1. List of cluster variables in $\mathscr{C}_1$ for exceptional types

We provide a complete list of cluster variables of $\mathscr{C}_1$ for exceptional types, along with the corresponding monomials and their images in the Verlinde ring here. An extended version of the list, including expressions of cluster variables as rational functions of the initial variables, is available at `https://github.com/chlee-0/verlinde_cluster`. For instance, for type E_6, refer to `https://github.com/chlee-0/verlinde_cluster/blob/main/cluster_variables_E6.txt`, among others. The height function used here is the one defined at the beginning of Section 5.

Cluster variable	**Dominant monomial**	**Verlinde image**
x_1	$Y_{1,-1}$	V_1
x_2	$Y_{1,-3}Y_{1,-1}$	V_3
x_3	$Y_{3,0}$	V_2+V_6
x_4	$Y_{3,-2}Y_{3,0}$	V_8
x_5	$Y_{4,1}$	$V_0+V_4+2V_5$
x_6	$Y_{4,-1}Y_{4,1}$	V_0
x_7	$Y_{2,2}$	V_0+V_5
x_8	$Y_{2,0}Y_{2,2}$	V_0
x_9	$Y_{5,2}$	V_1+V_7
x_{10}	$Y_{5,0}Y_{5,2}$	V_3
x_{11}	$Y_{6,3}$	V_2
x_{12}	$Y_{6,1}Y_{6,3}$	V_8
x_{13}	$Y_{3,-2}Y_{4,1}$	V_2
x_{14}	$Y_{5,0}Y_{6,3}$	V_4
x_{15}	$Y_{2,2}Y_{3,-2}Y_{6,3}$	V_7+V_8
x_{16}	$Y_{3,-2}Y_{6,3}$	V_7+V_8
x_{17}	$Y_{1,-3}Y_{6,3}$	V_4
x_{18}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{6,3}$	V_2+V_6
x_{19}	$Y_{2,2}Y_{4,-1}Y_{6,3}$	V_2
x_{20}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}$	V_2+V_6
x_{21}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}$	$V_0+V_4+2V_5$
x_{22}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{5,2}$	V_3+V_6
x_{23}	$Y_{2,2}Y_{3,-2}$	V_2
x_{24}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}$	V_4+V_5
x_{25}	$Y_{3,-2}Y_{5,2}$	V_4
x_{26}	$Y_{1,-3}$	V_1
x_{27}	$Y_{6,1}$	V_2
x_{28}	$Y_{3,-2}$	V_2+V_6
x_{29}	$Y_{4,-1}Y_{5,2}$	V_1
x_{30}	$Y_{2,0}$	V_0+V_5

x_{31}	$Y_{4,-1}$	$V_0 + V_4 + 2V_5$
x_{32}	$Y_{4,-1}Y_{6,3}$	$V_2 + V_6$
x_{33}	$Y_{2,2}Y_{4,-1}$	$V_0 + V_5$
x_{34}	$Y_{1,-3}Y_{3,0}$	V_4
x_{35}	$Y_{1,-3}Y_{2,2}$	V_1
x_{36}	$Y_{2,2}Y_{4,-1}Y_{5,2}$	V_1
x_{37}	$Y_{1,-3}Y_{2,2}Y_{6,3}$	$V_4 + V_5$
x_{38}	$Y_{1,-3}Y_{2,2}Y_{5,2}$	$V_2 + V_3 + 2V_6$
x_{39}	$Y_{2,2}Y_{3,-2}Y_{5,2}$	$V_4 + V_5$
x_{40}	$Y_{1,-3}Y_{5,2}$	$V_3 + V_6$
x_{41}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{6,3}$	$V_4 + V_5$
x_{42}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{6,3}$	$V_1 + 2V_7 + V_8$
x_{43}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{5,2}Y_{6,3}$	$V_1 + V_7$
x_{44}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{5,2}Y_{6,3}$	$V_0 + V_4 + 2V_5$
x_{45}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{5,2}$	$V_1 + V_7$
x_{46}	$Y_{5,0}$	$V_1 + V_7$
x_{47}	$Y_{1,-3}Y_{4,1}$	$V_1 + V_7$
x_{48}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}$	$V_0 + V_4 + 2V_5$

Table 1: Cluster variables with corresponding dominant monomials and their images under φ_{k} in $\mathscr{C}_1$ of type E_6.

