Session 2 - Let's Do Some Maths!
UCAS Program 2020

Chloe Lau

August 2020

1 Why Maths?

Computer Science is half a Maths degree, it basically is a subset of Maths.

So you might ask: what sort of maths do you do at university?

$$
\begin{aligned}
\mathit{Add}(3,4) &\triangleq \mathit{Add}(\mathit{Succ}(\mathit{Succ}(\mathit{Succ}(0))),\ \mathit{Succ}(\mathit{Succ}(\mathit{Succ}(\mathit{Succ}(0))))) \\
&= \mathit{Succ}(\mathit{Add}(\mathit{Succ}(\mathit{Succ}(0)),\ \mathit{Succ}(\mathit{Succ}(\mathit{Succ}(\mathit{Succ}(0)))))) \\
&= \mathit{Succ}(\mathit{Succ}(\mathit{Add}(\mathit{Succ}(0),\ \mathit{Succ}(\mathit{Succ}(\mathit{Succ}(\mathit{Succ}(0))))))) \\
&= \mathit{Succ}(\mathit{Succ}(\mathit{Succ}(\mathit{Add}(0,\ \mathit{Succ}(\mathit{Succ}(\mathit{Succ}(\mathit{Succ}(0)))))))) \\
&= \mathit{Succ}(\mathit{Succ}(\mathit{Succ}(\mathit{Succ}(\mathit{Succ}(\mathit{Succ}(\mathit{Succ}(0))))))) \qquad \triangleq 7
\end{aligned}
$$

A typical lemma in Discrete Maths.

Okay that is a joke that our lecturer included, but it still makes logical sense.

$\exists x \exists y \exists z(\texttt{bought}(x,y) \wedge \texttt{bought}(x,z) \wedge \neg(y=z))$
'There are x, y, z such that x bought y, x bought z, and y is not z.'
'Something bought at least two different things.'

$\forall x(\exists y \exists z(\texttt{bought}(x,y) \wedge \texttt{bought}(x,z) \wedge \neg(y=z)) \rightarrow x = \texttt{Tony})$
'For all x, if x bought two different things then x is equal to Tony.'
'Anything that bought two different things is Tony.'
Care: it doesn't say Tony did buy 2 things, just that no one else did.

A snippet of my Logic Lecture Slides

Yes, sometimes we have to do translations to English too!

They are all Maths!

2 The Real Maths at This Stage

Applying to Computer Science, you have to prepare for **a lot of** admission tests:

- CSAT - Cambridge
- TMUA - Cambridge and maybe Warwick
- MAT - Oxford and Imperial Maths
- STEP - Imperial **Computing or JMC** (almost guaranteed!), Imperial Maths, Cambridge Maths and CS)

Being mathy is a feature of a good Computer Scientist, but as a good Computer Scientist, being mathy isn't they only feature you need to carry.

What do you think is the most important to a Mathematician or Computer Scientist? Why do you think so?

What are your strongest parts in Mathematics? What do you find the most interesting in Mathematics? Theory? Statistics? Mechanics? Discrete?

How much maths have you done up till now? How confident are you with solving the questions?

How do you tackle these problems in an admission test? How would you tackle them in an interview otherwise?

Let's try some questions:

2.1 MAT 2010, Q5

This question concerns calendar dates of the form

$$d_1 d_2 / m_1 m_2 / y_1 y_2 y_3 y_4$$

in the order day/month/year.

The question specifically concerns those dates which contain no repetitions of a digit. For example, the date 23/05/1967 is one such date but 07/12/1974 is not such a date as both $1 = m_1 = y_1$ and $7 = d_2 = y_3$ are repeated digits.

We will use the Gregorian Calendar throughout (this is the calendar system that is standard throughout most of the world; see below.)

(i) Show that there is no date with no repetition of digits in the years from 2000 to 2099.

(ii) What was the last date before today with no repetition of digits? Explain your answer.

(iii) When will the next such date be? Explain your answer.

(iv) How many such dates were there in years from 1900 to 1999? Explain your answer.

[The Gregorian Calendar uses 12 months, which have, respectively, 31, 28 or 29, 31, 30, 31, 30, 31, 31, 30, 31, 30 and 31 days. The second month (February) has 28 days in years that are not divisible by 4, or that are divisible by 100 but not 400 (such as 1900); it has 29 days in the other years (leap years).]

Some space for scribbles:

2.2 STEP 2016, Paper 1, Q7

The set S consists of all the positive integers that leave a remainder of 1 upon division by 4. The set T consists of all the positive integers that leave a remainder of 3 upon division by 4.

(i) Describe in words the sets $S \cup T$ and $S \cap T$.

(ii) Prove that the product of any two integers in S is also in S. Determine whether the product of any two integers in T is also in T.

(iii) Given an integer in T that is not a prime number, prove that at least one of its prime factors is in T.

(iv) For any set X of positive integers, an integer in X (other than 1) is said to be X-*prime* if it cannot be expressed as the product of two or more integers *all in* X (and all different from 1).

(a) Show that every integer in T is either T-prime or is the product of an odd number of T-prime integers.

(b) Find an example of an integer in S that can be expressed as the product of S-prime integers in two distinct ways. [Note: $s_1 s_2$ and $s_2 s_1$ are not counted as distinct ways of expressing the product of s_1 and s_2.]

Some space for scribbles: