UNIVERSITY of VIRGINIA | DARDEN SCHOOL of BUSINESS

Week 3: Common Pricing Metrics: Elasticities

Video 1: Intro to Common Pricing Metrics: Elasticities

Common Pricing Metrics: Elasticities

- Regressions
- Common elasticities
  - Price
  - Cross-price
  - Income
- Drivers of price elasticity
  - Market share
  - Competitors' prices
- Price optimization

By the end of this module you'll be able to…

- Estimate regression models
- Describe and calculate three types of elasticities
  - Price
  - Cross-price
  - Income
- Explain the drivers of price elasticity and their impact
- Perform a price optimization based on demand models

Price Elasticities

Why Measure Price Elasticity?

- Provides a standard way to predict quantity reactions to price.
- Can enable us to better understand the competitive structure of the marketplace.
- **Price elasticity** is defined as %ΔQ/%ΔP
  - The percent change in quantity divided by the percent change in price.
- **Cross-price elasticity** is the percent change in *my quantity* divided by the change in *some other price*.

Why Measure Price Elasticity?

- **Income elasticity** is the percent change in quantity divided by the percent change in per capita disposable income.

Our Scanner Data

- Monthly sales and price data on different cuts of beef and chicken.
- Data is a national sample collected by the U.S. Department of Agriculture (2001-2007).
- Also contains monthly data on per capita disposable income (Economagic)

*Data and initial modeling information obtained from Hayes, Fred H. and Stephen A. DeLurgio, "Where's The Beef: Statistical Demand Estimation Using Supermarket Scanner Data," Journal of Case Research in Business and Economics.

Linear Model

$$Q_x = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + e_i$$

Q_x = an index of beef quantities (base year = 2001)

α = constant (equals quantity of X when all other variables =0)

X_1 = Px_1, the "own" price of a given type of beef
$[B_1 = \Delta Q_x / \Delta Px_1]$

X_2 = Px_2, the price of a related good, chicken
$[B_2 = \Delta Q_X / \Delta Px_2]$

X_3 = measure of disposable (after-tax) income (Inc)
$B_3 = \Delta Q_X / \Delta \text{Inc}$

X_4 = trend variable (1, 2, 3 … n)

e_i = error term

Linear Model

- Initial analysis uses quantities and prices per pound for chuck roast.
- Same for chicken.

Estimated Model

	Coefficients	Standard Error	t Stat
Intercept	779.4412709	234.3797806	3.325548
Chuck Price	-50.1604464	16.51476771	-3.03731
Chicken Price	-53.6349433	46.5686921	-1.15174
Income	-0.05444669	0.022800774	-2.38793
Trend	1.6709003	0.67842569	2.462908

$$Q_x = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + e_i$$

What can we do with this?

- Calculate price elasticity, cross price elasticity and income elasticity.
- Use the estimated model as a demand function.
- Compute the optimal price of this cut of beef under different scenarios.

Let's do that!

- **Price elasticity** is defined as %ΔQ/%ΔP
  - $(\Delta Q \div Q)/(\Delta P \div P)$
  - Rearranging: $\Delta Q/\Delta P * P/Q$
  - This is a nice formulation because you know $\Delta Q/\Delta P$.

Estimated Model

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Price Elasticity

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  - Rearranging: $\Delta Q/\Delta P * P/Q$
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- So $E = -50.16 * P/Q$ (use the means)

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Means	107.5455	2.47
	ChuckQ	ChuckP

$$Q_x = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + e_i$$

Let's do that!

- **Price elasticity** is defined as %ΔQ/%ΔP
  - $(\Delta Q \div Q)/(\Delta P \div P)$
  - Rearranging: $\Delta Q/\Delta P * P/Q$
  - This is a nice formulation because you know $\Delta Q/\Delta P$.
- So $E = -50.16 * P/Q$ (use the means)
- $E = -50.16 * 2.47/107.55 = \mathbf{-1.15}$
- A 10% change in price will be associated with an 11.5% change in quantity.