Cluster variable	**Dominant monomial**	**Verlinde image**
x_1	$Y_{1,-1}$	$V_0 + V_2$
x_2	$Y_{1,-3}Y_{1,-1}$	V_0
x_3	$Y_{3,0}$	$V_0 + 2V_2 + V_4$
x_4	$Y_{3,-2}Y_{3,0}$	V_0
x_5	$Y_{4,1}$	$2V_0 + 3V_2 + 3V_4 + V_5$
x_6	$Y_{4,-1}Y_{4,1}$	V_0
x_7	$Y_{2,2}$	$V_1 + V_3$
x_8	$Y_{2,0}Y_{2,2}$	V_5
x_9	$Y_{5,2}$	$2V_1 + 2V_3$
x_{10}	$Y_{5,0}Y_{5,2}$	V_5
x_{11}	$Y_{6,3}$	$V_0 + V_2 + V_4$
x_{12}	$Y_{6,1}Y_{6,3}$	V_0
x_{13}	$Y_{7,4}$	V_1
x_{14}	$Y_{7,2}Y_{7,4}$	V_5
x_{15}	$Y_{5,0}Y_{6,3}$	V_1
x_{16}	$Y_{5,0}Y_{7,4}$	$V_2 + V_4 + V_5$
x_{17}	$Y_{3,-2}Y_{4,1}$	$V_0 + V_2$

x_{18}	$Y_{2,2}Y_{3,-2}Y_{7,4}$	V_0+V_2
x_{19}	$Y_{1,-3}Y_{2,2}Y_{7,4}$	$V_0+V_2+V_4$
x_{20}	$Y_{1,-3}Y_{7,4}$	V_1
x_{21}	$Y_{1,-3}Y_{2,2}$	V_1
x_{22}	$Y_{6,1}Y_{7,4}$	V_1
x_{23}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{5,2}Y_{7,4}$	$2V_1+2V_3$
x_{24}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{5,2}Y_{7,4}$	V_1+V_3
x_{25}	$Y_{1,-3}Y_{5,2}$	V_1+V_3
x_{26}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{5,2}$	$V_0+V_2+V_4$
x_{27}	$Y_{2,2}Y_{4,-1}Y_{5,2}$	V_0+V_2
x_{28}	$Y_{3,-2}Y_{5,2}$	V_1
x_{29}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{5,2}Y_{6,3}$	$V_0+2V_2+V_4$
x_{30}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{5,2}$	V_0+V_2
x_{31}	$Y_{7,2}$	V_1
x_{32}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}$	$2V_1+2V_3$
x_{33}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}$	$V_0+V_2+V_4$
x_{34}	$Y_{1,-3}Y_{6,3}$	V_0+V_2
x_{35}	$Y_{2,2}Y_{4,-1}Y_{6,3}$	V_1
x_{36}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{6,3}$	V_1+V_3
x_{37}	$Y_{2,2}Y_{3,-2}Y_{6,3}$	V_1+V_3
x_{38}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{6,3}Y_{7,4}$	$V_0+2V_2+V_4$
x_{39}	$Y_{3,-2}Y_{6,3}$	V_0+V_2
x_{40}	$Y_{3,-2}Y_{7,4}$	V_1+V_3
x_{41}	$Y_{2,0}$	V_1+V_3
x_{42}	$Y_{4,-1}Y_{7,4}$	$2V_1+2V_3$
x_{43}	$Y_{1,-3}$	V_0+V_2
x_{44}	$Y_{3,-2}$	$V_0+2V_2+V_4$
x_{45}	$Y_{4,-1}$	$2V_0+3V_2+3V_4+V_5$
x_{46}	$Y_{4,-1}Y_{5,2}$	V_1
x_{47}	$Y_{4,-1}Y_{6,3}$	$V_0+V_2+V_4$
x_{48}	$Y_{2,2}Y_{4,-1}$	V_1+V_3
x_{49}	$Y_{1,-3}Y_{3,0}$	V_0+V_2
x_{50}	$Y_{1,-3}Y_{2,2}Y_{6,3}$	$2V_1+2V_3$
x_{51}	$Y_{1,-3}Y_{4,1}$	$V_0+2V_2+V_4$
x_{52}	$Y_{1,-3}Y_{2,2}Y_{5,2}$	$2V_0+3V_2+3V_4+V_5$
x_{53}	$Y_{6,1}$	$V_0+V_2+V_4$
x_{54}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}$	$2V_1+2V_3$
x_{55}	$Y_{1,-3}Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}$	$2V_0+3V_2+3V_4+V_5$
x_{56}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{6,3}Y_{7,4}$	$V_0+V_2+V_4$
x_{57}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{7,4}$	$V_0+2V_2+V_4$
x_{58}	$Y_{2,2}Y_{4,-1}Y_{7,4}$	V_0+V_2

