Cost-Sensitive Label Embedding for Multi-Label Classification

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Multi-Label Classification (MLC)

Multi-label classification

- an extension of the multiclass classification
- allow instance with multiple associated classes

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Example: image tag with (dog, cat, rabbit, shark)

image				
tag	{ dog, cat }	{ dog }	{ dog, cat, rabbit }	{ shark }
label	$(1, 1, 0, 0)$	$(1, 0, 0, 0)$	$(1, 1, 1, 0)$	$(0, 0, 0, 1)$

Multi-Label Classification (MLC)

Notation

- feature vector (image): $\mathbf{x} \in \mathcal{X} \subseteq \mathbb{R}^d$
- label vector (tag): $\mathbf{y} \in \mathcal{Y} \subseteq \{0, 1\}^K$

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Multi-label classification (MLC)

- given training instances $\mathcal{D} = \{(\mathbf{x}^{(n)}, \mathbf{y}^{(n)})\}_{n=1}^{N}$
- learn a **predictor** h from $\mathcal{D}$
- for testing instance $(\mathbf{x}, \mathbf{y})$, prediction $\tilde{\mathbf{y}} = h(\mathbf{x})$
- let the prediction $\tilde{\mathbf{y}}$ is close to ground truth $\mathbf{y}$

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Evaluation of closeness

- cost function $c(\mathbf{y}, \tilde{\mathbf{y}})$: the penalty of predicting $\mathbf{y}$ as $\tilde{\mathbf{y}}$
- Hamming loss, 0/1 loss, Rank loss, F1 score(loss), Accuracy score(loss)

Cost-Sensitive Multi-Label Classification (CSMLC)

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Cost-sensitive multi-label classification (CSMLC)

- given training instances $\mathcal{D} = \{(\mathbf{x}^{(n)}, \mathbf{y}^{(n)})\}_{n=1}^{N}$ and cost function c
- learn a **predictor** h from both $\mathcal{D}$ and c
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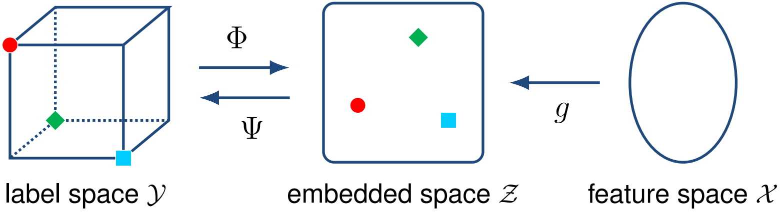
Label Embedding



Training stage

- embedding function Φ: label vector $\mathbf{y} \rightarrow$ embedded vector $\mathbf{z}$
- train a regressor g from $\{(\mathbf{x}^{(n)}, \mathbf{z}^{(n)})\}_{n=1}^{N}$

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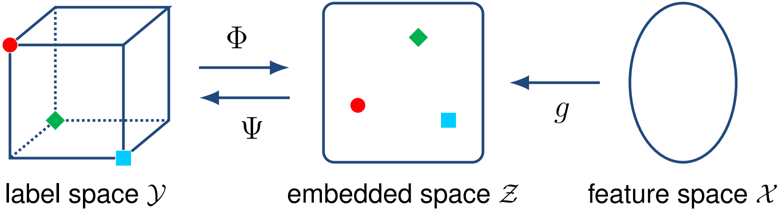

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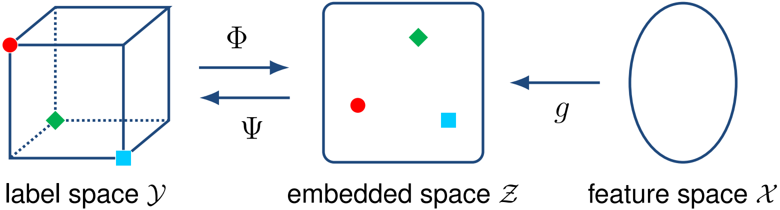
Predicting stage

- for testing instance $\mathbf{x}$, predicted embedded vector $\tilde{\mathbf{z}} = g(\mathbf{x})$
- decoding function Ψ: predicted embedded vector $\tilde{\mathbf{z}} \rightarrow$ predicted label vector $\tilde{\mathbf{y}}$