Cross-Price Elasticity

Cross-Price Elasticity

- **Cross-price elasticity** is defined as %ΔQ/%ΔP
- $E_c = \Delta Q / \Delta P_{\circ} * P_{\circ} / Q$

Estimated Model

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$$Q_x = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + e_i$$

Cross-Price Elasticity

- **Cross-price elasticity** is defined as %ΔQ/%ΔP
- $E_c = \Delta Q/\Delta P_{\circ} * P_{\circ}/Q$
- So $E_c = -53.63 * P_{\circ}/Q$ (use the means)
- $E = -53.63 * (1.695/107.55) = -.84$
- The negative sign indicates that these products are complements [not substitutes].
- A 10% increase in the price of chicken will decrease our demand by about 8.4%

Understanding Competitive Structure

Cross-Elasticity of Row Brand with Respect to Column Brand

	Aquafresh	Colgate	Crest	Mentadent
Aquafresh	-4.2	1.0	0.8	2.0
Colgate	1.2	-3.5	1.5	1.5
Crest	1.0	2.5	-3.2	1.8
Mentadent	1.8	1.2	1.3	-4.0

Cross-Price Elasticities in Practice

	Aquafresh	Colgate	Crest	Mentadent
Aquafresh	-4.2	1.0	0.8	2.0
Colgate	1.2	-3.5	1.5	1.5
Crest	1.0	2.5	-3.2	1.8
Mentadent	1.8	1.2	1.3	-4.0

- Which brand is most sensitive to its own price?
- Who is Crest's main competitor?
- Can underlying preferences be detected?
  - We would expect asymmetric cross-price elasticities between Audi and VW.

Income Elasticity

Income Elasticity

- **Income elasticity** is defined as %ΔQ/%ΔI
- $E_I = \Delta Q/\Delta I * I/Q$

Estimated Model

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$$Q_x = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + e_i$$

Income Elasticity

- **Income elasticity** is defined as %ΔQ/%ΔI
- $E_I = \Delta Q/\Delta I * I/Q$
- So $E_i = -0.54 * I/Q$
- $E\_i = -0.054 * 9252.85/107.55 = -4.646$
- The negative sign indicates that this product is an inferior good.
- A 10% increase in disposable income will decrease our demand by about 46%.

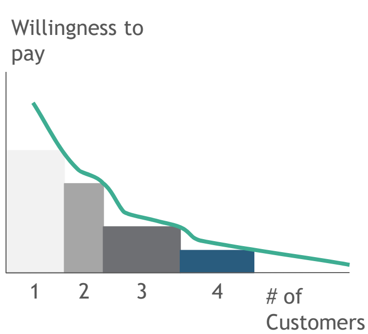
Interpreting Income Elasticity

Range of Income Elasticity	Interpretation
E < 0	Inferior good
E = 0	Sticky good
0 < E < 1	Necessity
E > 1	Luxury

