Master Operations Research and Combinatorial Optimization
Master Informatique

Multi-actor optimization algorithm to meet energy and environmental challenges at a regional scale

Chloé DANIEL

July 4, 2025

Research project performed at:
Commissariat à l'énergie atomique et aux énergies alternatives (CEA)

Under the supervision of:
Florent MONTIGNAC, CEA
Thibaut WISSOCQ, CEA
Pimprenelle PARMENTIER, CEA
Yoann JOVET, CEA

Defended before a jury composed of:
Nadia BRAUNER, Professor, UGA, G-SCOP
Van-Dat CUNG, Professor, UGA, G-SCOP
Florent MONTIGNAC, CEA
Thibaut WISSOCQ, CEA
Yoann JOVET, CEA

Abstract

This report presents a multi-actor optimization framework for the design and operation of an energy supply network, with a specific application to low-carbon hydrogen. It focuses on fairness among the network's actors and the trade-off between economic profit and carbon impact. A mixed-integer linear model was developed to represent the infrastructure components and their interactions. Each actor's objective is normalized into a satisfaction function based on upper and lower objective values. To ensure equitable compromise solutions among potentially conflicting objectives, two strategies were tested: goal programming and the lexicographic max-min approach. While goal programming may advantage some actors over others, the lexicographic max-min method produces interpretable and balanced results with acceptable satisfaction levels for all actors. The method also enables the exploration of lower CO_2 emissions through controlled relaxation of economic deterioration. This structured approach offers a decision-support tool for designing and operating networks in a way that aligns with decarbonization goals and accounts for the interests of multiple stakeholders.

Résumé

Ce rapport présente une méthodologie d'optimisation multi-acteurs pour la conception et l'utilisation d'un réseau de distribution d'énergie, avec un cas appliqué à l'hydrogène bas carbone. Il met l'accent sur l'équité entre les acteurs du réseau ainsi que sur les compromis entre rentabilité économique et impact carbone. Un modèle linéaire mixte a été développé pour représenter les composants du réseau ainsi que leurs interactions. L'objectif de chaque acteur est normalisé sous forme d'une fonction de satisfaction, définie à partir de bornes supérieures et inférieures de ses résultats possibles Afin d'assurer des compromis équitables entre des objectifs potentiellement contradictoires, deux approches ont été testées : la programmation par objectifs et la méthode lexicographique max-min. Tandis que la programmation par objectifs peut favoriser certains acteurs au détriment d'autres, la méthode max-min lexicographique fournit des résultats interprétables et équilibrés, avec des niveaux de satisfaction acceptables pour l'ensemble des acteurs. Cette méthode permet également d'explorer des scénarios à plus faibles émissions de CO_2 grâce à une relaxation contrôlée de la performance économique. Cette approche constitue un outil d'aide à la décision pour la conception et l'utilisation de réseaux compatibles avec des objectifs de décarbonation et tenant compte des intérêts de l'ensemble des parties prenantes.

Contents

— 1 —

Context of the internship

This internship was conducted within the French Alternative Energies and Atomic Energy Commission (CEA), specifically at the "Laboratoire des Systèmes Énergétiques pour les Territoires" (LSET), which is part of the "Département Thermique Conversion Hydrogène" (DTCH) under the LITEN division "Laboratoire d'Innovation pour les Technologies des Énergies Nouvelles".
CEA-LITEN is one of the main research institutions in Europe dedicated to the development of low-carbon energy technologies. While being focussed on academic research, it has the particularity to have strong ties to the industry. Its mission is to accelerate the energy transition through several innovations related to energy like hydrogen or solar energy.
The LSET laboratory focuses on the development and use of decision-support tools and energy systems at a territorial scale. This laboratory works is multidisciplinary and involves people working on process engineering, environmental assessment, economics, chemistry, operations research and more. Its projects aim to support public and private stakeholders in the design of resilient, low-carbon energy infrastructures adapted to local needs.
It is a geographically distributed team, with part of the people based in Grenoble and the other part at "Institut National de l'Énergie Solaire" (INES) in Le Bourget-du-Lac.

Throughout the internship, I had the opportunity to collaborate closely with Florent Montignac, Thibaut Wissocq, Pimprenelle Parmentier and Yoann Jovet. All of them helped me navigate the topic I was assigned and guided me with their own expertise.

— 2 —
Introduction

2.1 Global warming and Hydrogen

The earliest signs of global warming can be traced back to the beginning of the Industrial Revolution in the late 18th century. With the advancement of the steam engine, mechanization and factories rapidly developing, it is a period marked by the large-scale use of fossil fuels, particularly coal.
Concerns about climate gradually rose during the early 20th century, but the scientific consensus was far from settled. The idea that Earth's climate was warming due to human activity remained controversial:

"Although the earth is growing warmer, scientists are not unanimously agreed as to the cause"
- The New York Times, 1956 [1]

Now climate science, supported by decades of empirical data and studies, has firmly established that global warming is caused by the accumulation of greenhouse gases, particularly carbon dioxide (CO_2), resulting primarily from the combustion of fossil fuels [2].
Since the pre-industrial era, the atmospheric concentration of CO_2 has increased significantly and in parallel, the global average temperature has risen by about 1.1°C, with projections showing potentially catastrophic outcomes if emissions are not drastically reduced [3].
The industrial sector has played a central role in this trend. It was responsible for about 25% of global CO_2 emissions in 2022, making it one of the most significant contributors to climate change [4].

Global warming and thus the reduction of CO_2 emissions represents one of the most pressing challenges of the 21st century, with widespread consequences for ecosystems, economies, and human societies.
Replacing the use of fossil fuels, limited in resource and responsible for high carbon emissions, with greener and more sustainable alternatives is of utmost importance. Among the alternative energy solutions under consideration for decarbonising industry, hydrogen has recently emerged as a promising option.
The principal reason hydrogen use is under investigation is that its combustion does not produce carbon dioxide but only water vapour. Hydrogen is an energy vector, meaning it can be used to store and transport energy, and later be converted in other usable forms such as heat, electricity or mechanical work.

However, hydrogen cannot be extracted directly from the ground like fossil fuels can be and so it has to be produced differently. Several production pathways exist for manufacturing hydrogen, with different environmental impacts.

Amongst those pathways [5], we define the most commonly used ones:

- Electrolysis: The hydrogen is made by using electricity from various sources, including renewable energies (such as solar or wind power) to electrolyse water. Electrolysers use an electrochemical reaction to split water into its components of hydrogen and oxygen:

$$2H_2O \rightarrow 2H_2 + O_2$$

  The electrolysis reaction itself does not generate any greenhouse gases. However, indirect emissions can occur depending on the carbon intensity of the electricity used to power the process.

- Steam Methane Reforming (SMR):This process brings together natural gas and heated water in the form of steam. However it produces a lot of CO_2 along H_2 production: about 10 $kgCO_2/kgH_2$ [6].

- SMR with carbon capture and storage: The hydrogen is also produced via SMR but it includes the use of carbon capture and storage (CCS)* to trap and store the carbon emitted. The hydrogen produced using this method is called "low-carbon hydrogen".

- Gasification: The hydrogen is made by burning coal and is the most environmentally damaging

** Carbon capture and storage : The CO_2 is captured during the industrial process, compressed ans transported and finally injected into geological storage (salt caverns, saline aquifers, deplete oil and gas reservoirs...)*

We can see that only certain production pathway of hydrogen are of interest if the goal is to use it with the objective to support a sustainable energy transition.
Thus, it was decided to focus particularly on hydrogen produced via electrolysis using various sources of electricity (renewable or not) and hydrogen produced via SMR with CCS to study the implementation and operation of a low-carbon hydrogen distribution network.

2.2 Motivation behind the Study

A hydrogen distribution network is composed of the infrastructures and actors involved in the production, transport, storage, and consumption of hydrogen (see Figure 2.1 for an example). In particular, a low-carbon hydrogen distribution network refers to a system in which hydrogen is produced with minimal CO_2 emissions and delivered efficiently to clients, such as industrial facilities. When looking to study such a network, two approaches can be considered:

- Centralised approach: The network is represented as a single, unified decision-making entity with a unique objective. The goal is to find a solution that is optimal over the whole network assuming one single global objective and perfect cooperation of the whole system. This approach is ideal but not realistic in an industrial ecosystem. It doesn't take into account any personal interest and encompass the whole supply chain: from production to consumption.

- Multi-actor approach: It is taken into consideration that the network is not operated by a single entity, but by multiple-stakeholders that interacts between themselves. They all have their own objectives, preferences, and operational constraints. These actors can compete or cooperate.

Through the rest of the study, we adopt a multi-actor perspective as this best describes the real-world behaviour of such system. It is highly unlikely that a single company will maintain a monopoly over the whole hydrogen supply chain and market.
Modelling a network that is composed of several entities with possibly conflicting objective brings a new layer of complexity to the representation of the network. Additionally to solving the distribution of physical flux and dimensioning of infrastructures, it is now necessary to take into account the interactions among the actors and the interests of each one.

With this in mind, we formulate the following research question:

How can we model and optimise a low-carbon hydrogen distribution network while taking into account the individual objectives and constraints of each actor involved in the network?

To support this approach and better position our work, we begin with a literature review. The goal is to explore how hydrogen networks are currently being studied, with a particular focus on approaches that consider multiple actors. This allows us to gain insight into the different modelling choices and methodologies that have been proposed, and to identify which ones offer interesting perspectives or tools that could be relevant for our study.

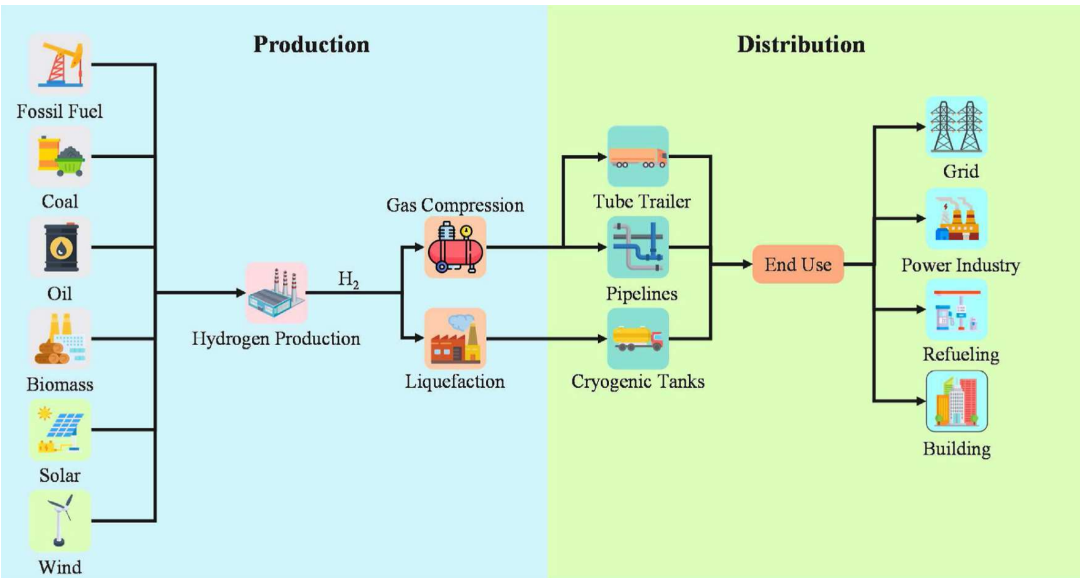


Figure 2.1: Schematic representation of a hydrogen supply chain (emphasis on transport) [7]

— 3 —

Literature review

3.1 Game theory

Game theory offers a formal framework for analysing interactions among rational, self-interested actors. Originating in economics [8], it has since been widely applied in energy system modelling [9] [10] [11], where it provides tools to capture both competitive dynamics and cooperative planning among agents. Game-theoretic models are particularly suitable for representing multi-stakeholder decision processes in energy markets, supply chains, and infrastructure development [12].
Game theory can be broadly classified into non-cooperative (competitive) and cooperative games. A more detailed overview of game theory classification and resolution methods associated based on Wang et al.'s work [13] can be found in Figure 3.1

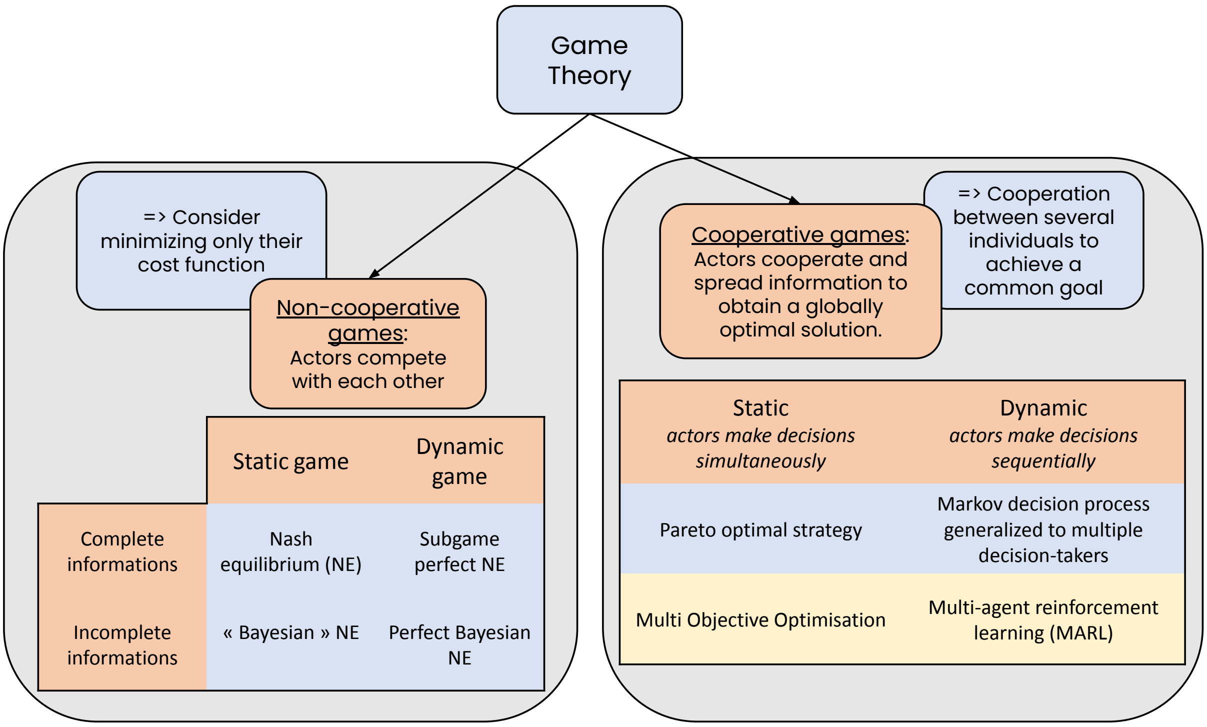


Figure 3.1: Game theory classification and resolution methods

3.1.1 Competitive behaviour: Non-Cooperative Game theory

In non-cooperative games, each actor optimizes their individual objective without coordination. Common frameworks include Nash equilibrium models, Stackelberg games (leader-follower), and Cournot competition. These approaches have been used to model competition in hydrogen supply chains [10] and other more general supply chains [14].
While non-cooperative models offer realism in a modern competitive liberal market, they often result in bi-level or multi-level programming formulations. In these formulations, the decision problem of one actor is constrained by the optimal response of another one, and in multi-level models, this hierarchy extends to multiple actors. These formulations are typically non-convex and non-linear due to the nested structure of decision-making and the interdependence of actors' strategies, which often involve discrete variables and complementarity constraints [15]. This brings various limitations for large-scale networks (computability, complexity, guarantee of optimality,...). Furthermore, competitive equilibria may not be Pareto-optimal, leading to inefficiencies and social welfare losses.

