2] → 1, c[94, 0] → 1, c[94, 1] → 1,
  c[94, 2] → 2, c[97, 0] → 1, c[97, 1] → 1, c[97, 2] → 1, c[102, 0] → 1, c[102, 1] → 2,
  c[102, 2] → 2, c[108, 0] → 1, c[108, 1] → 1, c[108, 2] → 2, c[111, 0] → 2, c[111, 1] → 3,
  c[111, 2] → 3, c[112, 0] → 1, c[112, 1] → 1, c[112, 2] → 2, c[116, 0] → 1, c[116, 1] → 1,
  c[116, 2] → 2, c[119, 0] → 1, c[119, 1] → 2, c[119, 2] → 2, c[120, 0] → 2, c[120, 1] → 3,
  c[120, 2] → 4, c[123, 0] → 1, c[123, 1] → 2, c[123, 2] → 2, c[124, 0] → 2,
  c[124, 1] → 3, c[124, 2] → 4, c[126, 0] → 2, c[126, 1] → 3, c[126, 2] → 4,
  c[127, 0] → 4, c[127, 1] → 5, c[127, 2] → 6, c[128, 0] → 2, c[128, 1] → 3,
  c[128, 2] → 3, c[129, 0] → 2, c[129, 1] → 3, c[129, 2] → 4, c[133, 0] → 8,
  c[133, 1] → 8, c[133, 2] → 12, c[135, 0] → 4, c[135, 1] → 5, c[135, 2] → 6};
Print["number of known variables before:", Length@knownsols];
equations = coeffList;
equations = equations /. knownsols;
equations = DeleteDuplicates@ Select[equations, Max[exponents[#]] == 1 &];
Print["number of equations used:", Length[equations]];
equations = Map[# == 0 &, equations];
equations = Join[equations, knownsols /. Rule → Equal];
solution = Solve[equations, unknowns];
If[Length[solution] == 1,
  newSols = Select[solution[[1]], NumericQ[#[[2]]] &];
  knownsols = Union[knownsols, newSols];
  Print["number of known variables after: ", {Length[#], #} &@knownsols];
]
]

```

number of known variables before:216

number of equations used:2386

number of known variables after:

```
{336, {c[20, 0] → 1, c[20, 1] → 1, c[20, 2] → 1, c[21, 0] → 1, c[21, 1] → 1, c[21, 2] → 1,
  c[22, 0] → 8, c[22, 1] → 8, c[22, 2] → 12, c[23, 0] → 1, c[23, 1] → 0, c[23, 2] → 1, c[24, 0] → 1,
  c[24, 1] → 2, c[24, 2] → 2, c[25, 0] → 1, c[25, 1] → 0, c[25, 2] → 1, c[26, 0] → 1, c[26, 1] → 1,
  c[26, 2] → 2, c[27, 0] → 1, c[27, 1] → 1, c[27, 2] → 2, c[28, 0] → 2, c[28, 1] → 3, c[28, 2] → 4,
  c[29, 0] → 1, c[29, 1] → 1, c[29, 2] → 1, c[30, 0] → 1, c[30, 1] → 0, c[30, 2] → 1, c[31, 0] → 2,
  c[31, 1] → 3, c[31, 2] → 3, c[32, 0] → 1, c[32, 1] → 0, c[32, 2] → 1, c[33, 0] → 1, c[33, 1] → 1,
  c[33, 2] → 2, c[35, 0] → 1, c[35, 1] → 2, c[35, 2] → 2, c[36, 0] → 4, c[36, 1] → 5, c[36, 2] → 6,
  c[38, 0] → 2, c[38, 1] → 3, c[38, 2] → 3, c[39, 0] → 1, c[39, 1] → 0, c[39, 2] → 1, c[40, 0] → 1,
