Tables and results

Item 1) Implement the Explicit Midpoint Method (EMP)

EMP.py

Item 2) Implement the Implicit Midpoint Method (IMP)

IMP.py

Item 3) Implement the Non-Standard Implicit Midpoint Method (NSIMP)

NSIMP.py

Item 4) All three methods are second order and they do not differ much in terms of computational cost; demonstrate this numerically. I expect several tables with results.

For different time lapse, and steps, we have,

By Explicit Midpoint Method

n	h	Run time	Rel. Error
100	0.050	0.0156	8.4008e-02
250	0.020	0.0468	2.2921e-02
500	0.010	0.0624	4.1966e-02
1000	0.005	0.1404	4.8404e-02
2000	0.003	0.2496	5.0935e-02

By Implicit Midpoint Method

n	h	Run time	Rel. Error
100	0.050	0.0312	1.1235e-02
250	0.020	0.1092	3.6040e-02
500	0.010	0.2184	4.4923e-02
1000	0.005	0.3588	4.9095e-02
2000	0.003	0.2496	5.0935e-02

By Non-Standard Implicit Midpoint Method

n	h	Run time	Rel. Error
100	0.050	0.0624	4.8671e-02
250	0.020	0.1560	2.8294e-02
500	0.010	0.3900	1.9808e-02
1000	0.005	0.7176	1.5603e-02
2000	0.003	0.2496	5.0935e-02

Item 5) For $\gamma = 0$, the implicit methods are equivalent, and we expect they are superior to the explicit method; check this numerically using long time simulations with various h, ω, x_0.

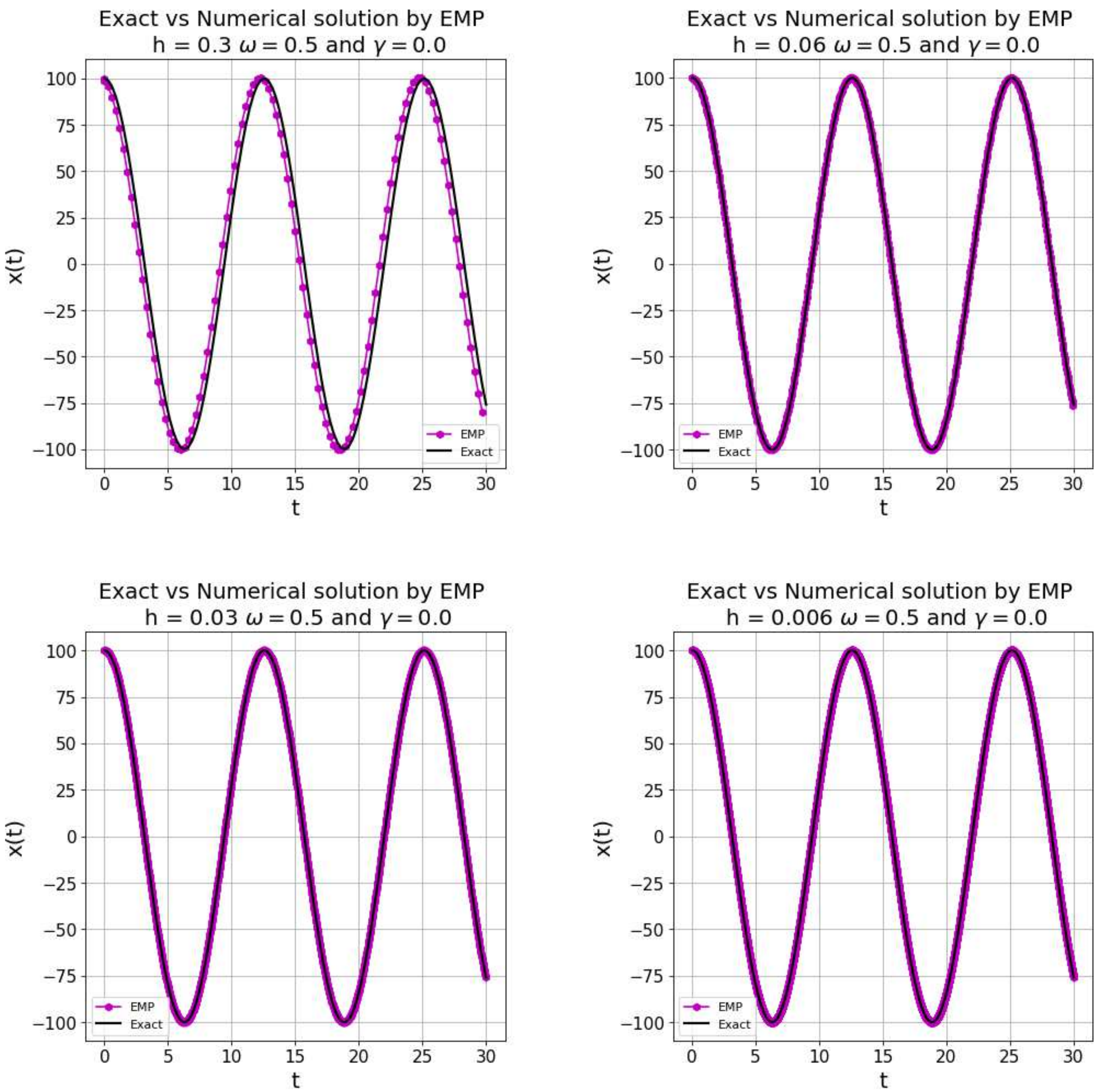


Figure 1: (X vs t). Exact vs Numerical solution by EMP, (1) with $h = 0.1$, (2) with $h = 0.02$, (3) with $h = 0.01$, and (4) with $h = 0.002$. $\omega = 0.5$ and $\gamma = 0$, in color black the exact solution. t in $[0, 30]$

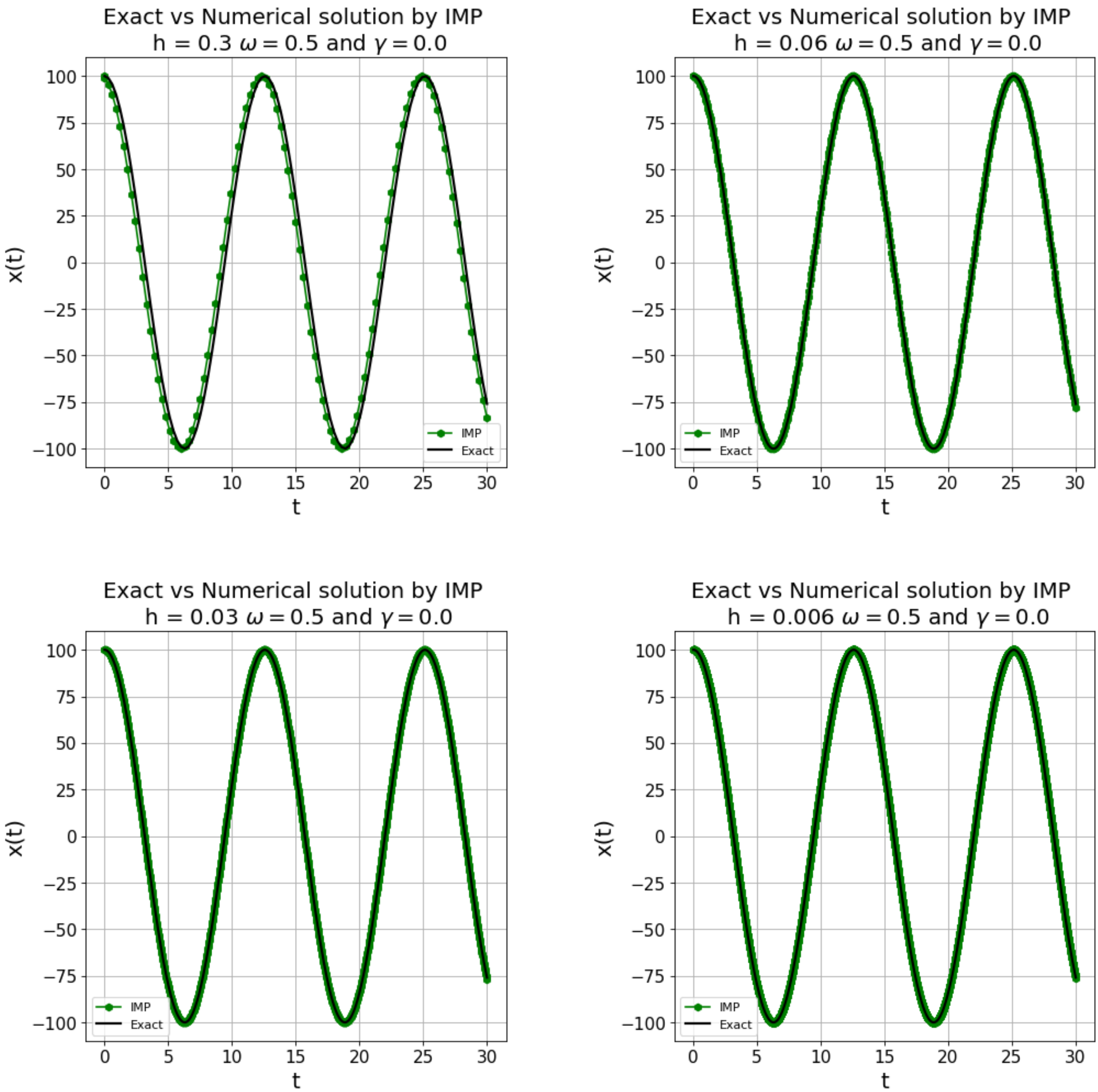


Figure 2: (X vs t). Exact vs Numerical solution by IMP, (1) with $h = 0.1$, (2) with $h = 0.02$, (3) with $h = 0.01$, and (4) with $h = 0.002$. $\omega = 0.5$ and $\gamma = 0$, in color black the exact solution. t in $[0, 30]$

