Project 2

2024-09-23

5.12. Refer to Flavor deterioration Problem 5.4. Find $(X'X)^{-1}$

```
Y <- c(7.8, 9.0, 10.2, 11.0, 11.7)
Y <- as.matrix(Y)

X <- c(8,4,0,-4,-8)
X <- as.matrix(X)
X <- cbind(rep(1,5), X)


Yt <- t(Y)

YtY <- Yt %*% Y



Xt <- t(X)

XtX <- Xt %*% X


XtXinvert <- solve(XtX) # getting the inverse of transpose of X * X
XtXinvert
```

```
##      [,1]    [,2]
## [1,]  0.2 0.00000
## [2,]  0.0 0.00625
```

```
XtXinvert %*% XtX # Double Checking the inverse is correct
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
```

5.13. Refer to Consumer finance Problem 5.5. Find $(X'X)^{-1}$

```
# Getting the X vector
x <- c(4, 1, 2, 3, 3, 4)
x <- as.matrix(x)
x <- cbind(rep(1,6), x)

# Getting the transpose of x

xt <- t(x)
```

```r
# Getting the Y vector

y <- c(16, 5, 10, 15, 13, 22)
y <- as.matrix(y)


# Calculating matrix multiplication to get the vector of estimated regression coeffecients

xtx <- xt %*% x # transpose of X * X



xtxinvert <- solve(xtx) # getting the inverse of transpose of X * X

xtxinvert
```

```
##            [,1]       [,2]
## [1,]  1.3414634 -0.4146341
## [2,] -0.4146341  0.1463415
```

```r
xtxinvert %*% xtx # Double Checking the inverse is correct
```

```
##      [,1]         [,2]
## [1,]    1 3.552714e-15
## [2,]    0 1.000000e+00
```

Chapter 5: Question 18

Part A: State the above in matrix notation.

$$W = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{pmatrix} \tag{1}$$

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Part B: Find the expectation of the random vector W.

$$EW = E\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{4}[EY_1 & EY_2 & EY_3 & EY_4] \\ \frac{1}{2}[EY_1 & EY_2 & EY_3 & EY_4] \end{pmatrix} \tag{2}$$

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Part C: Find the variance-covariance matrix of W.

$$\sigma^2 W = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \sigma^2\{Y_1\} & \sigma\{Y_1, Y_2\} & \sigma\{Y_1, Y_3\} & \sigma\{Y_1, Y_4\} \\ \sigma\{Y_2, Y_1\} & \sigma^2\{Y_2\} & \sigma\{Y_2, Y_3\} & \sigma\{Y_2, Y_4\} \\ \sigma\{Y_3, Y_1\} & \sigma\{Y_3, Y_2\} & \sigma^2\{Y_3\} & \sigma\{Y_3, Y_4\} \\ \sigma\{Y_4, Y_1\} & \sigma\{Y_4, Y_2\} & \sigma\{Y_4, Y_3\} & \sigma^2\{Y_4\} \end{pmatrix} \begin{pmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} \end{pmatrix} \tag{3}$$

3

Chapter 5: Question 24 Refer to Consumer finance Problems 5.5 and 5.13.

Using the Data from Question 5.13 From earlier in this project

A. Using matrix methods, obtain the following: (1) vector of estimated regression coefficients, (2) vector of residuals, (3) SSR, (4) SSE, (5) estimated variance-covariance matrix of b, (6) point estimate of E{Yh} when X/r = 4, (7) s2{pred} when XIz = 4.

1. Vector of Estimated Regression Coefficients

```
xty <- xt %*% y
b <- xtxinvert %*% xty
b
```

```
##           [,1]
## [1,] 0.4390244
## [2,] 4.6097561
```

```
# Double Checking the vector of estimated regression coefficient
finance_model <- lm(y ~ x)

summary_finance_model <- summary(finance_model)
summary_finance_model
```

```
##
## Call:
## lm(formula = y ~ x)
##
## Residuals:
##        1        2        3        4        5        6
## -2.87805 -0.04878  0.34146  0.73171 -1.26829  3.12195
##
## Coefficients: (1 not defined because of singularities)
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.4390     2.6087   0.168  0.87452
## x1                NA         NA      NA       NA
## x2            4.6098     0.8616   5.350  0.00589 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.252 on 4 degrees of freedom
## Multiple R-squared:  0.8774, Adjusted R-squared:  0.8467
## F-statistic: 28.62 on 1 and 4 DF,  p-value: 0.005886
```

2 Vector of Residuals

```
e <- y - x%*%b # Obtaining the vector of residuals
e
```

```
##             [,1]
## [1,] -2.87804878
## [2,] -0.04878049
## [3,]  0.34146341
```

```
## [4,]  0.73170732
## [5,] -1.26829268
## [6,]  3.12195122
```

```
finance_model$residuals #Double checking the vector of residuals is accurate
```

```
##           1           2           3           4           5           6
## -2.87804878 -0.04878049  0.34146341  0.73170732 -1.26829268  3.12195122
```