  c[40, 1] → 1, c[40, 2] → 2, c[41, 0] → 8, c[41, 1] → 8, c[41, 2] → 12, c[42, 0] → 1, c[42, 1] → 2,
  c[42, 2] → 2, c[43, 0] → 4, c[43, 1] → 5, c[43, 2] → 6, c[45, 0] → 2, c[45, 1] → 3, c[45, 2] → 4,
  c[46, 0] → 2, c[46, 1] → 3, c[46, 2] → 3, c[47, 0] → 1, c[47, 1] → 0, c[47, 2] → 1, c[48, 0] → 1,
  c[48, 1] → 1, c[48, 2] → 2, c[49, 0] → 1, c[49, 1] → 2, c[49, 2] → 2, c[50, 0] → 8, c[50, 1] → 8,
  c[50, 2] → 12, c[51, 0] → 4, c[51, 1] → 5, c[51, 2] → 6, c[52, 0] → 1, c[52, 1] → 1,
  c[52, 2] → 1, c[53, 0] → 2, c[53, 1] → 3, c[53, 2] → 4, c[54, 0] → 2, c[54, 1] → 3, c[54, 2] → 3,
  c[55, 0] → 1, c[55, 1] → 0, c[55, 2] → 1, c[56, 0] → 1, c[56, 1] → 1, c[56, 2] → 2, c[57, 0] → 1,
  c[57, 1] → 2, c[57, 2] → 2, c[58, 0] → 1, c[58, 1] → 1, c[58, 2] → 1, c[59, 0] → 1, c[59, 1] → 1,
  c[59, 2] → 1, c[60, 0] → 1, c[60, 1] → 0, c[60, 2] → 1, c[61, 0] → 1, c[61, 1] → 0, c[61, 2] → 1,
  c[62, 0] → 2, c[62, 1] → 3, c[62, 2] → 3, c[63, 0] → 1, c[63, 1] → 2, c[63, 2] → 2, c[64, 0] → 1,
  c[64, 1] → 1, c[64, 2] → 2, c[65, 0] → 1, c[65, 1] → 1, c[65, 2] → 1, c[66, 0] → 1, c[66, 1] → 1,
  c[66, 2] → 2, c[67, 0] → 1, c[67, 1] → 0, c[67, 2] → 1, c[68, 0] → 1, c[68, 1] → 0, c[68, 2] → 1,
  c[69, 0] → 2, c[69, 1] → 3, c[69, 2] → 4, c[70, 0] → 2, c[70, 1] → 3, c[70, 2] → 3, c[71, 0] → 1,
  c[71, 1] → 0, c[71, 2] → 1, c[72, 0] → 1, c[72, 1] → 1, c[72, 2] → 1, c[73, 0] → 1, c[73, 1] → 2,
  c[73, 2] → 2, c[74, 0] → 1, c[74, 1] → 1, c[74, 2] → 2, c[75, 0] → 1, c[75, 1] → 1, c[75, 2] → 1,
  c[76, 0] → 1, c[76, 1] → 2, c[76, 2] → 2, c[77, 0] → 2, c[77, 1] → 3, c[77, 2] → 4, c[78, 0] → 1,
  c[78, 1] → 0, c[78, 2] → 1, c[79, 0] → 1, c[79, 1] → 1, c[79, 2] → 2, c[80, 0] → 1, c[80, 1] → 1,
  c[80, 2] → 1, c[81, 0] → 1, c[81, 1] → 1, c[81, 2] → 1, c[82, 0] → 4, c[82, 1] → 5, c[82, 2] → 6,
  c[83, 0] → 2, c[83, 1] → 3, c[83, 2] → 3, c[84, 0] → 1, c[84, 1] → 0, c[84, 2] → 1, c[85, 0] → 1,
  c[85, 1] → 2, c[85, 2] → 2, c[86, 0] → 1, c[86, 1] → 2, c[86, 2] → 2, c[87, 0] → 1, c[87, 1] → 1,
  c[87, 2] → 1, c[88, 0] → 1, c[88, 1] → 1, c[88, 2] → 1, c[89, 0] → 2, c[89, 1] → 3, c[89, 2] → 4,
  c[90, 0] → 1, c[90, 1] → 2, c[90, 2] → 2, c[91, 0] → 2, c[91, 1] → 3, c[91, 2] → 3, c[92, 0] → 4,
  c[92, 1] → 5, c[92, 2] → 6, c[93, 0] → 8, c[93, 1] → 8, c[93, 2] → 12, c[94, 0] → 1,
  c[94, 1] → 1, c[94, 2] → 2, c[95, 0] → 2, c[95, 1] → 3, c[95, 2] → 3, c[96, 0] → 4, c[96, 1] → 5,
