Journal of Shanghai University (English Edition), 2003, 7(3): 275~279
Article ID: 1007-6417(2003)03-0275-05

Study on Theoretical Modeling of Semi-Active Electro-Rheological Fluid Damper

ZHANG Hua-Liang(张华良)[1], *RAO Zhu-Shi*(饶柱石)[2], *FU Zhi-Fang*(傅志方)[2]
1. *School of Electromechanical Engineering and Automation, Shanghai University, Shanghai* 200072, *China*
2. *State Key Laboratory of Vibration, Shock & Noise, Shanghai Jiaotong Univeristy, Shanghai* 200030, *China*

Abstract This paper emphases on analyzing and investigating the mechanical behavior of electro-rheological fluid (ERF) semi-active damper. Theoretical model was developed to describe the relationship between electric field and the resistance force of ERF flowing through two parallel plane electrodes. In the model, the pressure drop along electrodes was supposed to consist of two parts: one related with viscosity and the other related with dynamic yield shear stress. The concept of yield stress influence factor was developed in deriving the theoretical formula for calculating the pressure drop in the damper. The influences of some other factors, such as, non-ideal Newtonian fluid and temperature have also been taken into account. Numerical and experimental work have been performed to prove the validity of the proposed model. The comparison of both results shows that the developed model is quite effective and practicable.

Key words electro-rheological(ER) fluid, damper, modeling.

1 Introduction

Electro-rheological (ER) fluid is a kind of smart material that undergoes large changes in yield stress as electric field is applied, and appears as an instantaneous increased resistance in flow. It has the features of easy control, fast reaction and low power consumption, and so is extremely useful for vibration control. So far as today, the controllable ER fluids have received considerable attention as components of engineering devices such as valves, clutches, brakes, dampers, suspensions, and engine mounts. Although efforts to use these fluids in different devices have increased rapidly, the modeling and systematic design aspects have lagged in terms of effectiveness and creativity, To some extent, this can be attributed to the complex nonlinear behavior of this fluid.

In Ref. [1], G M Kamath investigated the behavior of semi-active ERF dampers. N M Wereley[2,3] studied the modeling of ER and Magneto-rheological (MR) dampers with idealized hysteresis method, and nonlinear quasi-steady ERF and MRF damper models were developed using an idealized Bingham plastic shear flow mechanism. Chen Suhuan[4] discussed the response of the dynamic damping force and phenomenological modeling of ERF with spectral analysis method. Yao Guozhi[5], Huang Zhenyu[6] investigated the dynamic characteristics of ERF damper and its application for vibration control by experiment method. These researches are very useful for us to understand the dynamic characteristics of ERF. But, the theoretical modeling and systematic design of ERF damper have not tended to ripe. Many theoretical and practical problems still need to be studied. The existed research results are deficient in expression being complex, physical meaning being unclear, and some parameters of the model being very difficult to determine. It can not meet the practical needs of structure design and vibration control. To establish a more effective model to well describe the dynamic properties of ERF damper has already become a hot topic. This is also the main task of this paper.

ERF damper operation involves three basic modes: shear mode, flow mode, and squeeze mode. Comparing with the shear mode, that is of small damping force, and the squeeze mode, that is limited in its displacement, the flow mode has gained much more wide application. Hereby, the flow mode is the main inter-

Received Sep. 24, 2002; Revised Nov. 29, 2002
Project supported by the Foundation of Shanghai Municipal Commission of Education (202126)
ZHANG Hua-Liang, Ph. D., Asso. Prof.
E-mail: zhl_shd@sina.com

est of our study.

2 Theoretical Modeling

The operation mechanism of flow mode is shown in Fig. 1. Two plane electrodes are parallel and fixed, and the ER fluids flow in the gap. Changing the electric field applied on the electrodes can control the flow resistance of ERF. There must exist a pressure drop along the electrodes when the ERF is flowing. For zero electric field, the ideal ERF can be considered as Newtonian fluid. Because of the small Reynolds number, the stable laminar flow occurs in the gap and the flow belongs to Poiseuille flow. The velocity profile of ERF flowing in the gap is parabolic. As the electric field is applied, the flow traits of ERF will change correspondingly. In this case, it no longer belongs to Newtonian fluid, its behavior can be described by Bingham plastic model. The velocity profile is not simply parabolic and a so-called "plug flow" will appear in the middle of the gap, as shown in Fig. 2, where u, v denote the velocity in x and y direction respectively, h is the plug thickness, and H is the distance between two electrodes. The plug fluid behavior likes a rigid solid because the shear stress is below the yield stress. This makes the flow resistance increase. Therefore, the controlled damping force of ERF can be achieved by making use of this kind of behavior.