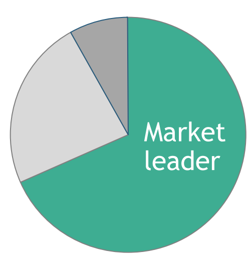
Drivers of Price Elasticity

Main factors that determine price elasticity

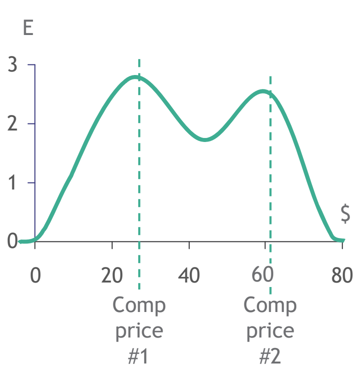
1 Shape of demand curve



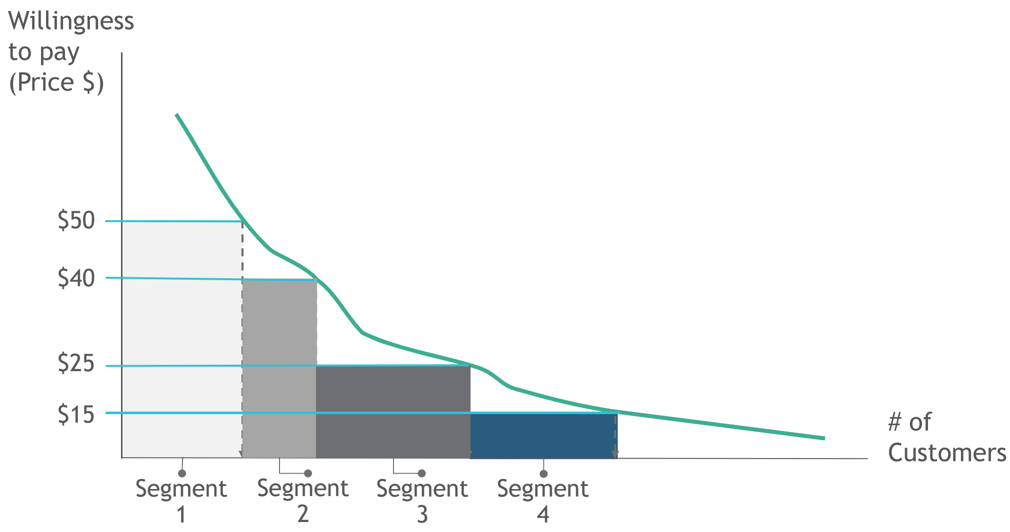
2 Market structure and share



3 Distribution of competitor prices

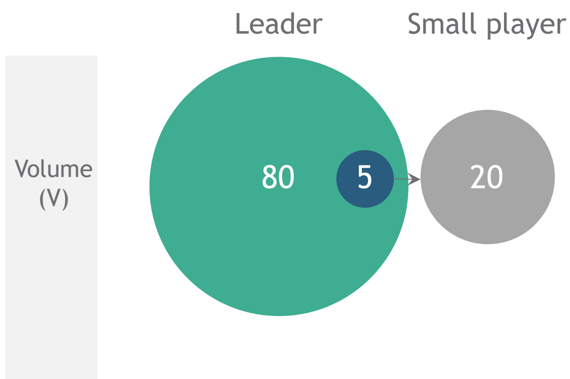


1 Shape of the demand curve



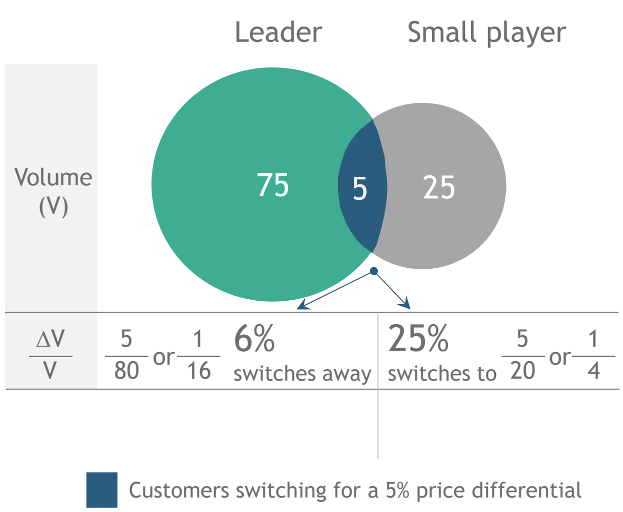
Drivers of Price Elasticity: Market Share