3. SSR

SSR is Total Sum of Squares - SSE

```
yt <- t(y)
j <- matrix(1, nrow = length(y), ncol = length(y))
ssto <- yt %*% y - (1 / length(y)) * yt %*% j %*% y
ssto
```

```
##       [,1]
## [1,] 165.5
```

```
et <- t(e)
sse <- et%*%e
sse
```

```
##          [,1]
## [1,] 20.29268
```

```
bt <- t(b)
ssecheck <- yt%*%y - bt%*%xt%*%y
ssecheck
```

```
##          [,1]
## [1,] 20.29268
```

```
ssr = ssto - sse
ssr
```

```
##          [,1]
## [1,] 145.2073
```

```
checkssr <- bt%*%xt%*%y - (1/length(y))%*%yt%*%j%*%y
checkssr
```

```
##          [,1]
## [1,] 145.2073
```

4. SSE

```
et <- t(e)
sse <- et%*%e
sse
```

```
##          [,1]
## [1,] 20.29268
```

```
bt <- t(b)
ssecheck <- yt%*%y - bt%*%xt%*%y
ssecheck
```

```
##          [,1]
```

```
## [1,] 20.29268
```

5. Estimated Variance-Covariance matrix of b

```r
mse <- sse/(nrow(y)-2)
mse <- as.numeric(mse)


varmatrix <- mse*xtxinvert
varmatrix
```

```
##           [,1]       [,2]
## [1,]  6.805473 -2.1035098
## [2,] -2.103510  0.7424152
```

6. Point Estimate of $E\{Y_h\}$ when $X_h = 4$

```r
x_h <- c(1, 4)
y_h_estimate <- x_h %*% b
y_h_estimate
```

```
##          [,1]
## [1,] 18.87805
```

```r
checkingy_h_estimate <- 0.4390244 + 4.6097561*4
checkingy_h_estimate
```

```
## [1] 18.87805
```

7. s^2 {pred} when $X_h = 4$

```r
x_h <- matrix(c(1, 4), nrow = 2, ncol = 1)

s2y_hat_4 <- mse * (1 + (t(x_h) %*% xtxinvert %*% x_h))

s2y_hat_4
```

```
##          [,1]
## [1,] 6.929209
```

```r
checks2y_hat_4 <- mse * (1 + t(x_h) %*% xtxinvert %*% x_h)

# Print the result
checks2y_hat_4
```

```
##          [,1]
## [1,] 6.929209
```

B. From your estimated variance-covariance matrix in part (A.5), obtain the following:

1. $s\{b_0, b_1\}$

```r
sd2b <- diag(varmatrix)
```

```r
sdb <- sqrt(sd2b)
sdb
```

```
## [1] 2.6087301 0.8616352
```

2. $s^2\{b_0\}$

```r
sd2b0 <- varmatrix[1,1]
sd2b0
```

```
## [1] 6.805473
```

3. $s\{b_1\}$

```r
sdb1 <- sqrt(varmatrix[2,2])
sdb1
```

```
## [1] 0.8616352
```

C. Find the hat matrix H.

```r
H <- x %*% xtxinvert %*% xt
H
```

```
##             [,1]       [,2]       [,3]      [,4]      [,5]        [,6]
## [1,]  0.36585366 -0.1463415 0.02439024 0.1951220 0.1951220  0.36585366
## [2,] -0.14634146  0.6585366 0.39024390 0.1219512 0.1219512 -0.14634146
## [3,]  0.02439024  0.3902439 0.26829268 0.1463415 0.1463415  0.02439024
## [4,]  0.19512195  0.1219512 0.14634146 0.1707317 0.1707317  0.19512195
## [5,]  0.19512195  0.1219512 0.14634146 0.1707317 0.1707317  0.19512195
## [6,]  0.36585366 -0.1463415 0.02439024 0.1951220 0.1951220  0.36585366
```

D. Find $s^2\{e\}$

```r
I <- diag(length(y))

sd2e <- mse * (I-H)
sd2e
```

```
##            [,1]       [,2]       [,3]       [,4]       [,5]       [,6]
## [1,]  3.2171327  0.7424152 -0.1237359 -0.9898870 -0.9898870 -1.8560381
## [2,]  0.7424152  1.7323022 -1.9797739 -0.6186794 -0.6186794  0.7424152
## [3,] -0.1237359 -1.9797739  3.7120761 -0.7424152 -0.7424152 -0.1237359
## [4,] -0.9898870 -0.6186794 -0.7424152  4.2070196 -0.8661511 -0.9898870
## [5,] -0.9898870 -0.6186794 -0.7424152 -0.8661511  4.2070196 -0.9898870
## [6,] -1.8560381  0.7424152 -0.1237359 -0.9898870 -0.9898870  3.2171327
```