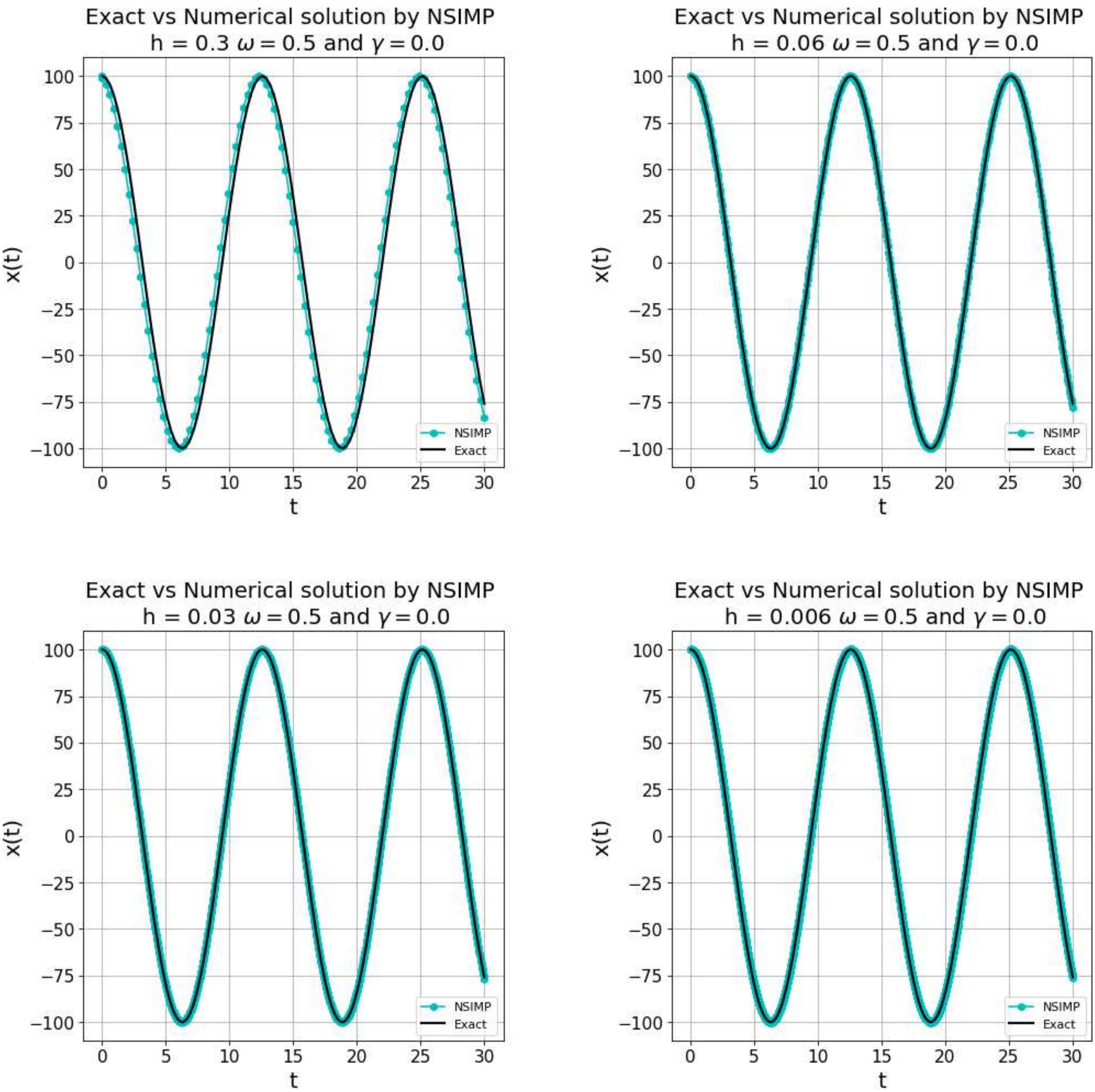


Figure 3: (X vs t). Exact vs Numerical solution by NSIMP, (1) with $h = 0.1$, (2) with $h = 0.02$, (3) with $h = 0.01$, and (4) with $h = 0.002$. $\omega = 0.5$ and $\gamma = 0$, in color black the exact solution. t in $[0, 30]$

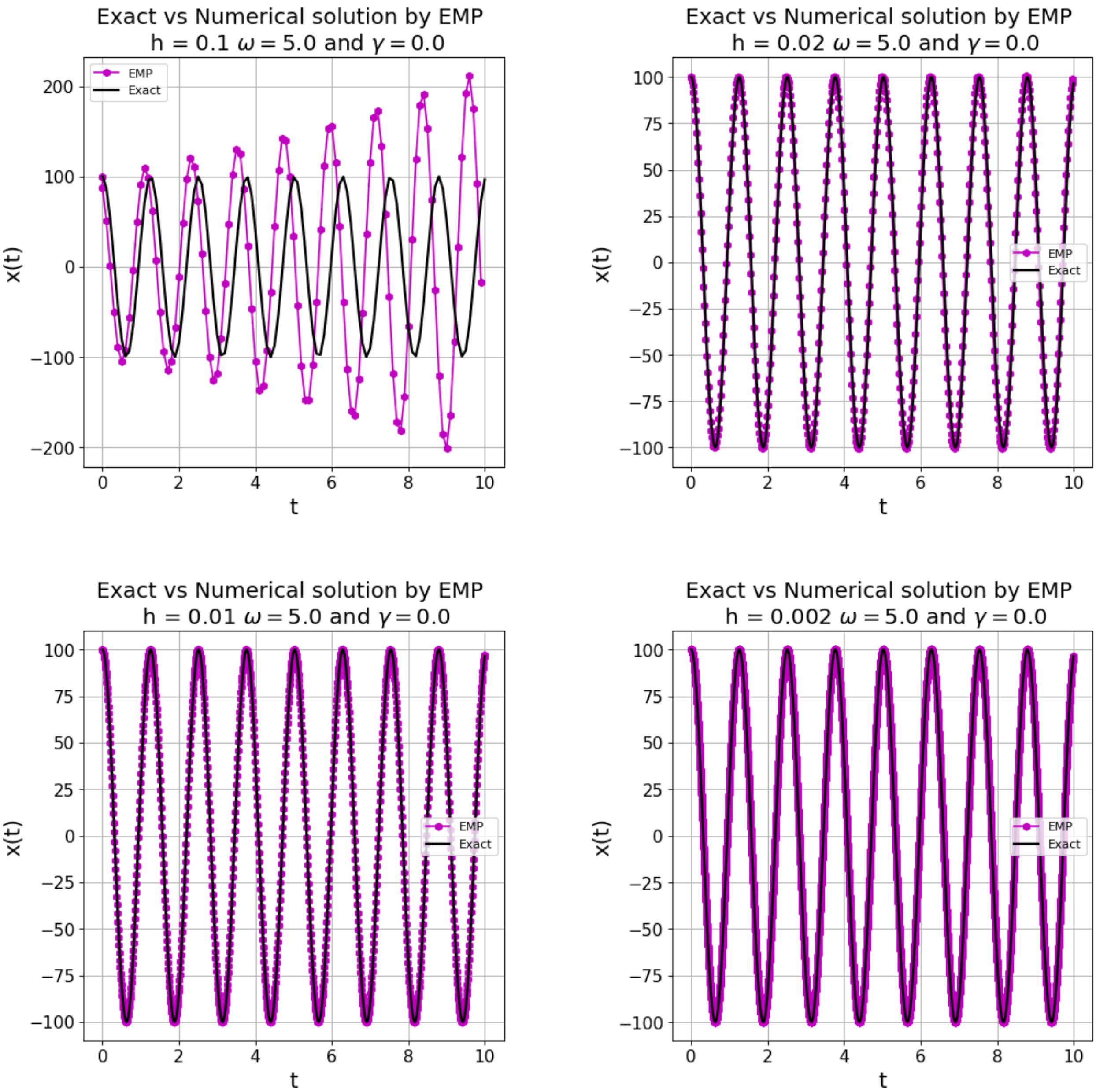


Figure 4: (X vs t). Exact vs Numerical solution by EMP, (1) with $h = 0.1$, (2) with $h = 0.02$, (3) with $h = 0.01$, and (4) with $h = 0.002$. $\omega = 5.0$ and $\gamma = 0$, in color black the exact solution. t in $[0, 10]$

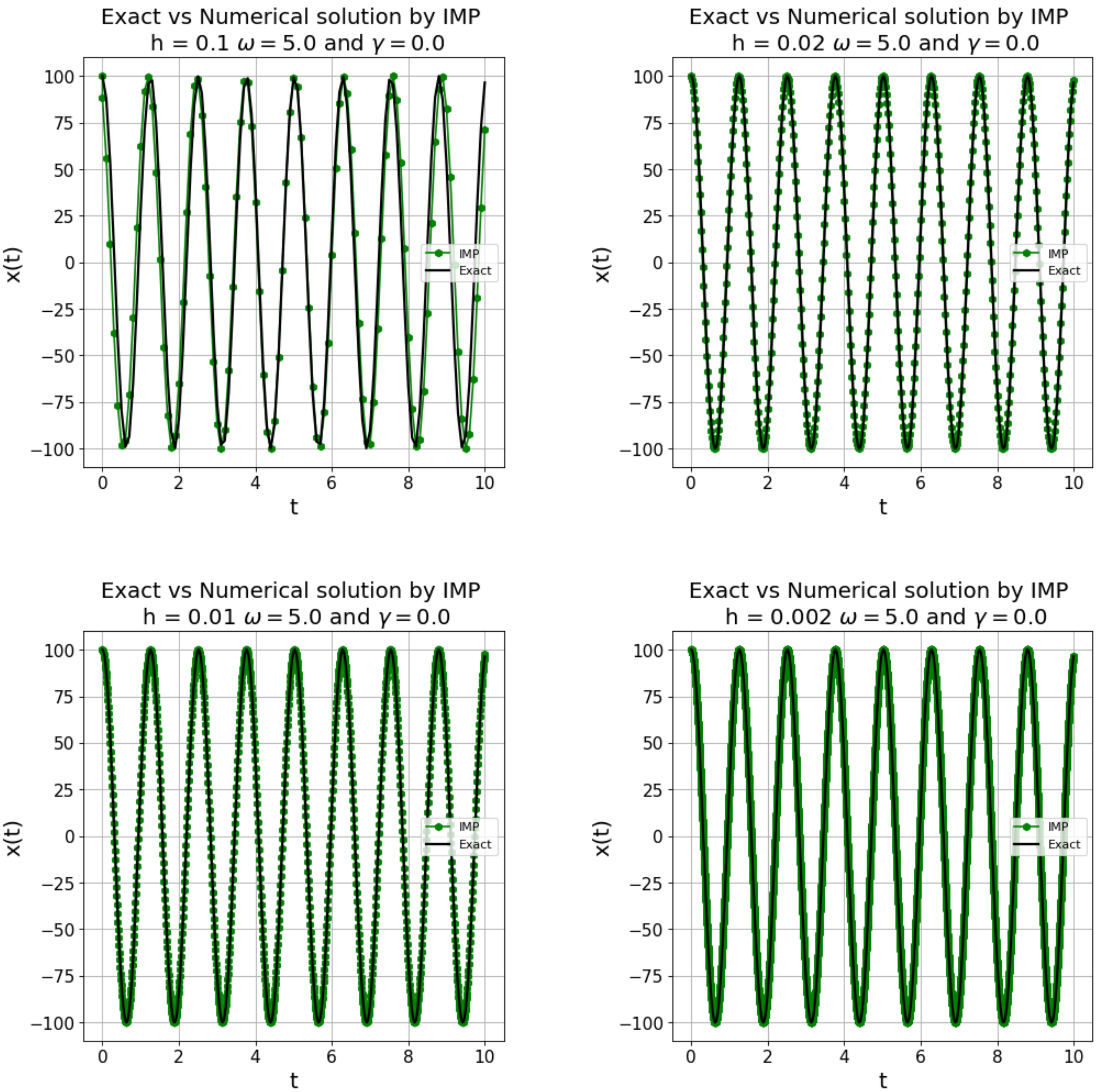


Figure 5: (X vs t). Exact vs Numerical solution by IMP, (1) with $h = 0.1$, (2) with $h = 0.02$, (3) with $h = 0.01$, and (4) with $h = 0.002$. $\omega = 5.0$ and $\gamma = 0$, in color black the exact solution. t in $[0, 60]$

