Model for i^{th} county:

$$
\begin{aligned}
R_0(t)_i &= \beta_0 + b_{1i} + \beta_\phi(t-d_1)X_1(d_1) + \beta_\pi(t-d_2)X_2(d_2) + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_6X_6 + \beta_7X_7 \\
&= \beta_0 + b_{1i} + \sum_{d_1=t_0}^{t} \beta_\phi(t-d_1)X_1(d_1) + \sum_{d_2=t_0}^{t} \beta_\pi(t-d_2)X_2(d_2) + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_6X_6 + \beta_7X_7 \\
&= \beta_0 + b_{1i} + \sum_{d_1=t_0}^{t} \{[\phi_1 B_{-1}(t-d_1) + ... + \phi_j B_{j-2}(t-d_1)]X_1(d_1)\} \\
&+ \sum_{d_2=t_0}^{t} \{[\pi_1 B_{-1}(t-d_2) + ... + \pi_j B_{k-2}(t-d_2)]X_2(d_2)\} + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_6X_6 + \beta_7X_7
\end{aligned}
$$

- X1(t)-(time-varying effect/variable): # of tests done in this county over time
- X2(t)-(time-varying effect/variable): currently social-distancing 0 or 1
- X3: population density
- X4: social-distancing adherence rate (use the pdf Dr.Wang sent)
- X5: % of essential workers
- X6: % of poverty
- X7: % of college degree of higher
- X8: ... hospital capacity maybe

Step 1: Estimate R_0 for each county in the US

Four methods:
https://bmcmedinformdecismak.biomedcentral.com/articles/10.1186/1472-6947-12-147

- By attack rate

  $$R_0 = -\frac{log(1-\frac{1-AR}{S_0})}{AR-(1-S_0)}$$

  S_0: the proportion of susceptible population in the beginning
  Assuming a closed population

- By exponential growth

  $$R_0 = \frac{1}{M(-r)}$$

  Where M is the mgf of the generation time distribution, and r is the per capita change in number of new cases per unit of time

- By maximum likelihood

$$l(R) = \sum_{t=1}^{T} log(\frac{e^{-\mu_t}\mu_t^{N_t}}{N_t!})$$

where,

$$\mu_t = R\sum_{i=1}^{t} N_{t-1}w_i$$

w: generation time distribution w
Assuming that the number of secondary cases caused by an index case is Poisson distributed with expected value R

- By sequential Bayesian

$$P(R \mid N_0, ..., N_{t+1}) = \frac{P(N_{t+1} \mid R, N_0, ..., N_t)P(R \mid N_0, ..., N_t)}{P(N_0, ..., N_{t+1})}$$

It does not account for susceptible depletion; it implicitly uses an exponential distribution for the generation time; and assumes random mixing in the population

Step 2: Estimate time-varying effect by ML using restricted cubic spline

This is a function of d_1

$$\beta_\phi(t-d_1) = \phi_1 B_{-1}(t-d_1) + ... + \phi_j B_{j-2}(t-d_1)$$

This is a function of d_2

$$\beta_\pi(t-d_2) = \pi_1 B_{-1}(t-d_2) + ... + \pi_j B_{k-2}(t-d_2)$$

Step 3: Estimate fixed effect by OLS

Step 4: Estimate random intercept by BLUP