# *AMA563*

Assignment 1. Due: Oct 13, 2021 (Wednesday), 5:00pm.

You should submit your answers electronically through Blackboard or physically to the General Office (TU Core) by the time shown above. If you hand this assignment late, a mark of 0 will be recorded, unless you make a case for leniency which is accepted by the lecturer.

This coursework tests your ability on manipulating data set and handling time series concept. When you work out the questions, you are expected to express how you get the numerical answers clearly.

**1.** Suppose $X$ follows an exponential distribution with pdf $f_X(x) = \lambda e^{-\lambda x}$ on $x > 0$, and zero elsewhere. Find the moment generating function of $X$. Find the expectation of $E\frac{1}{\sqrt{X}}$.

**2.** Let $X_1, \ldots, X_n$ be a random sample from the pdf $f(x|\theta) = \frac{1}{\theta-1}$ on $1 \leq x \leq \theta$ and zero elsewhere, with $\theta > 1$.

a. Find the method of moments estimator $\widehat{\theta}_{\mathsf{MME}}$ for $\theta$.

b. Find the maximum likelihood estimator $\widehat{\theta^2}_{\mathsf{MLE}}$ for $\theta^2$.

c. Find the bias of $\widehat{\theta^2}_{\mathsf{MLE}}$.

**3.** Let $X_1, \ldots, X_n$ be a random sample from the pdf $f(x|\theta) = \theta(1-x)^{\theta-1}$, $0 < x < 1$, $\theta > 0$, zero elsewhere.

a. Find the method of moments estimator $\widehat{\theta}_{\mathsf{MME}}$ for $\theta$.

b. Find the maximum likelihood estimator $\widehat{\theta}_{\mathsf{MLE}}$ for $\theta$.

c. Compute the Fisher information $I(\theta)$.

**4.** Let $X_1, \ldots, X_n$ be a random sample from the pdf $f(x|\theta) = -(\theta+\theta^2)(1+x)^{\theta-1}x$, $-1 < x < 0$, $\theta > 0$, zero elsewhere.

a. Find the method of moments estimator $\widehat{\theta}_{\mathsf{MME}}$ for $\theta$.

b. Find the maximum likelihood estimator $\widehat{\theta}_{\mathsf{MLE}}$ for $\theta$.

c. Compute the minimum possible variance of all the unbiased estimators for $\theta^2$.

**5.** Let $X_1, \ldots, X_n$ be a random sample from $\mathsf{Poisson}(\lambda)$.

a. Find the maximum likelihood estimator $\widehat{\lambda}_{\mathsf{MLE}}$ for $\lambda$.

b. Compute the Fisher information $I(\lambda)$.

c. Compute the bias of $(\overline{X})^2$ as an estimator for $\lambda^2$.

d. Compute the minimum possible variance of all the unbiased estimators for $\frac{1}{\lambda}$.

** END **