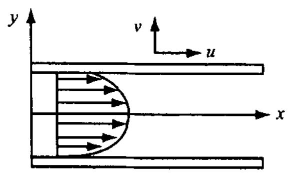

Fig. 1 Velocity profile of ERF with zero electric field

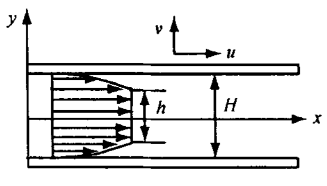

Fig. 2 Velocity profile of ERF in electrode gap with electric field

Assume that the flow is quasi-steady and fully developed, the damper components are massless, there is no friction between non-fluid components, and quasi-steady force is applied to the damper. For ERF flowing between parallel electrodes, the velocity is only the function of coordinate y (as shown in Fig. 1). The zero point of coordinate y is located at the middle of the gap. Hence, we have

$$\frac{\partial u}{\partial x}=0,\quad \frac{\partial^2 u}{\partial x^2}=0,\quad v=0$$

The simplified N-S equation is

$$\frac{\mathrm{d}^2 u}{\mathrm{d}y^2}=\frac{1}{\mu}\frac{\mathrm{d}p}{\mathrm{d}x} \tag{1}$$

where, $\mathrm{d}p/\mathrm{d}x$ is pressure gradient along coordinate x, μ is viscosity. The boundary conditions are

$$u(H/2)=0,\quad \frac{\partial u}{\partial y}(H/2)=0 \tag{2}$$

From Eq. (1) and (2), the velocity profile can be derived as

$$u=-\frac{1}{8\mu}\frac{\mathrm{d}p}{\mathrm{d}x}(H^2-4y^2) \tag{3}$$

The average velocity can then be obtained as

$$u_m=\frac{2}{H}\int_0^{H/2}u\,\mathrm{d}y=-\frac{1}{8H\mu}\int_0^{H/2}\frac{\mathrm{d}p}{\mathrm{d}x}(H^2-4y^2)\mathrm{d}y=-\frac{H^2}{12\mu}\frac{\mathrm{d}p}{\mathrm{d}x} \tag{4}$$

Suppose the width of electrode is b, the volume flux q between the electrodes is

$$q=bHu_m=-\frac{bH^3}{12\mu}\frac{\mathrm{d}p}{\mathrm{d}x} \tag{5}$$

The negative sign in Eq. (5) expresses that the pressure decreases along axial x gradually.

Assume pressure p varies linearly along the length L of the electrode, *i.e.*,

$$\frac{\Delta P_\mu}{L}=\frac{\mathrm{d}p}{\mathrm{d}x}$$

where ΔP_μ is the pressure drop due to viscosity. Hence, Eq. (5) can be rewritten as :

$$\Delta P_\mu=\frac{12\mu Lq}{bH^3} \tag{6}$$

As mentioned above, when electric field is applied, the velocity profile of ERF flowing in the gap is no longer simple parabolic and the "plug flow" appears. At the same time, its thickness h will become wider as the electric field increases. When h equals to H, the flow will stop, due to the "fiber structure" effect of ERF as the electric field is applied. It is known that the resistance of ERF damper can be expressed by pressure drop ΔP, and the relationship between ERF yield stress τ_y and the applied electric field E, $i.e.$, $\tau_y = f(E)$, can be obtained by experiment. If the relationship between ΔP and τ_y can be find out, the relationship between damping force and electric field is ready to achieve.