3.1.2 Cooperative Behaviour: Multi-Objective Optimization

In contrast, cooperative games assume that actors are willing to collaborate to identify outcomes that benefit the group. This is particularly relevant in policy-driven or regulated contexts, such as national hydrogen strategies or shared distribution infrastructure.
One common modelling approach is multi-objective optimization (MOO), where the goals of multiple actors are explicitly represented as separate objectives. Solutions are obtained via Pareto front exploration, aggregation methods, or interactive techniques such as the NIMBUS [16] or NAUTILUS method [17]. MOO has been widely applied in the design of sustainable energy systems [18], including hydrogen-based energy islands [19] or eco-industrial networks [20], and process integration [11]. A key limitation of traditional MOO is that it often assumes a single decision-maker. Techniques such as fuzzy MOO or multi-actor multi-criteria decision analysis (MCDA) address this by incorporating stakeholder preferences and uncertainty [21] [20].
Another reason to favour collaborative models is explainability. Solutions derived from cooperative MOO frameworks can be visualized, interpreted, and adjusted, which enhances transparency and legitimacy [22]. In contrast, solutions from complex non-linear bilevel games may lack interpretability for non-experts. If the stakeholder fail to be convinced by the solution, it makes it impossible to implement.
Finally, collaborative approaches are computationally less demanding. While still complex, they allow the use of linear or convex formulations, in contrast to bilevel non-cooperative games, which often require heuristic or evolutionary algorithms with no guaranty of optimality [15].

3.2 Multi-objective optimisation

3.2.1 What is multi-objective optimisation?

When looking to make the best decision in the design and operation of a complex distribution network, mathematical modelling present a framework to represent real life processes in a

structured form.
A widely used approach is linear programming where the system is represented using a set of continuous decision variables, linear constraints, and a linear objective function that needs to be either maximized or minimized [23]. These linear programs can be efficiently solved using mathematical solvers (such as Cplex or Gurobi) that make use of specialized algorithms such as the Simplex method or interior-point methods. Not only are those methods used to find feasible solutions to solve the implemented system, but the solutions found are mathematically proved to be optimal.

As mentioned in the previous section, we focus here on the optimisation of a network composed of cooperative actors. The method employed to achieve this is multi-objective optimisation. Unlike classical optimization, where the aim is to minimize (or maximize) a global function, multi-objective optimization contains a set of objective functions to be optimized over the same set of variables and constraints.

General formulation of an optimisation problem:

Single-objective :	Multi-objective (m objectives):
$$\begin{aligned} z = min \quad & f(X) \\ st. & \\ & g_j(X) \geq 0 \qquad j = 1,2,\cdots,q \\ & h_k(X) = 0 \qquad k = 1,2,\cdots,p \end{aligned}$$	$$\begin{aligned} z_i =& min \quad f_i(X) && i = 1,\cdots,m \\ st. & && \\ & g_j(X) \geq 0 && j = 1,2,\cdots,q \\ & h_k(X) = 0 && k = 1,2,\cdots,p \\ Z =& \{z_i \quad \mid \quad i \in \{1,\cdots,m\}\} && \end{aligned}$$

where $X = [x_1, x_2, \cdots, x_n] \in S$ is a vector of n decision variables and S is the set of feasible solution in the decision space.
The expressions $g_j(X)$ et $h_k(X)$ are respectively the inequality and equality constraints.
Z is the set of feasible solutions in the objective space.

These different objective functions can be contradictory and come into conflict, so that there is no optimal solution to all of them. In multi-objective optimization, we therefore don't try to find a single optimal solution, but rather a set of non-dominated solutions: the set of Pareto-optimal solutions. Solving a multi-objective problem thus involves finding a Pareto-optimal solution that best satisfies the preferences of a decision-maker (an external actor tasked with selecting a compromise solution). A solution is considered Pareto-optimal if it is impossible to improve the value of one objective function without deteriorating at least one other.
The following terms are defined [24]:

- <u>Domination</u>: A solution $Z^u = (z_i^u, \cdots, z_m^u)$ dominates a solution $Z^v = (z_i^v, \cdots, z_m^v)$ if and only if $\forall i \in \{1, \cdots, m\} \quad z_i^u \leq z_i^v$ and $\exists i \in \{1, \cdots, m\} \quad z_i^u < z_i^v$.

- <u>Pareto Optimality</u>: A solution $X^* \in S$ is Pareto optimal if and only if there does not exist another solution $X \in S$ such that X dominates X^*.

- The set of Pareto optimal solution is called the <u>Pareto set</u>, noted as P^*. Its image is noted PF^* and called <u>Pareto front</u> or the compromise surface. (See Figure 3.2)

- The Ideal point $Z^I = (z_1^I, \cdots, z_m^I)$ is the vector composed of the best objective values.
- The Nadir point $Z^N = (z_1^N, \cdots, z_m^N)$ is the vector composed of the worst objective values over the Pareto Front.
- The worst point is the vector composed of the worst objective values over the whole objective space.

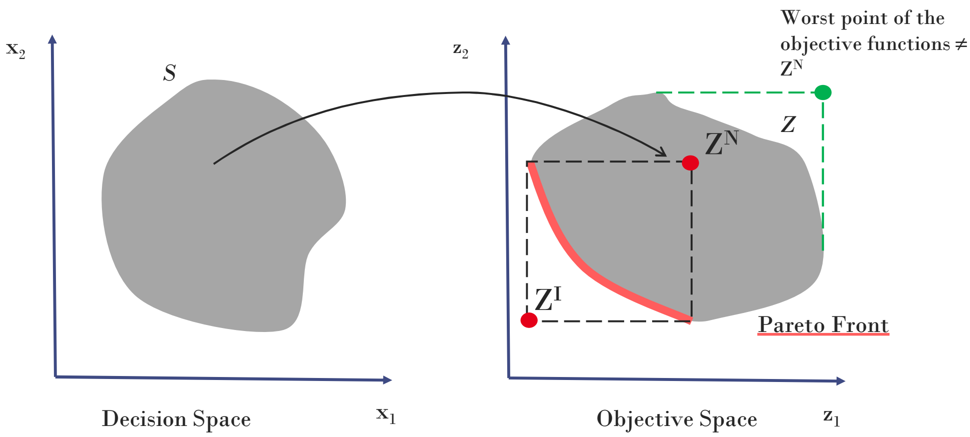


Figure 3.2: Graphical representation of the Pareto Front [25]

3.2.2 Categories of resolution methods

As all Pareto solutions are equivalent to each other, it is necessary to call on an external decision-maker to select a solution. Depending on the resolution stage at which the external decision-maker intervenes, we can distinguish three categories of multi-objective resolution methods.

- A Priori: The decision-maker expresses his global preferences prior to resolution, and then a solution that satisfies these preferences is chosen. The advantage of this approach is the relatively low complexity of the resolution. However, determining global preferences can be complicated. (e.g. Goal Programming)
- A Posteriori: First, an approximation of the Pareto front is calculated before selecting a solution of interest to the decision-maker. This approach is advantageous for the decision-maker, who has a global view of the problem, but generating the full Pareto front is very costly.
- Interactive: In this solution method, the decision maker determines these preferences as he goes along. The decision-maker doesn't have to have global preferences from the outset, and only part of the solution set is generated, reducing complexity.

Lexicographic optimisation

One of the methods traditionally used in Multi-objective optimisation is lexicographic optimisation. A focus is made on this particular method as its comprehension is needed for one of the

following sections (3.4.2).
In lexicographic optimisation, the decision maker ranks the objectives by order of priority and then solves the optimisation problem sequentially, starting with the highest-priority objective. Once an optimal value is found for the first objective, this value is imposed as a constraint while solving for the next objective in the priority list, and so on. Because the preference information is provided before the optimisation process starts, this method is categorised as an "A priori" approach.

Consider the general multi-objective optimisation problem with m objectives:

$$min \quad f_i(X) \quad i = 1, \cdots, m$$

with $X \in S$.
Let L be the ordered list containing the objective function index by order of priority.

Then the lexicographic optimisation process is as described in Algorithm 1.
The final solution computed is said to be lexicographically optimal. To note that a lexicographically optimal solution is necessarily Pareto optimal.

for $i \in L$ **do**
  Solve:
  $z_i = min \quad f_i(X)$
  $\qquad st. \quad X \in S$
  Add the constraint:
  $f_i(X) = z_i$
end

Algorithm 1: Lexicographic optimisation procedure

3.3 Normalisation : A Problem of Scale Coherency

When comparing multiple objective, it is common that they differ in magnitude.
The producers do not have the same scale of production and cannot expect the same benefit, the cost of production of hydrogen widely varies with the method of production, and the objective between consumers and producers are also very different: while consumer want to minimize the price at which 1 kg of hydrogen is bought, producers want to maximize their *total* benefit. Normalisation thus becomes necessary to ensure that the objective can be properly and meaningfully compared. Failing to properly normalize the different objective functions can lead to bad quality solution [26].

With the previously defined ideal and nadir points, we define the normalised objective vector as:

$$\tilde{z}_i(x) = \frac{f_i(x) - z_i^I}{z_i^N - z_i^I}, \quad \forall i \in \{1, \cdots, m\}.$$

After normalisation, each objective satisfies:

$$0 \leq \tilde{z}_i(x) \leq 1$$

where $\tilde{z}_i = 0$ corresponds to the ideal point, and $\tilde{z}_i = 1$ to the nadir point.
It is therefore fairly straightforward to see that the quality of the normalisation, solely depends on the quality of the ideal and nadir point value.

Computing the ideal point is a trivial process. Let us define (P_q) the mono-objective optimisation program of the q^{th} objective function of the original multi-objective problem:

$$\begin{aligned} &min \quad f_q(X) \\ st. \quad &X \in S \end{aligned}$$

Solving (P_q) yields z_q^I [27].
Therefore, we only need to solve the m optimisation problems (P_q) with $q \in \{1, \cdots, m\}$ to compute all the coordinates of the ideal point.

3.4 The Nadir Point: Estimation Challenges and Methods

While the ideal point can be obtained by individually optimising each objective, computing the exact nadir point is more difficult. It requires knowledge of the entire Pareto front, and is often infeasible in problems with more than three objectives. Thus, we need to compute an approximation of this point. This approximation must be not too far from the actual value to keep the quality of the normalisation. Moreover, the Nadir point can be used in more than just normalisation. It gives us information on the Pareto set and can also be used as a reference or even starting point in Evolutionary Multi-objective Optimization (EMO) [17].
However, even computing an estimation of the Nadir point is a difficult task. As noted by Ehrgott [27], "estimating the nadir point is a challenging and unsolved computing problem in case of more than two objectives." Metev [28] further explains that even for linear multi-objective programs, the set of efficient solutions is non-convex, making the determination of the nadir vector non-trivial.
To compute an estimation of the Nadir point, several heuristics have been proposed.

3.4.1 Estimation methods

- **Payoff Table Method**: Each objective is minimised independently, and the corresponding values of the other objectives are kept. The nadir estimate is then taken as the worst value observed for each objective. However, since each optimal solution may not be unique, worse Pareto optimal solutions that are not captured by this method may exist. Therefore, it often underestimates the true nadir and cannot be trusted as a reliable approximation. [29]

- **Evolutionary Approaches**: Evolutionary algorithms, such as the Non-dominated Sorting Genetic Algorithm II (NSGA-II), can be used to explore the whole Pareto front. Once the Pareto Front has been generated, one can keep the worst value for each objective to get an approximate of the solution. These methods are particularly useful in high-dimensional objective spaces but may still face challenges related to convergence speed and solution diversity.
  Deb et al.[30] proposed modifications to NSGA-II that emphasize the exploration of

boundary solutions, thereby improving the estimation of the nadir point.
Bechikh et al.[24] used mobile reference points to iteratively compute a better estimate of the nadir by exploring the neighbourhood of previously found extreme solutions.

3.4.2 Exact methods

It is important to note that in the case of two objectives, lexicographic optimisation is all that is needed to compute the Nadir point.

Let $x^{1,2}$ and $x^{2,1}$ be two lexicographically optimal solutions obtained by fixing $L = [1,2]$ and $L = [2,1]$ respectively.
Then $Z^N = (f_1(x^{2,1}), f_2(x^{1,2}))$.

In the case of three objectives, Ehrgott and Tenfelde-Podehl [27] have developed a general method to compute the Nadir point using the points obtained from the sub-problem containing only two objectives.

— 4 —

Presentation of the case study: Multi-actor low-carbon hydrogen distribution network

During this internship, a scenario was designed to serve as a basis for studying and implementing a multi-actor optimization methodology. The choices made to design this model were made to be able to generalize the methodology, to be able to re-use it in other different context.
This case study is set within the broader context of decarbonizing the industrial sector through the development of regional low-carbon hydrogen production and distribution networks.

4.1 Presentation of the case study

In this scenario, the objective is to optimize the infrastructure (production unit sizing) and operational flows within a system that includes:

- Hydrogen producers via electrolysis
- Hydrogen producers via SMR with carbon capture and storage
- Hydrogen consumers with different demand profiles

While the scenario is inspired from real-world projects, the data used does not originate from a single unified project. However, it is internally consistent and sufficient to support the analysis of the resulting system behaviour.
We suppose that the market price between every producers and consumers is know.

4.1.1 Actors of the networks

Production via Electrolysis

The model includes two hydrogen producers using electrolysis and an hydrogen storage. They differ by the sourcing of their electricity.

One of the two actors can only supply its demand in electricity (used to produce hydrogen) with the electricity grid. The characteristics of this grid are :

- *Consistent supply*: Electricity can always be bought from the grid and is not limited in supply.

- *Fluctuating price*: The price of electricity changes every hour.
- *Varying level of CO_2 emissions*: The CO_2 generated by the production of the grid varies every hour. This comes from the fact that different methods are used to produce this electricity depending on the time of the year and of the day (burning fossil fuels, nuclear power, etc...)

The second producer of hydrogen via electrolysis can also be supplied by the grid but also by a second source: renewable energy. This has been set to a photovoltaic plant of 8MW. The characteristics of this plant are:

- *Fluctuating supply*: The amount of electricity available from PV is always changing. There are even times when the supply is non-existent.
- *Constant price*: The price is fixed at 100 €/MWh
- *Constant* and *low carbon* emission

For these producers, their decisions lie first in the sourcing of the electricity. When should they purchase electricity? From which source? The second decision is in the dimensioning of their infrastructure: the size of their electrolyser and the size of their storage.

Production via SMR

There is a single hydrogen producer using SMR. The energy necessary for the production of their hydrogen is gas. The price of gas is defined as constant, as is its carbon content and its availability. Thus the decision lies elsewhere: the dimensioning of their CO_2 capture device.
Production via steam methane reforming has a high level of CO_2 emissions. Thus, to be able to enter the market of "low-carbon" hydrogen, these kinds of producers need to invest in a system that capture the CO_2 for they are to remain below a certain level of CO_2 emission per kg of hydrogen produced.

Hydrogen consumer

For the consumers of hydrogen, two profile have been selected. The first one, an industrial, has a fluctuating but generally high demand, ranging from 0 to about 90kgH_2/h with a mean of 22.8 kgH_2 per hour. The second one has a lower but constant demand of 6.25 kgH_2/h.