  c[96, 2] → 6, c[97, 0] → 1, c[97, 1] → 1, c[97, 2] → 1, c[98, 0] → 2, c[98, 1] → 3, c[98, 2] → 4,
  c[99, 0] → 8, c[99, 1] → 8, c[99, 2] → 12, c[100, 0] → 2, c[100, 1] → 3, c[100, 2] → 3,
  c[101, 0] → 4, c[101, 1] → 5, c[101, 2] → 6, c[102, 0] → 1, c[102, 1] → 2, c[102, 2] → 2,
  c[103, 0] → 2, c[103, 1] → 3, c[103, 2] → 4, c[104, 0] → 4, c[104, 1] → 5, c[104, 2] → 6,
  c[105, 0] → 2, c[105, 1] → 3, c[105, 2] → 4, c[106, 0] → 8, c[106, 1] → 8, c[106, 2] → 12,
  c[107, 0] → 4, c[107, 1] → 5, c[107, 2] → 6, c[108, 0] → 1, c[108, 1] → 1, c[108, 2] → 2,
  c[109, 0] → 2, c[109, 1] → 3, c[109, 2] → 3, c[110, 0] → 2, c[110, 1] → 3, c[110, 2] → 4,
  c[111, 0] → 2, c[111, 1] → 3, c[111, 2] → 3, c[112, 0] → 1, c[112, 1] → 1, c[112, 2] → 2,
  c[115, 0] → 2, c[115, 1] → 3, c[115, 2] → 3, c[116, 0] → 1, c[116, 1] → 1, c[116, 2] → 2,
  c[117, 0] → 8, c[117, 1] → 8, c[117, 2] → 12, c[118, 0] → 8, c[118, 1] → 8, c[118, 2] → 12,
  c[119, 0] → 1, c[119, 1] → 2, c[119, 2] → 2, c[120, 0] → 2, c[120, 1] → 3, c[120, 2] → 4,
  c[121, 0] → 4, c[121, 1] → 5, c[121, 2] → 6, c[122, 0] → 8, c[122, 1] → 8, c[122, 2] → 12,
  c[123, 0] → 1, c[123, 1] → 2, c[123, 2] → 2, c[124, 0] → 2, c[124, 1] → 3, c[124, 2] → 4,
  c[125, 0] → 8, c[125, 1] → 8, c[125, 2] → 12, c[126, 0] → 2, c[126, 1] → 3, c[126, 2] → 4,
  c[127, 0] → 4, c[127, 1] → 5, c[127, 2] → 6, c[128, 0] → 2, c[128, 1] → 3, c[128, 2] → 3,
  c[129, 0] → 2, c[129, 1] → 3, c[129, 2] → 4, c[130, 0] → 8, c[130, 1] → 8, c[130, 2] → 12,
  c[131, 0] → 4, c[131, 1] → 5, c[131, 2] → 6, c[132, 0] → 8, c[132, 1] → 8, c[132, 2] → 12,
  c[133, 0] → 8, c[133, 1] → 8, c[133, 2] → 12, c[134, 0] → 4, c[134, 1] → 5, c[134, 2] → 6,
  c[135, 0] → 4, c[135, 1] → 5, c[135, 2] → 6, c[136, 0] → 8, c[136, 1] → 8, c[136, 2] → 12}}
```

verification of the solution

```

In[ ]:= solution = {c[20, 0] → 1, c[20, 1] → 1, c[20, 2] → 1, c[21, 0] → 1, c[21, 1] → 1,
  c[21, 2] → 1, c[22, 0] → 8, c[22, 1] → 8, c[22, 2] → 12, c[23, 0] → 1, c[23, 1] → 0,
  c[23, 2] → 1, c[24, 0] → 1, c[24, 1] → 2, c[24, 2] → 2, c[25, 0] → 1, c[25, 1] → 0,
  c[25, 2] → 1, c[26, 0] → 1, c[26, 1] → 1, c[26, 2] → 2, c[27, 0] → 1, c[27, 1] → 1,
  c[27, 2] → 2, c[28, 0] → 2, c[28, 1] → 3, c[28, 2] → 4, c[29, 0] → 1, c[29, 1] → 1,
  c[29, 2] → 1, c[30, 0] → 1, c[30, 1] → 0, c[30, 2] → 1, c[31, 0] → 2, c[31, 1] → 3,