x_{59}	$Y_{5,0}$	$2V_1 + 2V_3$
x_{60}	$Y_{2,2}Y_{3,-2}$	V_1
x_{61}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{7,4}$	$V_0 + V_2 + V_4$
x_{62}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}$	$V_0 + 2V_2 + V_4$
x_{63}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}$	$2V_0 + 3V_2 + 3V_4 + V_5$
x_{64}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{7,4}$	$2V_0 + 3V_2 + 3V_4 + V_5$
x_{65}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}$	$V_0 + V_2 + V_4$
x_{66}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{6,3}$	$2V_1 + 2V_3$
x_{67}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{7,4}$	$V_0 + 2V_2 + V_4$
x_{68}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{6,3}$	$V_1 + V_3$
x_{69}	$Y_{2,2}Y_{3,-2}Y_{5,2}$	$V_0 + 2V_2 + V_4$
x_{70}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{6,3}Y_{7,4}$	$V_0 + 2V_2 + V_4$
x_{71}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{7,4}$	$V_1 + V_3$
x_{72}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{6,3}Y_{7,4}$	$2V_1 + 2V_3$
x_{73}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}$	$2V_0 + 3V_2 + 3V_4 + V_5$
x_{74}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{7,4}$	$2V_1 + 2V_3$
x_{75}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{5,2}Y_{6,3}$	$2V_0 + 3V_2 + 3V_4 + V_5$
x_{76}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}$	$2V_0 + 3V_2 + 3V_4 + V_5$
x_{77}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{6,3}Y_{7,4}$	$2V_0 + 3V_2 + 3V_4 + V_5$

Table 2: Cluster variables with corresponding dominant monomials and their images under φ_{k} in $\mathscr{C}_1$ of type E_7.

Cluster variable	**Dominant monomial**	**Verlinde image**
x_1	$Y_{1,-1}$	$V_0 + V_1 + V_2$
x_2	$Y_{1,-3}Y_{1,-1}$	V_0
x_3	$Y_{3,0}$	$2V_0 + 3V_1 + 4V_2$
x_4	$Y_{3,-2}Y_{3,0}$	V_0
x_5	$Y_{4,1}$	$8V_0 + 8V_1 + 12V_2$
x_6	$Y_{4,-1}Y_{4,1}$	V_0
x_7	$Y_{2,2}$	$V_0 + 2V_1 + 2V_2$
x_8	$Y_{2,0}Y_{2,2}$	V_0
x_9	$Y_{5,2}$	$4V_0 + 5V_1 + 6V_2$
x_{10}	$Y_{5,0}Y_{5,2}$	V_0
x_{11}	$Y_{6,3}$	$2V_0 + 3V_1 + 3V_2$
x_{12}	$Y_{6,1}Y_{6,3}$	V_0
x_{13}	$Y_{7,4}$	$V_0 + V_1 + 2V_2$
x_{14}	$Y_{7,2}Y_{7,4}$	V_0
x_{15}	$Y_{8,5}$	$V_0 + V_2$
x_{16}	$Y_{8,3}Y_{8,5}$	V_0