4.2 Modelisation concept and decisions

The network/case study has been developed in generic and modular way, using pyomo package and python algorithms, in order to be adapted and implemented into CAIRN software: LSET laboratory's energetic optimisation software. (see Section 7.5)
First, on a global level, only the external flux, the ones between producer and consumers, are represented. As can be seen in Figure 4.1, only the interactions between actors are visible. This view takes complete abstraction of each actor's internal behaviour, the goal being : to be able to replace any of them by another one with a different internal behaviour.

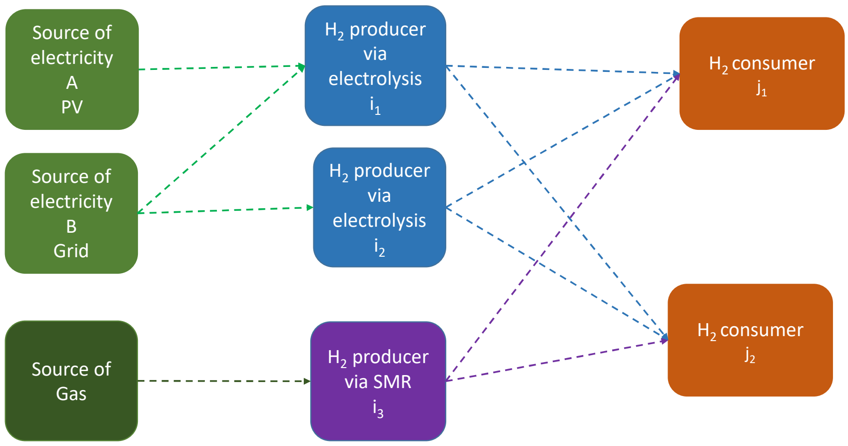


Figure 4.1: Global model view of the system

Then, each actor has its own modelling, independent of the network. Its internal behaviour is modelled. The behaviour of the electrolysis producer is represented in Figure 4.2, the SMR producer in Figure 4.3 and the consumer in Figure 4.4.

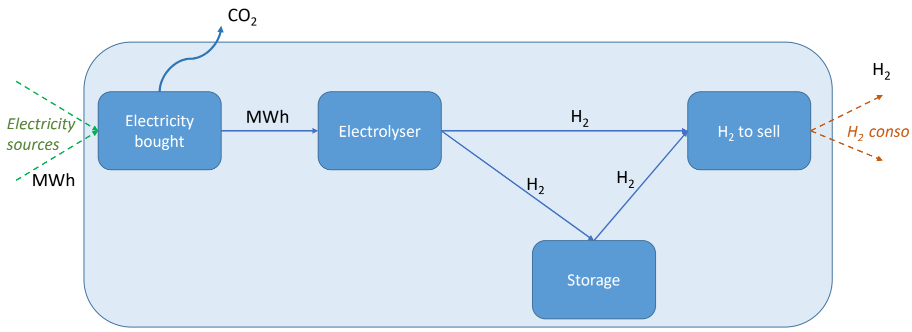


Figure 4.2: Internal model of an H_2 producer via electrolysis

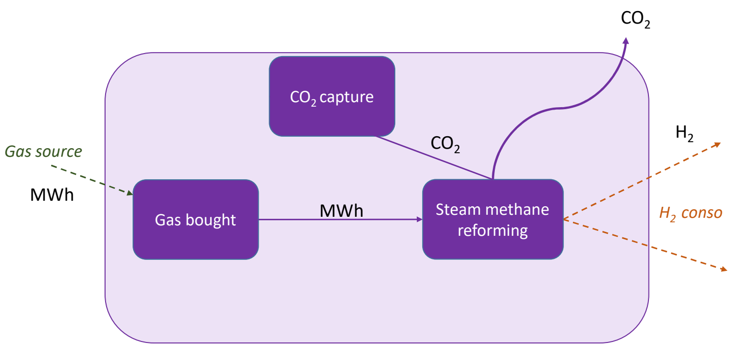


Figure 4.3: Internal model of an H_2 producer via SMR

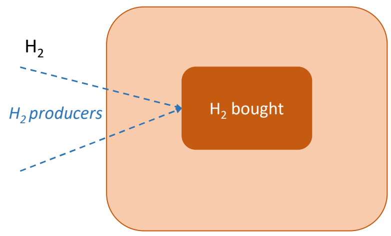


Figure 4.4: Internal model of an H_2 consumer

The details of the mathematical modelisation can be found in Annex A.

4.3 Constraint: a quick overview of the main constraints

In this section, we give a quick overview of the constraints that influence the system behaviour. The modelling constraints for the energy flux and economic flux will not be detailed, as they can be found in Annex A.3.

Demand:
One of the main constraints of this model is that the producers must satisfy the demand. This is especially important as sometimes producing energy may cost more than selling it. In this situation, the producers must find a way to mitigate losses.

Emission quota:
It is important to mention that since the system aims to help reduce carbon emissions, a constraint on the emission of CO_2 per kg of H_2 is implemented. However, this constraint has been modelled two ways, and they will be compared during the analysis of the results.

- Global constraint: over the optimisation horizon, the producer's emissions must, ***on average***, be below this quota.

$$\sum_t \mathbf{Impact\_prod_i(t)} \leq \text{Impact\_max} \cdot \sum_t \mathbf{Q\_H2\_prod_i(t)} \quad \forall i$$

- Hourly constraint: the producer's emissions must, ***at every time step*** (hour), be below this quota.

$$\mathbf{Impact\_prod_i(t)} \leq \text{Impact\_max} \cdot \mathbf{Q\_H2\_prod_i(t)} \quad \forall i \quad \forall t$$

In both design, producers have a quota of $3.5 kgCO_2/kgH_2$ that they must not go over.

NB: Through this report, decision variables are written in bold font.

— 5 —
Methodology and implementation

5.1 Initialisation of the model

The first step of the methodology implemented consists in designing the network with mathematical expressions.
For each actor, the decision variables and constraints needed to ensure the proper internal behaviour of their installations are set up. This includes the modelling of the physical flux of the different energies, the sizing of production facilities as well as the economic and environmental flux.

Once the internal behaviour of each actor is properly defined, it is necessary to link the actors through a new set of equations and constraints.

After formulating these, individual objective can be added into the model. However, instead of being directly assigned to the global objective function, these objectives are defined using auxiliary decision variables denoted by $\mathbf{f\_obj_i}$.
Let us define the objective of every actors:

- Producer i of H_2 via SMR

$$\begin{aligned}\mathbf{f\_obj_i} =& \mathbf{P\_energy\_total_i}\\ &+\mathbf{P\_CAPEX\_CCS_i} \cdot Time\_Horizon\\ &- \sum_{j \in consumers} \sum_t \mathbf{P\_H2\_sold_{ij}}(\mathbf{t})\end{aligned}$$

- Producer i of H_2 via electrolysis

$$\begin{aligned}\mathbf{f\_obj_i} =& \mathbf{P\_energy\_total_i}\\ &+\mathbf{P\_CAPEX\_Electrolyser_i} \cdot Time\_Horizon\\ &+\mathbf{P\_CAPEX\_Storage_i} \cdot Time\_Horizon\\ &- \sum_{j \in consumers} \sum_t \mathbf{P\_H2\_sold_{ij}}(\mathbf{t})\end{aligned}$$

The producers want to maximize their profit which is the income minus the production costs.
It can be seen that the CAPEX terms are multiplied by the time horizon. Although CAPEX are

traditionally considered as one-time investments and not modelled in this way, in this formulation it is converted from €/unit to €/unit/h. This transformation allows the analysis of shorter time horizons while keeping the same behaviour.

- Consumer j of H_2

$$\mathbf{f\_obj_j} = \frac{\sum_{i \in producers} \sum_t \mathbf{P\_H2\_sold_{ij}(t)}}{\sum_t Demand\_H2_j(t)}$$

The consumers want to buy hydrogen at the lowest price.

Once this is done, we can move on to the resolution of the multi-objective optimisation.

5.2 Mono-objective optimisation

First, to highlight once more the motivation behind the multi-actor approach, we conduct the optimisation as it is usually done: through optimising the whole system by minimising the overall costs with no regards for actors' personnal motivation.

For this, we define the global objective function as :

$$\min \sum_i \mathbf{f\_obj_i} + \left(\sum_j \mathbf{f\_obj_j} \cdot \sum_t Demand\_H2_j(t) \right)$$

It is important to note that here, none of the objective are normalised.
It is equivalent to minimizing the sum of energy and CAPEX costs of each producers.

5.3 Normalisation of the objectives

We have seen in Section 3.4 that to normalise the objective functions, it is necessary to compute the Nadir point.
However, for producers, we adopt a simplified yet pragmatic approach. We replace the Nadir point, that corresponded to a satisfaction of 0, by another more significant value : $\mathbf{f\_obj_i} = 0$. This means that a producer starts being satisfied when he starts earning money from its investments. The resulting satisfaction function is as depicted in figure 5.1.

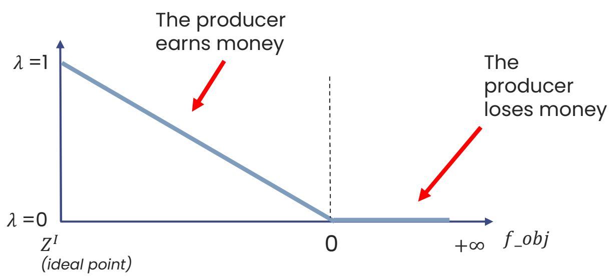


Figure 5.1: Satisfaction function of hydrogen producers. (λ is the satisfaction)

For the satisfaction of the hydrogen consumers, it was decided to fix a lower and an upper threshold. The lower bound corresponds to the price at which the consumers will be totally satisfied. Going lower than this price will not increase their satisfaction. The upper bound, on the opposite, is the maximum price at which the consumers will accept to buy hydrogen. The satisfaction is then 0.
The resulting satisfaction function will be as depicted in figure 5.2.

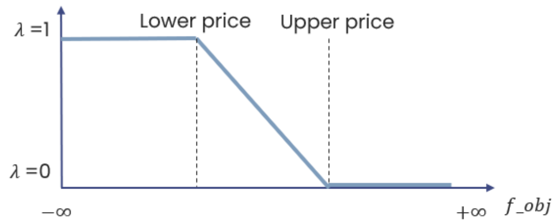


Figure 5.2: Satisfaction function of hydrogen consumers. (λ is the satisfaction)

The motivation behind this satisfaction function is that, on the contrary to producers, consumers do not focus on the overall cost but on the price of H_2 per kg. However, this is non-linear:

$$\sum_i \sum_t \frac{\mathbf{P\_H2\_sold_{ij}(t)}}{\mathbf{Q\_H2\_sold_{ij}(t)}}$$

As a result, this expression cannot be directly computed or used within the satisfaction function. This is why, to address this, bounds are defined prior to optimization. Section 7.1.1 presents an alternative attempt to handle the non-linearity, but this approach did not result in a usable solution.

5.3.1 Implementation of the satisfaction function

We define the value of the satisfaction (normalized) function as $\lambda_\mathbf{i}$ that is a decision variable in the model. $lower\_bound_i$ an $upper\_bound_i$ are defined before optimization and fixed as given data.
We want to obtain:

if $\mathbf{f\_obj_i} \leq lower\_bound_i$ **then**
  $\lambda_\mathbf{i} = 1$
end
else if $\mathbf{f\_obj_i} \geq upper\_bound_i$ **then**
  $\lambda_\mathbf{i} = 0$
end
else
  $\lambda_\mathbf{i} = \dfrac{\mathbf{f\_obj_i} - upper\_bound_i}{lower\_bound_i - upper\_bound_i}$
end

Algorithm 2: Satisfaction λ_i depending on the value of $\mathbf{f\_obj_i}$

To do that, we define $\mathbf{bin_i}$ a binary variable associated with each objective function such that $\mathbf{bin_i} = 1 \implies \mathbf{f\_obj_i} \geq upper\_bound_i$.

To model this implication, we need the following constraints:

- $1-\mathbf{bin_i} \geq \lambda_\mathbf{i} \geq 0$
- $\mathbf{f\_obj_i} \geq upper\_bound_i - M(1-\mathbf{bin_i})$
- $\lambda_\mathbf{i} <= \dfrac{\mathbf{f\_obj_i} - upper\_bound_i}{lower\_bound_i - upper\_bound_i} + M \cdot \mathbf{bin_i}$

with M = upper_bound$_i$ - ideal_point$_i$ (M needs to be big enough to activate the constraint but small enough to have a better efficiency)
Further details can be found in Annex D, Figure D.1.

5.4 Nadir Point

In our implementation, a score higher than 0 means a producer begins to generate profit. It is based on the assumption that if a producer projects to loose money, he will not be investing and entering the green hydrogen market.
However, in the objective of making a model that is general and applicable, it is still necessary to have the option to fix the lower threshold of satisfaction to an approximation of the Nadir point. Furthermore, other multi-objective optimisation method still require this point. This is why we implemented methods to compute the Nadir point.

5.4.1 Payoff table

The payoff-table method is based on independently minimising each actor's objective, storing the values of the others and selecting the worse among those. The process is as follows:

1. Each objective f_i is minimised independently by solving:

$$\min_X f_i(X) \quad \text{s.t. } X \in S$$

   which returns the Ideal value z_i^I and the associated solution X_i^*.

2. For each solution X_i^*, all other objective values $f_j(X_i^*)$, with $j \neq i$, are kept.

3. The Nadir point is approximated by selecting, for each objective j,

$$Z_j^N = \max_{i=1,\ldots,m} f_j(X_i^*).$$

Although computationally efficient, this method tends to over or underestimate the true Nadir point since the extreme solutions for individual objectives may not sufficiently cover the Pareto front. Since this study includes CAPEX investments (significantly larger than energy costs) for all actors that can be arbitrarily pushed to their maximum value by the solver, the results can be widely inaccurate.

Evolutionary Approximation via NSGA-II

To obtain a more representative coverage of the Pareto front, a second method based on the Non-dominated Sorting Genetic Algorithm II (NSGA-II) was tested. This algorithm explores the objective space through an evolutionary process designed to approximate the Pareto front efficiently. The main characteristics of the implementation are as follows:

- A population of candidate solutions is evolved over the full decision space X,
- Fitness evaluation is based on the complete objective vector $(f_1(X), \ldots, f_m(X))$,
- Diversity is maintained through the crowding-distance metric, which promotes boundary solutions and better front coverage.

After a sufficient number of generations, the approximate Pareto front $\mathrm{PF}_{\mathrm{approx}}$ is collected. The Nadir estimate is then defined as:

$$z_j^N = \max_{X \in \mathrm{PF}_{\mathrm{approx}}} f_j(X), \quad j = 1, \ldots, m.$$

While promising, this option is not yet accurately implemented as we will see in section 7.3.1.

5.5 Goal Programming

After defining the satisfaction function as described in section 5.3, we define the global objective function of the goal programming as:

$$\begin{aligned} \max \quad & \sum_i \lambda_i \\ & st. \quad X \in S \end{aligned}$$

This method is fairly similar to the classic mono-objective optimisation of section 5.2 but the objective are normalised, ensuring that each actor have a similar impact in the optimisation.

5.6 Lexicographic Max-Min Optimisation

In a similar way as goal programming, we define the global objective function as:

$$\begin{aligned} max \quad & min \quad \lambda_i \\ & st. \quad X \in S \end{aligned}$$

However, while the goal programming function guarantees Pareto Optimality, this objective function does not.

