

# **Interest Rates: APR and EAR**

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# Last Time

## Time Value of Money

- Intuition, tools and discounting
- Compounding
- Useful shortcuts
- Taxes
- Inflation





# This Time

## Interest Rates

- Interest rate quotes
- Non-annual cash flows and compounding





# APR & EAR

# Current 5-Year Jumbo CD Rates

Institution	APY	Rate
 ★★★★★	<b>2.40%</b> Tue Dec 16	<b>2.37%</b> Compounded daily
Maximize growth and savings. Member FDIC. Apply Now!		
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Difference between  
“Rate” and “APY”?

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Rate = APR or Annual Percentage Rate

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Rate = **APR** or **Annual Percentage Rate**

Measures amount of **simple interest** earned in a year

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Rate = APR or Annual Percentage Rate

Simple interest =  
interest earned  
*without* compounding

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Many bank quotes are in terms of APR

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APR typically *not* what we earn or pay

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APY = Annual  
Percentage Yield

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APY = **EAR** or  
**Effective Annual Rate**

**EAR** measures actual  
amount of interest  
earned/paid in year

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# **HOW ARE DIFFERENT RATES RELATED?**

**Lesson:** EAR is a discount rate

EAR is what matters for computing interest and discounting cash flows



**Lesson:** APR is *not* a discount rate.

APR is a means to an end. We use it to get a discount rate (e.g., EAR)

How do we get from an APR to an  
EAR (and vice versa)?

**Lesson:** The relation between APR and EAR is:


$$\begin{aligned} EAR &= \left(1 + \frac{APR}{k}\right)^k - 1 \\ &= (1 + i)^k - 1 \end{aligned}$$

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$$EAR = \left( 1 + \frac{APR}{k} \right)^k - 1$$
$$= (1 + i)^k - 1$$

*k* is the number of  
compounding periods  
per year

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$i$  is the periodic interest rate, or  
periodic discount rate

# Example

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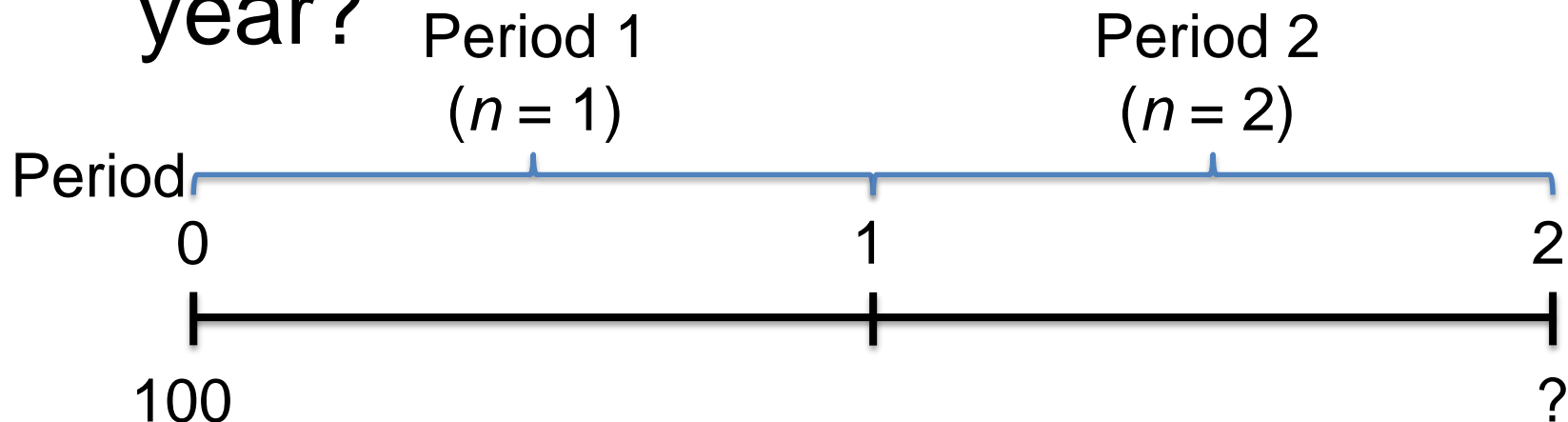
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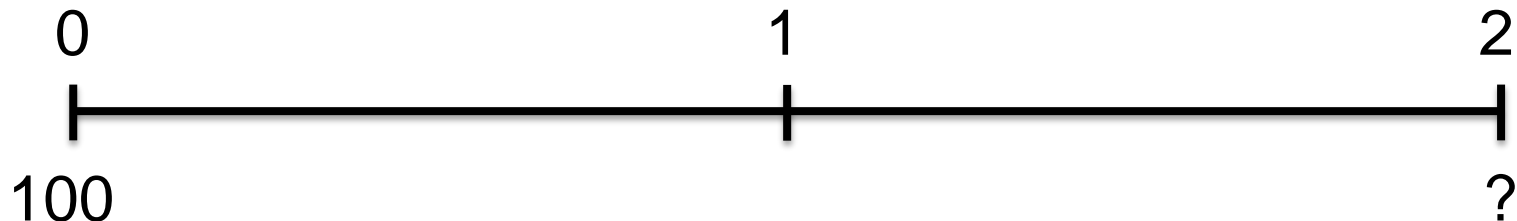


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$$i = \frac{APR}{k} = \frac{5\%}{2} = 2.5\%$$

Period



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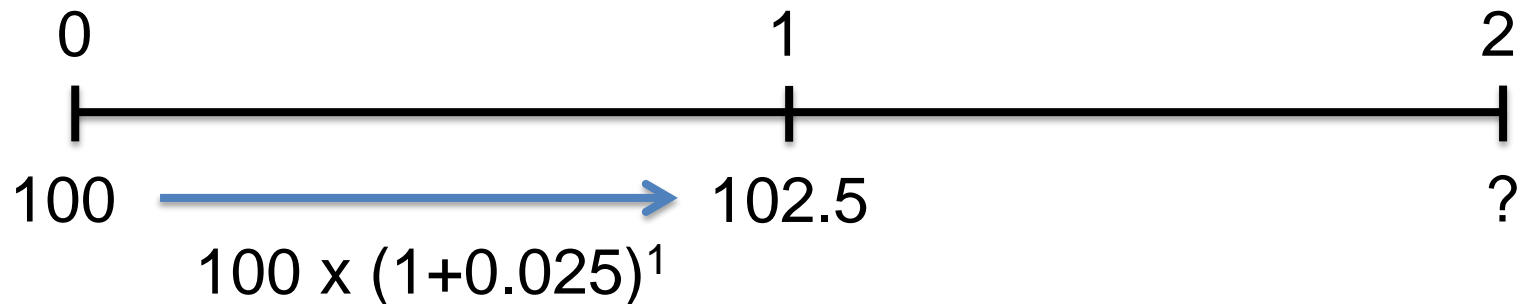


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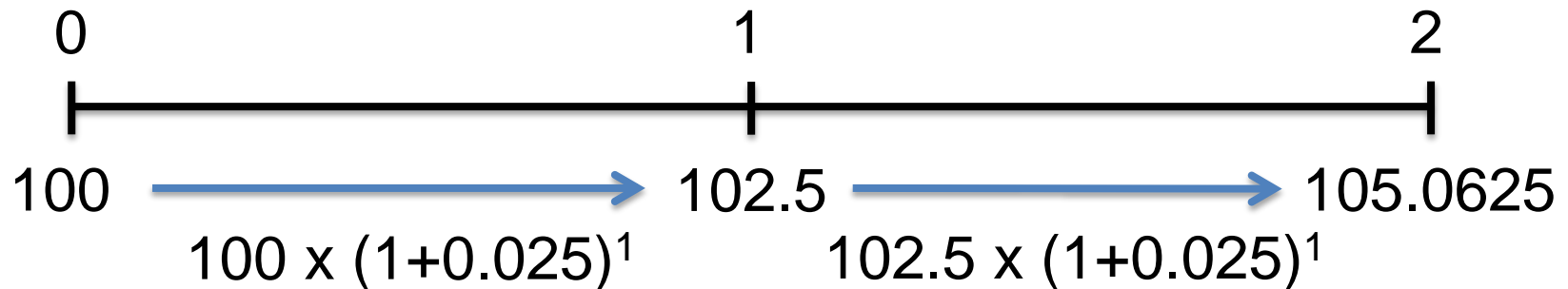