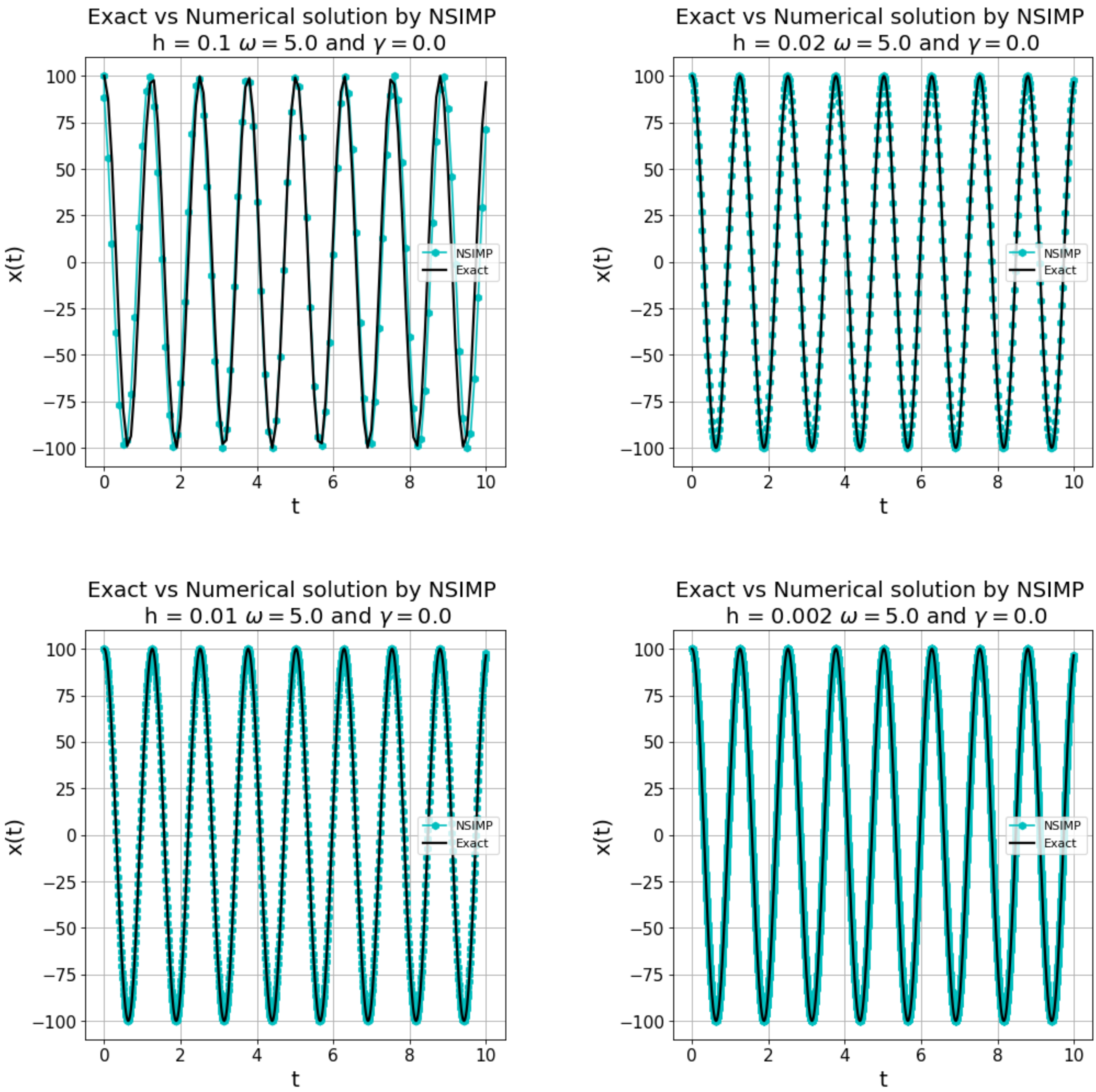


Figure 6: (X vs t). Exact vs Numerical solution by NSIMP, (1) with $h = 0.1$, (2) with $h = 0.02$, (3) with $h = 0.01$, and (4) with $h = 0.002$. $\omega = 5.0$ and γ= 0, in color black the exact solution. t in $[0, 60]$

Item 6) Experiment on the long-time accuracy of the methods for small values of $\gamma > 0$.

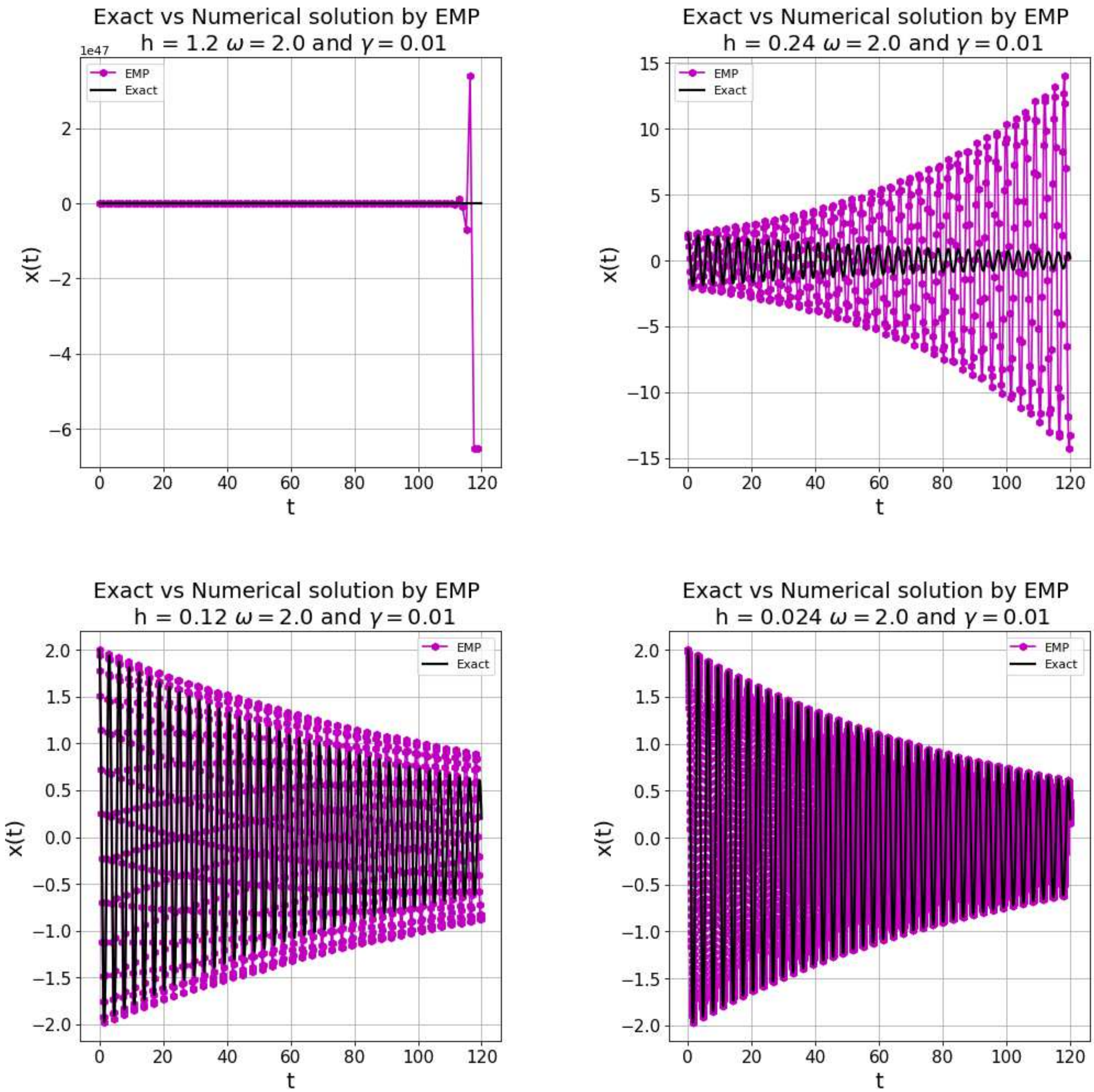


Figure 7: (X vs t). Exact vs Numerical solution by EMP, (1) with $h = 0.1$, (2) with $h = 0.02$, (3) with $h = 0.01$, and (4) with $h = 0.002$. $\omega = 2.0$ and $\gamma = 0.01$, in color black the exact solution. t in $[0, 120]$