Proof. Let us consider two actors A_1 and A_2 with respective satisfaction functions λ_1 and λ_2. We assume that we have two solutions X^1 and X^2 of a max-min program.

Solution	λ_1	λ_2
X^1	0.6	0.7
X^2	0.6	0.8

With the optimisation function as defined, both X^1 and X^2 can be selected as optimal solutions. However, it is easy to see that X^1 is not Pareto optimal since the objective function of A_2 can still be increased without decreasing A_1's. □

To solve this problem, we implement a lexicographic optimisation procedure (see algorithm 3) that refines the max-min approach and ensures Pareto optimality (a lexicographically optimal solution is Pareto Optimal). The idea is to iteratively maximise the satisfaction of the least satisfied actor, then set it as a constraint and continue optimising over the remaining actors.
This iterative lexicographic max-min method gives a fairer and more balanced allocation while guaranteeing Pareto optimality.

With I the list of actors to optimize
while $I \neq \emptyset$ **do**
  Solve:
  $z_i = max \quad min_{i \in I} \quad \lambda_i$
  $st. \quad X \in S$
  Add the constraint:
  $\lambda_i = z_i$ for i the least satisfied actor
  Remove the least satisfied actor from I
end

Algorithm 3: Lexicographic optimisation procedure of the max-min optimisation

5.7 Minimizing the CO_2 emission: deterioration of the economic objective

The primary goal in the implementation of a low-carbon hydrogen network is to ensure that hydrogen production does not lead to CO_2 emissions exceeding a set threshold. As such, carbon emissions have so far only been considered as a constraint and not an objective.
In this section, we extend the optimisation framework by adding the minimisation of CO_2 emissions as an objective (while keeping the previous CO_2 constraint).
Starting from the solution obtained through the lexicographic max-min optimisation, we add a final optimisation iteration:
The satisfaction λ_i of all actors are fixed at their optimal values obtained from the previous stage, and the new objective becomes the minimisation of total CO_2 emissions.

$$\begin{aligned} min \quad & \mathrm{CO_2\_emissions} \\ \text{s.t.} \quad & X \in S \\ & \lambda_i = z_i \quad \forall i \quad (a) \end{aligned}$$

We also implement a variation of this method that allows a deterioration of the economic objectives of the actors in order to achieve lower emissions.
Let d be the maximum allowed deterioration of each actor's satisfaction. ($0 \leq d \leq 1$)

We then relax the equality constraints (a) to the inequalities:

$$\lambda_i \geq z_i - d \quad \forall i$$

— 6 —

Results and interpretation

In this chapter, we present practical results of the different methods defined before.

To get the results obtained in this section, we fix the prices as defined in Table 6.1.

	P1_electrolysis(with PV)	P2_electrolysis	P3_SMR
C1_industrial	6 €	8 €	4.75 €
C2_mobility	10 €	11.4 €	7.6 €

Table 6.1: Price fixed between Producers and Consumers (€/kgH_2)

Even if these prices are not competitive compared to current market hydrogen costs, they are coherent with results obtained throughout this study and particularly levelised cost of hydrogen (LCOH).
These prices are not competitive since CCS and electrolysis are not yet fully mature technologies. Their costs remain high due to limited development and investment. However, we study them assuming that in the future, these technologies will become economically viable.
Therefore, direct comparison with today's market is not possible.

At the exception of section 6.3.1 and 6.3.2, the CO_2 emission quota constraint fixed is hourly.
The optimization horizon is one year (8736h).

We first present the results of the classical mono-objective optimisation. Next, we examine the outcomes of the goal programming approach. These results are then compared with those obtained using the lexicographic max-min strategy. Finally, we analyse the differences in CO_2 emissions across the various constraint types, as well as the relationship between economic degradation and environmental impact.

Further detailed results can be found in Annex C.

6.1 Mono-objective optimisation

When running the scenario given a unique objective function that minimizes the total costs, we obtain the results of Table 6.2. The *Objective function* column refers to each actor's individual objective. For producers, the values are expressed in euros: negative values indicate a profit, while positive values represent a loss. For consumers, the values correspond to the average price paid per kilogram of hydrogen purchased.

Here, only the SMR producer makes a profit. The proposed solution given is clearly not acceptable for the electrolysis producers as they will either not participate in the network, or worse, provoke financial losses.
These results motivate the multi-actor approach of this study.

Actor	Objective function
P1_electrolysis(with PV)	20934.1
P2_electrolysis	0
P3_SMR	-146690
C1_industrial	4.79
C2_mobility	7.6

Table 6.2: Objective functions of actors using mono-objective optimisation

6.2 Goal Programming

Using goal programming, we obtain the results of Table 6.3. A negative value indicate a profit.

Actor	Objective function	Satisfaction
P1_electrolysis(with PV)	-159946	0.65
P2_electrolysis	-26533	0.98
P3_SMR	-223646	0.39
C1_industrial	5	0.52
C2_mobility	10	0.49

Table 6.3: Objective functions of actors using goal programming. Negative value indicate a profit

With this method, we find a solution that is viable for every actor. Since they are all projecting to make profits, they are given the incentive to participate in the implementation of this network.
While this solution is usable and fairer than the mono-objective optimisation, we can still see disparities between the satisfaction of the actors. Comparing the satisfaction of actor P2 and P3, we see that P2 is favoured over P3 in this solution.
The disparities highlight potential negotiation issues when translating mathematical solutions into real-world contracts. This is what motivate the use of the next method.

6.3 Lexicographic Max-Min Optimisation

Applying the lexicographic max-min approach (see Section 5.6) gives the results in Table 6.4.

Actor	Objective function	Satisfaction
P1_electrolysis(with PV)	-128930	0.52
P2_electrolysis	-14209	0.52
P3_SMR	-297887	0.52
C1_industrial	5	0.52
C2_mobility	9	0.53

Table 6.4: Objective functions of actors using max-min

This confirms that max-min delivers balanced satisfaction across all actors. Figure 6.1 further highlights the fact that this method is more balanced than the goal programming. There are no actors favoured over the other. This makes it the most interesting option when the objective is to balance the profits amongst the actors of the network.

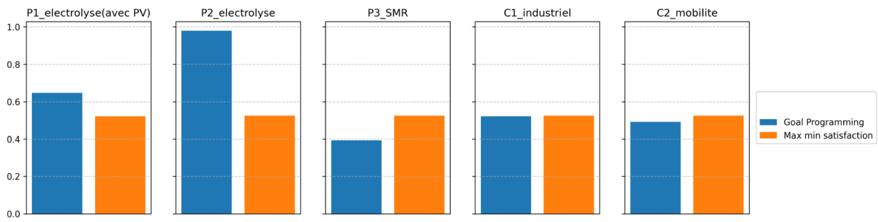


Figure 6.1: Comparison of the satisfaction of the different methods

6.3.1 Global or hourly CO_2 quota?

We make a comparison of the results obtained depending on the type of emission quota constraints used in the model, hourly or global. The comparison is made using the max-min method, as it is the fairest implemented.
When running the program, we noticed that the program using global constraints was taking significantly more time than the one with hourly constraints. The optimization that took about an hour in the hourly case was taking more than 6 hours with the global constraint.
This is because with the global constraint, the model is not constrained enough. The solver explores a much larger feasible space, leading to longer computation times.
Because of that, we conduct the comparison on a restrained time horizon of 3 months (2190h).

Actor	Objective function	Satisfaction
P1_electrolysis	-29121	0.51
P2_electrolysis	-8773	0.52
P3_SMR	-85698	0.52
C1_industrial	5	0.52
C2_mobility	10	0.52

Table 6.5: Global emission constraint

Actor	Objective function	Satisfaction
P1_electrolysis	-29862	0.52
P2_electrolysis	-4678	0.54
P3_SMR	-79163	0.52
C1_industrial	5	0.53
C2_mobility	9	0.54

Table 6.6: Hourly emission constraint

We see that P2, despite making less profit in the hourly scenario, observes an increase in satisfaction. This makes it clear that satisfaction is not comparable across scenarios. It is a metric of fairness between actors within the same scenario. Therefore, to compare the two instance, we look at the objective functions' values.
While the global constraint results in a solution that generates 42 593 kgCO_2, the hourly one generates 25 431 kgCO_2. The hourly quota halves the total amount of emissions compared to the global quota, but the second producer is impacted by this constraint change: its profit is divided by 2.
The second consumer has a lower objective function because the price at which he buys hydrogen is lower. This is because P3, who sells at a lower price, is favoured when the environmental is higher. However, P3 still makes less profit because the CAPEX costs of CCS are higher to respect the tighter constraint.

In general, we clearly see a trade-off between environmental objectives and economic gains. This will be further highlighted in the next section.

6.3.2 Deterioration of the objectives: an improved CO_2 impact

We previously described the process of deterioration to optimize CO_2 emissions (see Section 5.7), we will present the results of this method in this section. The time horizon is kept at three months to also observe the behaviour of the global emission constraint.

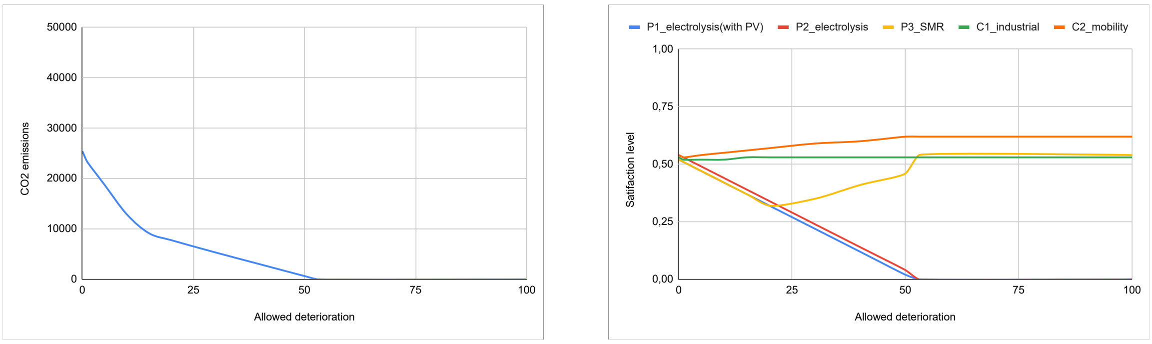
First, it is important to note that even with no tolerance for deterioration in the actors' objectives, simply performing a CO_2 minimisation step following the lexicographic max-min optimisation results in a significant reduction in emissions, without changing the satisfaction level of any actor.
This is shown in Table 6.7. While keeping the exact same objective function results, we obtain lower level of CO_2 emissions.

Constraint type	Without CO_2 optimisation	With CO_2 optimisation
Hourly CO_2 quota	27 199	25 431
Global CO_2 quota	104 443	42 593

Table 6.7: CO_2 emissions resulting from the optimisation with and without post-lexicographic CO_2 minimisation (in kg CO_2)

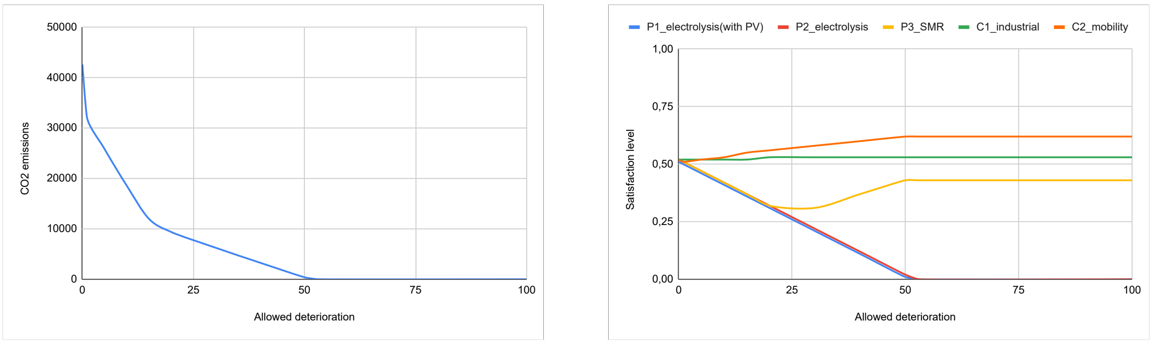
Therefore, the CO_2 emission optimisation is systematically included in the process as it has no impact on the economic objectives but has an impact on the environmental footprint of the network.

Fixing this step, we then observe how much allowing deterioration can improve the environmental impact of the network.
Figures 6.2 and 6.3 show how emissions vary as a function of the allowed economic deterioration.



Total emissions (kgCO_2) Satisfaction level of every actor

Figure 6.2: CO_2 emissions vs. economic deterioration using hourly quota constraint



Total emissions (kgCO_2) Satisfaction level of every actor

Figure 6.3: CO_2 emissions vs. economic deterioration using global quota constraint

We observe that the environmental gains are not linear. Small deterioration allowances show significant improvement in the CO_2 objective function. There is a cost-effective way to improve environmental impact. Beyond about 20% deterioration, further sacrifices produce way less CO_2 reductions, indicating the possibility of an optimal compromise range.

If the allowed economic deterioration exceeds the maximum satisfaction level of all actors, the optimisation becomes a simple CO_2 minimisation problem. In this extreme case, the network configuration that achieves the lowest possible emissions, theoretically zero, involves only the third producer (SMR), who operates with a profit of 80 847 €and a satisfaction score of 0.54. However, this outcome is biased: the SMR method will never truly reach zero in practice. The fact that a zero-emissions result is achieved while relying on a fossil-based process indicates a limitation of the model's physical realism. This shows the importance of continuously confronting optimisation results with physical constraints to ensure coherence with real-world feasibility.

— 7 —

Limitations and Openings

7.1 How to Fix a Market Price?

Since satisfaction in our model is based on profits, it depends critically on the price at which hydrogen is sold. Consequently, fixing an appropriate market price has a major impact on the optimization results. In practice, we have observed that fixing the price without proper consideration leads to meaningless outcomes. In this section, we discuss two approaches we explored to fix an appropriate market price in our mathematical program and why this approaches are still insufficient.

7.1.1 A Linear Relaxation of price × quantity: McCormick Envelopes

One of the first approaches we considered was to include the contract price $Contract\_Price_{i,j}$ between each producer and consumer as a decision variable in the model. However, doing so immediately introduces bilinear terms of the form

$$\mathbf{P\_H2\_sold_{i,j}(t) = Q\_H2\_sold_{i,j}(t) \times Contract\_Price_{i,j}}$$

This makes the optimization problem non-linear, which standard solvers can not or struggle to solve reliably. To resolve this, we investigated a common relaxation technique based on the McCormick envelopes [31].

The idea is to impose linear inequalities that bound $\mathbf{P\_H2\_sold_{i,j}(t)}$ from above and below.