x_{17}	$Y_{8,3}$	$V_0 + V_2$
x_{18}	$Y_{7,2}$	$V_0 + V_1 + 2V_2$
x_{19}	$Y_{6,1}$	$2V_0 + 3V_1 + 3V_2$
x_{20}	$Y_{2,2}Y_{4,-1}Y_{5,2}$	$V_0 + V_1 + V_2$
x_{21}	$Y_{1,-3}Y_{3,0}$	$V_0 + V_1 + V_2$
x_{22}	$Y_{1,-3}Y_{2,2}Y_{5,2}$	$8V_0 + 8V_1 + 12V_2$
x_{23}	$Y_{1,-3}Y_{2,2}$	$V_0 + V_2$
x_{24}	$Y_{1,-3}Y_{5,2}$	$V_0 + 2V_1 + 2V_2$
x_{25}	$Y_{7,2}Y_{8,5}$	$V_0 + V_2$
x_{26}	$Y_{6,1}Y_{8,5}$	$V_0 + V_1 + 2V_2$
x_{27}	$Y_{1,-3}Y_{2,2}Y_{8,5}$	$V_0 + V_1 + 2V_2$
x_{28}	$Y_{1,-3}Y_{4,1}$	$2V_0 + 3V_1 + 4V_2$
x_{29}	$Y_{3,-2}Y_{4,1}$	$V_0 + V_1 + V_2$
x_{30}	$Y_{1,-3}Y_{8,5}$	$V_0 + V_2$
x_{31}	$Y_{5,0}Y_{8,5}$	$2V_0 + 3V_1 + 3V_2$
x_{32}	$Y_{6,1}Y_{7,4}$	$V_0 + V_2$
x_{33}	$Y_{5,0}Y_{7,4}$	$V_0 + V_1 + 2V_2$
x_{34}	$Y_{2,0}$	$V_0 + 2V_1 + 2V_2$
x_{35}	$Y_{3,-2}Y_{8,5}$	$V_0 + 2V_1 + 2V_2$
x_{36}	$Y_{4,-1}Y_{8,5}$	$4V_0 + 5V_1 + 6V_2$
x_{37}	$Y_{3,-2}$	$2V_0 + 3V_1 + 4V_2$
x_{38}	$Y_{4,-1}Y_{7,4}$	$2V_0 + 3V_1 + 3V_2$
x_{39}	$Y_{5,0}Y_{6,3}$	$V_0 + V_2$
x_{40}	$Y_{4,-1}Y_{6,3}$	$V_0 + V_1 + 2V_2$
x_{41}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{8,5}$	$8V_0 + 8V_1 + 12V_2$
x_{42}	$Y_{2,2}Y_{4,-1}$	$V_0 + 2V_1 + 2V_2$
x_{43}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{7,4}$	$4V_0 + 5V_1 + 6V_2$
x_{44}	$Y_{1,-3}$	$V_0 + V_1 + V_2$
x_{45}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{8,5}$	$2V_0 + 3V_1 + 4V_2$
x_{46}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{6,3}$	$2V_0 + 3V_1 + 3V_2$
x_{47}	$Y_{4,-1}Y_{5,2}$	$V_0 + V_2$
x_{48}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}$	$V_0 + V_1 + 2V_2$
x_{49}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{7,4}$	$V_0 + 2V_1 + 2V_2$
x_{50}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{7,4}Y_{8,5}$	$8V_0 + 8V_1 + 12V_2$
x_{51}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{6,3}Y_{8,5}$	$4V_0 + 5V_1 + 6V_2$
x_{52}	$Y_{2,2}Y_{4,-1}Y_{8,5}$	$V_0 + V_1 + V_2$
x_{53}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{7,4}Y_{8,5}$	$2V_0 + 3V_1 + 4V_2$
x_{54}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{8,5}$	$2V_0 + 3V_1 + 3V_2$
x_{55}	$Y_{2,2}Y_{3,-2}$	$V_0 + V_2$
x_{56}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{8,5}$	$V_0 + V_1 + 2V_2$
x_{57}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{6,3}Y_{8,5}$	$V_0 + 2V_1 + 2V_2$