  c[31, 2] → 3, c[32, 0] → 1, c[32, 1] → 0, c[32, 2] → 1, c[33, 0] → 1, c[33, 1] → 1,
  c[33, 2] → 2, c[35, 0] → 1, c[35, 1] → 2, c[35, 2] → 2, c[36, 0] → 4, c[36, 1] → 5,
  c[36, 2] → 6, c[38, 0] → 2, c[38, 1] → 3, c[38, 2] → 3, c[39, 0] → 1, c[39, 1] → 0,
  c[39, 2] → 1, c[40, 0] → 1, c[40, 1] → 1, c[40, 2] → 2, c[41, 0] → 8, c[41, 1] → 8,
  c[41, 2] → 12, c[42, 0] → 1, c[42, 1] → 2, c[42, 2] → 2, c[43, 0] → 4, c[43, 1] → 5,
  c[43, 2] → 6, c[45, 0] → 2, c[45, 1] → 3, c[45, 2] → 4, c[46, 0] → 2, c[46, 1] → 3,
  c[46, 2] → 3, c[47, 0] → 1, c[47, 1] → 0, c[47, 2] → 1, c[48, 0] → 1, c[48, 1] → 1,
  c[48, 2] → 2, c[49, 0] → 1, c[49, 1] → 2, c[49, 2] → 2, c[50, 0] → 8, c[50, 1] → 8,
  c[50, 2] → 12, c[51, 0] → 4, c[51, 1] → 5, c[51, 2] → 6, c[52, 0] → 1, c[52, 1] → 1,
  c[52, 2] → 1, c[53, 0] → 2, c[53, 1] → 3, c[53, 2] → 4, c[54, 0] → 2, c[54, 1] → 3,
  c[54, 2] → 3, c[55, 0] → 1, c[55, 1] → 0, c[55, 2] → 1, c[56, 0] → 1, c[56, 1] → 1,
  c[56, 2] → 2, c[57, 0] → 1, c[57, 1] → 2, c[57, 2] → 2, c[58, 0] → 1, c[58, 1] → 1,
  c[58, 2] → 1, c[59, 0] → 1, c[59, 1] → 1, c[59, 2] → 1, c[60, 0] → 1, c[60, 1] → 0,
  c[60, 2] → 1, c[61, 0] → 1, c[61, 1] → 0, c[61, 2] → 1, c[62, 0] → 2, c[62, 1] → 3,
  c[62, 2] → 3, c[63, 0] → 1, c[63, 1] → 2, c[63, 2] → 2, c[64, 0] → 1, c[64, 1] → 1,
  c[64, 2] → 2, c[65, 0] → 1, c[65, 1] → 1, c[65, 2] → 1, c[66, 0] → 1, c[66, 1] → 1,
  c[66, 2] → 2, c[67, 0] → 1, c[67, 1] → 0, c[67, 2] → 1, c[68, 0] → 1, c[68, 1] → 0,
  c[68, 2] → 1, c[69, 0] → 2, c[69, 1] → 3, c[69, 2] → 4, c[70, 0] → 2, c[70, 1] → 3,
  c[70, 2] → 3, c[71, 0] → 1, c[71, 1] → 0, c[71, 2] → 1, c[72, 0] → 1, c[72, 1] → 1,
  c[72, 2] → 1, c[73, 0] → 1, c[73, 1] → 2, c[73, 2] → 2, c[74, 0] → 1, c[74, 1] → 1,
  c[74, 2] → 2, c[75, 0] → 1, c[75, 1] → 1, c[75, 2] → 1, c[76, 0] → 1, c[76, 1] → 2,
  c[76, 2] → 2, c[77, 0] → 2, c[77, 1] → 3, c[77, 2] → 4, c[78, 0] → 1, c[78, 1] → 0,
  c[78, 2] → 1, c[79, 0] → 1, c[79, 1] → 1, c[79, 2] → 2, c[80, 0] → 1, c[80, 1] → 1,
  c[80, 2] → 1, c[81, 0] → 1, c[81, 1] → 1, c[81, 2] → 1, c[82, 0] → 4, c[82, 1] → 5,
  c[82, 2] → 6, c[83, 0] → 2, c[83, 1] → 3, c[83, 2] → 3, c[84, 0] → 1, c[84, 1] → 0,
  c[84, 2] → 1, c[85, 0] → 1, c[85, 1] → 2, c[85, 2] → 2, c[86, 0] → 1, c[86, 1] → 2,