2 | Market share is an effective predictor of price elasticity



- Smaller player reduces price by 5%
- 5% of leader's share migrates

2 | Market share is an effective predictor of price elasticity



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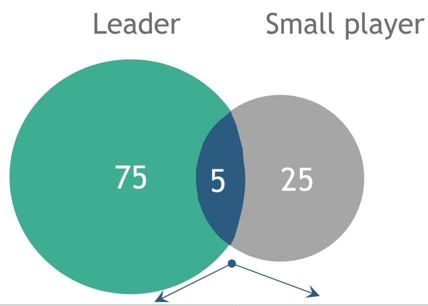
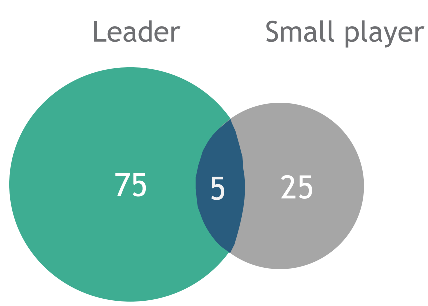


Elasticity (ε) =		Leader	Small player
	$\frac{\Delta V}{V}$	6% switches away	25% switches to
	$\frac{\Delta P}{P}$	5% price change	5% price change

Customers switching for a 5% price differential

2 Market share is an effective predictor of price elasticity



Elasticity (ε) =		Leader	Small player
	$\frac{\Delta V}{V}$	6% switches away	25% switches to
	$\frac{\Delta P}{P}$	5% price change	5% price change
Elasticity (ε)		1.2	5

Key takeaways

Established, large players are less elastic

- Tend to price at a premium
- Select the price umbrella

Small competitors have higher elasticities

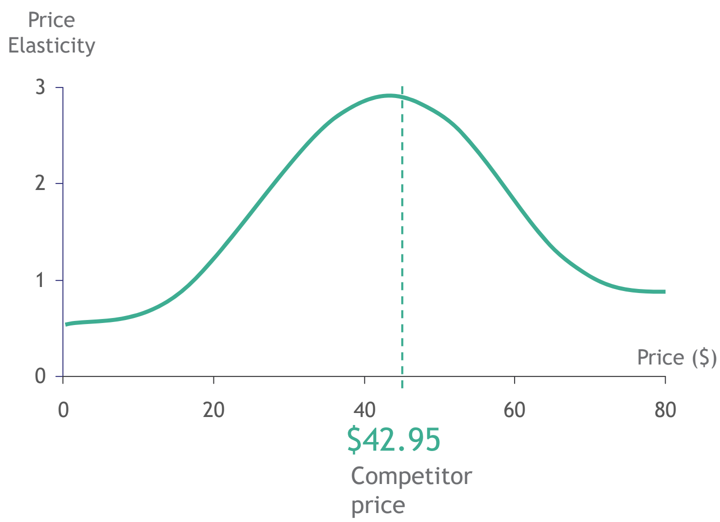
- Tend to undercut market leaders
- Have opportunity to gain share

Small channels tend to be more elastic

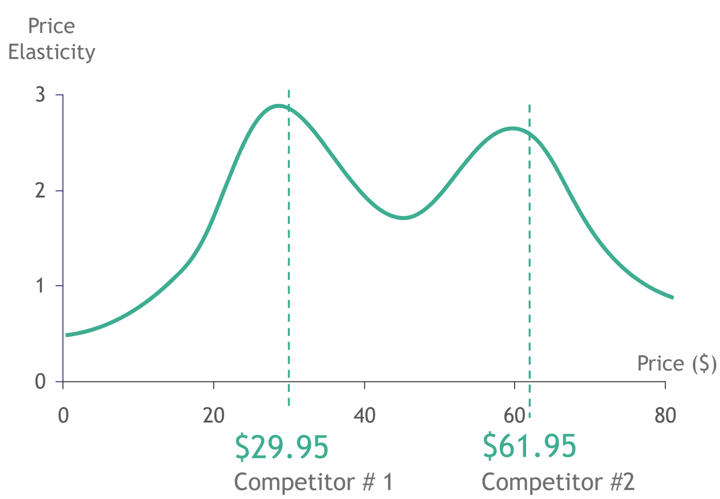
- Online channel often has lower prices…