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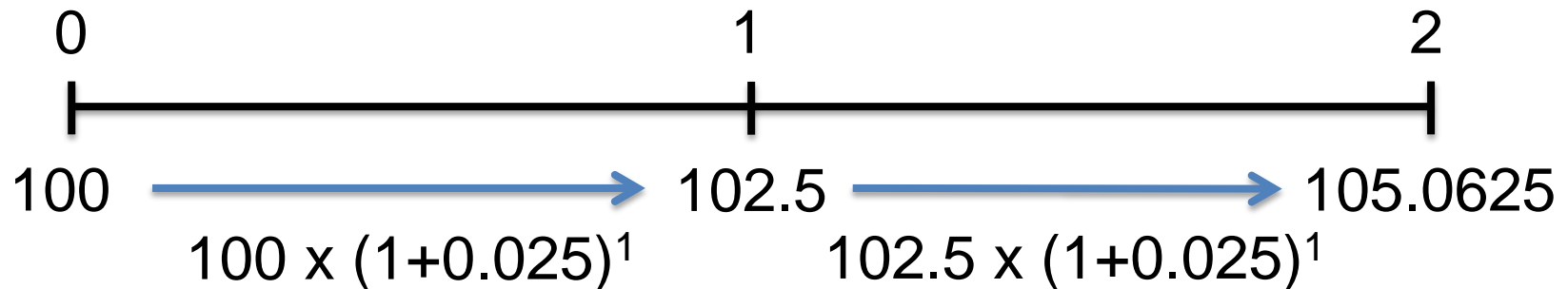


# Example

- Invest \$100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

$$FV_2(100) = 100 \times (1 + 0.025)^2 = 105.0625$$

Period



# Example

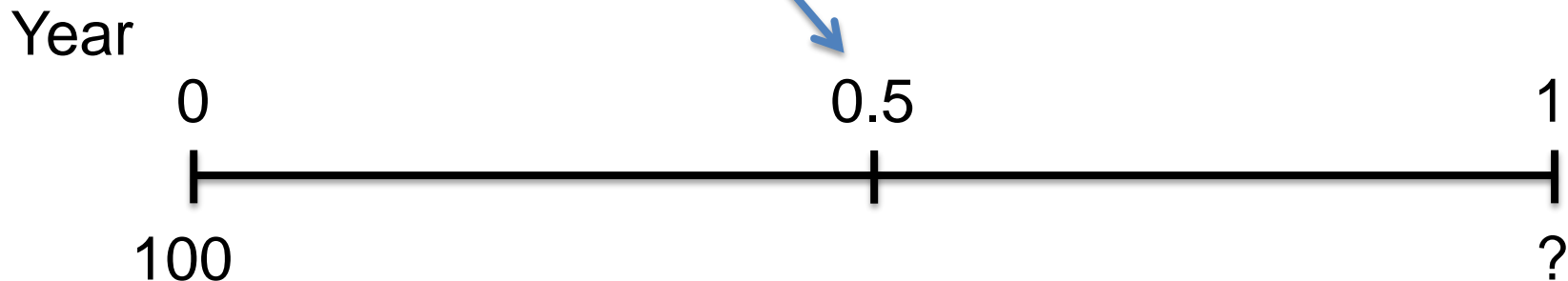
- Invest \$100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