Fig.3 demonstrates a micro-element of ERF, where dx and dy denote the length and the height of the element respectively. From the force equilibrium, we can obtain

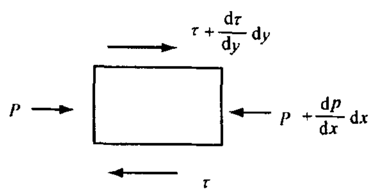

Fig.3 Micro-element of ERF

$$\frac{d\tau}{dy}dydx = -\frac{dp}{dx}dxdy \tag{7}$$

By integrating, we have

$$\tau = -\frac{dp}{dx}y \tag{8}$$

Because symmetry, we only discuss the upper half part of the gap.

In case of $0 < y < h/2$, there exists $u \equiv u_c$, where u_c is the velocity of "plug flow". When $y = h/2, u = u_c$, at this point the flow state changes from "plug flow" to laminar flow. From Eq.(8) we have

$$\tau = \tau_y = -\frac{dp}{dx}\frac{h}{2} \tag{9}$$

When $h/2 < y < H/2$, the ERF flows in laminar flow and the shear stress can be simply given as

$$\tau = -\mu\frac{du}{dy} \tag{10}$$

Substituting Eq. (8) into Eq. (10), integrating and solving for u gives

$$u - u_c = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - \frac{h^2}{4}\right) \tag{11}$$

From the boundary condition we know that at point $y = H/2$ there exists $u = 0$. Then the velocity of "plug flow" is as follows:

$$u_c = -\frac{1}{8\mu}\frac{dp}{dx}(H^2 - h^2) \tag{12}$$

Substituting Eq.(12) into Eq.(11), the velocity profile of laminar flow can be obtained as

$$\begin{aligned} u &= u_c + \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - \frac{h^2}{4}\right) \\ &= -\frac{1}{2\mu}\frac{dp}{dx}\left(\frac{H^2}{4} - y^2\right) \end{aligned} \tag{13}$$

Then the average velocity u_m can be given as fallowing:

$$\begin{aligned} u_m &= \frac{2}{H}\int_0^{H/2} u\,dy = \frac{2}{H}\left(\int_0^{h/2} u_c dy + \int_{h/2}^{H/2} u\,dy\right) \\ &= -\frac{1}{12\mu H}\frac{dp}{dx}(H^3 - h^3) \end{aligned} \tag{14}$$

The volume flux q is

$$q = bHu_m = -\frac{b}{12\mu}\frac{dp}{dx}(H^3 - h^3) \tag{15}$$

Combining Eq.(9) with Eq.(15) and simplifying, it can be given that

$$\left(\frac{dp}{dx}\right)^3 + \frac{12\mu q}{bH^3}\left(\frac{dp}{dx}\right)^2 + \frac{8\tau_y^3}{H^3} = 0 \tag{16}$$

Recall that $\frac{\Delta P}{L} = -\frac{dp}{dx}$, Eq.(16) can be rewritten as

$$(\Delta P)^3 - \frac{12\mu qL}{bH^3}(\Delta P)^2 - \left(\frac{2L}{H}\tau_y\right)^3 = 0 \tag{17}$$

Eq. (17) is a nonlinear cubic equation in ΔP. When volume flux q is given, the relationship between ΔP and τ_y is determined.

3 Determination of ΔP

Based on the theoretical analysis and experimental investigation, the pressure drop along electrodes can be considered to be composed of two parts: one related with viscosity of the fluid, and the other related with dynamic yield shear stress. When $E = 0$, ΔP is fully induced by viscosity, denoted as ΔP_μ. When $E > 0$, ΔP consists of two parts: ΔP_μ and ΔP_τ, where ΔP_τ is related with yield stress as electric field is applied. Hence,

$$\Delta P = \Delta P_\mu + \Delta P_\tau \tag{18}$$

From Eq. (16)

$$\Delta P_\mu = \frac{12\mu Lq}{bH^3}$$

Rewriting Eq.(15) as

$$q = \frac{b}{12\mu}\frac{\Delta P}{L}(H^3 - h^3) = \frac{bH^3}{12\mu}\frac{\Delta P}{L} - \frac{bh^3}{12\mu}\frac{\Delta P}{L} \tag{19}$$