  c[86, 2] → 2, c[87, 0] → 1, c[87, 1] → 1, c[87, 2] → 1, c[88, 0] → 1, c[88, 1] → 1,
  c[88, 2] → 1, c[89, 0] → 2, c[89, 1] → 3, c[89, 2] → 4, c[90, 0] → 1, c[90, 1] → 2,
  c[90, 2] → 2, c[91, 0] → 2, c[91, 1] → 3, c[91, 2] → 3, c[92, 0] → 4, c[92, 1] → 5,
  c[92, 2] → 6, c[93, 0] → 8, c[93, 1] → 8, c[93, 2] → 12, c[94, 0] → 1, c[94, 1] → 1,
  c[94, 2] → 2, c[95, 0] → 2, c[95, 1] → 3, c[95, 2] → 3, c[96, 0] → 4, c[96, 1] → 5,
  c[96, 2] → 6, c[97, 0] → 1, c[97, 1] → 1, c[97, 2] → 1, c[98, 0] → 2, c[98, 1] → 3,
  c[98, 2] → 4, c[99, 0] → 8, c[99, 1] → 8, c[99, 2] → 12, c[100, 0] → 2, c[100, 1] → 3,
  c[100, 2] → 3, c[101, 0] → 4, c[101, 1] → 5, c[101, 2] → 6, c[102, 0] → 1, c[102, 1] → 2,
  c[102, 2] → 2, c[103, 0] → 2, c[103, 1] → 3, c[103, 2] → 4, c[104, 0] → 4, c[104, 1] → 5,
  c[104, 2] → 6, c[105, 0] → 2, c[105, 1] → 3, c[105, 2] → 4, c[106, 0] → 8, c[106, 1] → 8,
  c[106, 2] → 12, c[107, 0] → 4, c[107, 1] → 5, c[107, 2] → 6, c[108, 0] → 1,
  c[108, 1] → 1, c[108, 2] → 2, c[109, 0] → 2, c[109, 1] → 3, c[109, 2] → 3, c[110, 0] → 2,
  c[110, 1] → 3, c[110, 2] → 4, c[111, 0] → 2, c[111, 1] → 3, c[111, 2] → 3, c[112, 0] → 1,
  c[112, 1] → 1, c[112, 2] → 2, c[115, 0] → 2, c[115, 1] → 3, c[115, 2] → 3, c[116, 0] → 1,
  c[116, 1] → 1, c[116, 2] → 2, c[117, 0] → 8, c[117, 1] → 8, c[117, 2] → 12,
  c[118, 0] → 8, c[118, 1] → 8, c[118, 2] → 12, c[119, 0] → 1, c[119, 1] → 2,
  c[119, 2] → 2, c[120, 0] → 2, c[120, 1] → 3, c[120, 2] → 4, c[121, 0] → 4, c[121, 1] → 5,
  c[121, 2] → 6, c[122, 0] → 8, c[122, 1] → 8, c[122, 2] → 12, c[123, 0] → 1,
  c[123, 1] → 2, c[123, 2] → 2, c[124, 0] → 2, c[124, 1] → 3, c[124, 2] → 4, c[125, 0] → 8,
  c[125, 1] → 8, c[125, 2] → 12, c[126, 0] → 2, c[126, 1] → 3, c[126, 2] → 4,

```

```
c[127, 0] → 4, c[127, 1] → 5, c[127, 2] → 6, c[128, 0] → 2, c[128, 1] → 3,
c[128, 2] → 3, c[129, 0] → 2, c[129, 1] → 3, c[129, 2] → 4, c[130, 0] → 8,
c[130, 1] → 8, c[130, 2] → 12, c[131, 0] → 4, c[131, 1] → 5, c[131, 2] → 6,
c[132, 0] → 8, c[132, 1] → 8, c[132, 2] → 12, c[133, 0] → 8, c[133, 1] → 8,
c[133, 2] → 12, c[134, 0] → 4, c[134, 1] → 5, c[134, 2] → 6, c[135, 0] → 4,
c[135, 1] → 5, c[135, 2] → 6, c[136, 0] → 8, c[136, 1] → 8, c[136, 2] → 12};
coeffList /. solution
```

[illegible]

[illegible]

[illegible]

final output

```
In[ ]:= {x[extractindex[#]], # /. variableToVerlindeImage /. solution /.