Given:

- $p^L_{i,j,t} \leq \mathbf{Contract\_Price_{i,j}} \leq p^U_{i,j,t}$
- $q^L_{i,j,t} \leq \mathbf{Q\_H2\_sold_{i,j}(t)} \leq q^U_{i,j,t}$

The McCormick relaxation for $\mathbf{P\_H2\_sold_{i,j}(t) = Q\_H2\_sold_{i,j}(t) \times Contract\_Price_{i,j}}$ adds the four constraints:

- $\mathbf{P\_H2\_sold_{i,j}(t)} \geq p^L_{i,j,t} \cdot \mathbf{Q\_H2\_sold_{i,j}(t)} + q^L_{i,j,t} \cdot \mathbf{Contract\_Price_{i,j}} - p^L_{i,j,t} \cdot q^L_{i,j,t}$
- $\mathbf{P\_H2\_sold_{i,j}(t)} \geq p^U_{i,j,t} \cdot \mathbf{Q\_H2\_sold_{i,j}(t)} + q^U_{i,j,t} \cdot \mathbf{Contract\_Price_{i,j}} - p^U_{i,j,t} \cdot q^U_{i,j,t}$
- $\mathbf{P\_H2\_sold_{i,j}(t)} \leq p^U_{i,j,t} \cdot \mathbf{Q\_H2\_sold_{i,j}(t)} + q^L_{i,j,t} \cdot \mathbf{Contract\_Price_{i,j}} - p^U_{i,j,t} \cdot q^L_{i,j,t}$

- $\mathbf{P\_H2\_sold_{i,j}(t)} \leq p^L_{i,j,t} \cdot \mathbf{Q\_H2\_sold_{i,j}(t)} + q^U_{i,j,t} \cdot \mathbf{Contract\_Price_{i,j}} - p^L_{i,j,t} \cdot q^U_{i,j,t}$

Any feasible solution obtained by this linear relaxation lies within the convex hull of the following rectangle: $[p^L_{i,j,t}, p^U_{i,j,t}] \times [q^L_{i,j,t}, q^U_{i,j,t}]$. Tightening the bounds theoretically reduces the relaxation gap.
To evaluate the quality of the solution obtained by this relaxation, we implemented these constraints.

Given that $p^L_{i,j,t} = q^L_{i,j,t} = 0$, $p^U_{i,j,t} = Worst\_price_j$ and $q^U_{i,j,t} = Demand\_H2_j(t)$, we obtain:

- $\mathbf{P\_H2\_sold_{i,j}(t)} \geq 0$

- $\mathbf{P\_H2\_sold_{i,j}(t)} \geq Worst\_price_j \cdot \mathbf{Q\_H2\_sold_{i,j}(t)} + Demand\_H2_j(t) \cdot \mathbf{Contract\_Price_{i,j}} - Worst\_price_j \cdot Demand\_H2_j(t)$

- $\mathbf{P\_H2\_sold_{i,j}(t)} \leq Worst\_price_j \cdot \mathbf{Q\_H2\_sold_{i,j}(t)}$

- $\mathbf{P\_H2\_sold_{i,j}(t)} \leq Demand\_H2_j(t) \cdot \mathbf{Contract\_Price_{i,j}}$

Running the problem on a optimization horizon of 1000 hours, we obtained the results displayed in Table 7.1 and 7.2.

	P1	**P2**	**P3**
C1	10	x	6,9
C2	20	20	0,12

Table 7.1: Average value of $\mathbf{Q\_H2\_sold_{i,j}(t)} \times \mathbf{Contract\_Price_{i,j}}$

	P1	**P2**	**P3**
C1	3,7	0	6,97
C2	20	20	0,6

Table 7.2: Value of the variable $\mathbf{Contract\_Price_{i,j}}$

These numerical results showed no promise of finding tight McCormick relaxations that are both computationally efficient (the computation time greatly increased with the introduction of these constraints) and economically explainable.
The pricing obtained is clearly unrealistic: no producer in a real world market would simultaneously sell at 7 €/kgH_2 to a client while selling at 0.6 €/kgH_2 to another customer. Such solutions, though mathematically valid under the relaxation, can not be used in a real world scenario. Therefore, even if we improve the quality of the relaxation, the fundamental issue of interpretability will remain. Thus, we decided to not to investigate bilinear relaxations for pricing further.

7.1.2 A Simplified Supply and Demand Law

As is emphasized with the previous method, the key issue with fixing a market price is the coherency with traditional market mechanism. A fundamental concept in determining a market price in economy is the law of supply and demand with which an equilibrium price can be found. Instead of including the price directly in the optimization, we experimented with a pre-processing approach that mimics a simple supply-demand adjustment process. It works as follows:

1. Each producer i submits a selling price range $\left[p_i^{\min}, p_i^{\max}\right]$. This range represents the minimum acceptable price per kilogram of hydrogen the producer accepts to sell and a highest realistic price at which he could still manage to sell the hydrogen produced.

2. Each consumer j submits a buying price range $\left[p_j^{\min}, p_j^{\max}\right]$. This range represents the ideal price they wish to purchase hydrogen at and the maximum amount they are willing to pay for it.

3. For each producer-consumer pair (i, j), we compute the crossing point of their price interval $p_{i,j}$ and that point is fixed as the contract price between the two of them.

4. If there is no crossing point between the producer and the consumer, no exchange will happen between the two actors.

The procedure can be graphically represented for better comprehension as seen in Figure 7.1. With this method, we ensure that the price fixed between producers and consumers is feasible

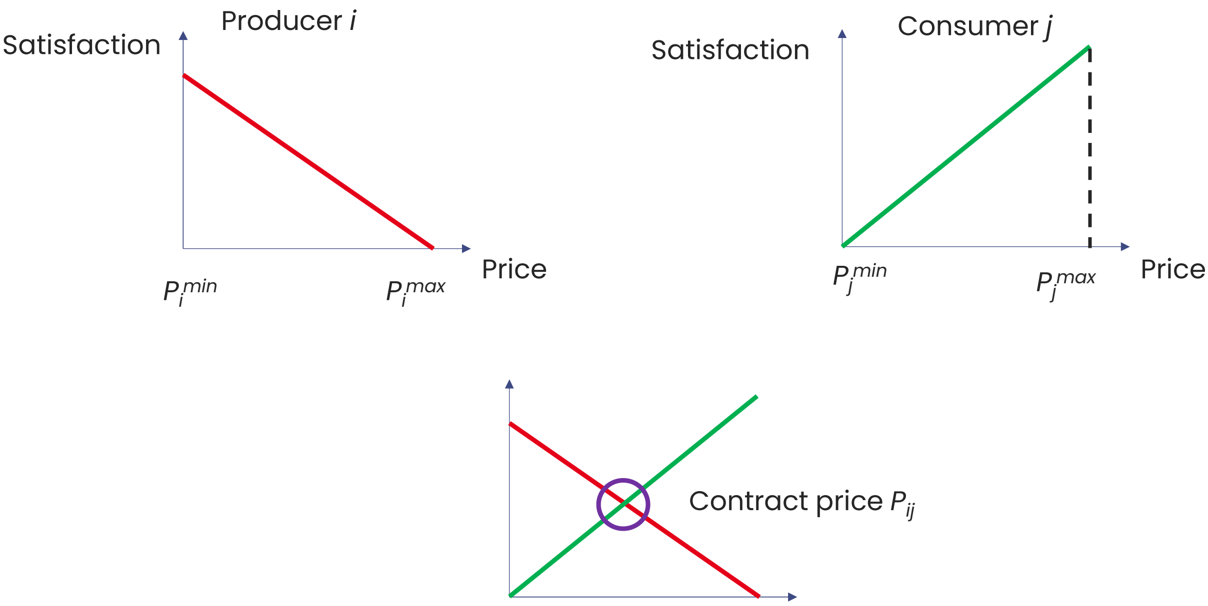


Figure 7.1: "Equilibrium" price between producers and consumers

and acceptable to both actors. However, there is no empirical justification that picking the mid-point of the simplified linear intersection yields the “true” market equilibrium price, especially when supply and demand curves are non-linear and competitivity has to be taken into account. Because of the lack of empirical baking, this supply-demand crossing point methods remains experimental. As a result, we leave it as a possible direction for future work. Some options that could be considered are:

- Piecewise Linear Supply-Demand Approximations:
  The approximation of each producer’s supply curve and each consumer’s demand curve could be improved by a sequence of linear segments. Then, the intersection of these piecewise functions would yields a piecewise-linear equilibrium price that can be incorporated directly into the MILP.

- Walrasian auctions:
  A Walrasian market is an economic model in which buy and sell orders are aggregated and processed simultaneously to determine a market-clearing price. Also known as a call market, this mechanism computes the equilibrium price where supply matches demand. Implementing such a system could introduce a structured and economically grounded approach to internal price formation.

- Gupta and Singh's Market-Linked Take-or-Pay Energy Purchase Contracts [32]
  An alternative pricing mechanism could involve adopting structured contracts such as the market-linked take-or-pay schemes proposed by Gupta and Singh. These contracts incorporate market-based pricing while ensuring minimum revenue guarantees for producers, which may be particularly relevant for long-term hydrogen supply agreements.

7.2 The Interpretation of Satisfaction

In our study, the satisfaction metric is defined in a relative manner. It depends on the maximum feasible profit given the model constraints. Because of this, it is only meaningful within a single, fixed scenario. If any of the parameters such as costs, capacities, or demand change, then the maximum feasible profit will change, making cross-scenario comparisons meaningless.
The risk of biasing the satisfaction metric by not properly setting a parameter is high. There needs to be a fixed external benchmark.
Generally, this implies that the user needs to remain careful while interpreting the scenarios and conducting sensitivity analysis. The computed satisfaction values serve only as a within-scenario indicator of how "fair" or "equitable" the outcome is between actors.
Making comparisons across scenarios currently lies outside the scope of the proposed work.

7.3 Other Resolution Methods: A Fairer Result?

7.3.1 Nadir point: Evolutionary algorithm

In this section, we analyse the results obtained using the implemented NSGA-II algorithm.
This algorithm is intended to find an approximation of the Nadir point. Unfortunately, we observe that the results are not satisfactory and this algorithm requires more attention.

Since the true Nadir point is unknown (and genetic algorithms provide no guarantee of optimality) it is impossible to precisely evaluate the quality of the obtained solution.
To assess the quality of the Pareto front generated by NSGA-II, we compare its Ideal point estimation with the actual Ideal point value. Because the Ideal point is known, this comparison offers a reliable indication of whether the NSGA-II outputs are of acceptable quality. We assume that a poor estimation of the Ideal point likely implies a poor estimation of the Pareto front and of the Nadir point. On the other hand, a good estimation of the Ideal point increases confidence in the Nadir point accuracy.

As shown in Table 7.3, even for a small instance (a time horizon of 10 hours), the Ideal point estimated by NSGA-II is significantly different from its true value. As a result, we do not use these results, as the quality of the corresponding Nadir point is expected to be poor as well. It

Actor	Ideal point	NSGAII Ideal point estimation	Payoff table Nadir point estimation (see Section 5.4.1)	NSGAII Nadir point estimation
P1_electrolysis (with PV)	-699	-214	685	263
P2 _electrolysis	-699	-196	685	295
P3_SMR	-696	-535	548	0

Table 7.3: Ideal and Nadir point estimation

is important to note that the computation of the Nadir point does not affect the results used in this study. Because we assume that producers do not sell hydrogen at a loss, the normalisation of objectives is bounded by 0 rather than by the Nadir point.

Improving the accuracy of Nadir point estimation remains an open area for future work. A more reliable estimation would allow the methodology to be applied in more general settings, including cases where the bounds cannot be arbitrarily defined.

7.3.2 NAUTILUS

There exist other different methods to solve a multi-objective problem. Because we were focussed on interpretability, we implemented a-priory methods. However, other resolution techniques could be of interest. Most notably, the interactive methods and more precisely the NAUTILUS method.
The NAUTILUS method, developed by Miettinen et al. [17], shows promise because of its psychologically grounded approach to multi-objective decision-making.
The method is based on the assumption that past experiences affect decision makers' hopes and that people do not react symmetrically to gains and losses". Unlike other interactive methods such as NIMBUS, NAUTILUS starts from the Nadir point instead of the Ideal point. This design choice helps mitigate the anchoring effect, a bias where individuals give disproportionate weight to initial information. Starting from the ideal point, which is often unrealistic in practice, can make decision makers against accepting trade-offs. In contrast, beginning from the Nadir point encourages collaboration and gradual progress toward more balanced, acceptable solutions.

NAUTILUS works as an iterative, interactive process. At each step, the decision maker is presented with a set of solutions that improves from the current position. They can indicate preferences on how much to improve each objective, without needing to specify a precise trade-off. This preference is used to navigate toward more desirable areas of the Pareto front.
The use of NAUTILUS was considered promising due to its fairness-oriented approach and its flexibility in handling conflicting objectives. However, since the method depends on the computation of the Nadir point, and this estimation is not yet deemed satisfactory, we did not implement this method.
The integration of NAUTILUS remains as a possible future work.

7.4 Competitive Approach

In this work, we have focused on cooperative optimization, where each actor seeks to maximize their individual profit while keeping fairness in mind. While this allows for simpler formulation and resolution, it does not corresponds to real behaviours present in the liberal market.

An alternative modelling direction is to adopt a competitive framework, where actors behave strategically and optimize their decisions in response to others, without coordination and incentive to collaborate.

Game-theoretic models are commonly used for competitive scenarios:

- Cournot Competition: In this model, each actor decides how much to produce, assuming that the others' choices stay the same. The market price is then set based on the total quantity produced. This setup shows how producers influence each other and helps analyse how competition affects prices and fairness in the market.

- Stackelberg Game: A hierarchical model where one actor moves first and the others respond optimally. This can be used to represent asymmetrical market power, for example, a large hydrogen producer setting its strategy while smaller producers react. It generally leads to bi-level optimization problems.

- Bi-level or Multi-level Scenarios: These formulations model decision-making hierarchies explicitly. For instance, a government (upper level) may set policy or pricing constraints, while market participants (lower level) optimize their profits. These problems are typically non-convex and computationally challenging but can provide realistic representations of regulatory and competitive dynamics.

Competitive approaches introduce significant complexity, often resulting in non-convex, multi-level formulations. However, they also offer a more realistic perspective on decentralized decision-making and price formation. They are of particular interest when modelling large systems with multiple self-interested actors.
Further work could explore how such models compare to cooperative formulations in terms of outcomes and computational performance.

7.5 CAIRN integration

What is CAIRN?

CAIRN is an optimizer tool for energy system studies. It allows to perform techno-economic and environmental analysis of energy systems by optimization of both the sizing of power systems and the operation planning.
CAIRN is based on the Mixed Integer Linear Programming formalism (MILP).
CAIRN has been build to allow for several purposes :

- allow to build any architecture of multi-energy system with a verified library of components.

- allow non-specialists to build easily optimisation problems, and analyse the results.

Throughout this study, the methodology was developed with an emphasis on generality. It is not intended to be specific to the arbitrarily designed low-carbon hydrogen network but is meant to be reusable and adaptable to real-world projects.
Given that a lot of studies at the LSET laboratory are conducted using the CAIRN software, the potential for future integration of this methodology into the software has been considered and will be explored in future developments.

— 8 —
Conclusion

This internship focused on the development of a collaborative multi-actor optimization framework to support the planning and operation of energy distribution networks.
For this, a mixed-integer linear programming model was developed to represent the internal operations of a specific case applied to hydrogen. It depicts various hydrogen producers (electrolysis with grid and photovoltaic sourcing, SMR with carbon capture), as well as industrial and mobility-oriented consumers. Both economic and environmental objectives were considered, particularly regarding CO_2 emissions per kilogram of hydrogen produced.

For this methodology, normalized satisfaction functions were introduced, enabling each actor's performance to be expressed and making it possible to compare and balance the outcomes across the network. Two main multi-objective optimization methods were applied: goal programming, which maximizes the sum of individual satisfactions, and lexicographic max-min optimization, which aims for fairness by maximizing the satisfaction of the least-satisfied actors.