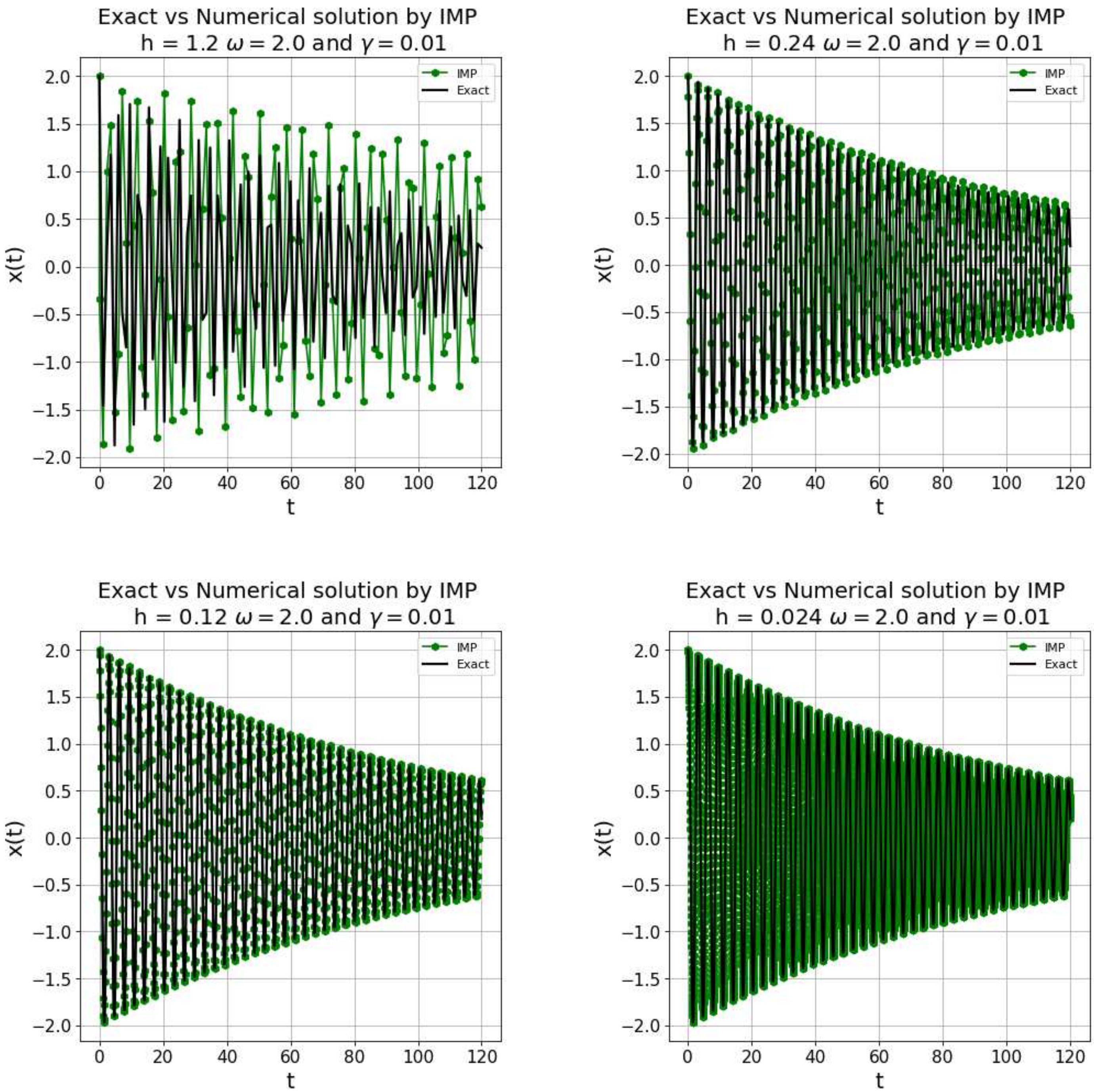


Figure 8: (X vs t). Exact vs Numerical solution by IMP, (1) with $h = 0.1$, (2) with $h = 0.02$, (3) with $h = 0.01$, and (4) with $h = 0.002$. $\omega = 2.0$ and $\gamma = 0.01$, in color black the exact solution. t in $[0, 120]$

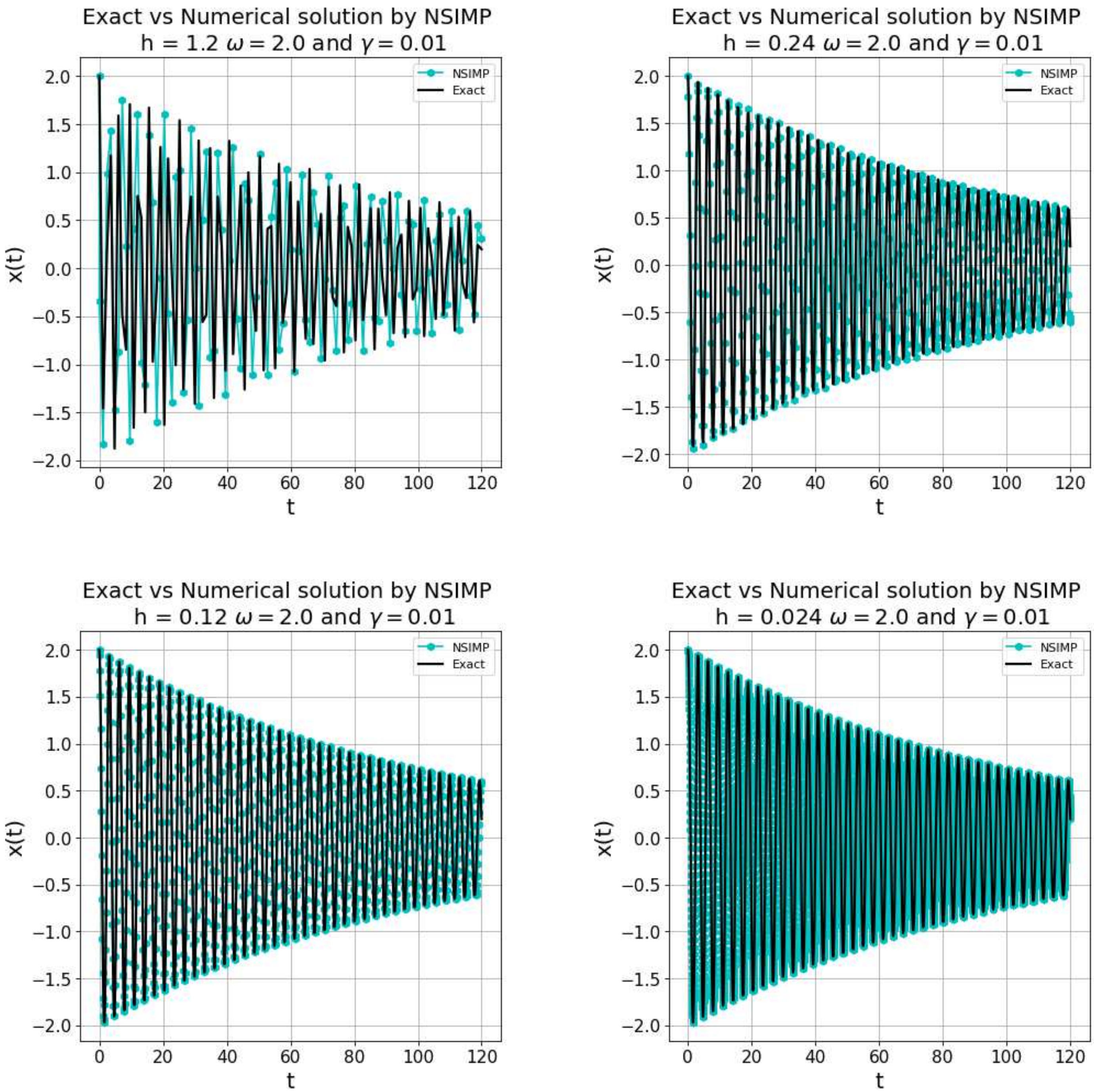


Figure 9: (X vs t). Exact vs Numerical solution by NSIMP, (1) with $h = 0.1$, (2) with $h = 0.02$, (3) with $h = 0.01$, and (4) with $h = 0.002$. $\omega = 2.0$ and $\gamma = 0.01$, in color black the exact solution. t in $[0, 120]$

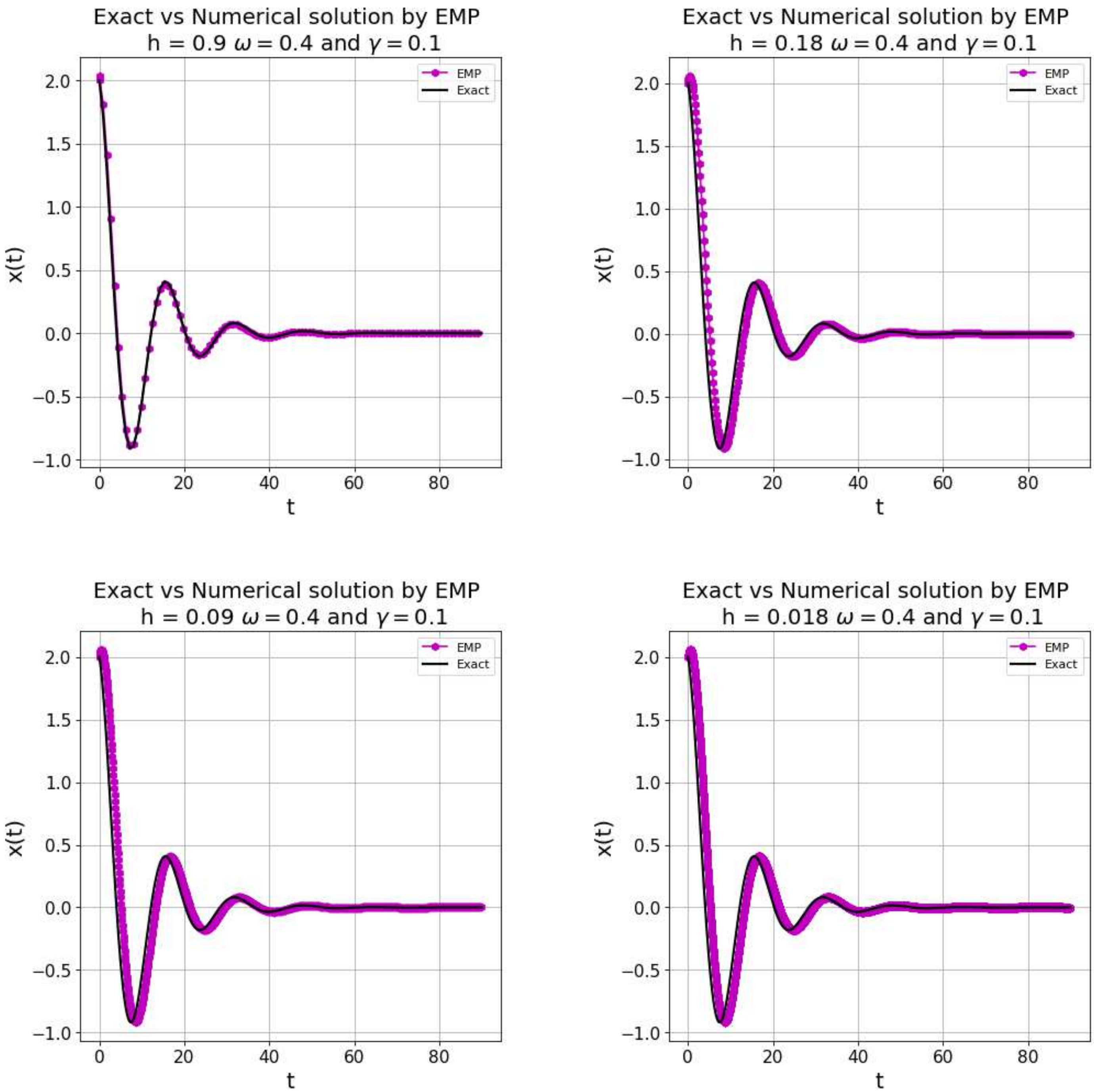


Figure 10: (X vs t). Exact vs Numerical solution by EMP, (1) with $h = 0.1$, (2) with $h = 0.02$, (3) with $h = 0.01$, and (4) with $h = 0.002$. $\omega = 0.4$ and $\gamma = 0.1$, in color black the exact solution. t in $[0, 90]$