Year



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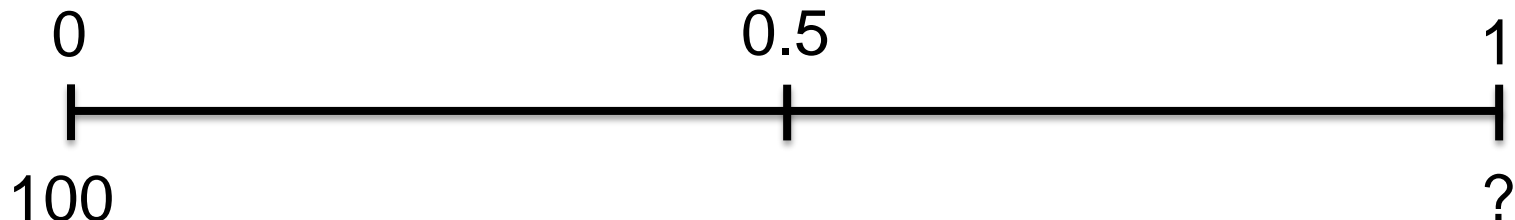


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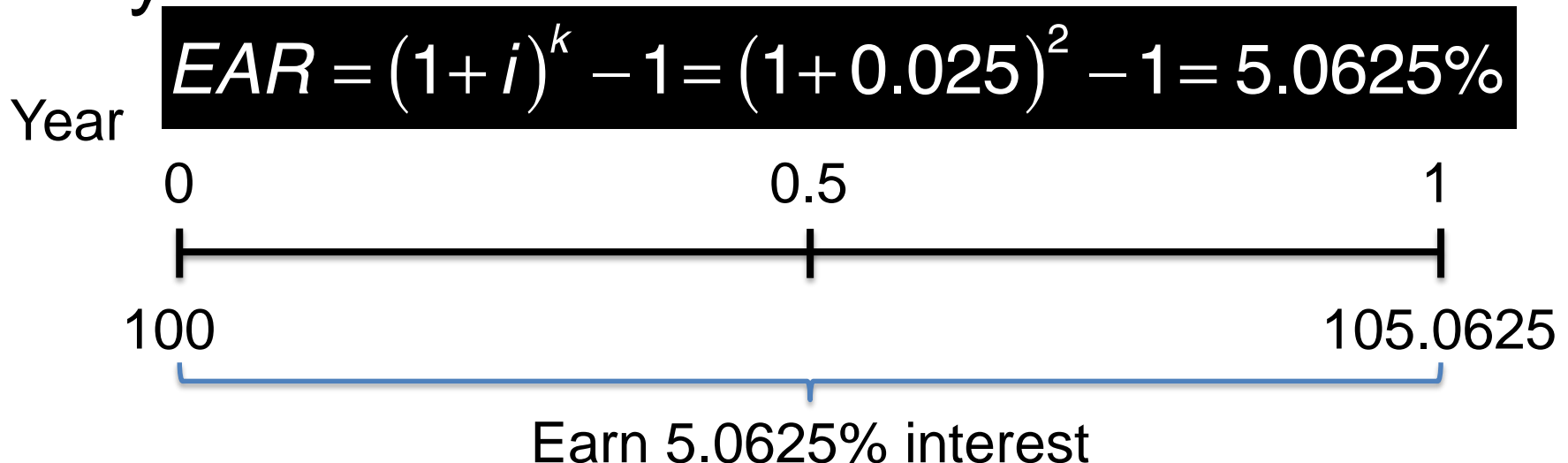
$$EAR = (1+i)^k - 1 = (1+0.025)^2 - 1 = 5.0625\%$$