Suppose the volume flux q in zero electric field is equal to the volume flux q as the electric field is applied. There exists

$$\begin{aligned} q &= \frac{bH^3}{12\mu L}(\Delta P_\mu + \Delta P_\tau) - \frac{bh^3}{12\mu L}(\Delta P_\mu + \Delta P_\tau) \\ &= \frac{bH^3}{12\mu L}\Delta P_\mu \end{aligned} \tag{20}$$

Simplifying Eq. (20) we have

$$H^3 \Delta P_\tau - h^3(\Delta P_\mu + \Delta P_\tau) = 0 \tag{21}$$

From Eq. (9) we have

$$h = \frac{2\tau_y L}{\Delta P_\mu + \Delta P_\tau}$$

Substituting it into Eq. (21) gives

$$\Delta P_\tau(\Delta P_\mu + \Delta P_\tau)^2 = \left(\frac{2L}{H}\tau_y\right)^3 \tag{22}$$

let

$$k = 2\left(\frac{\lambda}{\lambda + 1}\right)^{2/3}, \quad \lambda = \frac{\Delta P_\tau}{\Delta P_\mu} \tag{23}$$

Substituting (23) into (22), we obtain

$$\Delta P_\tau = k\frac{L}{H}\tau_y \tag{24}$$

Eq. (18) can then be rewritten as

$$\Delta P = \frac{12\mu q L}{bH^3} + k\frac{L}{H}\tau_y \tag{25}$$

here, factor k is related with the ratio of $\Delta P_\tau / \Delta P_\mu$, and is defined as yield stress influence factor. It is found by further analysis that, for ERF damper, factor k is also the function of piston moving velocity and electric field. Its value ranges from 0 to 2. This is quite different from the traditional concept in which the value is considered as a constant.

To determine the value of factor k, iterative method can be applied. Firstly, one initial value k is given, then the new value of k can be calculated by Eq. (23) and (24). By repeating this process, the result will converge quickly into its true value with any given precise. Once value k is determined, then ΔP can be calculated by Eq. (25). Numerical simulation demonstrated that the value of ΔP determined by this method is equal to that by Eq. (17), as shown in Fig. 4. While the method proposed here has clearer physic meaning and is more concise.

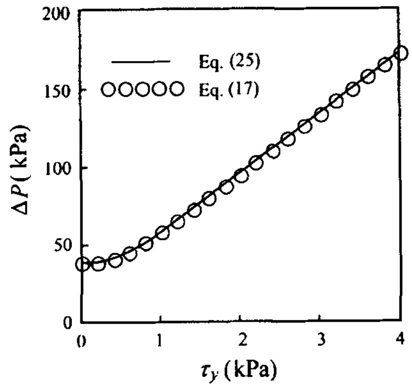

Fig. 4 Curve of pressure drop with shear yield stress

Considering the influence of some factors such as non-ideal Newtonian fluid, and the difference between dynamic yield stress and static yield stress of ERF, and the temperature that affects the mechanical properties of ERF damper, moreover, the influences of the structure factors such as elbow and small hole, two correcting factors can be introduced into Eq. (25), that is

$$\Delta P = k_\mu \Delta P_\mu + k_\tau \Delta P_\tau = k_\mu \frac{12\mu q L}{bH^3} + k_\tau \frac{k\tau_y L}{H} \tag{26}$$

where, k_μ is viscosity influence factor, k_τ is electric field influence factor. Both of them can be determined by experiment data.

4 Experimental Results

Fig. 5 is the schematic of a model ERF damper. The structure parameters are as follows: gap thickness $H = 1$ mm, piston rod diameter $d = 15$ mm, piston diameter $D = 50$ mm, the main components of ERF are silicon oil and starch with viscosity $\mu = 0.05$ Pa·s. The relationship between its static yield stress τ_y and electric field E is determined by experiment as follow:

$$\tau_y = 0.09E^2 + 0.16E + 0.18 \tag{27}$$

Fig. 6 is the schematic of experiment sets of ERF damper. The experiment results are shown in Fig. 7. In the figure, " * " stand for observed data, "—" stand for theoretical results. Here the viscosity influence factor and electric field influence factor are $k_\mu = 0.9$ and $k_\tau = 0.5$ respectively. It can be seen that both results are in good agreement.