Cost-Sensitive Label Embedding



Existing works

- **label embedding**: PLST, FaIE, RAkEL, ECC-based [Tai et al., 2012; Lin et al., 2014; Tsoumakas et al., 2011; Ferng et al., 2013]
- **cost-sensitivity**: CFT, PCC [Li et al., 2014; Dembczynski et al., 2010]
- **cost-sensitivity + label embedding**: no existing works

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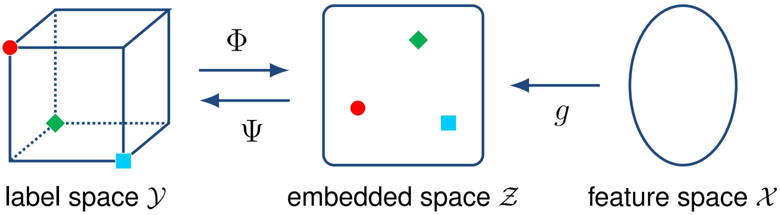

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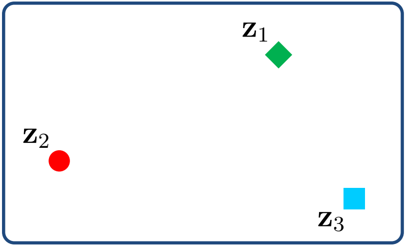
Cost-sensitive label embedding

- consider cost function c when designing embedding function Φ and decoding function Ψ (cost-sensitive embedded vectors $\mathbf{z}$)

Cost-Sensitive Label Embedding (Training)



embedded space $\mathcal{Z}$

Training stage

- distances between embedded vectors $\Leftrightarrow$ cost information
- larger (smaller) distance $d(\mathbf{z}_i, \mathbf{z}_j) \Leftrightarrow$ higher (lower) cost $c(\mathbf{y}_i, \mathbf{y}_j)$

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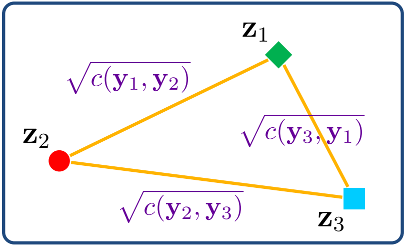

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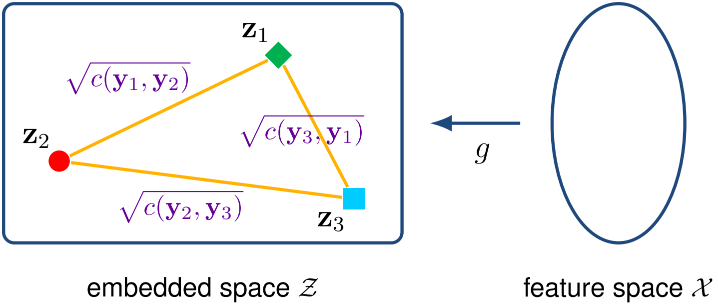

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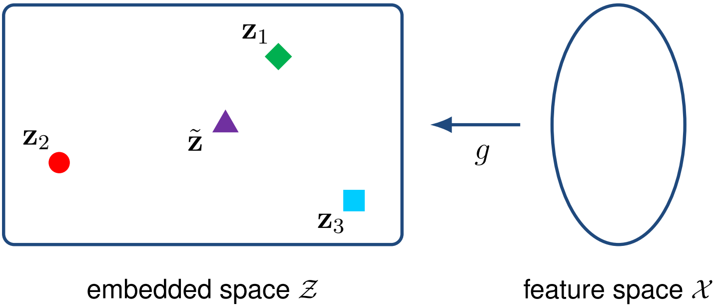

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- for testing instance $\mathbf{x}$, predicted embedded vector $\tilde{\mathbf{z}} = g(\mathbf{x})$