ringElement -> Identity} & /@ clustervariables
% //
```

TableForm

```
Out[ ]:= { {x[1], V0 + V1 + V2}, {x[2], V0}, {x[3], 2 V0 + 3 V1 + 4 V2}, {x[4], V0},
{x[5], 8 V0 + 8 V1 + 12 V2}, {x[6], V0}, {x[7], V0 + 2 V1 + 2 V2}, {x[8], V0},
{x[9], 4 V0 + 5 V1 + 6 V2}, {x[10], V0}, {x[11], 2 V0 + 3 V1 + 3 V2}, {x[12], V0},
{x[13], V0 + V1 + 2 V2}, {x[14], V0}, {x[15], V0 + V2}, {x[16], V0}, {x[17], V0 + V2},
{x[18], V0 + V1 + 2 V2}, {x[19], 2 V0 + 3 V1 + 3 V2}, {x[20], V0 + V1 + V2},
{x[21], V0 + V1 + V2}, {x[22], 8 V0 + 8 V1 + 12 V2}, {x[23], V0 + V2}, {x[24], V0 + 2 V1 + 2 V2},
{x[25], V0 + V2}, {x[26], V0 + V1 + 2 V2}, {x[27], V0 + V1 + 2 V2}, {x[28], 2 V0 + 3 V1 + 4 V2},
{x[29], V0 + V1 + V2}, {x[30], V0 + V2}, {x[31], 2 V0 + 3 V1 + 3 V2}, {x[32], V0 + V2},
{x[33], V0 + V1 + 2 V2}, {x[34], V0 + 2 V1 + 2 V2}, {x[35], V0 + 2 V1 + 2 V2},
{x[36], 4 V0 + 5 V1 + 6 V2}, {x[37], 2 V0 + 3 V1 + 4 V2}, {x[38], 2 V0 + 3 V1 + 3 V2},
{x[39], V0 + V2}, {x[40], V0 + V1 + 2 V2}, {x[41], 8 V0 + 8 V1 + 12 V2},
{x[42], V0 + 2 V1 + 2 V2}, {x[43], 4 V0 + 5 V1 + 6 V2}, {x[44], V0 + V1 + V2},
{x[45], 2 V0 + 3 V1 + 4 V2}, {x[46], 2 V0 + 3 V1 + 3 V2}, {x[47], V0 + V2},
{x[48], V0 + V1 + 2 V2}, {x[49], V0 + 2 V1 + 2 V2}, {x[50], 8 V0 + 8 V1 + 12 V2},
{x[51], 4 V0 + 5 V1 + 6 V2}, {x[52], V0 + V1 + V2}, {x[53], 2 V0 + 3 V1 + 4 V2},
{x[54], 2 V0 + 3 V1 + 3 V2}, {x[55], V0 + V2}, {x[56], V0 + V1 + 2 V2}, {x[57], V0 + 2 V1 + 2 V2},
{x[58], V0 + V1 + V2}, {x[59], V0 + V1 + V2}, {x[60], V0 + V2}, {x[61], V0 + V2},
{x[62], 2 V0 + 3 V1 + 3 V2}, {x[63], V0 + 2 V1 + 2 V2}, {x[64], V0 + V1 + 2 V2},
{x[65], V0 + V1 + V2}, {x[66], V0 + V1 + 2 V2}, {x[67], V0 + V2}, {x[68], V0 + V2},
{x[69], 2 V0 + 3 V1 + 4 V2}, {x[70], 2 V0 + 3 V1 + 3 V2}, {x[71], V0 + V2},
{x[72], V0 + V1 + V2}, {x[73], V0 + 2 V1 + 2 V2}, {x[74], V0 + V1 + 2 V2}, {x[75], V0 + V1 + V2},
{x[76], V0 + 2 V1 + 2 V2}, {x[77], 2 V0 + 3 V1 + 4 V2}, {x[78], V0 + V2}, {x[79], V0 + V1 + 2 V2},
{x[80], V0 + V1 + V2}, {x[81], V0 + V1 + V2}, {x[82], 4 V0 + 5 V1 + 6 V2},
{x[83], 2 V0 + 3 V1 + 3 V2}, {x[84], V0 + V2}, {x[85], V0 + 2 V1 + 2 V2},
{x[86], V0 + 2 V1 + 2 V2}, {x[87], V0 + V1 + V2}, {x[88], V0 + V1 + V2},
{x[89], 2 V0 + 3 V1 + 4 V2}, {x[90], V0 + 2 V1 + 2 V2}, {x[91], 2 V0 + 3 V1 + 3 V2},
{x[92], 4 V0 + 5 V1 + 6 V2}, {x[93], 8 V0 + 8 V1 + 12 V2}, {x[94], V0 + V1 + 2 V2},