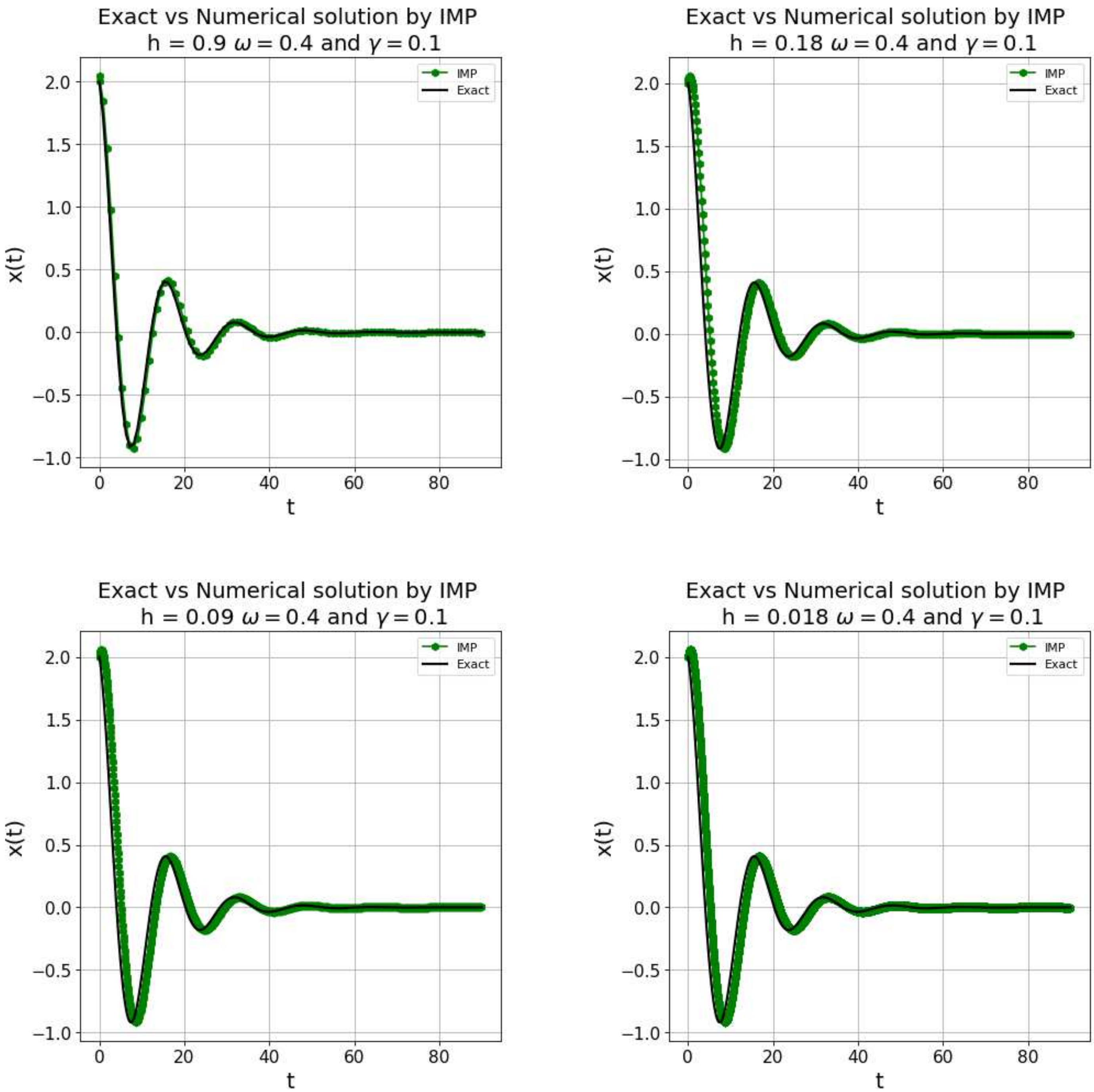


Figure 11: (X vs t). Exact vs Numerical solution by IMP, (1) with $h = 0.1$, (2) with $h = 0.02$, (3) with $h = 0.01$, and (4) with $h = 0.002$. $\omega = 0.4$ and $\gamma = 0.1$, in color black the exact solution. t in $[0, 90]$

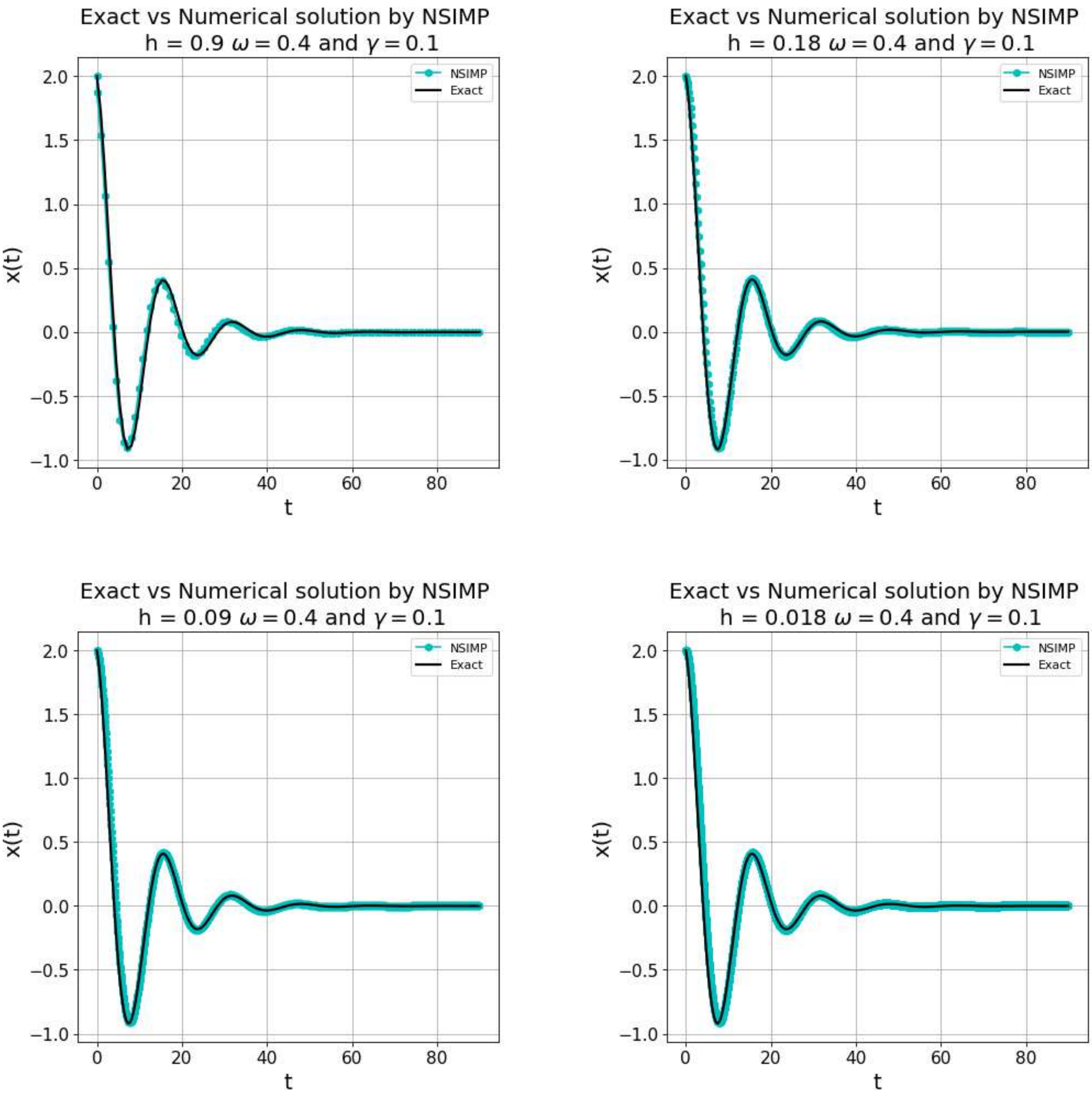


Figure 12: (X vs t). Exact vs Numerical solution by NSIMP, (1) with $h = 0.1$, (2) with $h = 0.02$, (3) with $h = 0.01$, and (4) with $h = 0.002$. $\omega = 0.4$ and $\gamma = 0.1$, in color black the exact solution. t in $[0, 90]$

Item 7: Run experiments to help you fully understand what happens to the approximations as γ increases.