x_{58}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{8,5}$	$V_0+V_1+V_2$
x_{59}	$Y_{3,-2}Y_{7,4}$	$V_0+V_1+V_2$
x_{60}	$Y_{3,-2}Y_{6,3}$	V_0+V_2
x_{61}	$Y_{2,2}Y_{3,-2}Y_{8,5}$	V_0+V_2
x_{62}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{6,3}Y_{7,4}Y_{8,5}$	$2V_0+3V_1+3V_2$
x_{63}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{7,4}Y_{8,5}$	$V_0+2V_1+2V_2$
x_{64}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{7,4}Y_{8,5}$	$V_0+V_1+2V_2$
x_{65}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{6,3}Y_{8,5}$	$V_0+V_1+V_2$
x_{66}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{7,4}Y_{8,5}$	$V_0+V_1+2V_2$
x_{67}	$Y_{1,-3}Y_{7,4}$	V_0+V_2
x_{68}	$Y_{2,2}Y_{4,-1}Y_{7,4}$	V_0+V_2
x_{69}	$Y_{1,-3}Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{6,3}Y_{7,4}Y_{8,5}$	$2V_0+3V_1+4V_2$
x_{70}	$Y_{1,-3}Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{7,4}Y_{8,5}$	$2V_0+3V_1+3V_2$
x_{71}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{5,2}$	V_0+V_2
x_{72}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{7,4}$	$V_0+V_1+V_2$
x_{73}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{6,3}Y_{7,4}$	$V_0+2V_1+2V_2$
x_{74}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{8,5}$	$V_0+V_1+2V_2$
x_{75}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{6,3}$	$V_0+V_1+V_2$
x_{76}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{7,4}$	$V_0+2V_1+2V_2$
x_{77}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}$	$2V_0+3V_1+4V_2$
x_{78}	$Y_{2,2}Y_{4,-1}Y_{6,3}$	V_0+V_2
x_{79}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}$	$V_0+V_1+2V_2$
x_{80}	$Y_{2,2}Y_{3,-2}Y_{7,4}$	$V_0+V_1+V_2$
x_{81}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{7,4}$	$V_0+V_1+V_2$
x_{82}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}Y_{8,5}$	$4V_0+5V_1+6V_2$
x_{83}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}$	$2V_0+3V_1+3V_2$
x_{84}	$Y_{3,-2}Y_{5,2}$	V_0+V_2
x_{85}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{8,5}$	$V_0+2V_1+2V_2$
x_{86}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{6,3}Y_{8,5}$	$V_0+2V_1+2V_2$
x_{87}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{5,2}Y_{8,5}$	$V_0+V_1+V_2$
x_{88}	$Y_{1,-3}Y_{6,3}$	$V_0+V_1+V_2$
x_{89}	$Y_{1,-3}Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{8,5}$	$2V_0+3V_1+4V_2$
x_{90}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{6,3}Y_{8,5}$	$V_0+2V_1+2V_2$
x_{91}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{8,5}$	$2V_0+3V_1+3V_2$
x_{92}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{3,-2}Y_{4,-1}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}Y_{8,5}$	$4V_0+5V_1+6V_2$
x_{93}	$Y_{1,-3}Y_{1,-3}Y_{2,2}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{3,-2}Y_{4,-1}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}Y_{7,4}Y_{8,5}$	$8V_0+8V_1+12V_2$
x_{94}	$Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{6,3}Y_{7,4}$	$V_0+V_1+2V_2$
x_{95}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}$	$2V_0+3V_1+3V_2$
x_{96}	$Y_{1,-3}Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}Y_{8,5}$	$4V_0+5V_1+6V_2$
x_{97}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{6,3}$	$V_0+V_1+V_2$
x_{98}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{8,5}$	$2V_0+3V_1+4V_2$