The results show that the lexicographic max-min approach provides a more equitable and interpretable solution, where all actors benefit at comparable levels. This balanced outcome is particularly valuable in collaborative contexts where incentive to cooperate is necessary for the success of a proposed solution.
An additional step was implemented to further reduce CO_2 emissions after the main economic optimization. This step fixes actors' satisfaction levels and then minimizes emissions, or allows a controlled deterioration of satisfaction to improve environmental outcomes. The findings show that even small economic deterioration can lead to significant environmental gains, making this a tool to balance climate goals with economic feasibility.

Overall, this work provides an adaptable decision-support tool that can assist decision makers in designing low-carbon energy networks. It also presents future extensions, such as integrating more realistic pricing mechanisms or interactive optimization techniques.

This study contributes to the ongoing energy transition and the need to lower the carbon footprint.

Bibliography

[1] Waldemar Kaempffert. Science in review. *The New York Times*, page 191, Oct. 1956. ProQuest Historical Newspapers: The New York Times (1851–2003).

[2] United Nations. Key findings on climate science. Online article, 2023. `https://www.un.org/en/climatechange/science/key-findings`.

[3] Intergovernmental Panel on Climate Change (IPCC). *Climate Change 2023: Synthesis Report. Contribution of Working Groups I, II and III to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*. Intergovernmental Panel on Climate Change, Geneva, Switzerland, 2023. `https://www.ipcc.ch/report/ar6/syr/`.

[4] International Energy Agency (IEA). Global co2 emissions by sector, 2019–2022. Online chart, 2023. `https://www.iea.org/data-and-statistics/charts/global-co2-emissions-by-sector-2019-2022`.

[5] National Grid. The hydrogen colour spectrum. *nationalgrid*, Feb. 2023. `https://www.nationalgrid.com/stories/energy-explained/hydrogen-colour-spectrum`.

[6] Bertrand Charmaison. Comment estimer les coûts de production de l'hydrogène bas carbone. *La Revue de l'Énergie n° 663*, 2022.

[7] Wisam Alsaba, Saad Ali Al-Sobhi, and Muhammad Abdul Qyyum. Recent advancements in the hydrogen value chain: Opportunities, challenges, and the way forward–middle east perspectives. *International Journal of Hydrogen Energy*, 48(68):26408–26435, 2023. `https://www.sciencedirect.com/science/article/pii/S0360319923024564`.

[8] John von Neumann, Oskar Morgenstern, and Ariel Rubinstein. *Theory of Games and Economic Behavior (60th Anniversary Commemorative Edition)*. Princeton University Press, 1944. `http://www.jstor.org/stable/j.ctt1r2gkx`.

[9] Mukesh Gautam and Mohammed Benidris. Coalitional game theory in power systems: Applications, challenges, and future directions. *In Proceedings of the 2023 IEEE Texas Power and Energy Conference (TPEC)*, Feb. 2023.

[10] Jose M. Flores-Perez, Catherine Azzaro-Pantel, Antonin Ponsich, and Alberto A. Aguilar Lasserre. Stackelberg game design and operation of a non-cooperative bi-level

h2 supply chain under cournot equilibrium. In *Computer Aided Chemical Engineering*, volume 51, pages 391–396. Elsevier, 2022. https://linkinghub.elsevier.com/retrieve/pii/B9780323958790500667.

[11] Fabian Lechtenberg, Lluc Aresté-Saló, Antonio Espuña, and Moisès Graells. Cooperative multi-actor multi-criteria optimization framework for process integration. *Applied Energy*, 377:124581, 2025. https://www.sciencedirect.com/science/article/pii/S0306261924019640.

[12] Qi Feng and Lauren Xiaoyuan Lu. Supply chain contracting under competition: Bilateral bargaining vs. stackelberg. *Production and Operations Management*, 22(3):661–675, May 2013. https://journals.sagepub.com/doi/10.1111/j.1937-5956.2012.01417.x.

[13] Jianrui Wang, Yitian Hong, Jiali Wang, Jiapeng Xu, Yang Tang, Qing-Long Han, and Jurgen Kurths. Cooperative and competitive multi-agent systems: From optimization to games. *IEEE/CAA Journal of Automatica Sinica*, 9(5):763–783, May 2022. https://ieeexplore.ieee.org/document/9763486/.

[14] Dajun Yue and Fengqi You. Game-theoretic modeling and optimization of multi-echelon supply chain design and operation under stackelberg game and market equilibrium. *Computers & Chemical Engineering*, 71:347–361, Dec. 2014. https://linkinghub.elsevier.com/retrieve/pii/S0098135414002567.

[15] Stephan Dempe, Vyacheslav Kalashnikov, Gerardo A. Pérez-Valdés, and Nataliya Kalashnykova. *Bilevel Programming Problems: Theory, Algorithms and Applications to Energy Networks*. Energy Systems. Springer Berlin Heidelberg, 2015. https://link.springer.com/10.1007/978-3-662-45827-3.

[16] Kaisa Miettinen and Marko M. Mäkelä. NIMBUS — interactive method for nondifferentiable multiobjective optimization problems. In Mehrdad Tamiz, editor, *Multi-Objective Programming and Goal Programming*, volume 432, pages 50–57. Springer Berlin Heidelberg, 1996. http://link.springer.com/10.1007/978-3-642-87561-8_5.

[17] Kaisa Miettinen, Petri Eskelinen, Francisco Ruiz, and Mariano Luque. NAUTILUS method: An interactive technique in multiobjective optimization based on the nadir point. *European Journal of Operational Research*, 206(2):426–434, Oct. 2010. https://linkinghub.elsevier.com/retrieve/pii/S0377221710001785.

[18] Azadeh Maroufmashat, Sourena Sattari, Ramin Roshandel, Michael Fowler, and Ali Elkamel. Multi-objective optimization for design and operation of distributed energy systems through the multi-energy hub network approach. *Industrial & Engineering Chemistry Research*, 55(33):8950–8966, Aug. 2016. https://pubs.acs.org/doi/10.1021/acs.iecr.6b01264.

[19] Li Li, Jing Wang, Xiaoyi Zhong, Jian Lin, Nianyuan Wu, Zhihui Zhang, Chao Meng, Xiaonan Wang, Nilay Shah, Nigel Brandon, Shan Xie, and Yingru Zhao. Combined multi-objective optimization and agent-based modeling for a 100% renewable island energy system considering power-to-gas technology and extreme weather conditions. *Applied En-*

ergy, 308:118376, Feb. 2022. https://linkinghub.elsevier.com/retrieve/pii/S0306261921016172.

[20] Kathleen B. Aviso, Raymond R. Tan, and Alvin B. Culaba. Designing eco-industrial water exchange networks using fuzzy mathematical programming. *Clean Technologies and Environmental Policy*, 12(4):353–363, Aug. 2010. http://link.springer.com/10.1007/s10098-009-0252-1.

[21] C. Banville, M. Landry, J.-M. Martel, and C. Boulaire. A stakeholder approach to MCDA. *Systems Research and Behavioral Science*, 15:15–32, 1998.

[22] Marco Cinelli, Stuart R. Coles, and Kerry Kirwan. Analysis of the potentials of multi criteria decision analysis methods to conduct sustainability assessment. *Ecological Indicators*, 46:138–148, 2014. https://www.sciencedirect.com/science/article/pii/S1470160X14002647.

[23] Nadia Brauner and Aurélie Lagoutte. Linear programming. Online course, 2021. Operations Research I course slides, Caseine platform, https://moodle.caseine.org/pluginfile.php/20662/mod_resource/content/7/Slides_En_LP.pdf.

[24] Slim Bechikh, Lamjed Ben Said, and Khaled Ghedira. Estimating nadir point in multi-objective optimization using mobile reference points. In *IEEE Congress on Evolutionary Computation*, pages 1–9. IEEE, July 2010. http://ieeexplore.ieee.org/document/5586203/.

[25] Laurent Lemarchand. Optimisation multi objectifs (moo). Lecture notes, 2018. https://labsticc.univ-brest.fr/~lemarch/FR/Cours/algoP4moo.pdf.

[26] Linjun He, Hisao Ishibuchi, Anupam Trivedi, Handing Wang, Yang Nan, and Dipti Srinivasan. A survey of normalization methods in multiobjective evolutionary algorithms. *IEEE Transactions on Evolutionary Computation*, 25(6):1028–1048, Dec. 2012. https://ieeexplore.ieee.org/document/9419072/.

[27] Matthias Ehrgott and Dagmar Tenfelde-Podehl. Nadir values: Computation and use in compromise programming. *European Journal of Operational Research (submitted)*, 2000. https://d-nb.info/1026779278/34.

[28] Boyan Metev and Vassil Vassilev. A method for nadir point estimation in MOLP problems. *Cybernetics and Information Technologies*, 2003. https://cit.iict.bas.bg/CIT_03/v3-2/15-24.pdf.

[29] Heinz Isermann and Ralph E. Steuer. Computational experience concerning payoff tables and minimum criterion values over the efficient set. *European Journal of Operational Research*, 33(1):91–97, 1988. https://www.sciencedirect.com/science/article/pii/0377221788902573.

[30] Kalyanmoy Deb, Kaisa Miettinen, and Shamik Chaudhuri. Toward an estimation of nadir objective vector using a hybrid of evolutionary and local search approaches. *IEEE Transactions on Evolutionary Computation*, 14(6):821–841, Dec. 2010. http://ieeexplore.ieee.org/document/5557782/.

[31] Alain Lambert. Programmation mathématique avancée – cours 1, 2020. https://cedric.cnam.fr/~lamberta/MPRO/PMA/PMA_cours1.pdf.

[32] Gaurav Gupta and Anoop Singh. Market-linked take-or-pay energy purchase contracts for green hydrogen supply chains: A stackelberg game approach. SSRN working paper. Available at: https://ssrn.com/abstract=5031841, 2024.

— A —

Modelisation of the scenario

A.1 Data

A.1.1 Energy sources

- Production_elec^e(t) : Quantity of electricity e available at time t (MWh)
  - PV : PV 8MW of Lloseta (Spain)
  - Electricity grid : no upper bound
- Price_elec^e(t) : Price of electricity e at time t (€/MWh)
  - Cost of PV electricity : 100 €/MWh $\forall t$ (deliberately high price to create arbitrary situations between the grid and low-cost, low-carbon intermittent PV)
  - Price of electricity from grid e: data price French grid 2022
- Price_gas(t) : Price of gas 50 €/MWh $\forall t$
- Impact_elec$^e(t)$: Impact of grid electricity at time t. (kgCO_2/MWh)
  - PV : 26 $\forall t$ (source : Electricity maps)
  - Grid electricity : data impact French grid 2022

A.1.2 Producers

- Size_SMR$_i$: 1 000 000 MW (virtually no upper bound) $\forall i \in$ P_SMR
- Size_max_electrolyser$_i$: 10 MW $\forall i \in$ P_electrolyser
- Size_max_storage$_i$: 1000 kgH_2 $\forall i \in$ P_electrolyser
- Size_max_CCS$_i$: 1000 kgCO_2 $\forall i \in$ P_SMR
- Efficiency_electrolyser$_i$: 20 kgH_2/MWh $\forall i$
- Efficiency_SMR$_i$: 20 kgH_2/MWh (for a production cost of 2.5€/kgH_2) $\forall i$

  Capex in €/unit:

- $CAPEX\_electrolyser_i$: 600 000 €/MW $\forall i$
  Alkaline electrolysis 30 bar - 2030 [6]
- $CAPEX\_storage_i$: 1000 €/kgH_2 $\forall i$
- $CAPEX\_CCS_i$: 4 800 €/kgCO_2 $\forall i$
- $Life\_electrolyser_i$: 10 years $\forall i$
- $Life\_storage_i$: 10 years $\forall i$
- $Life\_CCS_i$: 10 years $\forall i$

  Capex in €/unit/h :

- $CAPEX\_t\_electrolyser_i$: $CAPEX\_electrolyser_i$ / (8760 * $Life\_electrolyser_i$) $\forall i$
- $CAPEX\_t\_storage_i$: $CAPEX\_storage_i$ / (8760 * $Life\_storage_i$) $\forall i \forall i$
- $CAPEX\_t\_CCS_i$:$CAPEX\_CCS_i$ / (8760 * $Life\_CCS_i$) $\forall i$ $\forall i$
- $Impact\_SMR_i$: 10 kgCO_2/kgH_2 [6] $\forall i$
- Impact_max : Maximal CO_2 impact autorized by kgH_2 to be certified "low-emission" hydrogen
  3.5 kgCO_2/kgH_2

A.1.3 Consumers

- $Demand\_H2_j$: H_2 demand of client j at time t (kgH_2)
  - Consumer j_1: data from a previous project (IMAGHyNE)
  - Consumer j_2: 6.25 $\forall t$
- $Price\_sold\_H2_{i,j}$: Price of hydrogen sold between producer i and consumer j (€/kgH_2)

	P1_electrolysis(with PV)	P2_electrolysis	P3_SMR
C1_industrial	6	8	4.75
C2_mobility	10	11.4	7.6

A.2 Decision variables

A.2.1 Flow variables

- $\mathbf{Q\_energy_i^e(t)}$: Quantity of energy coming from source e used by producer i. (MWh)
- $\mathbf{Q\_energy\_total_i(t)}$: Total quantity of energy used by producer i. (MWh)
- $\mathbf{Q\_H2\_prod_i(t)}$: Quantity of H_2 produced at time t by producer i. (kgH_2)
- $\mathbf{Q\_H2\_stock_i(t)}$: Quantity of H_2 in storage of producer i a time t. (kgH_2)
- $\mathbf{Q\_H2\_init\_stock_i}$: Initial H_2 quantity in the storage of producer i. (kgH_2)
- $\mathbf{Q\_H2\_stock\_in_i(t)}$: Quantity of H_2 coming in the storage of producer i at time t. (kgH_2)
- $\mathbf{Q\_H2\_stock\_out_i(t)}$: Quantity of H_2 coming out of the storage of producer i at time t. (kgH_2)
- $\mathbf{Q\_H2\_to\_sell_i(t)}$: Quantity of H_2 sold on the market by the producer i at time t (before splitting it between consumers). (kgH_2)
- $\mathbf{Q\_H2\_sold_{i,j}(t)}$: Quantity of H_2 sold by producer i to consumer j at time t. (kgH_2)

A.2.2 Sizing variables

- $\mathbf{Size\_electrolyser_i}$: Size of the electrolyser of producer i. (MW)
- $\mathbf{Size\_storage_i}$: Size of the storage of producer i. (kgH_2)
- $\mathbf{Size\_CCS_i}$: Size of the carbon capture and storage system of producer i ($kgCO_2$)

A.2.3 Economic variables

- $\mathbf{P\_energy\_total_i}$: Total price of the energy consumed by producer i. (€)
- $\mathbf{P\_CAPEX\_electrolyser_i}$: Investment cost of producer i for its electrolyser per hour. (€/h)
- $\mathbf{P\_CAPEX\_storage_i}$: Investment cost of producer i for its storage per hour. (€/h)
- $\mathbf{P\_CAPEX\_CCS_i}$: Investment cost of producer i for its carbon capture and storage system per hour. (€/h)
- $\mathbf{P\_H2\_sold_{i,j}(t)}$: Revenue from H_2 sales to consumer j by producer i at time t. (€)

A.2.4 Environmental variables

- $\mathbf{Impact\_prod_i(t)}$: CO_2 impact of producer i at time t. ($kgCO_2$)
- $\mathbf{Emission\_SMR_i(t)}$: CO_2 emissions generated by the steam reforming of producer i at time t. ($kgCO_2$)
- $\mathbf{CCS_i(t)}$: CO_2 emissions captured by the CCS system of the SMR producer i at time t. ($kgCO_2$)