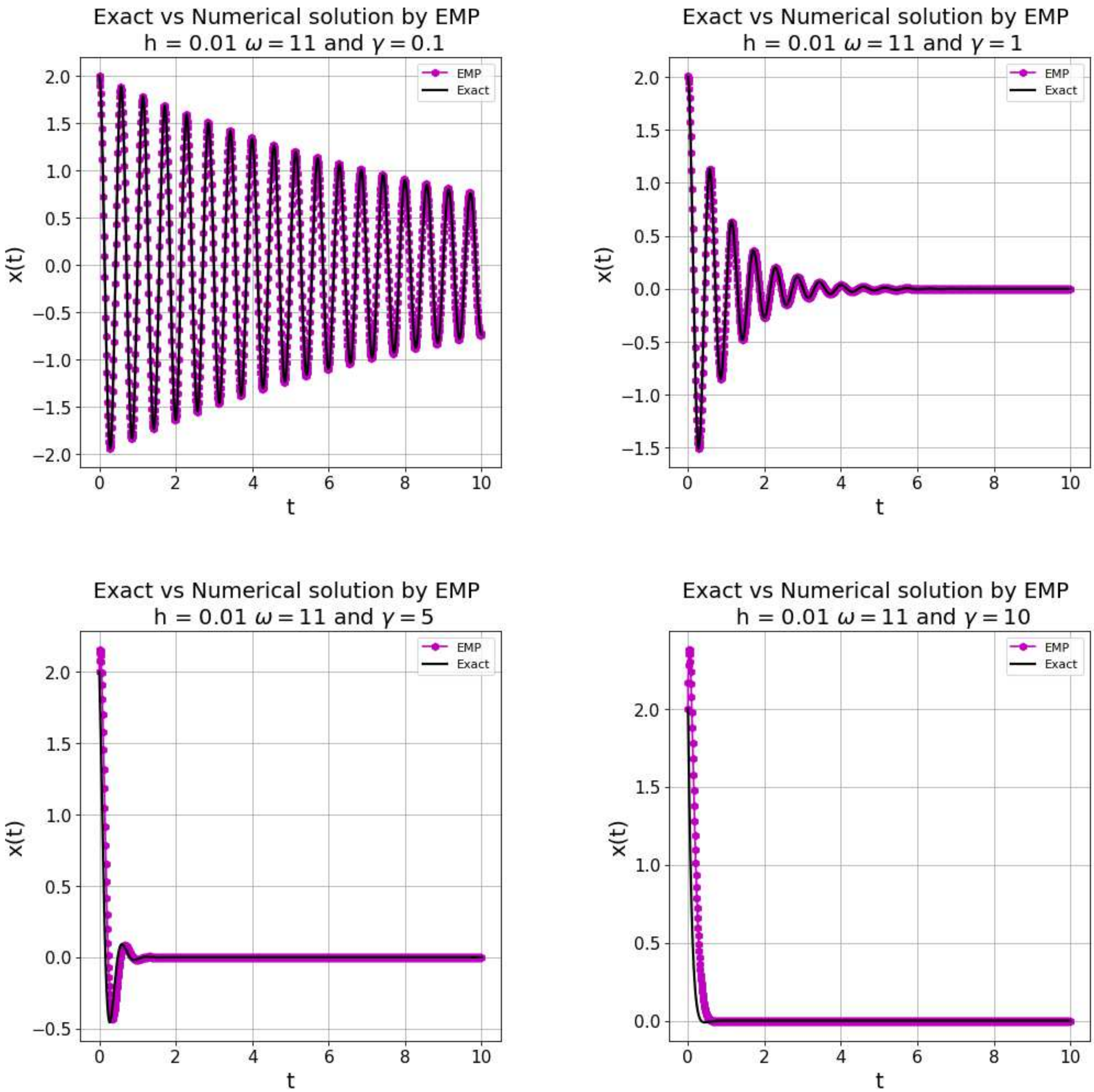


Figure 13: (X vs t). Exact vs Numerical solution by EMP, (1) with $\gamma = 0.1$, (2) with $\gamma = 1$, (3) with $\gamma = 5$, and (4) with $\gamma = 10$. $\omega = 11$ and $h = 0.01$, in color black the exact solution. t in $[0, 10]$

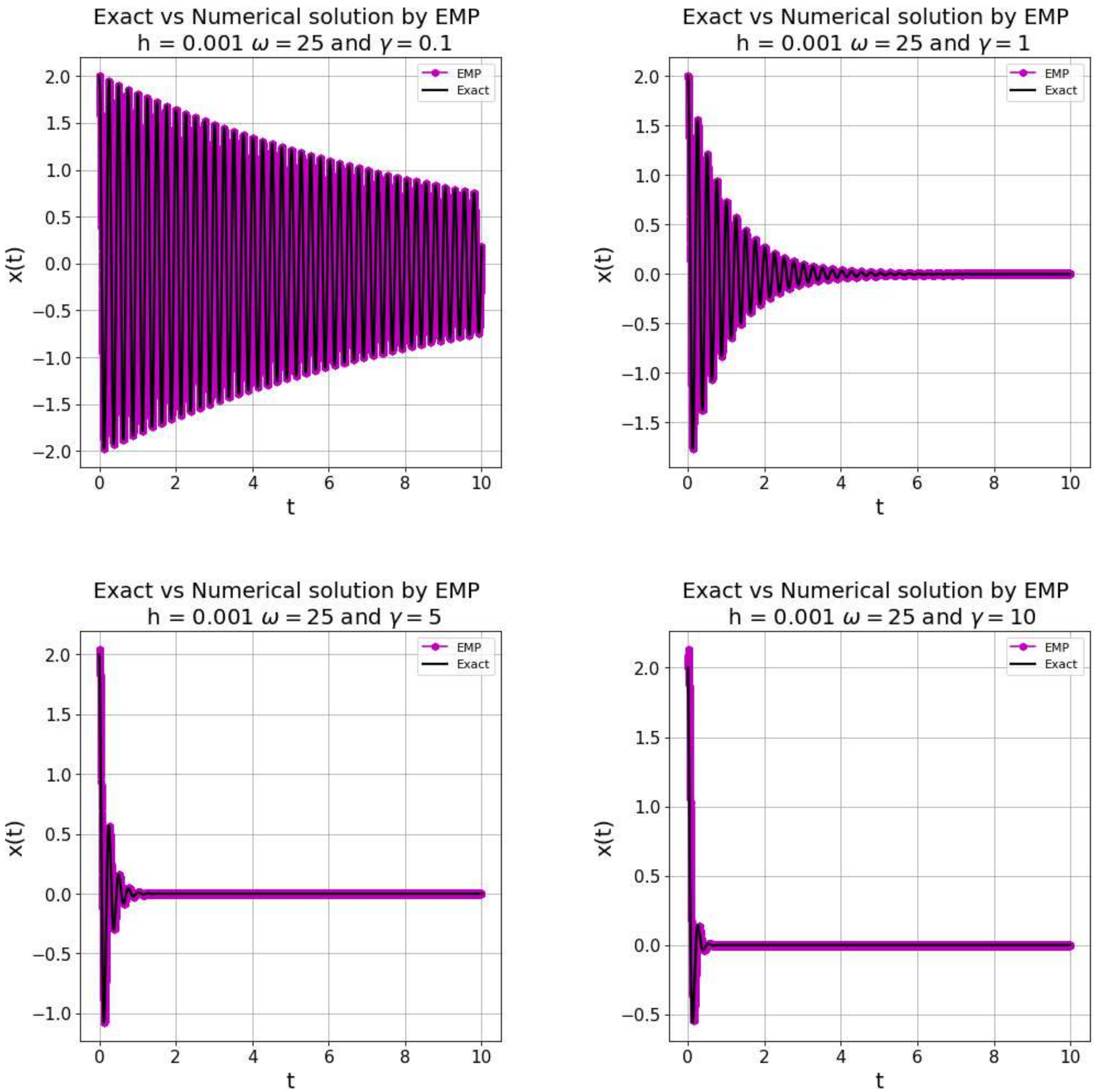


Figure 14: (X vs t). Exact vs Numerical solution by EMP, (1) with $\gamma = 0.1$, (2) with $\gamma = 1$, (3) with $\gamma = 5$, and (4) with $\gamma = 10$. $\omega = 25$ and $h = 0.001$, in color black the exact solution. t in $[0, 10]$

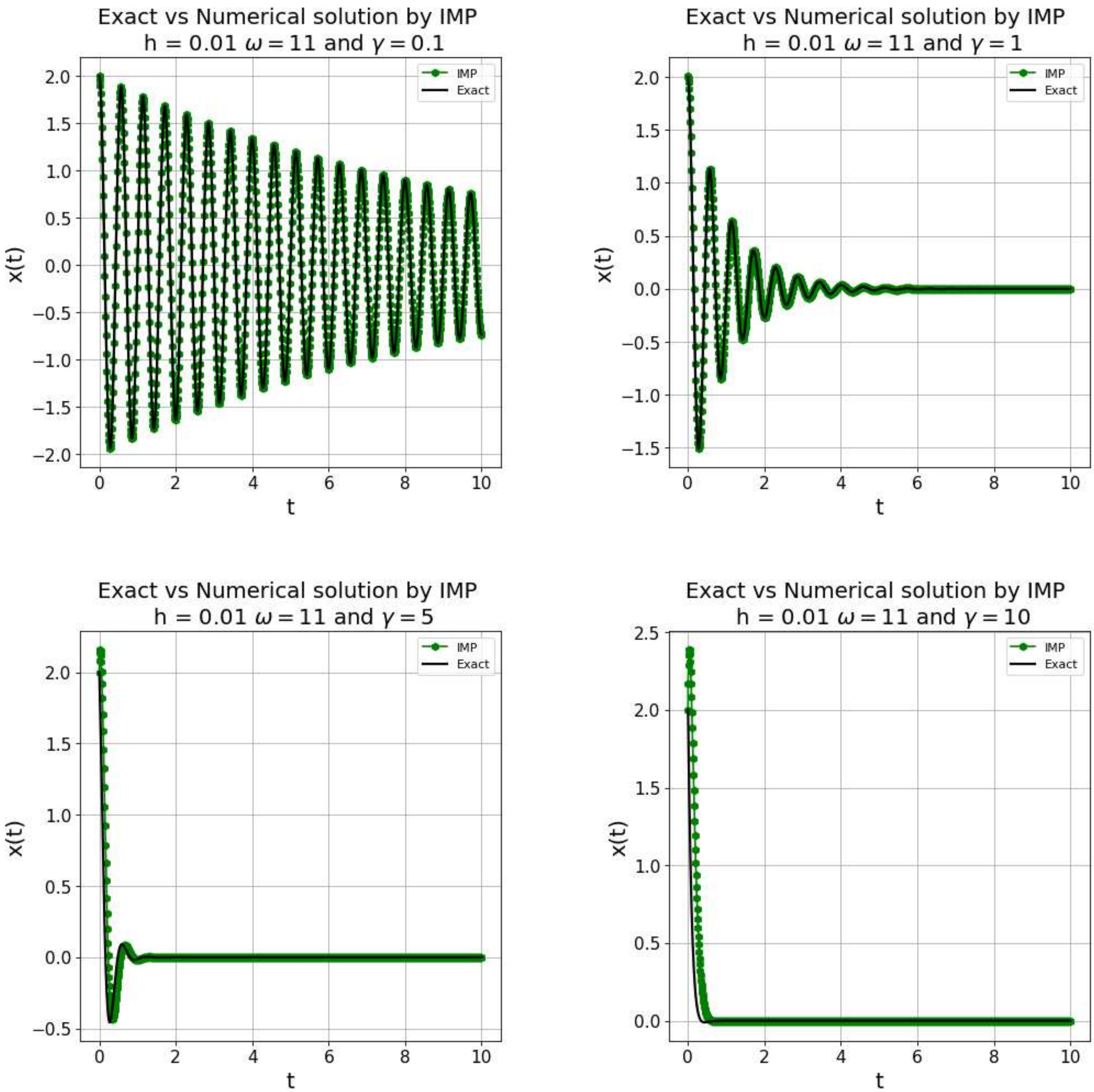


Figure 15: (X vs t). Exact vs Numerical solution by IMP, (1) with $\gamma = 0.1$, (2) with $\gamma = 1$, (3) with $\gamma = 5$, and (4) with $\gamma = 10$. $\omega = 11$ and $h = 0.01$, in color black the exact solution. t in $[0, 10]$