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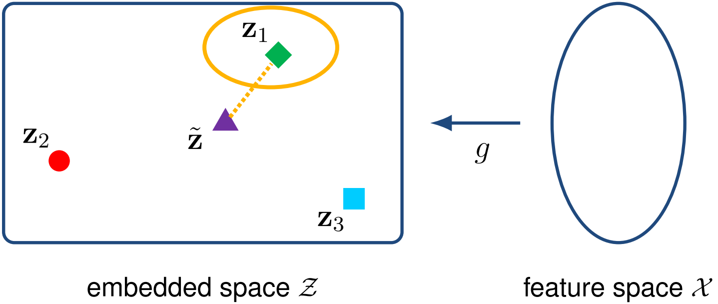

Predicting stage

- for testing instance $\mathbf{x}$, predicted embedded vector $\tilde{\mathbf{z}} = g(\mathbf{x})$
- find nearest embedded vector $\mathbf{z}_q$ of $\tilde{\mathbf{z}}$
- cost-sensitive prediction $\tilde{\mathbf{y}} = \mathbf{y}_q$

Cost-Sensitive Label Embedding (Theorem)

Theorem

$$c(\mathbf{y}, \tilde{\mathbf{y}}) \leq 5\Bigg(\underbrace{\Big(d(\mathbf{z}, \mathbf{z}_q) - \sqrt{c(\mathbf{y}, \tilde{\mathbf{y}})}\Big)^2}_{\text{embedding error}} + \underbrace{d(\mathbf{z}, \tilde{\mathbf{z}})^2}_{\text{regression error}}\Bigg)$$

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Optimization

- embedding error $\rightarrow$ multidimensional scaling (manifold learning)
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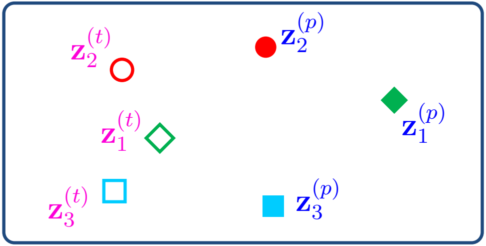
Challenge

- **asymmetric cost function** vs. **symmetric distance**?
- $c(\mathbf{y}_i, \mathbf{y}_j) \neq c(\mathbf{y}_j, \mathbf{y}_i)$ vs. $d(\mathbf{z}_i, \mathbf{z}_j)$

Mirroring Trick



embedded space $\mathcal{Z}$

- two roles of $\mathbf{y}$: **ground truth role** $\mathbf{z}_i^{(t)}$ and **prediction role** $\mathbf{z}_i^{(p)}$

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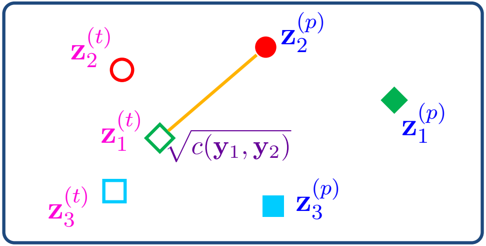

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- predict $\mathbf{y}_i$ as $\mathbf{y}_j \Rightarrow \sqrt{c(\mathbf{y}_i, \mathbf{y}_j)}$ for $\mathbf{z}_i^{(t)}$ and $\mathbf{z}_j^{(p)}$

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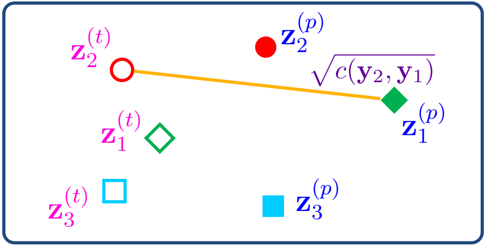

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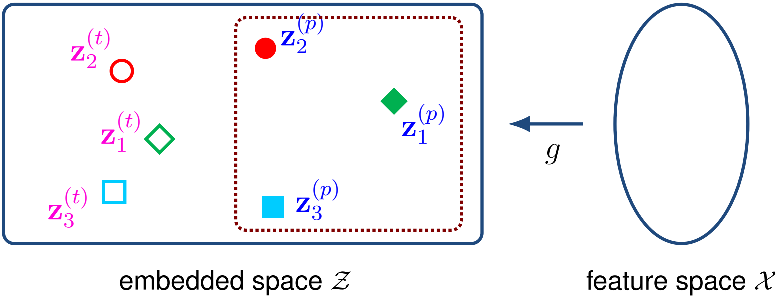