{x[95], 2 V0 + 3 V1 + 3 V2}, {x[96], 4 V0 + 5 V1 + 6 V2}, {x[97], V0 + V1 + V2},
{x[98], 2 V0 + 3 V1 + 4 V2}, {x[99], 8 V0 + 8 V1 + 12 V2}, {x[100], 2 V0 + 3 V1 + 3 V2},
{x[101], 4 V0 + 5 V1 + 6 V2}, {x[102], V0 + 2 V1 + 2 V2}, {x[103], 2 V0 + 3 V1 + 4 V2},
{x[104], 4 V0 + 5 V1 + 6 V2}, {x[105], 2 V0 + 3 V1 + 4 V2}, {x[106], 8 V0 + 8 V1 + 12 V2},
{x[107], 4 V0 + 5 V1 + 6 V2}, {x[108], V0 + V1 + 2 V2}, {x[109], 2 V0 + 3 V1 + 3 V2},
{x[110], 2 V0 + 3 V1 + 4 V2}, {x[111], 2 V0 + 3 V1 + 3 V2}, {x[112], V0 + V1 + 2 V2},
{x[113], 4 V0 + 5 V1 + 6 V2}, {x[114], 8 V0 + 8 V1 + 12 V2}, {x[115], 2 V0 + 3 V1 + 3 V2},
{x[116], V0 + V1 + 2 V2}, {x[117], 8 V0 + 8 V1 + 12 V2}, {x[118], 8 V0 + 8 V1 + 12 V2},
{x[119], V0 + 2 V1 + 2 V2}, {x[120], 2 V0 + 3 V1 + 4 V2}, {x[121], 4 V0 + 5 V1 + 6 V2},
{x[122], 8 V0 + 8 V1 + 12 V2}, {x[123], V0 + 2 V1 + 2 V2}, {x[124], 2 V0 + 3 V1 + 4 V2},
{x[125], 8 V0 + 8 V1 + 12 V2}, {x[126], 2 V0 + 3 V1 + 4 V2}, {x[127], 4 V0 + 5 V1 + 6 V2},
{x[128], 2 V0 + 3 V1 + 3 V2}, {x[129], 2 V0 + 3 V1 + 4 V2}, {x[130], 8 V0 + 8 V1 + 12 V2},
{x[131], 4 V0 + 5 V1 + 6 V2}, {x[132], 8 V0 + 8 V1 + 12 V2}, {x[133], 8 V0 + 8 V1 + 12 V2},
{x[134], 4 V0 + 5 V1 + 6 V2}, {x[135], 4 V0 + 5 V1 + 6 V2}, {x[136], 8 V0 + 8 V1 + 12 V2} }
```

Out[]:=TableForm=

x[1]	V0 + V1 + V2
x[2]	V0
x[3]	2 V0 + 3 V1 + 4 V2
x[4]	V0
x[5]	8 V0 + 8 V1 + 12 V2
x[6]	V0
x[7]	V0 + 2 V1 + 2 V2
x[8]	V0
x[9]	4 V0 + 5 V1 + 6 V2
x[10]	V0

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x[11] 2 V0 + 3 V1 + 3 V2
x[12] V0
x[13] V0 + V1 + 2 V2
x[14] V0
x[15] V0 + V2
x[16] V0
x[17] V0 + V2
x[18] V0 + V1 + 2 V2
x[19] 2 V0 + 3 V1 + 3 V2
x[20] V0 + V1 + V2
x[21] V0 + V1 + V2
x[22] 8 V0 + 8 V1 + 12 V2
x[23] V0 + V2
x[24] V0 + 2 V1 + 2 V2
x[25] V0 + V2
x[26] V0 + V1 + 2 V2
x[27] V0 + V1 + 2 V2
x[28] 2 V0 + 3 V1 + 4 V2
x[29] V0 + V1 + V2
x[30] V0 + V2
x[31] 2 V0 + 3 V1 + 3 V2
x[32] V0 + V2
x[33] V0 + V1 + 2 V2
x[34] V0 + 2 V1 + 2 V2
x[35] V0 + 2 V1 + 2 V2
x[36] 4 V0 + 5 V1 + 6 V2
x[37] 2 V0 + 3 V1 + 4 V2
x[38] 2 V0 + 3 V1 + 3 V2
x[39] V0 + V2
x[40] V0 + V1 + 2 V2
x[41] 8 V0 + 8 V1 + 12 V2
x[42] V0 + 2 V1 + 2 V2
x[43] 4 V0 + 5 V1 + 6 V2
x[44] V0 + V1 + V2
x[45] 2 V0 + 3 V1 + 4 V2
x[46] 2 V0 + 3 V1 + 3 V2
x[47] V0 + V2
x[48] V0 + V1 + 2 V2
x[49] V0 + 2 V1 + 2 V2
x[50] 8 V0 + 8 V1 + 12 V2