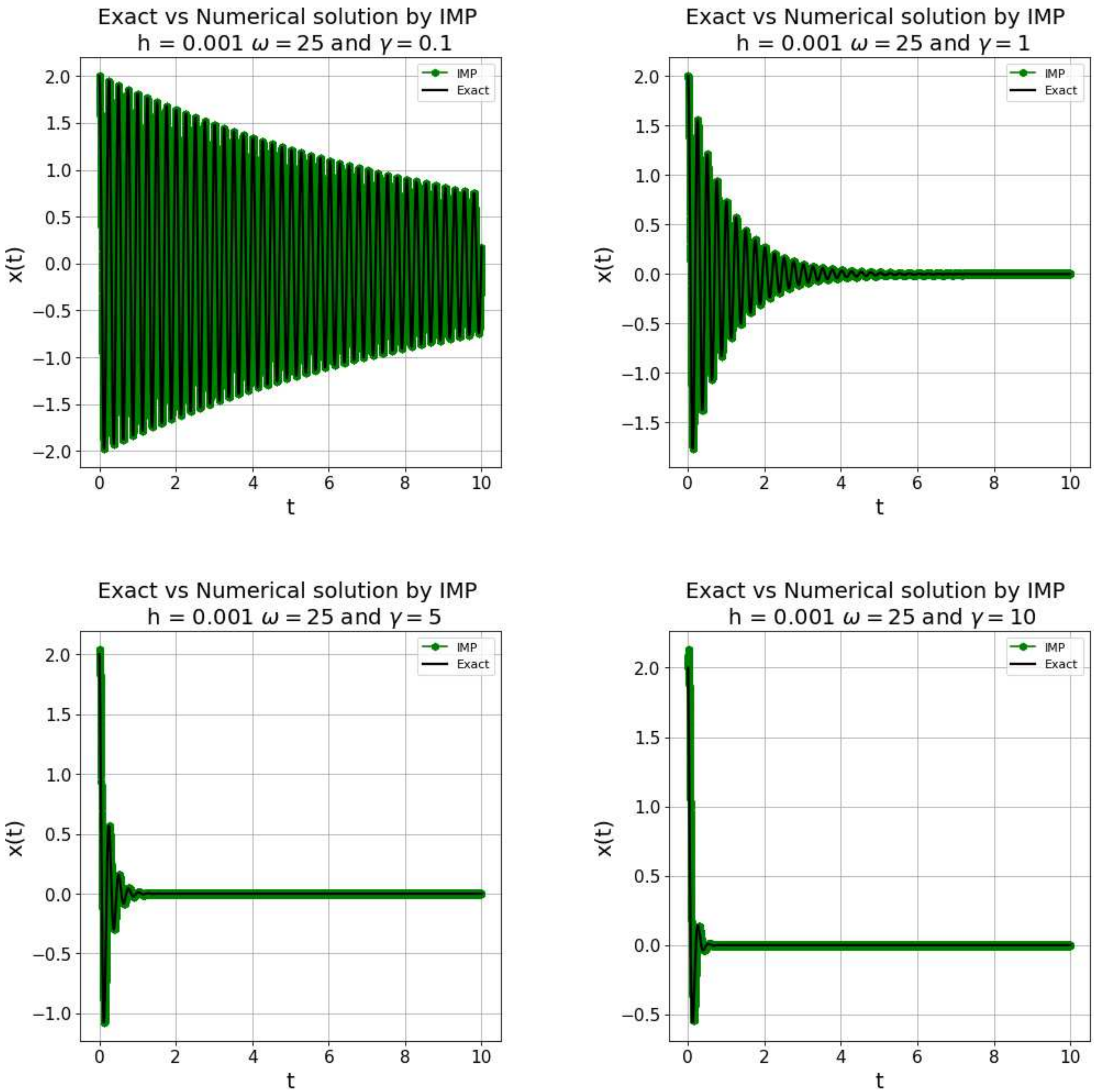


Figure 16: (X vs t). Exact vs Numerical solution by EMP, (1) with $\gamma = 0.1$, (2) with $\gamma = 1$, (3) with $\gamma = 5$, and (4) with $\gamma = 10$. $\omega = 25$ and $h = 0.001$, in color black the exact solution. t in $[0, 10]$

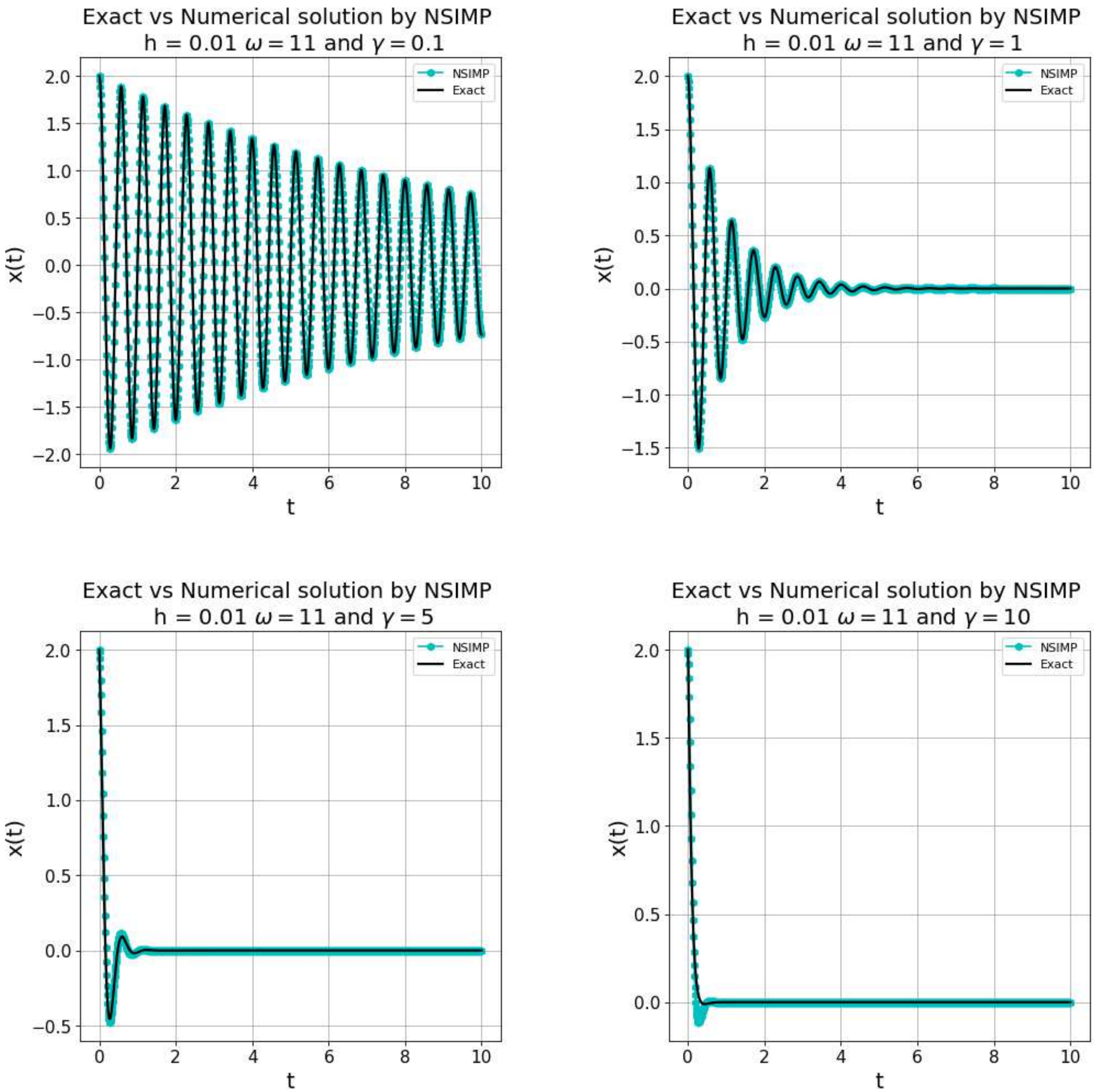


Figure 17: (X vs t). Exact vs Numerical solution by NSIMP, (1) with $\gamma = 0.1$, (2) with $\gamma = 1$, (3) with $\gamma = 5$, and (4) with $\gamma = 10$. $\omega = 11$ and $h = 0.01$, in color black the exact solution. t in $[0, 10]$