A.3 Constraints

A.3.1 Producer *i* via electrolysis

- Energy sources
  - Limited quantity of electricity

$$\sum_{i \in \text{Producers}} \mathbf{Q\_energy_i^e(t)} \leq \text{Production\_elec}^e(t) \quad \forall e \in \text{Electricity sources} \quad \forall t$$

- Flux
  - Quantity of electricity bought by the producer

$$\mathbf{Q\_energy\_total_i(t)} = \sum_{e \in \text{Electricity sources}} \mathbf{Q\_energy_i^e(t)} \quad \forall i \quad \forall t$$

  - H_2 quantity produced with the energy bought

$$\mathbf{Q\_H2\_prod_i(t)} = \mathbf{Q\_energy\_total_i(t)} \cdot \text{Efficiency\_electrolyser}_i \quad \forall i \quad \forall t$$

  - Sizing constraint

$$\mathbf{Q\_energy\_total_i(t)} \leq \mathbf{Size\_electrolyser_i} \quad \forall i \quad \forall t$$

  - H_2 quantity in storage

$$\mathbf{Q\_H2\_stock_i(0)} = \mathbf{Q\_H2\_init\_stock_i} + \mathbf{Q\_H2\_stock\_in_i(0)} - \mathbf{Q\_H2\_stock\_out_i(0)} \quad \forall i$$

$$\mathbf{Q\_H2\_stock_i(t)} = \mathbf{Q\_H2\_stock_i(t-1)} + \mathbf{Q\_H2\_stock\_in_i(t)} - \mathbf{Q\_H2\_stock\_out_i(t)} \forall i \quad \forall t \geq 1$$

  - Initial quantity of H_2 in stock (50% of storage capacity)

$$\mathbf{Q\_H2\_init\_stock_i} = 0.5 \cdot \mathbf{Size\_Storage_i} \quad \forall i$$

  - Final quantity of H_2 in stock

$$\mathbf{Q\_H2\_init\_stock_i} = \mathbf{Q\_H2\_stock_i(t_{max})} \quad \forall i$$

  - H_2 to sell

$$\mathbf{Q\_H2\_to\_sell_i(t)} = \mathbf{Q\_H2\_prod_i(t)} - \mathbf{Q\_H2\_stock\_in_i(t)} + \mathbf{Q\_H2\_stock\_out_i(t)} \quad \forall i \quad \forall t$$

  - Sizing constraint

$$\mathbf{Q\_H2\_stock_i(t)} \leq \mathbf{Size\_storage_i} \quad \forall i \quad \forall t$$

  - Quantity of H_2 sold

$$\mathbf{Q\_H2\_to\_sell_i(t)} = \sum_{j \in \text{consumers}} \mathbf{Q\_H2\_sold_{i,j}(t)} \quad \forall i \quad \forall t$$

- Max size of electrolyser (to bound the maximisation problem)

$$\mathbf{Size\_electrolyser_i} \leq \text{Size\_max\_electrolyser}_i \quad \forall i$$

- Max size of storage (to bound the maximisation problem)

$$\mathbf{Size\_storage_i} \leq \text{Size\_max\_storage}_i \quad \forall i$$

- Economic
  - Cost of H_2 production : electricity

$$\mathbf{P\_energy\_total_i(t)} = \sum_{e \in \text{Electricity sources}} \mathbf{Q\_energy_i^e(t)} \cdot \text{Price\_elec}^e(t) \quad \forall i \quad \forall t$$

  - Cost of H_2 production : CAPEX per hour

$$\mathbf{P\_CAPEX\_electrolyser_i} = \mathbf{Size\_electrolyser_i} \cdot \text{CAPEX\_t\_electrolyser}_i \quad \forall i$$

$$\mathbf{P\_CAPEX\_Storage_i} = \mathbf{Size\_Storage_i} \cdot \text{CAPEX\_t\_Storage}_i \quad \forall i$$

  - H_2 Sales Profit

$$\mathbf{P\_H2\_sold_{ij}(t)} = \mathbf{Q\_H2\_sold_{i,j}(t)} \cdot \text{Price\_sold\_H2}_{i,j} \quad \forall i \quad \forall j \quad \forall t$$

- Environnement
  - Carbon impact of H_2 producer via electrolysis (impact of used electricity)

$$\mathbf{Impact\_prod_i(t)} = \sum_{e \in \text{Electricity sources}} \mathbf{Q\_energy_i^e(t)} \cdot \text{Impact\_elec}^e(t) \quad \forall i \quad \forall t$$

  - Max carbon impact constraints per kgH_2
    * Global

$$\sum_t \mathbf{Impact\_prod_i(t)} \leq \text{Impact\_max} \cdot \sum_t \mathbf{Q\_H2\_prod_i(t)} \quad \forall i$$

    * **OR** Per time step (Hourly)

$$\mathbf{Impact\_prod_i(t)} \leq \text{Impact\_max} \cdot \mathbf{Q\_H2\_prod_i(t)} \quad \forall i \quad \forall t$$

A.3.2 Producer *i* via SMR

- Flux
  - Quantity of energy bought

$$\sum_{e \in \text{Energy sources}} \mathbf{Q\_energy_i^e(t)} = \mathbf{Q\_energy\_total_i(t)} \quad \forall i \quad \forall t$$

– Quantity of H_2 produced with the gas bought

$$\mathbf{Q\_H2\_prod_i(t)} = \mathbf{Q\_energy\_total_i(t)} \cdot \text{Efficiency\_SMR}_i \quad \forall i \quad \forall t$$

– Sizing Constraint

$$\mathbf{Q\_energy\_total_i(t)} \leq \text{Size\_SMR}_i \quad \forall i \quad \forall t$$

– Quantity of H_2 sold

$$\mathbf{Q\_H2\_prod_i(t)} = \sum_{j \in \text{consumers}} \mathbf{Q\_H2\_sold_{i,j}(t)} \quad \forall i \quad \forall t$$

$$\mathbf{Q\_H2\_to\_sell_i(t)} = \mathbf{Q\_H2\_prod_i(t)} \quad \forall i \quad \forall t$$

– Sizing of CCS system

$$\mathbf{CCS_i(t)} \leq \mathbf{Size\_CCS_i} \quad \forall i \quad \forall t$$

– Max size of CCS (to bound the maximisation problem)

$$\mathbf{Size\_CCS_i} \leq \text{Size\_max\_CCS}_i \quad \forall i$$

- Economic
  – Cost of H_2 production : gas

$$\mathbf{P\_energy\_total_i} = \sum_t \mathbf{Q\_energy\_total_i(t)} \cdot \text{Price\_gas}(t) \quad \forall i$$

  – Cost of H_2 production : CAPEX per hour

$$\mathbf{P\_CAPEX\_CCS_i} = \mathbf{Size\_CCS_i} \cdot \text{CAPEX\_t\_CCS}_i \quad \forall i$$

  – H_2 Sales Profit

$$\mathbf{P\_H2\_sold_{ij}(t)} = \mathbf{Q\_H2\_sold_{i,j}(t)} \cdot \text{Price\_sold\_H2}_{i,j} \quad \forall i \quad \forall t \quad \forall j$$

- Environnemental
  – CO_2 emissions from SMR

$$\mathbf{Emission\_SMR_i(t)} = \mathbf{Q\_H2\_prod_i(t)} \cdot \text{Impact\_SMR}_i \quad \forall i \quad \forall t$$

  – Carbon impact of H_2 producer i via SMR

$$\mathbf{Impact\_prod_i(t)} = \mathbf{Emission\_SMR_i(t)} - \mathbf{CCS_i(t)} \quad \forall i \quad \forall t$$

  – Max carbon impact constraints per kgH_2
    * Global

$$\sum_t \mathbf{Impact\_prod_i(t)} \leq \text{Impact\_max} \cdot \sum_t \mathbf{Q\_H2\_prod_i(t)} \quad \forall i$$

    * **OR** Per time step (Hourly)

$$\mathbf{Impact\_prod_i(t)} \leq \text{Impact\_max} \cdot \mathbf{Q\_H2\_prod_i(t)} \quad \forall i \quad \forall t$$

A.3.3 H_2 consumer j

- Flux

  - The demand is satisfied

$$\sum_{i \in \text{producers}} \mathbf{Q\_H2\_sold_{i,j}(t)} = \text{Demand\_H2}_j(t) \quad \forall j \quad \forall t$$

- Economic

  - Buying cost

$$\mathbf{P\_H2\_sold_{i,j}(t)} = \mathbf{Q\_H2\_sold_{i,j}(t)} \cdot \text{Price\_sold\_H2}_{\mathbf{i,j}} \quad \forall i \quad \forall j \quad \forall t$$

— B —

Code

We present here the code organisation and how the functions are called during the optimisation process.
The optimisation parameters chosen can be modified in the file *config.py* before running *py main.py*.
The calls made by the main function can be seen in Figure B.1.

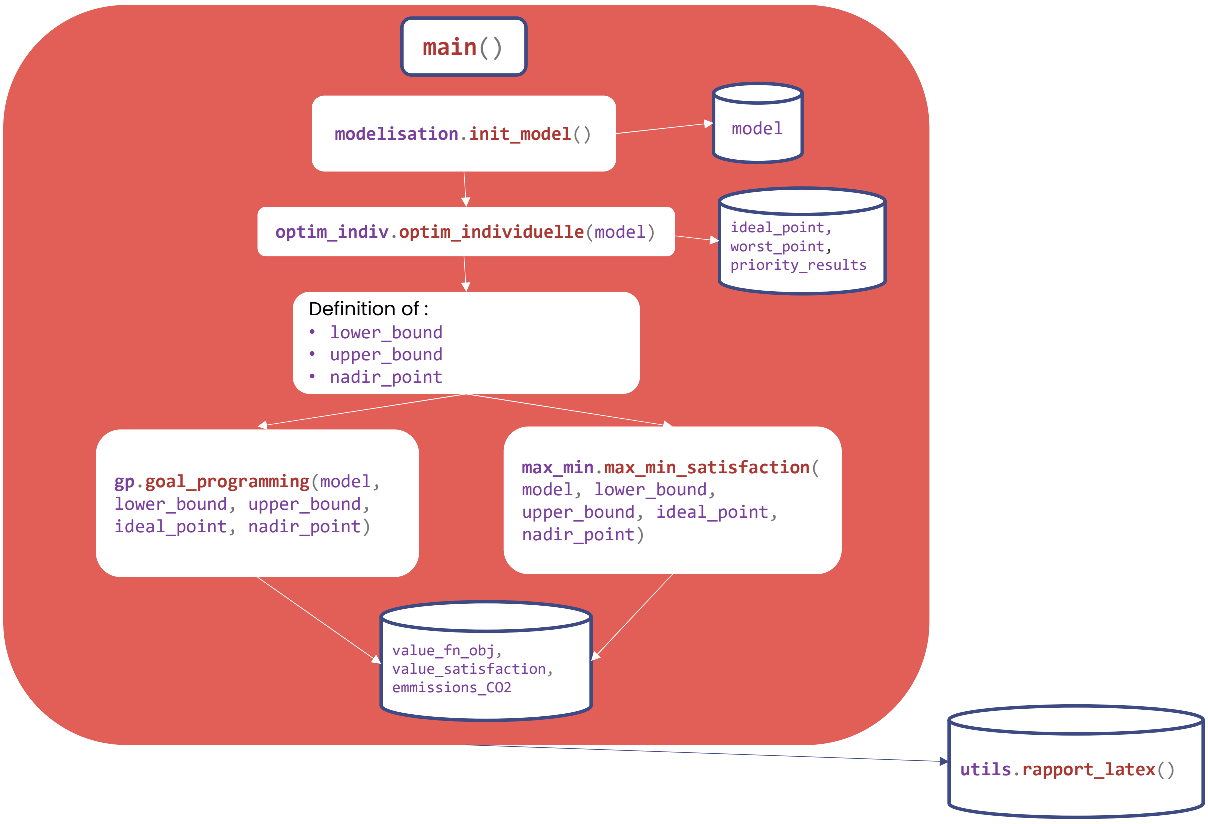


Figure B.1: main.py

The optimisation model is defined in python using the pyomo package in the file described by Figure B.2.

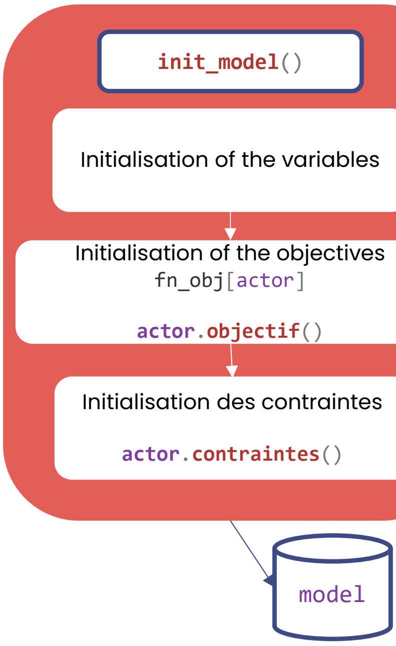


Figure B.2: Definition/modelisation.py

The optimisation process starts with sequential individual optimisation as shown in Figure B.3

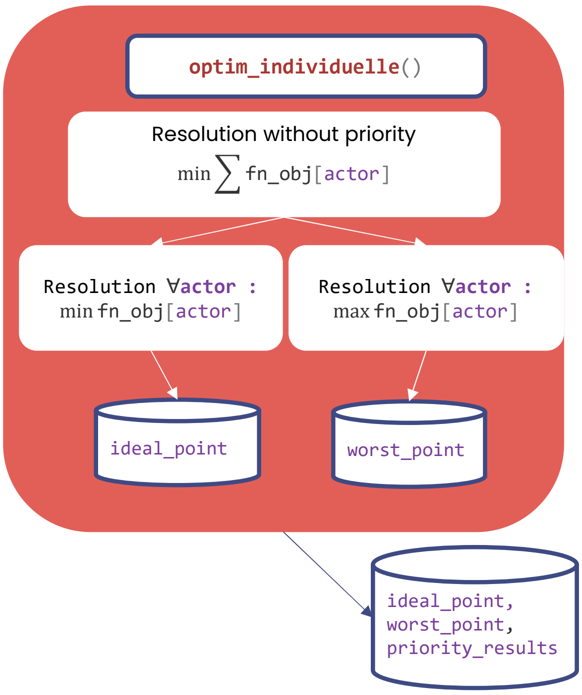


Figure B.3: Resolution/optim_individuelle.py

The goal programming process is as shown in Figure B.4.