Drivers of Price Elasticity: Competitors' Prices

Elasticity is higher around competitors' prices



Elasticity is higher around competitors' prices



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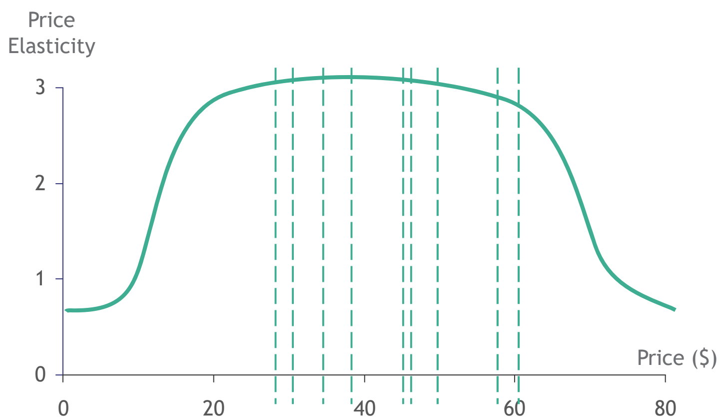
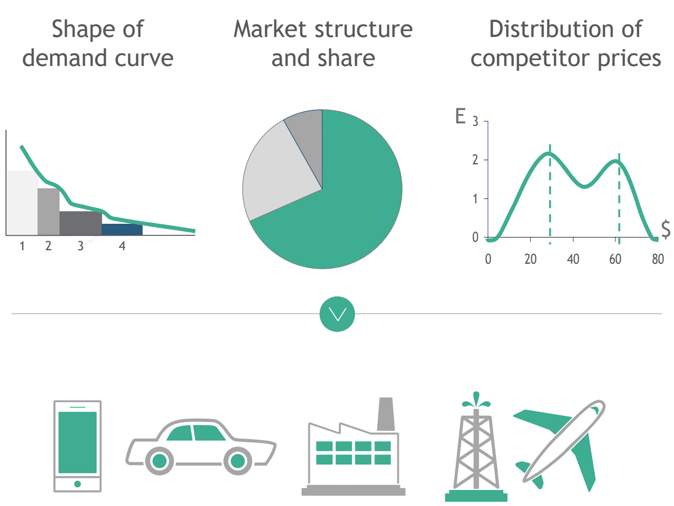


Many competitors at different prices

Drivers cause elasticities to vary across industries

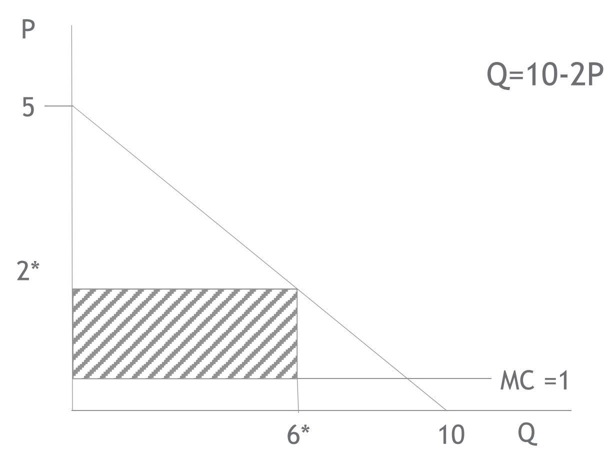


Taking a Derivative

- When there's a constant and a variable, the derivative is the constant.
- Derivative of a constant equals zero.
- Derivative of an exponent = exponent * constant – 1
- Partial derivatives (with more than one variable) take the derivative of z with respect to x. Then take the derivative again, but this time, take it with respect to y, and hold the x constant.