Year



# Example

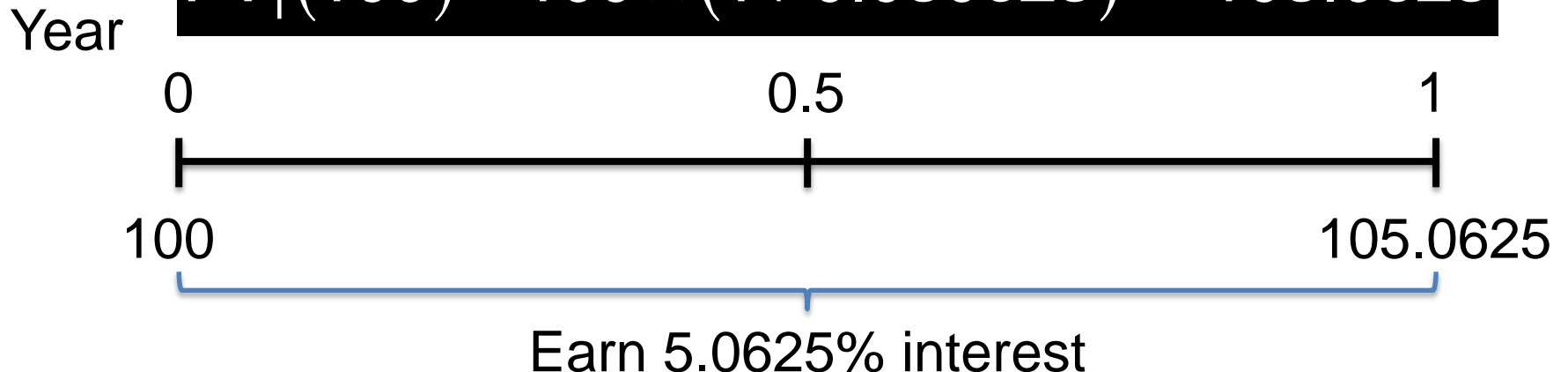
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- Invest \$100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

$$FV_1(100) = 100 \times (1 + 0.050625)^1 = 105.0625$$



**Lesson:** If you discount cash flows using EAR, then measure time in years. If you discount cash flows using periodic interest rate, then measure time in periods.

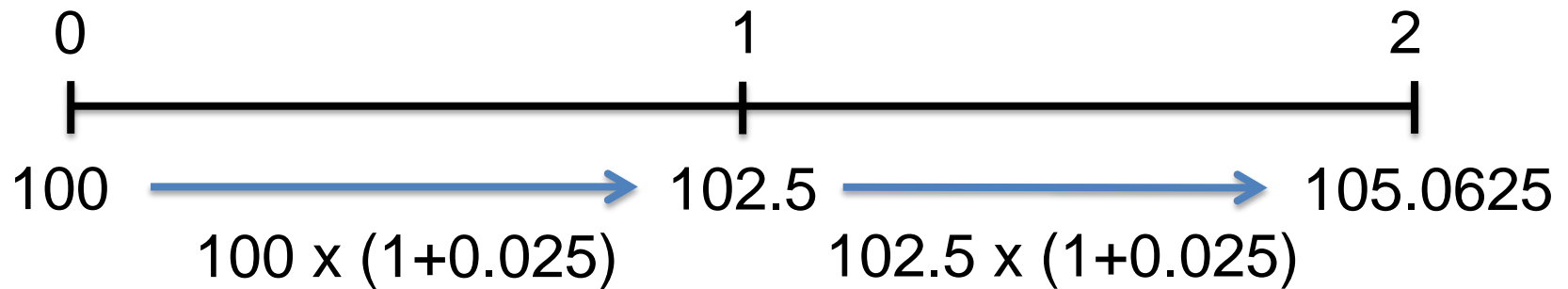
# Proof

$$\begin{aligned}(1 + EAR)^T &= \left(1 + (1 + i)^k - 1\right)^T \\&= \left((1 + i)^k\right)^T \\&= (1 + i)^{kT} \\&= (1 + i)^N\end{aligned}$$