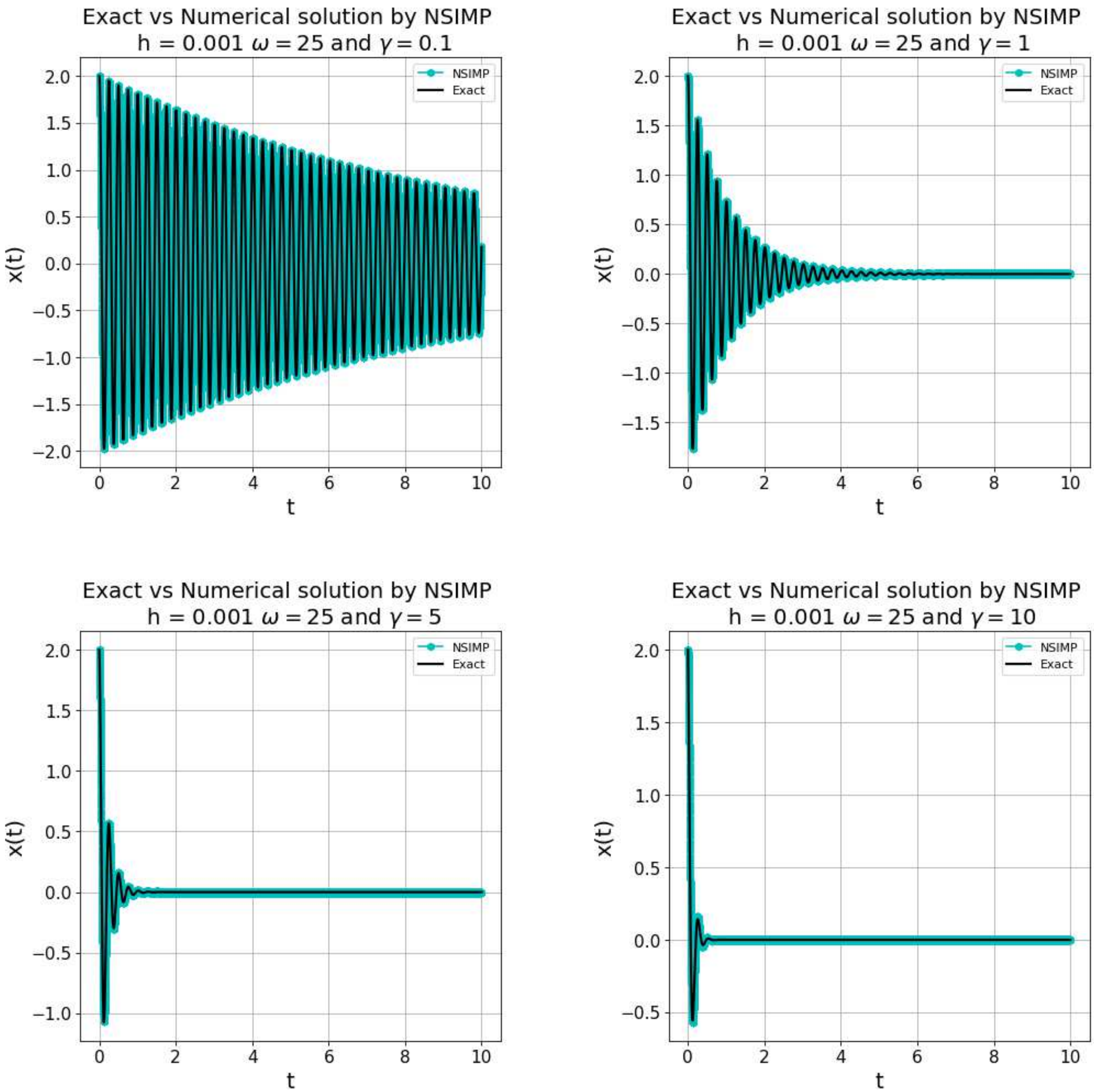
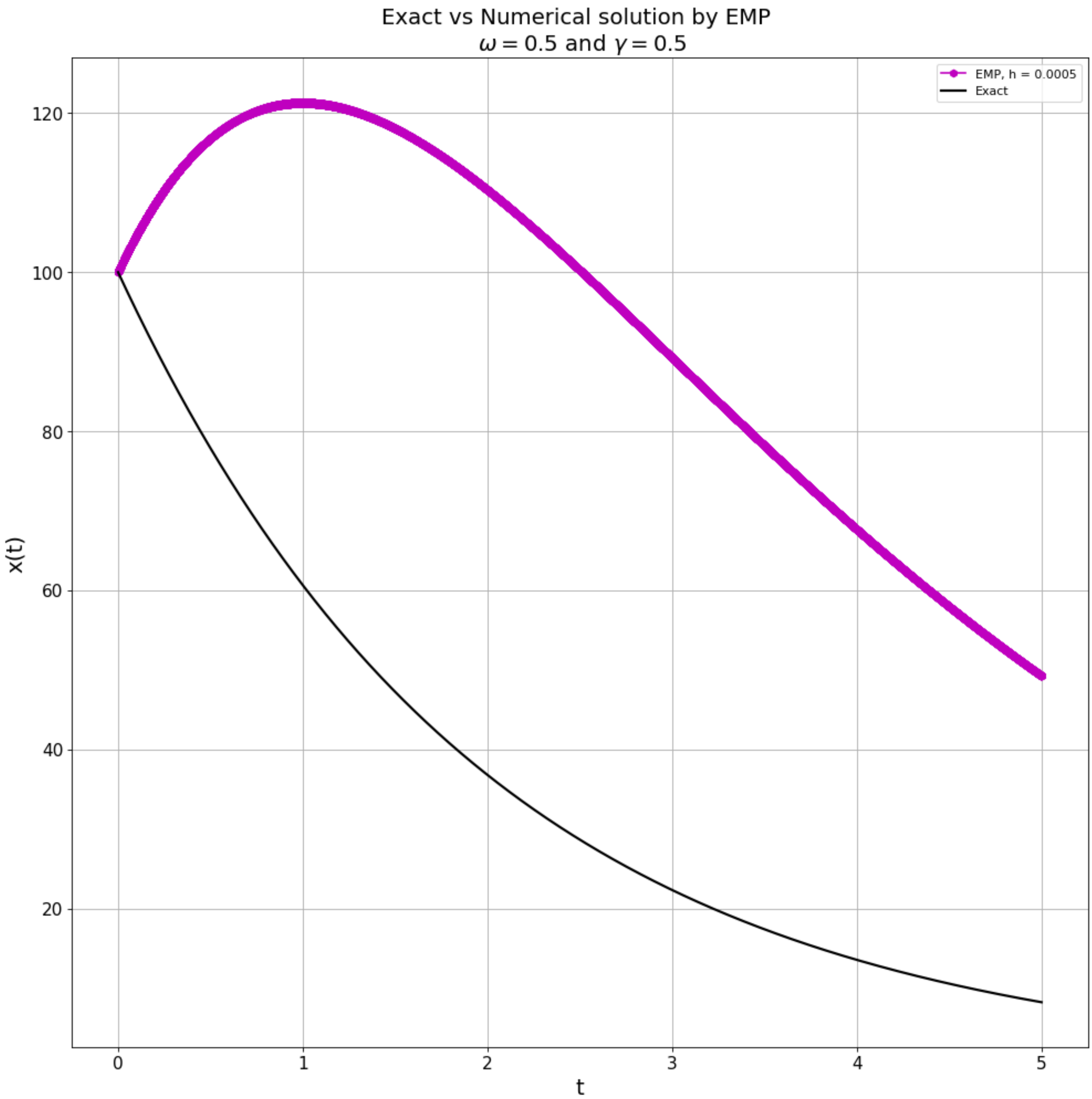


Figure 18: (X vs t). Exact vs Numerical solution by NSIMP, (1) with $\gamma = 0.1$, (2) with $\gamma = 1$, (3) with $\gamma = 5$, and (4) with $\gamma = 10$. $\omega = 25$ and $h = 0.001$, in color black the exact solution. t in $[0, 10]$

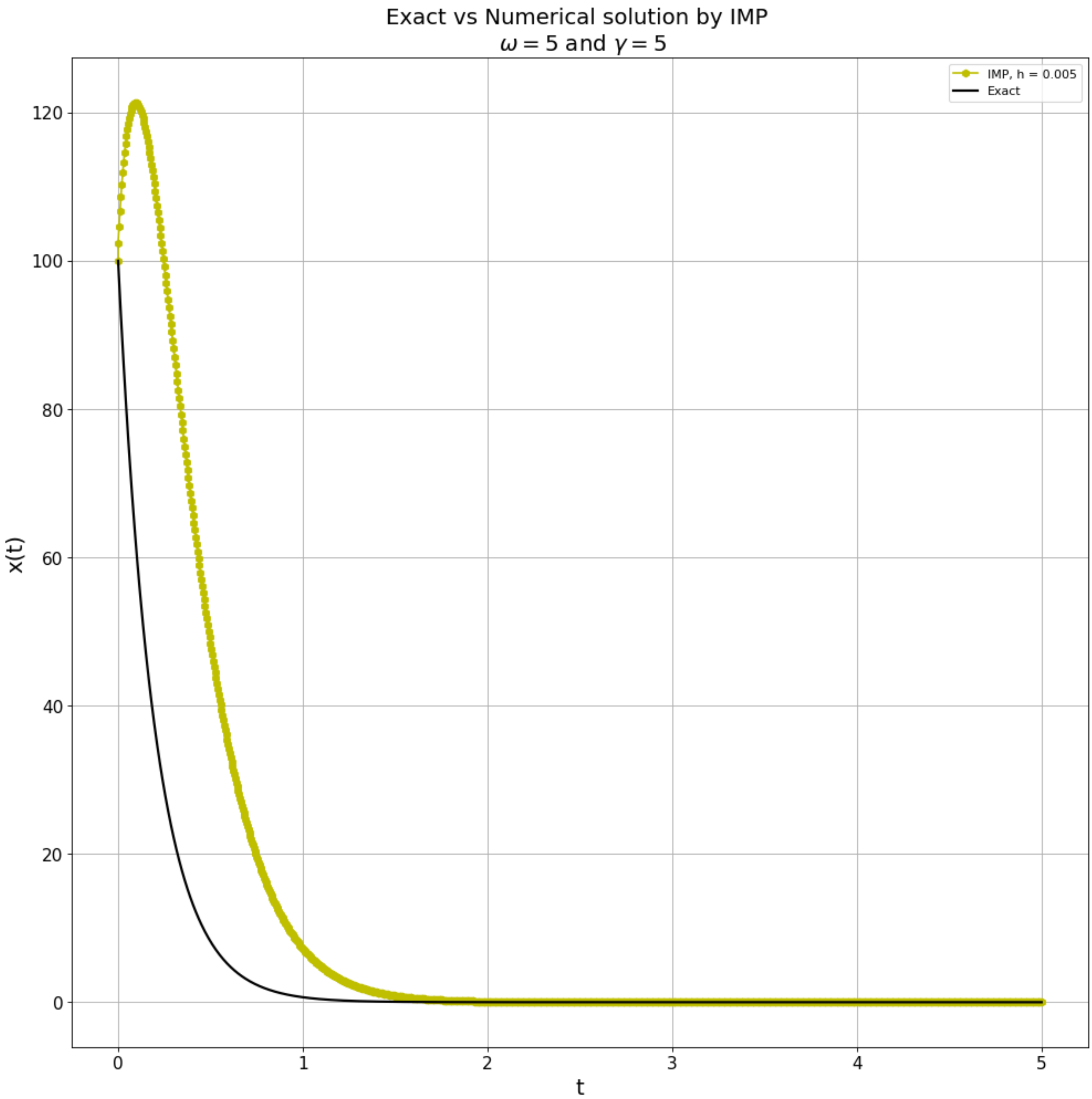
Item 8) If there is, in fact, a "breaking point" for any method as γ increases determine what it is and how it depends upon h or ω.

if $\gamma > \omega$, by definition not possible for each method and

if $\gamma = \omega$, the approximations are not good



Exact vs Numerical solution by EMP
$\omega = 0.5$ and $\gamma = 0.5$
EMP, h = 0.0005
Exact
x(t)
t
0
1
2
3
4
5
20
40
60
80
100
120



Exact vs Numerical solution by IMP
$\omega = 5$ and $\gamma = 5$
IMP, h = 0.005
Exact
x(t)
t
0
20
40
60
80
100
120
1
2
3
4
5