Price Optimization Considering Demand

The Primitives of Pricing



Using Demand Information

$\pi = (P - MC)Q$

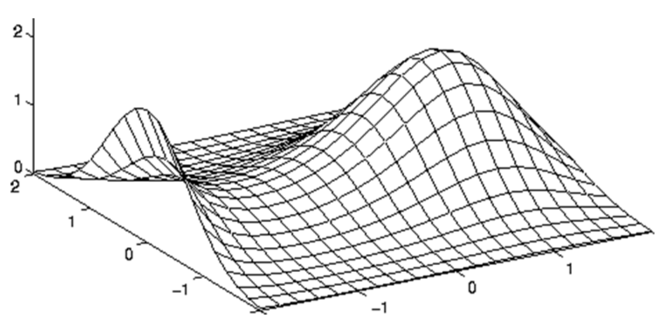
P = price, MC is marginal cost, and Q is quantity sold.

$\pi = (P - MC)(10 - 2P)$

$\pi = 10P - 2P^2 - 10MC + 2MCP$

Calculus

- Nobody freak out!



Using Demand Information

$$\pi = 10P - 2P^2 - 10MC + 2MCP$$

$$\frac{\partial \pi}{\partial P} = 0 \Rightarrow \text{Maximization!}$$

$$10 - 4P + 2MC = 0$$

Solve for P* -> The profit maximizing price.

Using Demand Information

$$P^* = \frac{10}{4} + \frac{MC}{2} \Rightarrow 2.5 + \frac{1}{2} = 3$$

Q* = 10 – 2 (3) = 4

Profit = 4*(3-1)=$8

Why is this harder in the real world?

- No one gives you demand.
- You might sell through someone who has their own incentives.
- Some things are illegal.

How do we get demand?

- Surveys (Zero Junk Mail)
- Conjoint Analysis (Portland Trailblazers)
- Purchase Data
  - Scanner data (Beef Sales)
  - Panel data
- Almost all CPG firm subscribe to scanner data services

Estimated Model

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$$Q_x = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + e_i$$

Estimated Model

$$Q_x = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + e_i$$

Q = dependent variable of quantity sold
a = constant (equals quantity of X when all other variables equal 0)
X_1 = my own price
X_2 = price of a related good
X_3 = measure of disposable income
X_4 = trend variable
e_1 = error term

Building the Demand Function

	Coefficients	Standard Error	t Stat
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$$Q_x = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + e_i$$

$$Q = 779.4 - 50.2 * P - 53.6 * P_C - .05 * Income + 1.67 * Trend$$

Price Optimization Example

Price Optimization

- The scenario:
  - You are the retailer.
  - You are setting the price for month 56, which is the month that occurs next after the end of our data.
  - You are going to price chicken at $1.75/pound.
  - The wholesale price of chuck is $1.50/pound.
  - Disposable income is stable around $10,000.

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Building the Demand Function

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$$Q = 779.4 - 50.2 * P - 53.6 * 1.75 - .05 * 10{,}000 + 1.67 * 56$$

Building the Demand Function

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$$Q = 779.4 - 50.2 * P - 93.8 - 500 + 93.52$$

Building the Demand Function

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$$Q = 279.12 - 50.2 * P$$

$$\pi = (P - C) * Q$$

$$\pi = (P - \$1.50) * (279.12 - 50.2 * P)$$

$$\pi = 279.12 * P - 50.2 * P^2 - 418.68 + 75.3 * P$$

$$\pi = -418.68 + 354.42 * P \; - 50.2 * P^2$$

Building the Demand Function

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$$\pi = -418.68 + 354.42 * P \; - 50.2 * P^2$$

$$\frac{\partial \pi}{\partial P} = 354.42 - 100.4P = 0$$

$$354.42 = 100.4P$$

$$P = \$3.53$$

Module Takeaways

Price Elasticity Key Points

- Regressions
- Common elasticities
  - Price
  - Cross-price
  - Income
- Drivers of price elasticity
  - Market share
  - Competitors' prices
- Price optimization