where  $N = kT = \#$  of periods

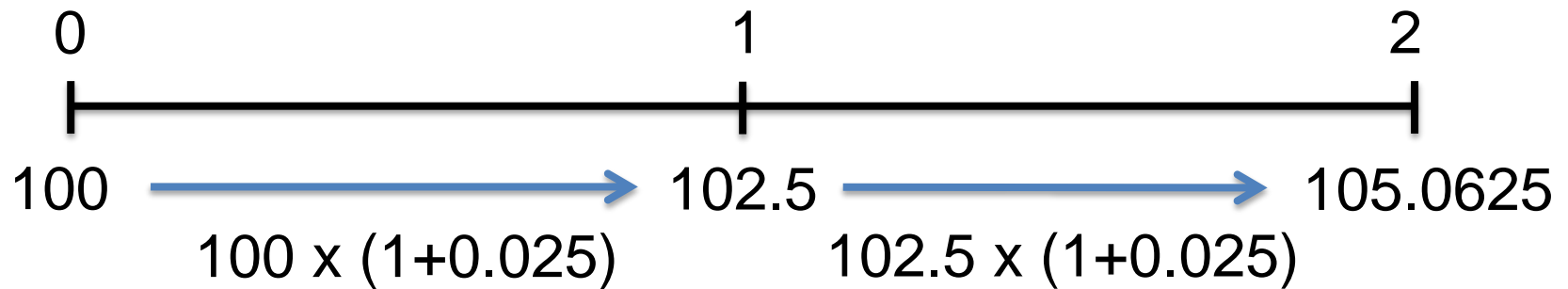
# Periods vs Years

Periods

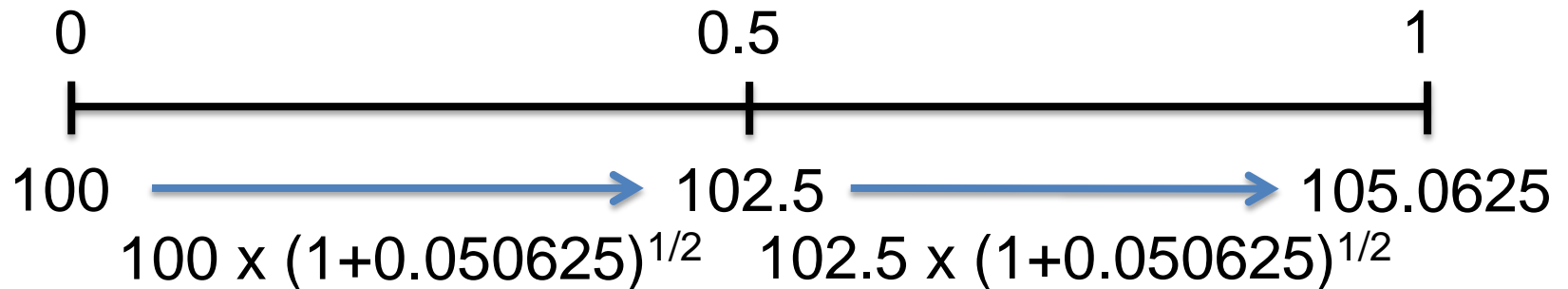


# Periods vs Years




## Periods



## Years



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$APR = 2.37\%$

$k = 365$  (or 360, 252)

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$$\text{APR} = 2.37\%$$

$$k = 365 \text{ (or 360, 252)}$$

$$\begin{aligned} \rightarrow i &= 2.37\% / 365 \\ &= 0.006714\% \end{aligned}$$

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$$k = 365 \text{ (or 360, 252)}$$

$$\begin{aligned} \rightarrow i &= 2.37\% / 365 \\ &= 0.006714\% \end{aligned}$$

$$\begin{aligned} \rightarrow \text{EAR} &= \\ &= (1 + 0.006714\%)^{365} - 1 \\ &= 2.398\% \end{aligned}$$

# Summary

# Lessons

- **EAR** is a discount rate
  - Measures cash flows in years
- **Period interest rate,  $i$ ,** is a discount rate
  - Measures cash flows in periods
- **APR** is not a discount rate

# Lessons

- Moving between **EAR** and **APR**

$$\begin{aligned} EAR &= \left( 1 + \frac{APR}{k} \right)^k - 1 \\ &= (1 + i)^k - 1 \end{aligned}$$

where

$i = APR / k$  and  $k = \#$  of periods per year

# Coming up next

- Interest Rates
  - Term Structure of interest rates and the yield curve