x[51] 4 V0 + 5 V1 + 6 V2
x[52] V0 + V1 + V2
x[53] 2 V0 + 3 V1 + 4 V2
x[54] 2 V0 + 3 V1 + 3 V2
x[55] V0 + V2
x[56] V0 + V1 + 2 V2
x[57] V0 + 2 V1 + 2 V2
x[58] V0 + V1 + V2
x[59] V0 + V1 + V2
x[60] V0 + V2
x[61] V0 + V2
x[62] 2 V0 + 3 V1 + 3 V2
x[63] V0 + 2 V1 + 2 V2
x[64] V0 + V1 + 2 V2
x[65] V0 + V1 + V2
x[66] V0 + V1 + 2 V2
x[67] V0 + V2
x[68] V0 + V2
x[69] 2 V0 + 3 V1 + 4 V2
x[70] 2 V0 + 3 V1 + 3 V2
x[71] V0 + V2
x[72] V0 + V1 + V2
x[73] V0 + 2 V1 + 2 V2
x[74] V0 + V1 + 2 V2

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x[75]      V0 + V1 + V2
x[76]      V0 + 2 V1 + 2 V2
x[77]      2 V0 + 3 V1 + 4 V2
x[78]      V0 + V2
x[79]      V0 + V1 + 2 V2
x[80]      V0 + V1 + V2
x[81]      V0 + V1 + V2
x[82]      4 V0 + 5 V1 + 6 V2
x[83]      2 V0 + 3 V1 + 3 V2
x[84]      V0 + V2
x[85]      V0 + 2 V1 + 2 V2
x[86]      V0 + 2 V1 + 2 V2
x[87]      V0 + V1 + V2
x[88]      V0 + V1 + V2
x[89]      2 V0 + 3 V1 + 4 V2
x[90]      V0 + 2 V1 + 2 V2
x[91]      2 V0 + 3 V1 + 3 V2
x[92]      4 V0 + 5 V1 + 6 V2
x[93]      8 V0 + 8 V1 + 12 V2
x[94]      V0 + V1 + 2 V2
x[95]      2 V0 + 3 V1 + 3 V2
x[96]      4 V0 + 5 V1 + 6 V2
x[97]      V0 + V1 + V2
x[98]      2 V0 + 3 V1 + 4 V2
x[99]      8 V0 + 8 V1 + 12 V2
x[100]     2 V0 + 3 V1 + 3 V2
x[101]     4 V0 + 5 V1 + 6 V2
x[102]     V0 + 2 V1 + 2 V2
x[103]     2 V0 + 3 V1 + 4 V2
x[104]     4 V0 + 5 V1 + 6 V2
x[105]     2 V0 + 3 V1 + 4 V2
x[106]     8 V0 + 8 V1 + 12 V2
x[107]     4 V0 + 5 V1 + 6 V2
x[108]     V0 + V1 + 2 V2
x[109]     2 V0 + 3 V1 + 3 V2
x[110]     2 V0 + 3 V1 + 4 V2
x[111]     2 V0 + 3 V1 + 3 V2
x[112]     V0 + V1 + 2 V2
x[113]     4 V0 + 5 V1 + 6 V2
x[114]     8 V0 + 8 V1 + 12 V2
x[115]     2 V0 + 3 V1 + 3 V2
x[116]     V0 + V1 + 2 V2
x[117]     8 V0 + 8 V1 + 12 V2
x[118]     8 V0 + 8 V1 + 12 V2
x[119]     V0 + 2 V1 + 2 V2
x[120]     2 V0 + 3 V1 + 4 V2
x[121]     4 V0 + 5 V1 + 6 V2
x[122]     8 V0 + 8 V1 + 12 V2
x[123]     V0 + 2 V1 + 2 V2
x[124]     2 V0 + 3 V1 + 4 V2
x[125]     8 V0 + 8 V1 + 12 V2
x[126]     2 V0 + 3 V1 + 4 V2
x[127]     4 V0 + 5 V1 + 6 V2
x[128]     2 V0 + 3 V1 + 3 V2
x[129]     2 V0 + 3 V1 + 4 V2
x[130]     8 V0 + 8 V1 + 12 V2
x[131]     4 V0 + 5 V1 + 6 V2
x[132]     8 V0 + 8 V1 + 12 V2
x[133]     8 V0 + 8 V1 + 12 V2
x[134]     4 V0 + 5 V1 + 6 V2
x[135]     4 V0 + 5 V1 + 6 V2
x[136]     8 V0 + 8 V1 + 12 V2

```