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- learn **regressor** g from $\mathbf{z}_i^{(p)}, \mathbf{z}_2^{(p)}, \ldots, \mathbf{z}_L^{(p)}$

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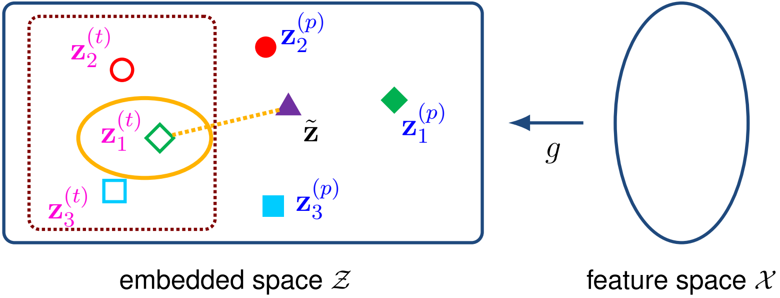

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- learn **regressor** g from $\mathbf{z}_i^{(p)}, \mathbf{z}_2^{(p)}, ..., \mathbf{z}_L^{(p)}$
- find **nearest embedded vector** of $\tilde{\mathbf{z}}$ from $\mathbf{z}_1^{(t)}, \mathbf{z}_2^{(t)}, ..., \mathbf{z}_L^{(t)}$

Candidate Set

Challenge

- label vector $\mathbf{y} \in \mathcal{Y} \subseteq \{0,1\}^K$
- 2^K possible label vectors (too many)
- what is the important(useful) label vectors?

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Candidate Set

- consider a **candidate set** $\mathcal{S}$ instead of $\mathcal{Y}$
- only label vectors in $\mathcal{S}$ are embedded
- $\mathcal{S}_{train}$ (all the label vectors in training set) is a reasonable choice

Cost-Sensitive Label Embedding with Multidimensional Scaling

Training stage of **CLEMS**

- given training instances $\mathcal{D} = \{(\mathbf{x}^{(n)}, \mathbf{y}^{(n)})\}_{n=1}^{N}$ and cost function c
- determine the candidate set $\mathcal{S}$
- find $\mathbf{z}_i^{(t)}$ and $\mathbf{z}_i^{(p)}$ for all $\mathbf{y}_i \in \mathcal{S}$ by **multidimensional scaling**
- $\Phi\colon \mathbf{y}_i \to \mathbf{z}_i^{(p)}$
- train a multi-target regressor g from $\{(\mathbf{x}^{(n)}, \Phi(\mathbf{y}^{(n)}))\}_{n=1}^{N}$

Cost-Sensitive Label Embedding with Multidimensional Scaling

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Predicting stage of **CLEMS**

- given the testing instance $(\mathbf{x}, \mathbf{y})$
- $\Psi(\cdot) = \Phi^{-1}($nearest neighbor$) = \Phi^{-1}\big(\operatorname{argmin} d(\mathbf{z}_i^{(t)}, \cdot)\big)$
- obtain the predicted embedded vector by $\tilde{\mathbf{z}} = g(\mathbf{x})$
- prediction $\tilde{\mathbf{y}} = \Psi(\tilde{\mathbf{z}})$

Experiments

Lists of experiments

- **comparison with label embedding algorithms**
  - LSDR algorithms (dim $\mathcal{Z} <$ dim $\mathcal{Y}$)
  - LSDE algorithms (dim $\mathcal{Z} \geq$ dim $\mathcal{Y}$)
- **comparison with cost-sensitive algorithms**
  - condensed filter tree (CFT) [Li et al., 2014]

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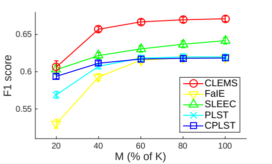
Settings