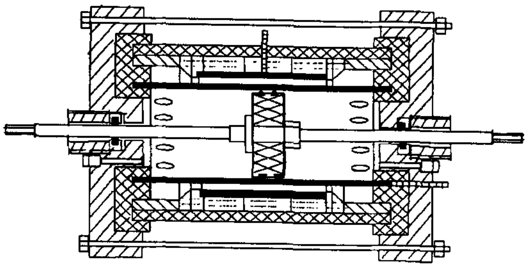

Fig. 5 Schematic of ERF damper structure

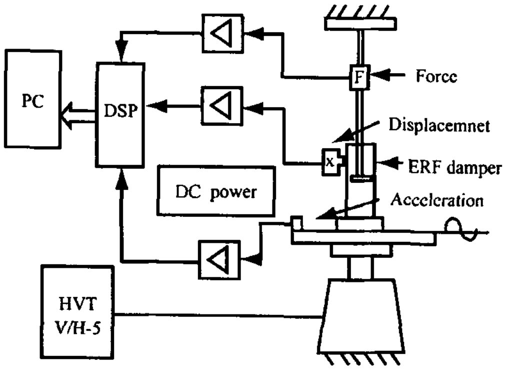

Fig. 6 Schematic of experiment sets of ERF damper

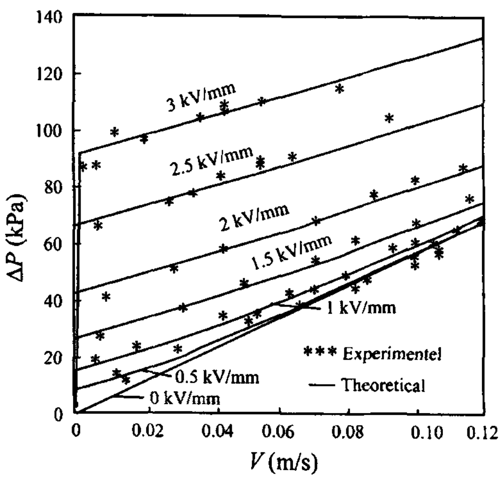

Fig. 7 Relationship of pressure drop *vs*. piston velocity and electric field intensity (H = 1 mm, L = 76 mm)

5 Conclusion

(1) A new dynamic model of ERF damper was devel-oped. The model has advantages of expression being concise, physics meaning being clear, and its parameters being easy to determine.

(2) The research results state that the yield stress influence factor is variable rather than constant. It is the function of velocity of ERF flow and electric field intensity, and its value ranges from 0 to 2.

(3) Two correcting factors were introduced into the model, so that the influence of some factors such as non-ideal Newtonian fluid, the difference between dynamic yield stress and static yield stress of ERF, and the temperature that affects the mechanical properties of ERF damper *etc*. can be taken into account.

(4) Both numerical and experimental work was carried out to prove the validity of the proposed model. The results show that the model developed in this approach is quite effective and practicable.

References

[1] Kamath G M, Hurt M K and Wereley N M. Analysis and testing of Bingham plastic behavior in semi-active electrorheological fluid dampers [J]. *Smart Materials and Structures*, 1996, **5**: 576 - 590.

[2] Wereley N M, Li Pang, and Kamath G M. Idealized hysteresis modeling of electrorheological and magnetorheological dampers [J]. *Journal of Intelligent Material Systems and Structures*, 1998, **9**: 642 - 649.

[3] Wereley N M and Li Pang. Nondimensional analysis of semi-active electrorheological and magnetorheological dampers using approximate parrel plate models [J]. *Smart Materials and Structures*. 1998, **7**: 732 - 743.

[4] Chen Suhuan and Liu Xinhui. Spectral analysis for response of the dynamic damping force and phenomenological modeling of electrorheological fluid[J]. *Smart Materials and Structures*, 1998, **8**: 499 - 503.

[5] Yao Guozhi, Guang Meng, and Tong Fang. The characteristics of electrorheological fluid and its application for vibration control [J]. *Journal of Machine Vibration*, 1995, **4**(4): 232 - 240.

[6] Huang Zhenyu. The Study for ERF Damper and Its Application in Vibration Control (D). Master dissertation, Shanghai Jiaotong University, 2000.2. (in Chinese)

(*Executive editor WANG Hai-Jiang*)