1 Uniform Distribution

Notation

$X \sim U(a,b)$ where $-\infty < a < b < \infty$.

Density Function

$f(x) = \frac{1}{b-a}$ for $x \in [a,b]$

Mean and Variance

$E[X] = \frac{1}{2}(a+b)$
$Var[X] = \frac{1}{12}(b-a)^2$

2 Binomial Distribution

Notation

$X \sim Bin(n,p)$

Density Function

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

Mean and Variance

$E(X) = np$, $Var(X) = np(1-p)$

3 Multinomial Distribution

Notation

$(X_1, ..., X_k) \sim Multinomial(n; p_1, ..., p_k)$

Density Function

$f(x_1, ..., x_k) = \binom{n}{x_1, ..., x_k} p_1^{x_1} ... p_k^{x_k}$ where $\binom{n}{x_1, ..., x_k} = \frac{n!}{x_1! ... x_k!}$

Mean and Variance

$E(X_i) = np_i$
$Var(X_i) = np_i(1-p_i)$
$Cov(X_i X_j) = -np_i p_j$ for $i \neq j$

4 Exponential Distribution

Waiting time. Continuous version of geometric distribution.

Notation

$X \sim Exp(1/\lambda)$

Density Function

$f(x) = \lambda e^{-\lambda x}$ for $x > 0$

Mean and Variance

$E(X) = 1/\lambda$, $Var(X) = 1/\lambda^2$

5 Beta Distribution

Notation

$Beta(\alpha, \beta)$ where $\alpha, \beta > 0$ are the shape parameters.

Density Function

$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$ where $B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$, for $x \in [0,1]$.

Mean and Variance

$E[X] = \frac{\alpha}{\alpha+\beta}$
$Var[X] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

Normal Approximation

For sufficiently large α, β, standardised X is approximately $N(0,1)$.

6 Dirichlet Distribution

This is a multivariate generalisation of the beta distribution, hence it may also known as the multivariate beta distribution (MBD).

Notation

$Dir(\alpha_1, \alpha_2, .., \alpha_n)$ where $\alpha_i > 0$.

Density Function

$f(x) = \frac{\Pi_{i=1}^n x_i^{\alpha_i - 1}}{B(\alpha_1, \alpha_2, .., \alpha_n)}$ where $B(\alpha_1, \alpha_2, ..., \alpha_n) = \frac{\Gamma(\alpha_1)\Gamma(\alpha_2)...\Gamma(\alpha_n)}{\Gamma(\alpha_1 + \alpha_2 + ... + \alpha_n)}$

Mean and Variance

$E[X_i] = \frac{\alpha_i}{\sum_{i=1}^n \alpha_i}$
$Var[X_i] = \frac{\tilde{\alpha}_i(1-\tilde{\alpha}_i)}{\alpha_0 + 1}$ where $\alpha_0 = \Sigma_{i=1}^n \alpha_i$ and $\tilde{\alpha}_i = \alpha_i / \alpha_0$

7 Gamma Distribution

Notation

$X \sim Gamma(\alpha, 1/\beta)$ where $\alpha, \beta > 0$

Density Function

$f(x) = (\frac{\beta^\alpha}{\Gamma(\alpha)}) x^{\alpha-1} e^{-\beta x}$, where $\Gamma(n) = \int_0^\infty x^{\alpha-1} e^{-x} dx$.
Note that $(\frac{\lambda^n}{\Gamma(n)})$ is constant wrt to x.

Mean and Variance

$E[X] = \alpha/\beta$
$Var[X] = \alpha/\beta^2$

Normal Approximation

When α is large, standardised X is approximately $N(0,1)$.

8 Inverse Gamma Distribution

Notation

$X \sim InverseGamma(\alpha, 1/\beta)$ where $\alpha, \beta > 0$

Density Function

$f(x) = (\frac{\beta^\alpha}{\Gamma(\alpha)}) x^{-\alpha-1} e^{-x/\beta}$, where $\Gamma(n) = \int_0^\infty x^{\alpha-1} e^{-x} dx$.

Mean and Variance

$E[X] = \frac{\beta}{\alpha-1}$ for $\alpha > 1$
$Var[X] = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$ for $\alpha > 2$

9 Normal Distribution

Notation

$N(\mu, \sigma^2)$ where μ is the mean, σ^2 is the variance

Density Function

$f(x) = \frac{1}{\sqrt{2\pi}\sigma} exp\{-\frac{((x-\mu)^2)}{2\sigma^2}\}$, $-\infty < x < \infty$.

10 Standardised t distribution

Notation

$X \sim t_v$ where $v > 0$ is the degrees of freedom

Density Function

$f(x) = (\frac{1}{Beta(v/2,1/2)}\frac{1}{\sqrt{v}})[1 + \frac{x^2}{v}]^{-\frac{v+1}{2}}$

Mean and Variance

$E(X) = 0$, $Var(X) = \frac{v}{v-2}$

11 Generalised t distribution

Notation

$X \sim t_v(\mu, \lambda^{-1})$ where $v > 0, \lambda > 0$.
v is the degrees of freedom, μ is the location parameter and λ is the precision parameter.
Note that $X = \mu + \lambda^{-1/2} t_v$

Density Function

$f(x) = (\frac{1}{Beta(v/2,1/2)}\frac{1}{\sqrt{v}})(\lambda^{1/2})[1 + \lambda\frac{(x-\mu)^2}{v}]^{-\frac{v+1}{2}}$

Mean and Variance

$E(X) = \mu$, $Var(X) = \frac{v}{v-2}\lambda^{-1}$

12 Pareto Distribution

Notation

$X \sim Pareto(m, a)$ where $m, a > 0$

Density Function

$f(x) = \frac{am^a}{x^{a+1}}$ for $x \geq m$
$F(x) = [1 - (\frac{m}{a})^a]$ for $x \geq m$

Mean, Mode and Variance

$E(X) = \begin{cases} \frac{am}{a-1} & \text{for } a > 1 \\ \infty & \text{for } a \leq 1 \end{cases}$
$Var(X) = \begin{cases} \frac{am^2}{(a-1)^2(a-2)} & \text{for } a > 2 \\ \infty & \text{for } a \leq 2 \end{cases}$
$Mode(X) = m$

13 Poisson Distribution

Notation

$X \sim Poisson(\lambda)$ where $\lambda > 0$

Density Function

$f(x) = e^{-\lambda}\frac{\lambda^x}{x!}$ for $x = 0, 1, 2, 3, ...$

Mean and Variance

$E(X) = Var(X) = \lambda$