x_{99}	$Y_{1,-3}Y_{1,-3}Y_{2,2}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{3,-2}Y_{4,-1}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}Y_{8,5}$	$8V_0+8V_1+12V_2$
x_{100}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{7,4}Y_{8,5}$	$2V_0+3V_1+3V_2$
x_{101}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{3,-2}Y_{4,-1}Y_{6,3}Y_{7,4}Y_{8,5}$	$4V_0+5V_1+6V_2$
x_{102}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{7,4}$	$V_0+2V_1+2V_2$
x_{103}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{7,4}Y_{8,5}$	$2V_0+3V_1+4V_2$
x_{104}	$Y_{1,-3}Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}Y_{8,5}$	$4V_0+5V_1+6V_2$
x_{105}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}Y_{8,5}$	$2V_0+3V_1+4V_2$
x_{106}	$Y_{1,-3}Y_{1,-3}Y_{2,2}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{3,-2}Y_{4,-1}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{6,3}Y_{7,4}Y_{8,5}$	$8V_0+8V_1+12V_2$
x_{107}	$Y_{1,-3}Y_{1,-3}Y_{2,2}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}Y_{8,5}$	$4V_0+5V_1+6V_2$
x_{108}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{6,3}Y_{7,4}$	$V_0+V_1+2V_2$
x_{109}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{6,3}Y_{7,4}Y_{8,5}$	$2V_0+3V_1+3V_2$
x_{110}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{7,4}Y_{8,5}$	$2V_0+3V_1+4V_2$
x_{111}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{5,2}Y_{8,5}$	$2V_0+3V_1+3V_2$
x_{112}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{5,2}$	$V_0+V_1+2V_2$
x_{113}	$Y_{5,0}$	$4V_0+5V_1+6V_2$
x_{114}	$Y_{4,-1}$	$8V_0+8V_1+12V_2$
x_{115}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}$	$2V_0+3V_1+3V_2$
x_{116}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}$	$V_0+V_1+2V_2$
x_{117}	$Y_{1,-3}Y_{1,-3}Y_{2,2}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}Y_{8,5}$	$8V_0+8V_1+12V_2$
x_{118}	$Y_{1,-3}Y_{1,-3}Y_{2,2}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{3,-2}Y_{4,-1}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}Y_{8,5}Y_{8,5}$	$8V_0+8V_1+12V_2$
x_{119}	$Y_{2,2}Y_{3,-2}Y_{6,3}$	$V_0+2V_1+2V_2$
x_{120}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{6,3}Y_{7,4}$	$2V_0+3V_1+4V_2$
x_{121}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{8,5}$	$4V_0+5V_1+6V_2$
x_{122}	$Y_{1,-3}Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}Y_{8,5}$	$8V_0+8V_1+12V_2$
x_{123}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{5,2}Y_{7,4}$	$V_0+2V_1+2V_2$
x_{124}	$Y_{1,-3}Y_{2,2}Y_{4,-1}Y_{5,2}Y_{6,3}$	$2V_0+3V_1+4V_2$
x_{125}	$Y_{1,-3}Y_{1,-3}Y_{2,2}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{3,-2}Y_{4,-1}Y_{4,-1}Y_{5,2}Y_{5,2}Y_{6,3}Y_{7,4}Y_{8,5}$	$8V_0+8V_1+12V_2$
x_{126}	$Y_{2,2}Y_{3,-2}Y_{5,2}$	$2V_0+3V_1+4V_2$
x_{127}	$Y_{1,-3}Y_{2,2}Y_{6,3}$	$4V_0+5V_1+6V_2$
x_{128}	$Y_{1,-3}Y_{2,2}Y_{7,4}$	$2V_0+3V_1+3V_2$
x_{129}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{6,3}Y_{7,4}$	$2V_0+3V_1+4V_2$
x_{130}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{4,-1}Y_{6,3}Y_{7,4}Y_{8,5}$	$8V_0+8V_1+12V_2$
x_{131}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{4,-1}Y_{5,2}Y_{7,4}Y_{8,5}$	$4V_0+5V_1+6V_2$
x_{132}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}$	$8V_0+8V_1+12V_2$
x_{133}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{5,2}Y_{6,3}$	$8V_0+8V_1+12V_2$
x_{134}	$Y_{1,-3}Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}$	$4V_0+5V_1+6V_2$
x_{135}	$Y_{1,-3}Y_{2,2}Y_{3,-2}Y_{5,2}Y_{7,4}$	$4V_0+5V_1+6V_2$
x_{136}	$Y_{1,-3}Y_{2,2}Y_{2,2}Y_{3,-2}Y_{3,-2}Y_{4,-1}Y_{4,-1}Y_{5,2}Y_{6,3}Y_{7,4}Y_{8,5}$	$8V_0+8V_1+12V_2$

Table 3: Cluster variables with corresponding dominant monomials and their images under φ_{k} in $\mathscr{C}_1$ of type E_8.