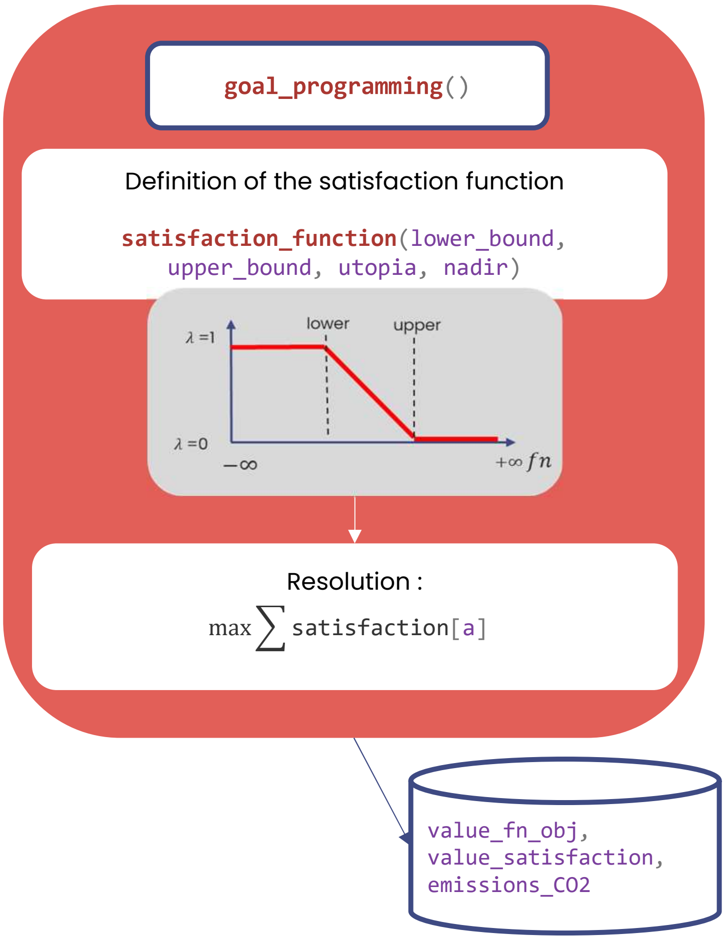


Figure B.4: Resolution/goal_programming.py

Finally, the max-min lexicographic process is as in Figure B.5

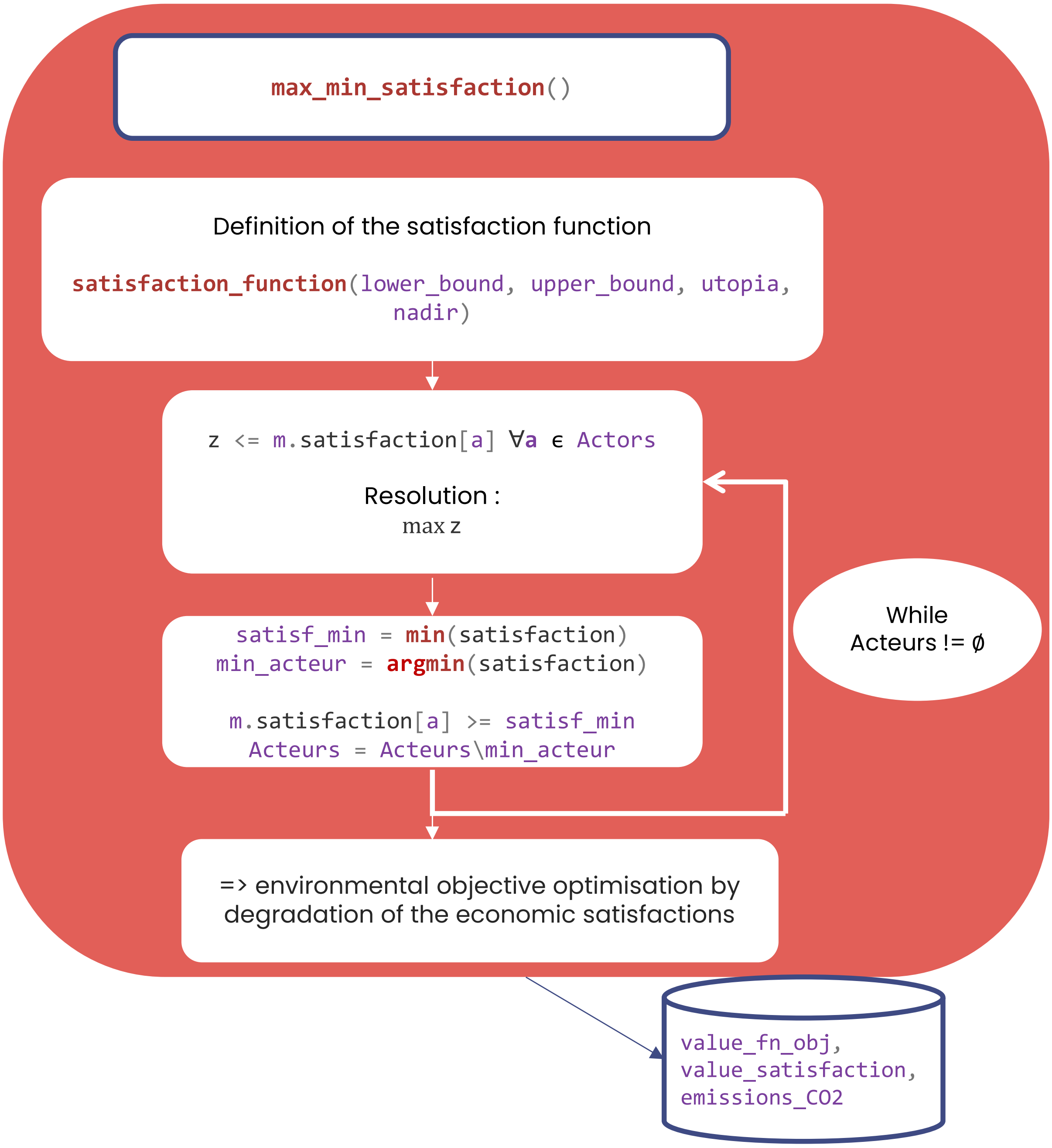


Figure B.5: Resolution/max_min_satisfaction.py

— C —
Detailed optimisation results

1 Informations

Horizon d'optimisation: 364.00 jours (8736h).
Acteurs du réseau: P1_electrolyse(avec PV), P2_electrolyse, P3_SMR, C1_industriel, C2_mobilite.
Nombre de variables: 419367.
Nombre de contraintes: 401898.
Les producteurs doivent respecter une contrainte CO_2 horaire.
Prix fixés entre Producteurs et consommateurs:

	P1_electrolyse(avec PV)	P2_electrolysis	P3_SMR
C1_industriel	6 €	8 €	4.75 €
C2_mobilité	10 €	11.4 €	7.6 €

2 Optimisations Individuelles

Temps d'éxécution: 712.86sec

2.1 Table de priorité

Dans le tableau de priorité, chaque ligne montre les résultats obtenus en priorisant l'acteur mentionné dans la première colonne.

	P1_electrolyse(avec PV)	P2_electrolyse	P3_SMR	C1_industriel	C2_mobilite
P1_electrolyse(avec PV)	-247150	703137	215483	5	10
P2_electrolyse	690576	-27077	-99449	5	9
P3_SMR	655924	706127	-567688	5	8
C1_industriel	594185	586506	-170322	5	8
C2_mobilite	790699	592860	-119331	5	8

2.2 Point significatifs

Acteur	Point Idéal	Point Nadir	Pire Point
P1_electrolyse(avec PV)	-247150	790699	2113917
P2_electrolyse	-27077	706127	964093
P3_SMR	-567688	215483	471187
C1_industriel	5	5	7
C2_mobilite	8	10	11

3 Goal Programming

Valeur de la fonction objective: 3.03
Temps d'éxécution: 864.25sec

3.1 Résultats généraux

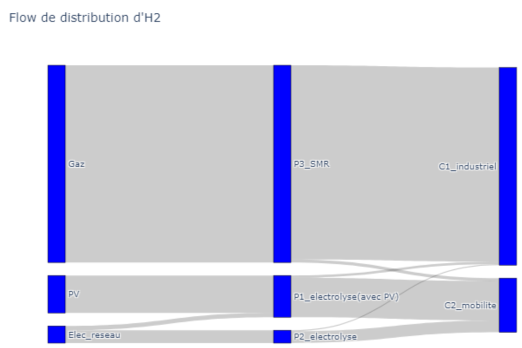
Acteur	Fonction objective	Satisfaction
P1_electrolyse(avec PV)	-159946	0.65
P2_electrolyse	-26533	0.98
P3_SMR	-223646	0.39
C1_industriel	5	0.52
C2_mobilite	10	0.49

3.2 Résultats Producteurs

Impact CO2 moyen : 2.6272 kgC02/kgH2

	P1_electrolyse(avec PV)	P2_electrolyse	P3_SMR
Qté. d'H2 prod - en kgH2	42063.37	12927.8	198819.42
Total d'achat d'énergie - en MWh	2103.17	646.39	9940.97
Cout total d'achat d'énergie - en EUR	203589.20	85814.52	497048.55
Total emission CO2 - en kgCO2	59960.72	33907.85	572943.96
Emission CO2 - en kgCO2/kgH2	1.43	2.62	2.88
Dim. Electrolyseur - en MW	0.65	0.49	
Taux d'utilisa° Electrolyseur	0.37	0.15	
Dim. Stockage - en kgH2	90.1	25.0	
Utilisa° Stockage - en #cycles	261.17	183.01	
Dim. Captage - en kgCO2			485.07
CO2 Capté - en kgCO2			1415250.24
CAPEX - en EUR	47662.0	31580.0	232197.0
Prix moyen achat énergie - en EUR/MWh	97.0	133.0	50.0
LCOH - en EUR/kgH2	5.98	9.09	3.67
Part de l'énergie dans le LCOH - en %	81	73	68

3.3 Sankey graph



4 Max min satisfaction

Valeur de la fonction objective: 109352.19
Temps d'éxécution: 1680.70sec

4.1 Résultats généraux

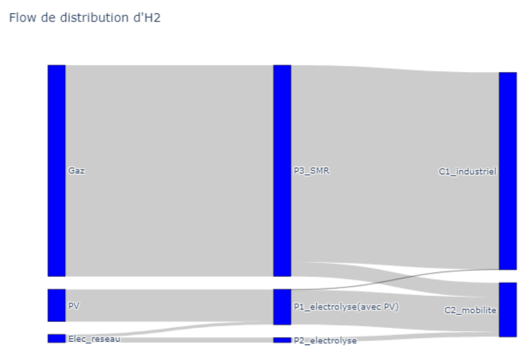
Acteur	Fonction objective	Satisfaction
P1_electrolyse(avec PV)	-128930	0.52
P2_electrolyse	-14209	0.52
P3_SMR	-297887	0.52
C1_industriel	5	0.52
C2_mobilite	9	0.53

4.2 Résultats Producteurs

Impact CO2 moyen : 0.4308 kgC02/kgH2

	P1_electrolyse(avec PV)	P2_electrolyse	P3_SMR
Qté. d'H2 prod - en kgH2	35566.69	4990.81	213253.09
Total d'achat d'énergie - en MWh	1778.33	249.54	10662.65
Cout total d'achat d'énergie - en EUR	173412.68	27354.69	533132.72
Total emission CO2 - en kgCO2	50032.55	12258.07	47061.57
Emission CO2 - en kgCO2/kgH2	1.41	2.46	0.22
Dim. Electrolyseur - en MW	0.43	0.26	
Taux d'utilisa° Electrolyseur	0.47	0.11	
Dim. Stockage - en kgH2	258.03	0.0	
Utilisa° Stockage - en #cycles	72.87	0	
Dim. Captage - en kgCO2			466.17
CO2 Capté - en kgCO2			2085469.30
CAPEX - en EUR	51649.0	15332.0	223148.0
Prix moyen achat énergie - en EUR/MWh	98.0	110.0	50.0
LCOH - en EUR/kgH2	6.35	8.57	3.55
Part de l'énergie dans le LCOH - en %	77	64	70

4.3 Sankey graph



5 Evolution de l'algorithme max min séquentiel

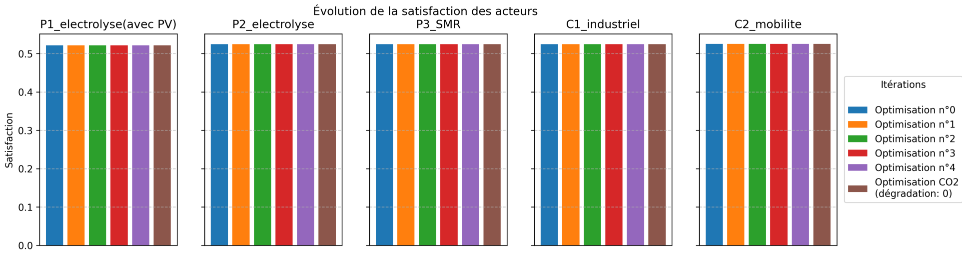


Figure 1: Comparaison de l'évolution de la satisfaction avec l'algorithme max min

6 Comparaison des méthodes

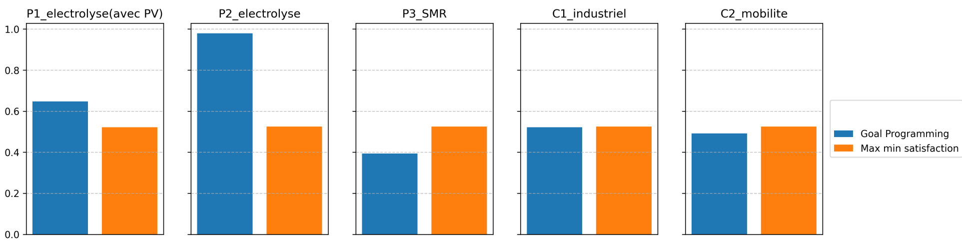
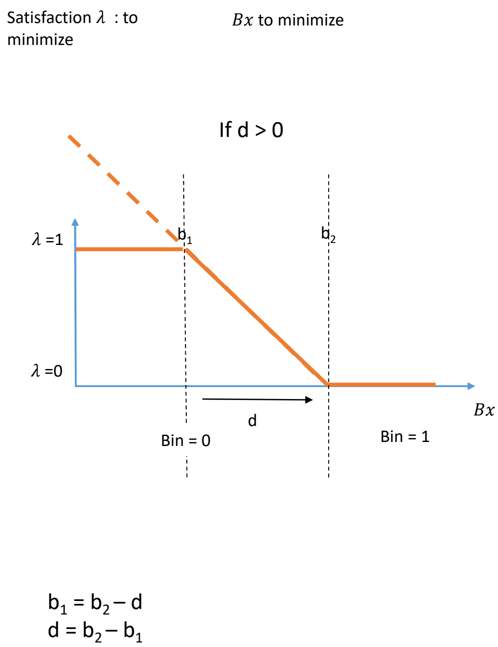


Figure 2: Comparaison of the satisfaction of the different methods

— D —

Miscellaneous



$(1\text{-}bin) \geq \lambda \geq 0$	If *bin*=0: $1 \geq \lambda \geq 0$
	If *bin*=1: λ=0
$Bx \geq b_2 - M(1 - bin)$	If *bin*=1: $Bx \geq b_2$
	If *bin*=0: $Bx \geq b_2 - M$ Good if $b_2 - M \leq$ Min value of Bx
$\lambda \leq \frac{Bx - b_2}{b_1 - b_2} + M \cdot bin$	If bin = 1: $\lambda \leq \frac{Bx - b_2}{b_1 - b_2} + M \cdot bin$ Ok if good M value
	If bin = 0: $\lambda \leq \frac{Bx - b_2}{b_1 - b_2}$ If Bx $\leq b_1$: $\frac{Bx-b_2}{b_1-b_2} \geq 1$ => $\lambda \leq$ C with C>1 OK since λ is maximised If $b_2 \geq$ Bx $\geq b_1$: $0 \leq \frac{Bx-b_2}{b_1-b_2} \leq 1$ Ok (it must be strictly greater than 0)

Figure D.1: Details on the modelisation of the membership function