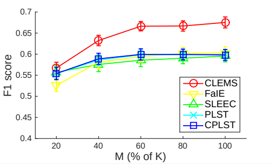
- 50% for training, 25% for validation, and 25% for testing
- base learner: random forest classifier or random forest regressor
- evaluation criteria
  - **F1 score** $\frac{2\|\mathbf{y} \cap \tilde{\mathbf{y}}\|_1}{\|\mathbf{y}\|_1 + \|\tilde{\mathbf{y}}\|_1}$ $(\uparrow)$
  - **Accuracy score** $\frac{\|\mathbf{y} \cap \tilde{\mathbf{y}}\|_1}{\|\mathbf{y} \cup \tilde{\mathbf{y}}\|_1}$ $(\uparrow)$
  - **Rank loss** $\sum_{\mathbf{y}[i] > \mathbf{y}[j]} (\llbracket \tilde{\mathbf{y}}[i] < \tilde{\mathbf{y}}[j] \rrbracket + \frac{1}{2} \llbracket \tilde{\mathbf{y}}[i] = \tilde{\mathbf{y}}[j] \rrbracket)$ $(\downarrow)$
- average results of 20 experiments

Comparison with LSDR Algorithms

F1 score (↑)



yeast



birds

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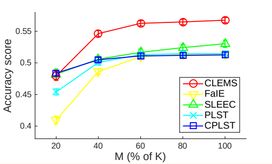

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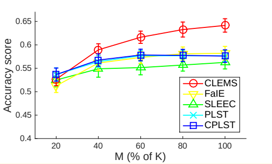
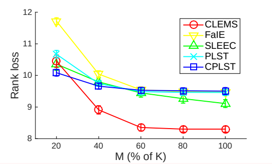


birds

Rank loss (↓)



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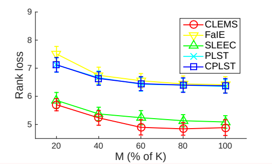
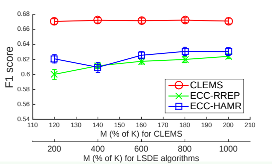


birds

CLEMS outperforms LSDR algorithms on all the cost functions!

Comparison with LSDE Algorithms

F1 score (↑)



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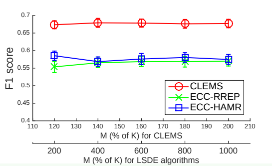

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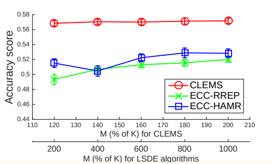

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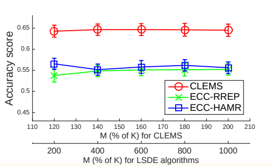

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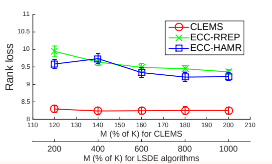

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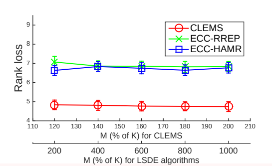

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Comparison with Cost-sensitive Algorithms

Table: Comparison with CFT

	F1 score (↑)		Accuracy score (↑)		Rank loss (↓)	
	CFT	CLEMS	CFT	CLEMS	CFT	CLEMS
emo.	0.640	**0.676**	0.557	**0.589**	1.563	**1.484**
scene	0.703	**0.770**	0.656	**0.760**	0.723	**0.672**
yeast	0.649	**0.671**	0.543	**0.568**	8.566	**8.302**
birds	0.601	**0.674**	0.586	**0.642**	4.908	**4.886**
med.	0.635	**0.814**	0.613	**0.786**	5.811	**5.170**
enron	0.557	**0.606**	0.448	**0.491**	**26.64**	29.40
CAL.	0.371	**0.419**	0.237	**0.273**	**1120.8**	1247.9
EUR.	0.456	**0.670**	0.450	**0.650**	129.53	**89.52**

CLEMS outperforms CFT in most of the cases!

Conclusion

- **algorithm design:** cost-sensitive label embedding algorithm (CLEMS)
  - mapping and nearest neighbor view for the efficient decoding function
  - embed the cost information in distance by multidimensional scaling
  - mirroring trick for the asymmetric cost function
  - candidate set to reduce the computational burden
- **theoretical guarantee:**
  - prove the upper bound of the predicted cost for CLEMS
- **empirical performance:**
  - CLEMS outperforms the existing LSDR and LSDE algorithms
  - CLEMS is better than the state-of-the-art cost-sensitive algorithms

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Thank you! Any question?